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Q. <1> Using Bisection Method, Find approximate root of equation $x^3-x^2+x-7=0$

$\Rightarrow$ Given: $x^3-x^2+x-7=0$

let $f(x)=x^3-x^2+x-7$

$x=0 \Rightarrow f(0)=-7$

$x=1 \Rightarrow f(1)=-6$

$x=2 \Rightarrow f(2)=-1<0$

$x=3 \Rightarrow f(3)=14>0$

The root lies in [2,3]

It 01: $a=2, b=3$

$t=\frac{a+b}{2}=\frac{2+3}{2}=2.5$

$f(t)=f(2.5)=4.875>0$

$f(2)=-1<0$

$\Rightarrow$ The root lies in [2, 2.5]

It 02: $a=2, b=2.5$

$t=\frac{2+2.5}{2}=2.25$

$f(2.25)=1.5781>0$

$f(2)=-1<0$

$\Rightarrow$ The root lies in [2, 2.25]

It 03: $a=2, b=2.25$

$t=\frac{2+2.25}{2}=2.125$

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$f(2.125) = 0.2050 > 0$
$f(2) = -1 < 0$
$\Rightarrow$ The root lies in $[2, 2.125]$

It 04 : $a=2$, $b=2.125$
$t = \frac{2+2.125}{2} = 2.0625$

$f(2.0625) = -0.4177 < 0$
$f(2.125) = 0.2050 > 0$
$\Rightarrow$ The root lies in $[2.0625, 2.125]$

It 05 : $a=2.0625$, $b=2.125$
$t = \frac{2.0625+2.125}{2} = 2.0937$

$f(2.0937) = -0.1119 < 0$
$f(2.125) > 0$
$\Rightarrow$ The root lies in $[2.0937, 2.125]$

It 06 : $a=2.0937$, $b=2.125$
$t = \frac{2.0937+2.125}{2} = 2.1093$

$f(2.1093) = 0.0447 > 0$
$f(2.0937) < 0$
$\Rightarrow$ The root lies in $[2.0937, 2.1093]$

It 07 : $a=2.0937$, $b=2.1093$
$t = \frac{2.0937+2.1093}{2} = 2.1015$

$f(2.1015) = -0.0339 < 0$
$f(2.1093) > 0$

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$\Rightarrow$ The root lies in $[2.1015, 2.1093]$

It 08 : $a = 2.1015$, $b = 2.1093$

$t = \frac{2.1015 + 2.1093}{2} = 2.1054$

$f(2.1054) = 0.0053 \approx 0$

$\Rightarrow$ The root of equation $x^3 - x^2 + x - 7 = 0$ is $x = 2.1054$

Q.2. Using Bisection Method, Find approximate value of $\sqrt{13}$ by performing 5 iteration.

