Q1. Name the physical quantity whose S.I. unit is JC^{-1}. Is it a scalar or a vector quantity?

(1) Current, vector

(2) Current, scalar

(3) Electrostatic potential, vector

(4) Electrostatic potential, scalar

Q2. Two vectors $\overrightarrow{\mathrm{a}}$ and $\overrightarrow{\mathrm{b}}$ are at an angle of 60° with each other. Their resultant makes an angle of 45° with $\overrightarrow{\mathrm{a}}$. If $|\overrightarrow{\mathrm{b}}|=2$ units, then $|\overrightarrow{\mathrm{a}}|$ is

(1) $\sqrt{3}$

(2) $\sqrt{3}-1$

(3) $\sqrt{3}+1$

(4) $\frac{\sqrt{3}}{2}$

Q3. If a parallelogram is formed with two sides represented by vectors $\vec{a}$ and $\vec{b}$, then $\vec{a}+\vec{b}$ represented the

(1) major diagonal when the angle between vectors is acute.

(2) minor diagonal when the angle between vectors is obtuse.

(3) Both of the above.

(4) None of the above.

Q4. Consider a vector $\vec{F}=4\hat{i}-3\hat{j}$. Another vector that is perpendicular to $\vec{F}$ is

(1) $4\hat{i}+3\hat{j}$

(2) $6\hat{i}$

(3) $7\hat{k}$

(4) $3\hat{i}-4\hat{j}$

Q5. Closeness of two or more measured values is called as

(1) accuracy

(2) precision

(3) error

(4) approximation

Q6. The least count of a measuring scale is 1 mm. The heights two students is measured with it. What will be the error in the difference between their heights?

(1) 1 mm

(2) 0 mm

(3) 2 mm

(4) 0.5 mm

Q7. The length of a rectangular plate is measured by a meter scale and is found to be 10.0 cm. Its width is measured by vernier calipers as 1.00 cm. The least count of the meter scale and vernier calipers are 0.1 cm and 0.01 cm respectively. In the previous question, what is the minimum possible error in area measurement?

(1) 02 cm^2

(2) 01 cm^2

(3) 03 cm^2

(4) 0

Q8. The most precise measurement among the following is

(1) 5.00 mm

(2) 5.00 cm

(3) 5.00 m

(4) 5.00 km

Q9. Find the relative error in P, if the physical quantity P is described by the relation $P=a^{\frac{1}{2}}b^{2}c^{3}d^{-4}$. It is given that the relative errors in the measurement of a, b, c and d respectively, are $2\%, 1\%, 3\%$ and 5%.

(1) 12%

(2) 8%

(3) 25%

(4) 32%

Q10. Calculate the fractional error $\left(\frac{\Delta x}{x}\right)$, if $x=a^{n}$

(1) $\pm\left(\frac{\Delta a}{a}\right)^{n}$

(2) $\pm n\left(\frac{\Delta a}{a}\right)$

(3) $\pm n\log_{\mathrm{e}}\frac{\Delta a}{a}$

(4) $\pm\frac{n\log\Delta a}{a}$

Q11. The relative error in the determination of the surface area of a sphere is α. The relative error in the determination of its volume is

(1) $\frac{3}{2}\alpha$

(2) $\frac{2}{3}\alpha$

(3) α

(4) $\frac{5}{2}\alpha$

Q12. The mass and volume of a body are found to be 5.00 ± 0.05 kg and $1.00 \pm 0.05\ \mathrm{m}^3$ respectively. Then the maximum possible percentage error in its density is

(1) 6%

(2) 3%

(3) 5%

(4) 7%

Answer Key

Q1 (4)	**Q2** (2)	**Q3** (3)	**Q4** (3)
Q5 (2)	**Q6** (3)	**Q7** (4)	**Q8** (1)
Q9 (4)	**Q10** (2)	**Q11** (1)	**Q12** (1)

Q1. The SI unit of electrostatic potential is joule per coulomb or J C^{-1}. It is a scalar quantity.

Q2.

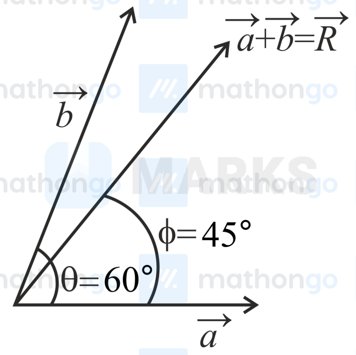
Given,

Magnitude of $\vec{b}$ is, $|\vec{b}|=2$ units.

Angle between the vectors is, $\theta=60^{\circ}$.

Angle between resultant and $\vec{a}$ is, $\phi=45^{\circ}$.

The above is shown in the below figure,



From parallelogram law, ϕ & θ is related as,

$\tan(\phi)=\frac{|\vec{b}|\sin(\theta)}{|\vec{a}|+|\vec{b}|\cos(\theta)}$.

Substituting all the corresponding given values in above relation, we have,

$\Rightarrow \tan(45^{\circ})=\frac{2\sin(60^{\circ})}{|\vec{a}|+2\cos(60^{\circ})}=\frac{\sqrt{3}}{a+1},\ \left(\because|\vec{a}|=a\right)$

$\Rightarrow 1=\frac{\sqrt{3}}{a+1}$

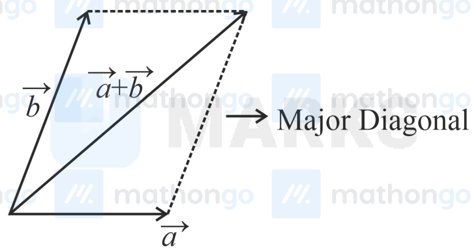
$\Rightarrow a+1=\sqrt{3}$

$\Rightarrow a=\sqrt{3}-1$.

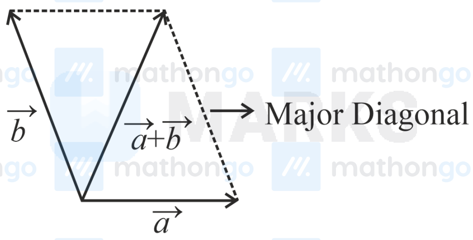
Q3.

If $\vec{a}$ & $\vec{b}$ represents side of parallelogram, then $\vec{a}+\vec{b}$ is represented by diagonal of parallelogram.

Case-(1), Angle between them is acute, then the addition is represented major diagonal shown as below,



Case-(2), Angle between them is obtuse, then the addition is represented minor diagonal shown as below,



$\vec{a}+\vec{b}$ →major diagonal, if angle between vectors is acute.

$\vec{a}+\vec{b}$ →minor diagonal, if angle between vectors is obtuse.

Q4.

Force F lies in the $x-y$ plane so a vector along z-axis will be perpendicular to F.

Q5.

Closeness of two or more measured values is called as precision. The precision reflects how reproducible measurements are, even if they are far from the accepted value. As much the measured values are close, more the measurement is accurate.

Q6.