$\Rightarrow$ Let $x = \sqrt{13}$

$x^2 = 13$

$x^2 - 13 = 0$

$\therefore f(x) = x^2 - 13$

$x = 0 \Rightarrow f(0) = -13$

$x = 1 \Rightarrow f(1) = -12$

$x = 2 \Rightarrow f(2) = -9$

$x = 3 \Rightarrow f(3) = -4 < 0$

$x = 4 \Rightarrow f(4) = 3 > 0$

$\Rightarrow$ The root lies in $[3, 4]$

It 01 : $a = 3$, $b = 4$

$t = \frac{a+b}{2} = \frac{3+4}{2} = 3.5$

$f(3.5) = -0.75 < 0$

$f(4) > 0$

$\Rightarrow$ The root lies in $[3.5, 4]$

It 02 : $a = 3.5$, $b = 4$

$t = \frac{3.5+4}{2} = 3.75$

$f(3.75) = 1.0625 > 0$

$f(3.5) < 0$

$\Rightarrow$ The root lies in $[3.5, 3.75]$

It 03 : $a = 3.5$, $b = 3.75$

$t = \frac{a+b}{2} = \frac{3.5+3.75}{2} = 3.625$

$f(3.625) = 0.1406 > 0$

$f(3.5) < 0$

$\Rightarrow$ The root lies in $[3.5, 3.625]$

It 04 : $a = 3.5$, $b = 3.625$

$t = \frac{3.5+3.625}{2} = 3.5625$

$f(3.5625) = -0.3085 < 0$

$f(3.625) > 0$

$\Rightarrow$ The root lies in $[3.5625, 3.625]$

It 05 : $a = 3.5625$, $b = 3.625$

$t = \frac{3.5625+3.625}{2} = 3.5937$

$f(3.5937) = -0.0853 < 0$

$f(3.625) > 0$

$\Rightarrow$ The root lies in $[3.5937, 3.625]$

It 06 : $a = 3.5937$, $b = 3.625$

$t = \frac{3.5937+3.625}{2} = 3.6093$

$f(3.6093) = 0.0270 \approx 0$

$\Rightarrow$ The root of $\sqrt{13}$ is 3.6093

Q.3. Using Regula-Falsi method, Find the root of equation $x^2 - \log_{10} x - 12 = 0$. Perform only 5 iterations.

$\Rightarrow$ Given : $x^2 - \log_{10} x - 12 = 0$

$f(x) = x^2 - \log_{10} x - 12$

$x = 1 \Rightarrow f(1) = -11$

$x = 2 \Rightarrow f(2) = -8.301$

$x = 3 \Rightarrow f(3) = -3.4771 < 0$

$x = 4 \Rightarrow f(4) = 3.3979 > 0$

$\Rightarrow$ The root lies in $[3, 4]$

It 01 : $a = 3$, $b = 4$, $f(3) = -3.4771$, $f(4) = 3.3979$

By Regula Falsi Method

$$c = \frac{a.f(b) - bf(a)}{f(b) - f(a)} \quad \text{--- (1)}$$

$$c = \frac{3.(3.3979) - 4(-3.4771)}{(3.3979) - (-3.4771)} = 3.5057$$

$f(c) = f(3.5057) = -0.2548 < 0$

$f(4) > 0$

$\Rightarrow$ The root lies in $[3.5057, 4]$

It 02 : $a = 3.5057$, $b = 4$, $f(a) = -0.2548$, $f(4) = 3.3979$

$$c = \frac{a.f(b) - bf(a)}{f(b) - f(a)} = \frac{3.5057(3.3979) - 4(-0.2548)}{(3.3979) - (-0.2548)}$$

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$c = 3.5401$

$f(3.5401) = -0.0167 < 0$

$f(4) > 0$

$\Rightarrow$ The root lies in $[3.5401, 4]$

It 03 : $a = 3.5401$, $b = 4$, $f(a) = -0.0167$, $f(b) = 3.3$
from ①

$$c = \frac{3.5401(3.3979) - 4(-0.0167)}{3.3979 - (-0.0167)}$$

$c = 3.5423$

$f(3.5423) = -0.00139 < 0$

$f(4) > 0$

$\Rightarrow$ The root lies in $[3.5423, 4]$

It 04 : $a = 3.5423$, $b = 4$, $f(a) = -0.00139$, $f(b) = 3.39$
from ①

$$c = \frac{3.5423(3.3979) - 4(-0.00139)}{3.3979 - (-0.00139)}$$

$c = 3.5424$

$f(3.5424) = -0.00069 < 0$

$f(4) > 0$

$\Rightarrow$ The root lies in $[3.5424, 4]$

It 05 : $a = 3.5424$, $b = 4$, $f(a) = -0.00069$
$f(b) = 3.3979$
from ①

$$c = \frac{3.5424(3.3979) - (-0.00069)4}{3.3979 - (-0.00069)}$$

$\boxed{c = 3.5424}$

Q.4. Us

$f(3.5424) = -0.00069 \approx 0$

$\therefore$ The root of equation $x^2 - \log_{10} x - 12 = 0$ is $x = 3.5424$

Q.4. Using Regula-Falsi method, Find the root of equations $x^3 - 3x + 4 = 0$, Perform only 5 iterations.

$\Rightarrow$ Given : $x^3 - 3x + 4 = 0$

$f(x) = x^3 - 3x + 4$

$x = 0 \Rightarrow f(0) = 4$

$x = 1 \Rightarrow f(1) = 2$

$x = -1 \Rightarrow f(-1) = 6$

$x = 2 \Rightarrow f(2) = 6$

$x = -2 \Rightarrow f(-2) = 2 > 0$

$x = 3 \Rightarrow f(3) = 22$

$x = -3 \Rightarrow f(-3) = -14 < 0$

$\Rightarrow$ The root lies in $[-3, -2]$

Regula Falsi Method formula,

$$c = \frac{a f(b) - b f(a)}{f(b) - f(a)} \quad \text{--- (1)}$$

It 01 : $a = -3$; $b = -2$; $f(a) = -14$; $f(b) = 2$

from (1)

$$c = \frac{-3(2) - (-2)(-14)}{2 - (-14)} = -2.125$$

$f(c) = f(-2.125) = 0.7792 > 0$

$f(-3) < 0$

$\Rightarrow$ The root lies in $[-3, -2.125]$

It 02 : $b=-2.125$, $a=-3$, $f(b)=0.7792$, $f(a)=-14$
from ①

$$c = \frac{-2.125(2)-(-2)(0.7792)}{2-(0.7792)} \quad \times$$

$$c = \frac{-3(0.7792)-(-2.125)(-14)}{0.7792-(-14)} = -2.1711$$

$f(-2.1711) = 0.2794 > 0$

$f(-3) < 0$

$\Rightarrow$ The root lies in $[-3, -2.1711]$

It 03 : $a=-3$, $b=-2.1711$, $f(a)=-14$, $f(b)=0.2794$
from ①

$$c = \frac{-3(0.2794)-(-2.1711)(-14)}{0.2794-(-14)} = -2.1873$$

$f(-2.1873) = 0.0972 > 0$

$f(-3) < 0$

$\Rightarrow$ The root lies in $[-3, -2.1873]$

It 04 : $a=-3$, $b=-2.1873$, $f(a)=-14$, $f(b)=0.0972$
from ①

$$c = \frac{-3(0.0972)-(-2.1873)(-14)}{0.0972-(-14)} = -2.1929$$

$f(-2.1929) = 0.0334 > 0$

$f(-3) < 0$

$\Rightarrow$ The root lies in $[-3, -2.1929]$

It 05 : $a=-3$, $b=-2.1929$, $f(a)=-14$, $f(b)=0.0334$

from ①

$$c=\frac{-3(0.0334)-(-2.1929)(-14)}{0.0334-(-14)}=-2.1948$$

$f(-2.1948)=0.0117\approx 0$

$\Rightarrow$ The root of equation $x^3-3x+4=0$ is $x=-2.1948$

Q.5. Using Regula-Falsi method, Find the root of equation $xe^x-1=0$. Perform only 5 iterations

$\Rightarrow$ Given : $xe^x-1=0$

$f(x)=xe^x-1$

$x=0 \Rightarrow f(0)=-1<0$

$x=1 \Rightarrow f(1)=1.7182>0$

$x=2 \Rightarrow f(2)=13.7781$

$\Rightarrow$ The root lies in $[0,1]$

It 01 : $a=0$, $b=1$, $f(a)=-1$, $f(b)=1.7182$

By Regula Falsi method,

$$c=\frac{a.f(b)-b\,f(a)}{f(b)-f(a)} \quad \text{——— ①}$$

$$c=\frac{0-1(-1)}{1.7182-(-1)}=0.3678$$

$f(c)=f(0.3678)=-0.4686<0$

$f(1)>0$

$\Rightarrow$ The root lies in $[0.3678, 1]$

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It 02 : $a = 0.3678$, $b = 1$, $f(a) = -0.4686$, $f(b) = 1.7182$
from (1)

$$c = \frac{0.3678(1.7182) - (1)(-0.4686)}{1.7182 - (-0.4686)}$$

$c = 0.5032$

$f(0.5032) = -0.1677 < 0$
$f(1) > 0$
$\Rightarrow$ The root lies in $[0.5032, 1]$

It 03 : $a = 0.5032$, $b = 1$, $f(a) = -0.1677$, $f(b) = 1.7182$
from (1)

$$c = \frac{0.5032(1.7182) - (1)(-0.1677)}{1.7182 - (-0.1677)}$$

$c = 0.5473$

$f(0.5473) = -0.0539 < 0$
$f(1) > 0$
$\Rightarrow$ The root lies in $[0.5473, 1]$

It 04 : $a = 0.5473$, $b = 1$, $f(a) = -0.0593$, $f(b) = 1.7182$
from (1)

$$c = \frac{0.5473(1.7182) - 1(-0.0593)}{1.7182 - (-0.0593}$$

$c = 0.5610$

$f(c) = f(0.5610) = -0.0168 < 0$
$f(1) > 0$
$\Rightarrow$ The root lies in $[0.5610, 1]$

It 05 : $a=0.5610$, $b=1$, $f(a)=-0.0168$, $f(b)=1.7182$

from (1)

$$c=\frac{0.5610(1.7182)-1(-0.0168)}{1.7182-(-0.0168)}$$

$c=0.5652$

$f(0.5652)=-0.0053\approx 0$

$\Rightarrow$ The root of equation $xe^x-1=0$ is $x=0.5652$

Q.6. ~~Per~~ Find approximate root of $x^3-x-1=0$ using secant method by performing 6 iteration.