Given,

The least count, $\Delta l = 1\ \text{mm}$

The error in the reading of the first student,

$\Delta l_1 = \Delta l = 1\ \text{mm}$

The error in the reading of the second student,

$\Delta l_2 = \Delta l = 1\ \text{mm}$

During addition or subtraction of two measurements, the error gets added up.

Hence, the net error becomes,

$\Delta l_{net} = \Delta l_1 + \Delta l_2 = 2\ \text{mm}$

Q7.

minimum possible error is always zero

Q8.

All given measurements are correct up to two decimal places.

As here $5.00\ \text{mm}$ has the smallest unit and the error in $5.00\ \text{mm}$ is least (commonly taken as $0.01\ \text{mm}$ if not specified), hence, $5.00\ \text{mm}$ is most precise.

Q9. $P = a^{\frac{1}{2}} b^2 c^3 d^{-4}$

$\left(\frac{\Delta p}{p}\right)_{max} = \frac{1}{2}\frac{\Delta a}{a} + 2\frac{\Delta b}{b} + 3\frac{\Delta c}{c} + 4\frac{\Delta d}{d}$

$100 \times \frac{\Delta P}{P} = \left(\frac{1}{2}\frac{\Delta a}{a} + 2\frac{\Delta b}{b} + 3\frac{\Delta c}{c} + 4\frac{\Delta d}{d}\right) \times 100$

$\%\ error\ in\ P = \left(\frac{1}{2} \times 2 + 2 \times 1 + 3 \times 3 + 4 \times 5\right)$

$= 32\%$

Q10. $x = a^n$

$\therefore \frac{\Delta x}{x} = \pm n\frac{\Delta a}{a}$

Q11. $\frac{\Delta s}{s} = 2 \times \frac{\Delta r}{r} \Rightarrow \frac{\Delta V}{V} = 3 \times \frac{\Delta r}{r}$

$\therefore \frac{\Delta V}{V} = \frac{3}{2}\frac{\Delta s}{s}$

$\frac{\Delta V}{V} = \frac{3}{2}\alpha$

Q12. $\frac{\Delta p}{p} = \frac{\Delta m}{m} + \frac{\Delta V}{V}$

$= \frac{0.05}{5} \times 100 + \frac{0.05}{1} \times 100$

$= 6$

$\frac{\Delta p}{p} = 6\%$

Q1. Which of the following is not the unit of energy?

(1) watt - hour

(2) electron - volt

(3) Nm

(4) $\mathrm{m}^2\ \mathrm{s}^{-2}$

Q2. Which of the following physical quantities has neither dimensions nor unit?

(1) Angle

(2) Luminous intensity

(3) Coefficient of friction

(4) Current

Q3. SI unit of universal gas constant is

(1) $\mathrm{Cal}^{\circ}\mathrm{C}^{-1}$

(2) Jmol^{-1}

(3) $\mathrm{Jmol}^{-1}\ \mathrm{K}^{-1}$

(4) Jkg^{-1}

Q4. The square root of the product of inductance and capacitance has the dimension of

(1) Length

(2) Mass

(3) Time

(4) Frequency

Q5. Units of a in Van der Waal's equation of state is

(1) $\mathrm{Nm}^4\ \mathrm{mol}^{-1}$

(2) $\mathrm{Nm}^2\ \mathrm{mol}^{-1}$

(3) $\mathrm{N}^2\ \mathrm{m}\ \mathrm{mol}^{-1}$

(4) $\mathrm{Nm}^4\ \mathrm{mol}^{-2}$

Q6. The terminal velocity v_r of a small steel ball of radius r falling under gravity through a column of a viscous liquid of coefficient of viscosity η depends on the mass of the ball m, acceleration due to gravity g, coefficient of viscosity η and radius r. Which of the following relations is dimensionally correct?

(1) $v_r \propto \frac{mgr}{\eta}$

(2) $v_r \propto mg\eta r$

(3) $v_r \propto \frac{mg}{r\eta}$

(4) $v_r \propto \frac{\eta mg}{r}$

Q7. The refractive index of a material is given by the equation $n = \frac{A+B}{\lambda^2}$, where A and B are constant. The dimensional formula for B is

(1) $\left[\mathrm{M}^0\ \mathrm{L}^2\ \mathrm{T}^{-1}\right]$

(2) $\left[\mathrm{M}^0\ \mathrm{L}^{-2}\ \mathrm{T}^0\right]$

(3) $\left[\mathrm{M}^0\ \mathrm{L}^2\ \mathrm{T}^{-2}\right]$

(4) $\left[\mathrm{M}^0\ \mathrm{L}^2\ \mathrm{T}^0\right]$

Q8. What is the dimensional formula of magnetic field?

(1) Dimensional formula: $\left[\mathrm{M}^1\ \mathrm{L}^0\ \mathrm{T}^{-2}\mathrm{I}^{-1}\right]$

(2) Dimensional formula: $\left[\mathrm{M}^4\ \mathrm{L}^0\ \mathrm{T}^{-6}\mathrm{I}^{-3}\right]$

(3) Dimensional formula: $\left[\mathrm{M}^{-2}\ \mathrm{L}^0\ \mathrm{T}^3\mathrm{I}^{-1}\right]$

(4) Dimensional formula: $\left[\mathrm{M}^{-6}\ \mathrm{L}^2\ \mathrm{T}^1\mathrm{I}^{-2}\right]$

Q9. In terms of resistance R and time T, the dimensions of ratio $\frac{\mu}{\epsilon}$ the permeability μ and permittivity ϵ is :

(1) $\left[\mathrm{RT}^{-2}\right]$

(2) $\left[\mathrm{R}^2\ \mathrm{T}^2\right]$

(3) $\left[\mathrm{R}^2\right]$

(4) $\left[\mathrm{R}^2\ \mathrm{T}^{-1}\right]$

Q10. The pairs having same dimensional formula

(1) Angular momentum, torque

(2) Torque, work

(3) Plank's constant, Boltzmann's constant

(4) Gas constant, pressure

Answer Key

Q1 (3)	**Q2** (3)	**Q3** (3)	**Q4** (3)
Q5 (4)	**Q6** (3)	**Q7** (4)	**Q8** (1)
Q9 (3)	**Q10** (2)		

Q1.

Energy is known as the capacity of performing work. It can occur in various possible ways, including kinetic, potential, chemical, electrical, thermal, radioactive, or other forms such as energy in the transition process through one body to the other i.e., work and heat.

The different units of energy in different unit systems are: Joule in S.I and MKS, Erg in CGS.

As we know, 1 Joule $= 1\ \text{kg m}^2\ \text{s}^{-2}$.

Energy is also expressed as $E = Power \times time$. Here, the unit of power is Watt and that of time is hour. Thus, its other unit will be $=$ Watt $-$ hour. It is equal to 1 Watt of output for 1 hour.

The quantity of kinetic energy obtained from a single electron accelerated from rest by an electrical potential gap in vacuum of one volt is an electron volt.

Work is expressed as

$\text{W} = \text{Force (Newton)} \times \text{displacement (metre)}$. Thus, the unit of work is Newton $\times$ metre $=$ N m.