$\Rightarrow$ Given : $x^3-x-1=0$

$f(x)=x^3-x-1$

$x=0$, $f(0)=-1$

$x=1$, $f(1)=-1<0$

$x=2$, $f(2)=5>0$

$\Rightarrow$ The root lies in $[1,2]$

It 01 : $x_0=1$, $x_1=2$, $f(x_0)=-1$, $f(x_1)=5$

By secant method,

$$x_2=x_1-\left[\frac{x_1-x_0}{f(x_1)-f(x_0)}\right]f(x_1) \quad \text{--- (1)}$$

$$=2-\left[\frac{2-1}{5-(-1)}\right]5=\frac{7}{6}$$

$\boxed{x_2=1.1666}$

$f(1.1666)=-0.5789<0$

$f(2)>0$

$\Rightarrow$ The root lies in $[1.1666, 2]$

It 02 : $x_0 = 1.1666$, $x_1 = 2$, $f(x_0) = -0.5789$

$f(x_1) = 5$

from (1)

$$x_2 = 2 - \left[\frac{2-1.1666}{5-(-0.5789)}\right] \times 5 = 1.2531$$

$f(1.2531) = -0.2854 < 0$

$f(2) > 0$

$\Rightarrow$ The root lies in $[1.2531, 2]$

It 03 : $x_0 = 1.2531$, $x_1 = 2$, $f(x_0) = -0.2854$

$f(x_1) = 5$

from (1)

$$x_2 = 2 - \left[\frac{2-1.2531}{5-(-0.2854)}\right] 5 = 1.2934$$

$f(x_2) = f(1.2934) = -0.1296 < 0$

$f(2) > 0$

$\Rightarrow$ The root lies in $[1.2934, 2]$

It 04 : $x_0 = 1.2934$, $x_1 = 2$, $f(x_0) = -0.1296$

$f(x_1) = 5$

from (1)

$$x_2 = 2 - \left[\frac{2.12934}{5-(-0.1296)}\right] 5 = 1.3112$$

$f(1.3112) = -0.0569 < 0$

$f(2) > 0$

$\Rightarrow$ The root lies in $[1.3112, 2]$

It 05 : $x_0 = 1.3112$, $x_1 = 2$, $f(x_0) = -0.0569$,
$f(x_1) = 5$

from ①

$$x_2 = 2 - \left[\frac{2 - 1.3112}{5 - (-0.0569)}\right] 5 = 1.3189$$

$$\boxed{x_2 = 1.3189}$$

$f(1.3189) = -0.0246 < 0$

$f(2) > 0$

$\Rightarrow$ The root lies in $[1.3189, 2]$

It 06 : $x_0 = 1.3189$, $x_1 = 2$, $f(x_0) = -0.0246$
$f(x_1) = 5$

from ①

$$x_2 = 2 - \left[\frac{2 - 1.3189}{5 - (-0.0246)}\right] 5 = 1.3223$$

$f(1.3223) = -0.0102 \approx 0$

$\Rightarrow$ The root of equation $x^3 - x - 1 = 0$ is $x = 1.3223$

Q.7. Find approximate root of $x^3 - 5x + 1 = 0$, using Secant Method by performing 5 iteration.