In general N m is used as unit of torque. Though it is dimensionally correct, we don't use it as unit of energy.

Q2.

Unit less physical quantities are always dimensionless. For example, refractive index and relative density. These quantities are the ratio of same physical quantities.

A dimensionless physical quantity may have a unit. For example, plane angle and solid angle. The unit of plane angle is radian but it is dimensionless.

Now,

Coefficient of friction $= \frac{\text{friction force}}{\text{Normal reaction}}$

$= \frac{[\text{MLT}^{-2}]}{[\text{MLT}^{-2}]} =$ no dimensions

Unit$= \frac{\text{N}}{\text{N}} =$ no unit

The coefficient of friction is the ratio of two forces. Hence, it is dimensionless. It also don't have any unit.

Q3.

The ideal gas equation is given by, $PV = nRT$, where P is pressure, V is volume, n is number of moles, R is gas constant and T is temperature. Gas constant is given by, $R = \frac{PV}{nT}$. After putting the units of all physical quantities. We get,

$\Rightarrow R = \frac{\text{N m}^{-2} \times \text{m}^3}{\text{mol} \times \text{K}}$

$\Rightarrow R = \frac{\text{N m}}{\text{mol} \times \text{K}}$ (N m $=$ J), so the unit of gas constant,

$R = \text{J mol}^{-1}\ \text{K}^{-1}$.

Q4. We know, $f = \frac{1}{2\pi\sqrt{LC}}$

Or $\sqrt{LC} = \frac{1}{2\pi f} =$ time

Thus, $\sqrt{LC}$ has the dimension of time.

Q5.

Van der Waals equation is given as $\left(P + \frac{an^2}{V^2}\right)(V - nb) = nRT$, here P is pressure, V is volume, T is temperature, n is the number of moles, and R is the universal gas constant.

The dimensional formula of pressure is

$P = \frac{\text{Force}}{\text{Area}} = \frac{[\text{MLT}^{-2}]}{[L^2]} = [\text{ML}^{-1}\ \text{T}^{-2}]$

The dimensional formula of the volume is $[L^3]$

As we know, the quantities with the same dimensions are added or subtracted. Thus, the dimension of $P =$the dimension of $\frac{an^2}{V^2}$.

Therefore, $a = \frac{PV^2}{n^2} = \frac{[\text{ML}^{-1}\text{T}^{-2}]\ [\text{L}^3]^2}{[\text{mol}]^2} = [\text{ML}^5\ \text{T}^{-2}\ \text{mol}^{-2}]$

Thus, the unit of constant a is $\text{N m}^4\ \text{mol}^{-2}$.

Q6.

According to question, terminal velocity (v_r) of ball depends on mass of ball (m), acceleration due to gravity (g), coefficient of viscosity (η) and radius of ball (r).

Using Dimensional analysis, we can write

$v_r \propto m^a g^b \eta^c r^d$

$\Rightarrow [v_r] = [m^a g^b \eta^c r^d]$

Substituting the dimensions of each term in the above expression we get

$\text{M}^0\text{L}^1\text{T}^{-1} = (\text{M}^1\text{L}^0\text{T}^0)^a (\text{M}^0\text{L}^1\text{T}^{-2})^b (\text{M}^1\text{L}^{-1}\text{T}^{-1})^c (\text{M}^0\text{L}^1\text{T}^0)^d$

Now equating the exponent of each of the fundamental dimensions, we get

$a + c = 0 \quad \ldots(1)$

$b + d - c = 1 \quad \ldots(2)$

$-2b - c = -1 \quad \ldots(3)$

Solving above system of equations, we get $a = b = 1$ and $c = d = -1$.

Hence, the dimensionally correct relation is $v_r \propto \frac{mg}{\eta r}$.

Q7. The given equation is

$n = \frac{A+B}{\lambda^2}$

Where A and B are constants.

By homogeneity principle the dimensions of all the terms on both sides should be same

$ie,\ [B] = [A] = [n\lambda^2]$

$\therefore \quad [B]=\left[\mathrm{M}^0\mathrm{L}^0\mathrm{T}^0\right]\left[\mathrm{L}^2\right]$

$=\left[\mathrm{M}^0\mathrm{L}^2\mathrm{T}^0\right]$

Q8. Units of magnetic field is Tesla has dimension of $MT^{-2}I^{-1}$ where M is mass, T is time, and I is electric current.

Q9. $\frac{\mu}{\in}=\frac{\left[\mathrm{MLT}^{-2}\mathrm{A}^{-2}\right]}{\left[\mathrm{M}^{-1}\mathrm{L}^{-3}\mathrm{T}^4\mathrm{A}^2\right]}$

$=\left[\mathrm{M}^2\mathrm{L}^4\mathrm{T}^{-6}\mathrm{A}^{-4}\right]$

$=\left[\mathrm{ML}^2\mathrm{T}^{-3}\mathrm{A}^{-2}\right]^2$

$=\mathrm{R}^2$

Q10.

We know,

$Work\ done = Force \times Displacement$

$Torque = force \times perpendicular\ distance\ between\ force\ and\ axis\ of\ rotation$

Dimension of Work= $\left[ML^2T^{-2}\right]$

Dimension of Torque $=\left[ML^2T^{-2}\right]$

Here it is clear that, dimension of work and dimension of torque are equal.

Q1. The position of a particle is given by $x=2\left(t-t^2\right)$, where t is expressed in seconds and x is in meter. The total distance travelled by the particle between t $=0$ to t $=1$ s is:

(1) 0 m

(2) 1 m

(3) 2 m

(4) $\frac{1}{2}$ m

Q2. A person travelling on a straight line without changing direction moves with a uniform speed v_1 for half distance and next half distance he covers with uniform speed v_2. The average speed v is given by

(1) $v=\frac{2 v_1 v_2}{v_1+v_2}$

(2) $v=\sqrt{V_1 V_2}$

(3) $\frac{V_1+V_2}{2}$

(4) $\frac{1}{V}=\frac{1}{V_1}+\frac{1}{V_2}$

Q3. A point moves with uniform acceleration and its initial speed and final speed are $1 \mathrm{~m} \mathrm{~s}^{-1}$ and $2 \mathrm{~m} \mathrm{~s}^{-1}$ respectively then, the space average of velocity over the distance moved is. (in ms^{-1}) :-

(1) $\frac{14}{9} \mathrm{~m} \mathrm{~s}^{-1}$

(2) $\frac{3}{2} \mathrm{~ms}^{-1}$

(3) $\frac{5}{2} \mathrm{~ms}^{-1}$

(4) None of these

Q4. A body starts from rest and moves with a constant acceleration. The ratio of distance covered in the nth second to the distance covered in n seconds is

(1) $\frac{2}{n}-\frac{1}{n^2}$

(2) $\frac{1}{n^2}-\frac{1}{n}$

(3) $\frac{2}{n^2}-\frac{1}{n}$

(4) $\frac{2}{n}+\frac{1}{n^2}$

Q5. Acceleration velocity graph of a particle moving in a straight line is as shown in figure. The slope of velocity-displacement graph

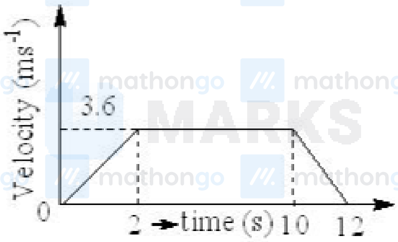
(1) increases linearly

(2) decreases linearly

(3) is constant

(4) increases parabolically

Q6. An elevator is going upward and the variation in the velocity of the elevator is as given in the graph. What is the height to which the elevator takes the passengers?



(1) 3.6 m

(2) 28.8 m

(3) 36.0 m

(4) 32.4 m

Q7. The displacement-time graph of two particles A and B are straight lines inclined at angles 30° and 60° with time axis. The ratio of $v_A: v_B$ is:

(1) $1: 2$

(2) $1: \sqrt{3}$

(3) $\sqrt{3}: 1$

(4) $1: 3$

Q8. If $a=\left(3 t^2+2 t+1\right) \mathrm{ms}^{-2}$ is the expression according to which the acceleration of a particle varies. Find a displacement of the particle after 2 s of start.

(1) 27 m

(2) $\frac{26}{3}$ m

(3) 30/7 m

(4) 26/2 m

Q9. A ball is dropped from a building of height 45 m. Simultaneously another ball is thrown up with a speed $40 \mathrm{~m} / \mathrm{s}$. Calculate the relative speed of the balls as a function of time.

(1) $40 \mathrm{~m} / \mathrm{s}$

(2) $30 \mathrm{~m} / \mathrm{s}$

(3) $20 \mathrm{~m} / \mathrm{s}$

(4) $10 \mathrm{~m} / \mathrm{s}$

Q10. A ball A is thrown vertically upward with speed u. At the same instant, another ball B is released from rest at height h. At time t, the speed of A relative to B is (Assume both are in the air)

(1) u

(2) $u-2gt$

(3) $\sqrt{u^2-2gh}$

(4) $u-gt$

Q11. A stone is thrown vertically upward with an initial speed u from the top of a tower, reaches the ground with a speed $3u$. The height of the tower is:

(1) $\frac{3u^2}{g}$

(2) $\frac{4u^2}{g}$

(3) $\frac{6u^2}{g}$

(4) $\frac{9u^2}{g}$

Q12. A young man of mass 60 kg stands on the floor of a lift which is accelerating downwards at $1\frac{\mathrm{m}}{\mathrm{s}^2}$ then the reaction of the floor of the lift on the man is (Take $g=9.8\frac{\mathrm{m}}{\mathrm{s}^2}$)

(1) 528 N

(2) 540 N

(3) 546 N

(4) None

Answer Key

Q1 (2)	**Q2** (1)	**Q3** (1)	**Q4** (1)
Q5 (3)	**Q6** (3)	**Q7** (4)	**Q8** (2)
Q9 (1)	**Q10** (2)	**Q11** (2)	**Q12** (1)

Q1.

Let the total distance be s between for $0 < t < 1$

We have

$x = 2\left(t - t^2\right)$

For $0 < t < \frac{1}{2}$

the displacement is $s_1 = \frac{1}{2}$

for $\frac{1}{2} < t < 1$

the displacement is $s_2 = \frac{-1}{2}$

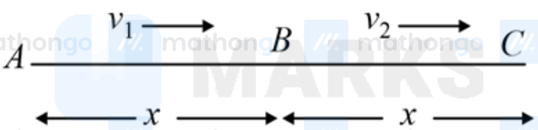
distance

$s = |s_1| + |s_2|$

$s = \frac{1}{2} + \frac{1}{2} = 1\ \mathrm{m}$

Q2.

Let the total distance be $2x$.



For the first half distance x, let time be t_1.

$\because$ time $= \frac{\text{distance}}{\text{speed}}$

$\therefore t_1 = \frac{x}{v_1} \ldots$ (i)

For second half distance x, let time be t_2.

$\because$ time $= \frac{\text{distance}}{\text{speed}}$

$\therefore t_2 = \frac{x}{v_2} \ldots$ (ii)

$\therefore$ Average speed for entire journey.

$v_{\text{av}} = \frac{\text{total distance}}{\text{total time}}$

$\Rightarrow v_{\text{av}} = \frac{2x}{\frac{x}{v_1} + \frac{x}{v_2}}$

$\Rightarrow v = \frac{2x}{x\left(\frac{v_1 + v_2}{v_1 v_2}\right)} \quad (\because v_{\text{av}} = v)$

$\Rightarrow v = \frac{2 v_1 v_2}{v_1 + v_2}$.

Note average speed is harmonic mean of given speeds.

Q3. $< v >_{\text{space}} = \frac{\int v dx}{\int dx}$

But $a = v\frac{\mathrm{dv}}{\mathrm{dx}}$

So $\mathrm{vdx} = \frac{v^2}{a}\ \mathrm{dv}$

and $dx = \frac{\mathrm{v}}{a}\ \mathrm{dv}$

So $\langle \mathrm{v} \rangle_{\text{space}} = \frac{\int \frac{\mathrm{v}^2}{\mathrm{a}} \mathrm{dv}}{\int \frac{\mathrm{v}}{\mathrm{a}} \mathrm{dv}} = \frac{\int_1^2 \mathrm{v}^2\, dv}{\int_1^2 \mathrm{vdv}} = \frac{\left[v^3/3\right]_1^2}{\left[v^2/2\right]_1^2} = \frac{14}{9}\ \mathrm{m\ s^{-1}}$

Q4.

Given initial velocity u=0 and constant acceleration.

-Distance travelled in nth second

$S_{nth} = u + \left(\frac{2n-1}{2}\right)a = 0 + \left(\frac{2n-1}{2}\right)a = \left(\frac{2n-1}{2}\right)a$

Distance travelled in n seconds is $S_n = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}an^2 = \frac{1}{2}an^2$

$\therefore \quad \frac{\mathrm{S_{nth}}}{\mathrm{S}_n} = \frac{(2\mathrm{n}-1)\mathrm{a}}{2} \times \frac{2}{\mathrm{an}^2}$

$= \frac{2}{\mathrm{n}} - \frac{1}{\mathrm{n}^2}$

Q5. From the acceleration-velocity graph, we have $\mathrm{a} = \mathrm{k}\ \mathrm{v}$ where k is a constant which represents the slope of the given line.

As, $\mathrm{a} = \mathrm{v}\frac{\mathrm{dv}}{\mathrm{ds}}$, so $\mathrm{v}\frac{\mathrm{dv}}{\mathrm{ds}} = \mathrm{kv}$

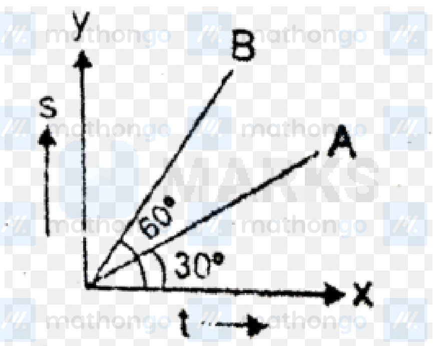
or $\frac{\mathrm{dv}}{\mathrm{ds}} = \mathrm{k} = \mathrm{a}$ constant

Thus, the slope of the velocity-displacement graph is the same as that of acceleration-velocity. Which is constant.