$\Rightarrow$ Given : $x^3 - 5x + 1 = 0$

$f(x) = x^3 - 5x + 1$

$x = 0$, $f(0) = 1$

$x = 1$, $f(1) = -3 < 0$

$x = 2$, $f(2) = -1 < 0$

$x = 3$, $f(3) = 13 > 0$

$\Rightarrow$ The root lies in $[2, 3]$

It 01 : $x_0 = 2$, $x_1 = 3$, $f(x_0) = -1$, $f(x_1) = 13$

By Secant Method,

$$x_2 = x_1 - \left[\frac{x_1 - x_0}{f(x_1) - f(x_0)}\right] f(x_1). \quad \text{—— (1)}$$

$$x_2 = 3 - \left[\frac{3-2}{13-(-1)}\right] 13 = 2.0714$$

$f(2.0714) = -0.4692 < 0$

$f(3) > 0$

$\Rightarrow$ The root lies in $[2.0714, 3]$

It 02 : $x_0 = 2.0714$, $x_1 = 3$, $f(x_0) = -0.4692$

$f(x_1) = 13$

from (1)

$$x_2 = 3 - \left[\frac{3-2.0714}{13-(-0.4692)}\right] 13 = 2.1037$$

$f(2.1037) = -0.2084 < 0$

$f(3) > 0$

$\Rightarrow$ The root lies in $[2.1037, 3]$

It 03 : $x_0 = 2.1037$, $x_1 = 3$, $f(x_0) = -0.2084$

$f(x_1) = 13$

from (1)

$$x_2 = 3 - \left[\frac{3-2.1037}{13-(-0.2084)}\right] 13 = 2.1178$$

$f(2.1178) = -0.0905 < 0$

$f(3) > 0$

$\Rightarrow$ The root lies in $[2.1178, 3]$

It 04 : $x_0 = 2.1178$, $x_1 = 3$, $f(x_0) = -0.0905$
$f(x_1) = 13$
from ①

$$x_2 = 3 - \left[\frac{3 - 2.1178}{13 - (-0.0905)}\right] 13 = 2.1238$$

$f(2.1238) = -0.0365 < 0$
$f(3) > 0$
$\Rightarrow$ The root lies in $[2.1238, 3]$

It 05 : $x_0 = 2.1238$, $x_1 = 3$, $f(x_0) = -0.0365$
$f(x_1) = 13$
from ①

$$x_2 = 3 - \left[\frac{3 - 2.1238}{13 - (-0.0365)}\right] 13 = 2.1262$$

$f(2.1262) = -0.0190 < 0$, $f(3) > 0$
$\Rightarrow$ The root of equation lies in $[2.1262, 3]$

It 06 : $x_0 = 2.1262$, $x_1 = 3$, $f(x_0) = -0.0190$,
$f(x_1) = 13$
from ①

$$x_2 = 3 - \left[\frac{3 - 2.1262}{13 - (-0.0190)}\right] 13 = 2.1274$$

$f(2.1274) = -0.0087 \approx 0$
$\Rightarrow$ The root of equation $x^3 - 5x + 1 = 0$ is
$x = 2.1274$

Q.8. Find approximate root of $x = \sqrt[3]{48}$ using Secant Method by performing 5 iteration.

$\Rightarrow$ Given : $x = \sqrt[3]{48}$

$x^3 = 48$

$x^3 - 48 = 0$

$f(x) = x^3 - 48$

$x = 0 \Rightarrow f(0) = -48$

$x = 1, \Rightarrow f(1) = -47$

$x = 2 \Rightarrow f(2) = -40$

$x = 3 \Rightarrow f(3) = -21 < 0$

$x = 4 \Rightarrow f(4) = 16 > 0$

$\Rightarrow$ The root lies in $[3, 4]$

It 01 : $x_0 = 3$, $x_1 = 4$, $f(x_0) = -21$, $f(x_1) = 16$

By Secant Method,

$$x_2 = x_1 - \left[\frac{x_1 - x_0}{f(x_1) - f(x_0)}\right] f(x_1) \quad \text{--- (1)}$$

$$= 4 - \left[\frac{4-3}{16-(-21)}\right] 16 = 3.5675$$

$f(x_2) = f(3.5675) = -2.5962 < 0$

$f(4) > 0$

$\Rightarrow$ The root lies in $[2.5675, 4]$.