Q6. Area under graph will give displacement

Height $= \frac{1}{2} \times 4 \times 3.6 + 3.6 \times 8 = 36\ \mathrm{m}$

Q7.



Given the angle of displacement -time graph of the curve $v_a = 30^\circ$.

The angle of displacement -time graph of the curve $V_b = 60^\circ$.

We know that the slope (tangent of the angle) gives the value of the velocity.

Therefore, The velocity of the first curve is,

$\mathrm{v_a} = \tan\ 30 = \frac{1}{\sqrt{3}}$.

The velocity of the second curve, $\mathrm{v_b} = \tan\ 60 = \sqrt{3}$

Therefore, the ratio is. $\frac{\mathrm{v_a}}{\mathrm{v_b}} = \frac{1/\sqrt{3}}{\sqrt{3}} = \frac{1}{3}$ or $\mathrm{v_a} : \mathrm{V_b} = 1 : 3$

Q8.

According to the data given,

$a=3t^2+2t+1$

We know that,

$a=\frac{dv}{dt}$

$\Rightarrow dv=a\cdot dt$

Integrating both sides with given limits,

$\int_0^t dv=\int_0^t\left(3t^2+2t+1\right)dt$

$[v]_0^v=3\left[\frac{t^3}{3}\right]_0^t+2\left[\frac{t^2}{2}\right]_0^t+[t]_0^t$

Using the property of definite integration,

$v=t^3+t^2+t$

From the definition of instantaneous velocity,

$v=\frac{ds}{dt}$

$\Rightarrow ds=v dt$

Putting the values with proper limits,

$\int_0^s ds=\int_0^2\left(t^3+t^2+t\right)dt$

$\Rightarrow [S]_0^s=\left[\frac{t^4}{4}+\frac{t^3}{3}+\frac{t^2}{2}\right]_0^2$

After 2 s of start the total displacement is, $S=\frac{26}{3}$

Q9. For the ball dropped from building, $u_1=0$

Velocity of the dropped ball after time t

$v_1=u_1+gt$

$v_1=gt$ (downward)

For the ball thrown up, $u_2=40$ m/s

Velocity of the ball after time t

$v_2=u_2-gt$

$=(40-gt)$ (upward)

$\therefore$ Relative velocity of one ball w.r.t another ball

$=v_1-v_2$

$=gt-[-(40-gt)]=40$ m/s

Q10. From equation of motion, we have

$v=u+gt$

where u is initial velocity and t is time.

For ball A, $v_A=u-gt$

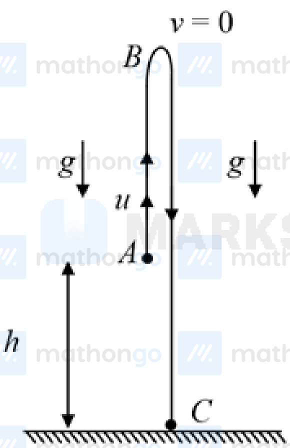
For ball B, $v_B=0+gt$

$\therefore$ Relative speed

$\Delta\vec{v}=\vec{v}_A-\vec{v}_B=(u-gt)\hat{j}-\left(+gt\,\hat{j}\right)=(u-2gt)\hat{j}$

Q11.

Let the height of the tower be h.



Taking $+ve$ $x-$axis along vertically upward direction, we have,

$u=u$

$a=-g$

$v=-3u$

$s=AC=-h$

As acceleration remains constant.

Now using $v^2=u^2+2as$, we get

$\Rightarrow(-3u)^2=(u)^2+2(-g)(-h)$

$\Rightarrow 9u^2=u^2+2gh$

$\Rightarrow 2gh=8u^2$

$\Rightarrow gh=4u^2$

$\Rightarrow h=\frac{4u^2}{g}$.

Q12. $R=m(g-a)$

Q1. A boat is sent across a river with a velocity of $8 \mathrm{~kmhr}^{-1}$. If the resultant velocity of the boat is $10 \mathrm{~kmhr}^{-1}$, then the velocity of the river is

(1) $12.8 \mathrm{~km} \mathrm{~h}^{-1}$

(2) $6 \mathrm{~km} \mathrm{~h}^{-1}$

(3) $8 \mathrm{~km} \mathrm{~h}^{-1}$

(4) $10 \mathrm{~km} \mathrm{~h}^{-1}$

Q2. Rain is falling vertically with a speed of $12 \mathrm{~m} \mathrm{~s}^{-1}$. A woman rides a bicycle with a speed of $12 \mathrm{~m} \mathrm{~s}^{-1}$ in east to west direction. What is the direction in which she should hold her umbrella?

(1) 30° towards West

(2) 45° towards West

(3) 30° towards East

(4) 45° towards East

Q3. A bus starts from rest and moves with constant acceleration $8 \mathrm{~m} \mathrm{~s}^{-2}$. At the same time, a car travelling with constant velocity $16 \mathrm{~m} \mathrm{~s}^{-1}$ overtakes and passes the bus. At what distance, bus overtakes the car?

(1) 24 m

(2) 32 m

(3) 60 m

(4) 64 m

Q4. The equation of projectile is $\mathrm{y}=\sqrt{3} \mathrm{x}-\frac{\mathrm{g}}{2} \mathrm{x}^2$. The angle of projection and initial velocity is -

(1) $30^{\circ}, 4 \mathrm{~m} / \mathrm{s}$

(2) $60^{\circ}, 2 \mathrm{~m} / \mathrm{s}$

(3) $60^{\circ}, 4 \mathrm{~m} / \mathrm{s}$

(4) $90^{\circ}, 4 \mathrm{~m} / \mathrm{s}$

Q5. A fighter plane, flying horizontally with a speed $360 \mathrm{~km} / \mathrm{h}$ at an altitude of $500 \mathrm{~m}$ drops a bomb for a target straight ahead of it on the ground. The bomb should be dropped at what approximate distance ahead of the target? Assume that acceleration due to gravity (g) is $10 \mathrm{~ms}^{-2}$. Also, neglect air drag.