It 02 : $x_0 = 3.5675$, $x_1 = 4$, $f(x_0) = -2.5962$

$f(x_1) = 16$

from (1)

$$x_2 = 4 - \left[\frac{4 - 3.5675}{16 - (-2.5962)}\right] 16 = 3.6278$$

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$f(3.6278) = -0.2547 < 0$

$f(4) > 0$

$\Rightarrow$ The root lies in $[3.6278, 4]$

It 03 : $x_0 = 3.6278$, $x_1 = 4$, $f(x_0) = -0.2547$

$f(x_1) = 16$

from ①

$$x_2 = 4 - \left[\frac{4 - 3.6278}{16 - (-0.2547)}\right] 16 = 3.6336$$

$f(3.6336) = -0.0254 < 0$

$f(4) > 0$

$\Rightarrow$ The root lies in $[3.6336, 4]$

It 04 : $x_0 = 3.6336$, $x_1 = 4$, $f(x_0) = -0.0254$

$f(x_1) = 16$

from ①

$$x_2 = 4 - \left[\frac{4 - 3.6336}{16 - (-0.0254)}\right] 16 = 3.6341$$

$f(3.6341) = -0.0055 < 0$

$f(4) > 0$

$\Rightarrow$ The root lies in $[3.6341, 4]$

It 05 : $x_0 = 3.6341$, $x_1 = 4$, $f(x_0) = -0.0055$, $f(x_1) = 16$

from ①

$$x_2 = 4 - \left[\frac{4 - 3.6341}{16 - (-0.0055)}\right] 16 = 3.6342$$

$f(3.6342) = -0.0016 \approx 0$

$\Rightarrow$ The root of equation $x^3 - 48 = 0$ is $x = 3.6342$

i.e. $x = \sqrt[3]{48}$ is 3.6342

Q.9. Compute the approximate root of equation $f(x) = x^3 - x - 1 = 0$, by using fixed point iteration method.

$\Rightarrow$ Given: $f(x) = x^3 - x - 1 = 0$

$x^3 - x - 1 = 0$

$x^3 = x + 1$

$x = \sqrt[3]{x+1}$

This is in the form of $x = \phi(x)$

So here $\phi(x) = \sqrt[3]{x+1}$

Let

$x = 0, \ f(0) = -1$

$x = 1, \ f(1) = -1 < 0$

$x = 2, \ f(2) = 5 > 0$

$\Rightarrow$ The root lies in $[1, 2]$

Let $x_0 = 1$

It 01: $x_1 = \phi(x_0) = \sqrt[3]{x_0 + 1}$

$x_1 = \sqrt[3]{1+1} \Rightarrow \boxed{x_1 = 1.2599}$

It 02: $x_2 = \phi(x_1) = \sqrt[3]{x_1 + 1}$

$x_2 = \sqrt[3]{1.2599 + 1} \Rightarrow \boxed{x_2 = 1.3122}$

It 03: $x_3 = \phi(x_2) = \sqrt[3]{x_2 + 1}$

$x_3 = \sqrt[3]{1.3122 + 1} \Rightarrow \boxed{x_3 = 1.3223}$

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It 04 : $x_4 = \phi(x_3) = \sqrt[3]{x_3+1}$

$x_4 = \sqrt[3]{1.3223+1} \Rightarrow \boxed{x_4 = 1.3242}$

It 05 : $x_5 = \phi(x_4) = \sqrt[3]{x_4+1}$

$x_5 = \sqrt[3]{1.3242+1} \Rightarrow \boxed{x_5 = 1.3246}$

$\Rightarrow$ The root of equation $f(x) = x^3 - x - 1$ is 1.3246

Q.10. Compute the approximate root of the equation $f(x) = 2x^3 - 2x - 5 = 0$, by using fixed point iteration method.

$\Rightarrow$ $f(x) = 2x^3 - 2x - 5 = 0$

$2x^3 - 2x - 5 = 0$

$2x^3 = 2x + 5$

$x = \sqrt[3]{\frac{2x+5}{2}}$

This is in the form of $x = \phi(x)$

So here $\phi(x) = \sqrt[3]{\frac{2x+5}{2}}$

$x = 0, \ f(0) = -5$

$x = 1, \ f(1) = -5 < 0$

$x = 2, \ f(2) = 7 > 0$

$\Rightarrow$ The root lies in $[1, 2]$

Let $x_0 = 1$

It 01 : $x_1 = \phi(x_0) = \sqrt[3]{\frac{2x_0+5}{2}} = \sqrt[3]{\frac{2(1)+5}{2}}$

$\boxed{x_1 = 1.5182}$

It 02 : $x_2 = \phi(x_1) = \sqrt[3]{\frac{2x_1+5}{2}} = \sqrt[3]{\frac{2(1.5182)+5}{2}}$