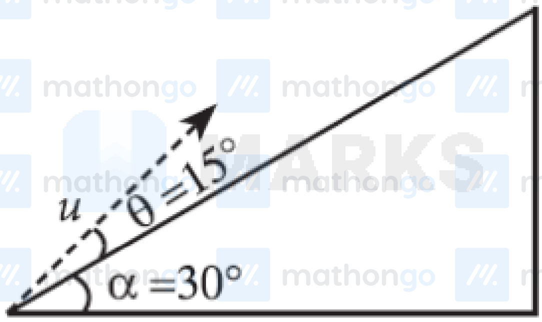
(1) 1000 m

(2) $50 \sqrt{5} \mathrm{~m}$

(3) $500 \sqrt{5} \mathrm{~m}$

(4) 866 m

Q6. A plane is inclined at an angle $\alpha=30^{\circ}$ with respect to the horizontal. A particle is projected with a speed $\mathrm{u}=2 \mathrm{~m} \mathrm{~s}^{-1}$, from the base of the plane, making an angle $\theta=15^{\circ}$ with respect to the plane as shown in the figure. The distance from the base, at which the particle hits the plane is close to: (Take $\mathrm{g}=10 \mathrm{~m} \mathrm{~s}^{-2}$)



(1) 20 cm

(2) 18 cm

(3) 14 cm

(4) 26 cm

Q7. A stone is thrown at an angle θ to the horizontal reaches a maximum heights H. then the time of flight of stone will be

(1) $\sqrt{\frac{2H}{g}}$

(2) $2\sqrt{\frac{2H}{g}}$
(3) $\frac{2\sqrt{2H}\sin\theta}{g}$
(4) $\frac{\sqrt{2H}\sin\theta}{g}$

Q8. A projectile is given an initial velocity of $\hat{i}+2\hat{j}$. The Cartesian equation of its path is: $\left(g=10m/s^2\right)$ (Here, $\hat{i}$ is unit vector along horizontal and $\hat{j}$ is unit vector vertically upwards)

(1) $y=2x-5x^2$
(2) $y=x-5x^2$
(3) $4y=2x-5x^2$
(4) $y=2x-25x^2$

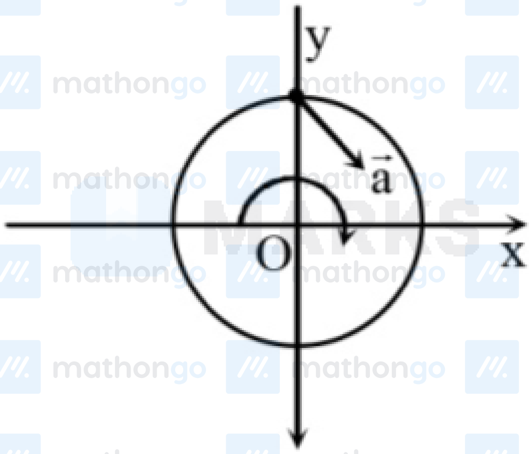
Q9. The trajectory of a projectile in a vertical plane is $y=ax-bx^2$, where a, b are constants, and x and y are respectively the horizontal and vertical distances of the projectile from the point of projection. Find the maximum height $H_{\max}$ attained by the projectile and the angle of projection from the horizontal.

(1) $H_{\max}=\frac{a^2}{4b}$
$=\tan^{-1}a$
(2) $H_{\max}=\frac{a}{2b}$
$=\tan^{-1}b$
(3) $H_{\max}=\frac{a^2}{2b}$
$\theta=\tan^{-1}\left(\sqrt{a^2+1}\right)$
(4) $H_{\max}=\frac{a}{4b^2}$
$\theta=\tan^{-1}a$

Q10. If the earth moves round the sun in a circular orbit of radius 150000000 km with an angular velocity of about $0.01745 \mathrm{rad day}^{-1}$. Its linear speed in kms^{-1} is approximately

(1) 30000000
(2) 1800
(3) 720
(4) 30

Q11. A body is moving is $x-y$ plane as shown in a circular path of radius 2 m. At a certain instant when the body is crossing the positive y-axis its acceleration is $(6\hat{\mathrm{i}}-8\hat{\mathrm{j}})\mathrm{ms}^{-2}$. Then its angular acceleration and angular velocity at this instant will be-



(1) $-3\hat{\mathrm{k}}\mathrm{rads}^{-2}$ and $-2\hat{\mathrm{k}}\mathrm{rads}^{-1}$ respectively
(2) $+3\hat{\mathrm{k}}\mathrm{rads}^{-2}$ and $+2\hat{\mathrm{k}}\mathrm{rads}^{-1}$ respectively
(3) $-4\hat{\mathrm{k}}\mathrm{rads}^{-2}$ and $-\sqrt{3}\hat{\mathrm{k}}\mathrm{rads}^{-1}$ respectively
(4) $+4\hat{\mathrm{k}}\mathrm{rads}^{-2}$ and $+\sqrt{3}\hat{\mathrm{k}}\mathrm{rads}^{-1}$ respectively

Q12. A long horizontal rod has a bead which can slide along its length and is initially placed at a distance L from one end A of the rod. The rod is set in angular motion about A with a constant angular acceleration α. If the coefficient of friction between the rod and bead is μ, and gravity is neglected, then the time after which the bead starts slipping is

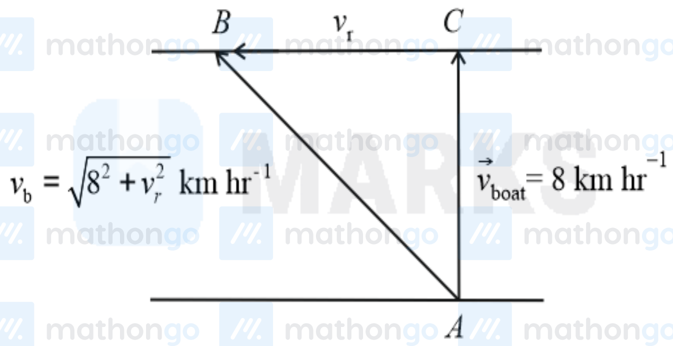
(1) $\sqrt{\frac{\mu}{\alpha}}$
(2) $\frac{\mu}{\sqrt{\alpha}}$
(3) $\frac{1}{\sqrt{\mu\alpha}}$

(4) Infinitesimal

Answer Key

Q1 (2)	**Q2** (2)	**Q3** (4)	**Q4** (2)
Q5 (1)	**Q6** (1)	**Q7** (2)	**Q8** (1)
Q9 (1)	**Q10** (4)	**Q11** (1)	**Q12** (1)

Q1.



Let the velocity of the river = $\vec{v}_R$

$\therefore \left|\vec{v}_{\text{boat}}\right| = 8 \text{ km h}^{-1}$

$\vec{v}_{bR} = \vec{v}_b - \vec{v}_R$

$\vec{v}_b = \vec{v}_{bR} + \vec{v}_R$

$\because \vec{v}_{bR} \perp \vec{v}_R$

$v_b = \sqrt{v_{bR}^2 + v_R^2}$

$v_b^2 = v_{bR}^2 + v_R^2$

$(10)^2 = (8)^2 + v_R^2$

$v_R = 6 \, \text{km h}^{-1}$

Q2.

To protect herself from the rain, the woman must hold her umbrella in the direction of the relative velocity of the rain with respect to the woman.

$\tan\theta = \frac{v_b}{v_r} = 1$

or, $\tan\theta = 1$

or, $\theta = 45^\circ$

Therefore, the direction in which she should hold her umbrella is 45° towards west.

Q3.

Let the bus overtakes the car after time t and at distance d.

Bus, $S = ut + \frac{1}{2}at^2$

$d = \frac{1}{2} \times 8 \times t^2 \ldots$(i)

Car: $S = vt$

$d = 16t \ldots$(ii)

by (i) and (ii)

$t = \text{sec}, \; d = 64$ m.

Q4. Equation of trajectory of a projectile

$y = (\tan\theta)x - \frac{g}{2u^2\cos^2\theta}x^2$ .... (i)

Given equation is

$y = \sqrt{3}\,x - \frac{g}{2}x^2$ .... (ii)

Comparing (1) and (2), we get

$\tan\theta = \sqrt{3} \Rightarrow \theta = 60^\circ$

And $2u^2\cos^2\theta = 2$

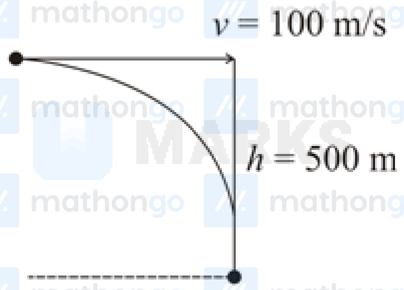
$\Longrightarrow u = \frac{1}{\cos\theta} = \frac{1}{\cos 60^\circ} = 2$ m/s

Q5.

Given, speed of fighter plane, $v = 360$ km/h

$= 360 \times \frac{5}{18}$ m/s $= 100$ m/s

Altitude, $h = 500$ m



Now, from equation of motion,

$h = ut + \frac{1}{2}gt^2$

$\Rightarrow \quad 500 = 0(t) + \frac{1}{2} \times 10 \times t^2 \begin{bmatrix} \because u = 0 \text{ and} \\ g = 10 \text{ m/s}^2 \end{bmatrix}$

$\Rightarrow \quad t^2 = 100 \Rightarrow t = 10$ s

$\therefore$ The bomb should be dropped at the distance,

$x = vt = 100 \times 10 = 1000$ m

Q6. Range $= \frac{2u^2 \sin\alpha \cos(\alpha+\theta)}{g\cos^2\theta}$

$\alpha =$ angle of inclination $= 30^\circ$

$\theta =$ angle of projection from inclination $= 15^\circ$

$u = 2$ m/s

$R = \frac{2(2)^2 \sin 15^\circ \cos(15^\circ + 30^\circ)}{g\cos^2 30^\circ} = \frac{8 \sin 15^\circ \cos 45^\circ}{g \cos^2 30^\circ}$

$\left[\sin 15^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}\right]$

$R = \frac{4}{5}\left(\frac{\sqrt{3}-1}{3}\right) \approx 0.2$ m

$R \approx 20$ cm

Q7. Maximum height $H = \frac{u^2 \sin^2\theta}{2g}$

and times of flight $T = \frac{2u\sin\theta}{g}$

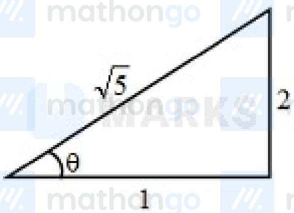
or $T^2 = \frac{4u^2\sin^2\theta}{g^2}$

$\therefore \frac{T^2}{H} = \frac{8}{g}$

or $T = \sqrt{\frac{8H}{g}} = 2\sqrt{\frac{2H}{g}}$

Q8.

$\tan\theta = \frac{u\sin\theta}{u\cos\theta} = \frac{2}{1}$



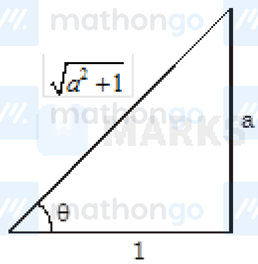
The desired equation is :

$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$

or $\quad y = x(2) - \frac{(10)\ (x)^2}{2\left(\sqrt{2^2+1^2}\right)^2 \left(\frac{1}{\sqrt{5}}\right)^2}$

or $\quad y = 2x - 5x^2$

Q9.



Trajectory of the projectile in a vertical plane -

$y = ax - bx^2$

Comparing it with equation of a projectile

$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$

$\Rightarrow \quad \tan\theta = a$ ....(i)

$\Rightarrow \theta = \tan^{-1}(a)$

and $\frac{g}{2u^2\cos^2\theta} = b$ .....(ii)

From (ii) $\frac{g}{2b^2\cos^2\theta} = \frac{g(a^2+1)}{2b}$ ....(iii)

$\left[\therefore \cos\theta = \frac{1}{\sqrt{a^2+1}}\right]$

Maximum height attained, $H_{\max} = \frac{u^2\sin^2\theta}{2a}$ .....(iv)

From (ii) $\frac{g}{2b} = u^2\cos^2\theta$

$\Rightarrow \frac{g}{2b} = u^2 - u^2\sin^2\theta$

$\Rightarrow u^2\sin^2\theta = \frac{g(a^2+1)}{2b} - \frac{g}{2b} = \frac{ga^2}{2b}$ ......(v)

From (iv) and (v) $H_{\max} = \frac{ga^2}{2b \times 2g} = \frac{a^2}{4b}$

Alternatively

$\frac{dy}{dx} = a - 2bx = 0$ (For Maximum Height)

$\Rightarrow x = \frac{a}{2h}$

Substituting the value of x in $y = ax - bx^2$ to find max height

$H_{\max} = a\left(\frac{a}{2b}\right) - b\left(\frac{a^2}{4b^2}\right) = \frac{a^2}{2b} - \frac{a^2}{4b} = \frac{a^2}{4b}$

Q10.

Given, angular velocity of earth,

$\omega = 0.01745$ rad day^{-1}

$= \frac{0.01745}{24 \times 60 \times 60}$ rad day^{-1} and orbital radius of

earth around the sun, $R = 15 \times 10^7$ km. Let the linear speed of earth be v. We know that,

linear speed, $v=\omega R$

$=\frac{0.01745}{24\times60\times60}\times15\times10^{7}=30\ \text{km s}^{-1}$.

Hence, linear speed of earth is $30\ \text{km s}^{-1}$.

Q11.