$\boxed{x_2 = 1.5898}$

It 03 : $x_3 = \phi(x_2) = \sqrt[3]{\frac{2x_2+5}{2}}$

$x_3 = \sqrt[3]{\frac{2(1.5898)+5}{2}} \Rightarrow \boxed{x_3 = 1.5991}$

It 04 :

$x_4 = \phi(x_3) = \sqrt[3]{\frac{2x_3+5}{2}}$

$x_4 = \sqrt[3]{\frac{2(1.5991)+5}{2}} \Rightarrow \boxed{x_4 = 1.6004}$

It 05 :

$x_5 = \phi(x_4) = \sqrt[3]{\frac{2x_4+5}{2}}$

$x_5 = \sqrt[3]{\frac{2(1.6004)+5}{2}} \Rightarrow \boxed{x_5 = 1.6005}$

$\Rightarrow$ The root of equation $f(x) = 2x^3 - 2x - 5$ is $x = 1.6005$.

Q.11. Using Newton-Raphson method to find root of equation $x^3-3x-5=0$

$\Rightarrow$ $f(x)=x^3-3x-5$

$x=0, \ f(0)=-5$
$x=1, \ f(1)=-7$
$x=2, \ f(2)=-3<0$
$x=3, \ f(3)=13>0$

Let $x_0=2$ as initial root

Here $f(x)=x^3-3x-5$
$f'(x)=3x^2-3$
$f(x_0)=f(2)=-3$
$f'(x_0)=f'(2)=9$

It 01:

By Newton-Raphson Method

$$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)} \quad \text{--- (1)}$$

$$x_1=x_0-\frac{f(x_0)}{f'(x_0)}=2-\frac{(-3)}{9}$$

$\boxed{x_1=2.3334}$

$f(x_1)=f(2.3334)=0.7046$
$f'(x_1)=f'(2.3334)=13.3342$

It 02: from (1)

$$x_2=x_1-\frac{f(x_1)}{f'(x_1)}=2.3334-\frac{0.7046}{13.3342}$$

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$$\boxed{x_2 = 2.2805}$$

$f(x_2) = 0.0186$
$f'(x_2) = 12.6020$

It 03 : from (1)

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 2.2805 - \frac{0.0186}{12.6020}$$

$$\boxed{x_3 = 2.2790}$$

$f(x_3) = -0.0002363$
$f'(x_3) = 12.5815$

It 04 : from (1)

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 2.2790 - \frac{(-0.0002363)}{12.5815}$$

$$\boxed{x_4 = 2.2790}$$

The root of the equation $x^3 - 3x - 5$ is $x = 2.2790$

Q.12. Use Newton-Raphson method to evaluate the value of $\sqrt[3]{20}$ upto 3 decimal places.

$\Rightarrow$ Let $x = \sqrt[3]{20}$

$x^3 = 20$

$x^3 - 20 = 0$

Here $f(x) = x^3 - 20$

$x = 0$, $f(0) = -20$

$x = 1$, $f(1) = -19$

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$x=2,\ f(2)=-12<0$
$x=3,\ f(3)=7<0$

Let $x_0=2$, as initial root
As $f(x)=x^3-20$
$f'(x)=3x^2$
$f(x_0)=-12$
$f'(x_0)=12$

It 01:
By using Newton-Raphson Method

$$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)} \quad \text{—— (1)}$$

$$x_1=x_0-\frac{f(x_0)}{f'(x_0)}=2-\frac{(-12)}{12}$$

$\boxed{x_1=3}$

$f(x_1)=7$
$f'(x_1)=27$

It 02: from (1)

$$x_2=x_1-\frac{f(x_1)}{f'(x_1)}=3-\frac{7}{27}$$

$\boxed{x_2=2.7407}$

$f(x_2)=0.5865$
$f'(x_2)=22.5343$

It 03: from (1)

$$x_3=x_2-\frac{f(x_2)}{f(x_2)}$$

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$$x_3 = 2.7407 - \frac{0.5865}{22.5343}$$

$\boxed{x_3 = 2.7146}$

$f(x_3) = 0.0040$

$f'(x_3) = 22.1071$

It ou $\because$ from (i)

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 2.7146 - \frac{0.0040}{22.1071}$$

$\boxed{x_4 = 2.7144}$

The root of equation $x^3 - 20 = 0$ is
$x = 2.7144$