Given that, the radius of the circular path is, $r=2$ m and the net acceleration of the particle is,

$\vec{a}=\left(6\hat{i}-8\hat{j}\right)\ \text{m s}^{-2}\quad\cdots(1)$

Compare the above equation with

$a=\left(a_t\hat{i}-a_r\hat{j}\right)\ \text{m s}^{-2}\quad\cdots(2)$

By comparing equation (1) & (2), we can write,

Radial acceleration as, $a_r=-8\hat{j}\ \text{m s}^{-2}$

Tangential acceleration as, $a_t=6\hat{i}\ \text{m s}^{-2}$

Relation between the tangential acceleration and angular acceleration is,

$|a_t|=6=r\alpha\ \Rightarrow\ 2\alpha=6$

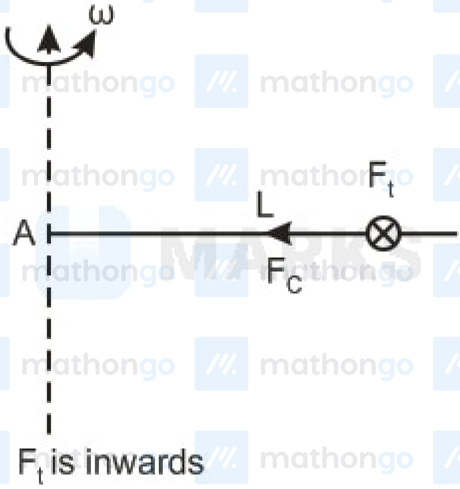
$\alpha=3\ \text{rad s}^{-2}\ \Rightarrow\ \alpha=-3\hat{k}\ \text{rad s}^{-2}$

Relation between the radial acceleration and angular velocity is,

$|a_r|=r\omega^2\ \Rightarrow\ \omega=\sqrt{\frac{|a_r|}{r}}\ \Rightarrow\ \omega=\sqrt{\frac{8}{2}}$

$\omega=2\ \text{rad s}^{-1}\ \Rightarrow\ \omega=-2\hat{k}\ \text{rad s}^{-1}$

Q12. Tangential force (F_t) of the bead will be given by the normal reaction (N), while centripetal force (F_c) is provided by friction (f_r) The bead starts sliding when the centripetal force is just equal to the limiting friction,



Therefore,

$F_t=ma=m\alpha L=N$

Therefore, limiting value of friction

Angular velocity at time t is

$\omega=\alpha t(fr)_{\max}=\mu N=\mu m\alpha L\quad\ldots$ (i)

Centripetal force at time t will be

$Fe=mL\omega2=mL\alpha2t2\ldots(ii)$

Equating equations (i) and (ii), we get,

$t=\sqrt{\frac{\mu}{\alpha}}$

For $t>F_c>(fr)_{\max}$,

i. e., the bead starts sliding.

In the figure, F_t is perpendicular to the paper inwards.

Q1. Match the following (Take the relative strength of the strongest fundamental forces in nature as one)

I	II
Fundamental forces in nature	Relative strength
(A) Strong nuclear force	$(e) 10^{-2}$
(B) Weak nuclear force	$(f) 1$
(C) Electromagnetic force	$(g) 10^{-13}$
(D) Gravitational force	$(h) 10^{-39}$

The correct match is:

(1) $\mathrm{A-f, B-h, C-e, D-g}$

(2) $\mathrm{A-h, B-f, C-e, D-g}$

(3) $A-f, B-g, C-e, D-h$

(4) $\mathrm{A-f, B-e, C-h, D-g}$

Q2. A monkey of mass 40 kg climbs on a massless rope which can stand a maximum tension of 500 N. In which of the following cases will the rope break? (Take $g = 10\ \mathrm{ms}^{-2}$). ||\||\||

(1) The monkey climbs up with an acceleration of $5\ \mathrm{m\ s}^{-2}$.

(2) The monkey climbs down with an acceleration of $5\ \mathrm{m\ s}^{-2}$.

(3) The monkey climbs up with a uniform speed of $5\ \mathrm{m\ s}^{-2}$.

(4) The monkey falls down the rope freely under gravity.

Q3. Match the following columns

	Column - I		Column - II
(a)	Inertia of motion and rest	(i)	Newton's third law
(b)	Measure of force	(ii)	Impulse
(c)	Relation between force and acceleration	(iii)	Newton's second law
(d)	Recoiling of gun	(iv)	Newton's first law

(1) a-ii; b-i; c-iii; d-iv

(2) $\mathrm{a-i; b-ii; c-iii; d-iv}$

(3) $a-iv; b-ii; c-iii; d-i$

(4) $a-iv; b-iii; c-ii; d-i$

Q4. Assertion: In the reference frame of centre of mass, net force acting on system is always zero.

Reason: A pseudo force given by $\vec{P}_{\mathrm{S}} = -m\vec{a}_{\mathrm{cm}}$ (where m is mass of system and $\vec{a}_{\mathrm{cm}}$ is acceleration of centre of mass)

acts on system which balances all the external forces.

(1) If both Assertion and Reason are true and the Reason is correct explanation of the Assertion.

(2) If both Assertion and Reason are true, but Reason is not correct explanation of the Assertion.

(3) If Assertion is true, but the Reason is false.

(4) If Assertion is false, but Reason is true.

Q5. The forces, which meet at one point but their lines of action do not lie in one plane, are called:-

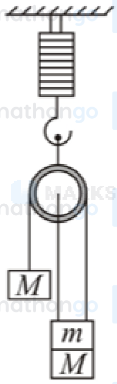
(1) Non-coplanar and concurrent force.

(2) Coplanar and concurrent forces.

(3) Non-coplanar and non-concurrent forces.

(4) Coplanar and non-concurrent forces.

Q6. The at wood machine shown is suspended from a spring balance. The mass on one hanger is M, that on the other is $(M+m)$. Suppose the heavier side (right side) hanger is fastened to the top of the pulley by a thread. The scale reads $(2M+m)g$. The thread is burned and the system accelerates. The reading of spring balance now will be



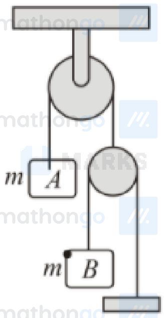
(1) the same as before.

(2) more than before.

(3) less than before.

(4) can't say.

Q7. In the figure shown, neglecting friction and mass of pulleys, what is the acceleration of mass B ?



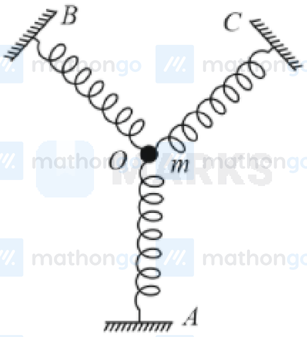
(1) $\frac{g}{3}$

(2) $\frac{5g}{2}$

(3) $\frac{2g}{2}$

(4) $\frac{2g}{5}$

Q8. A particle of mass m is attached to three identical springs A, B and C each of force constant k as shown in figure. If the particle of mass m is pushed slightly against the spring A and released, then the time period of oscillation is -



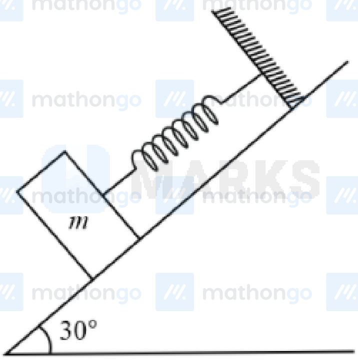
(1) $2\pi\sqrt{\frac{2m}{k}}$

(2) $2\pi\sqrt{\frac{m}{2k}}$

(3) $2\pi\sqrt{\frac{m}{k}}$

(4) $2\pi\sqrt{\frac{m}{3k}}$

Q9. A body of mass 5 kg is suspended by a spring balance on an inclined plane as shown in the figure. Measure the force applied on spring balance,



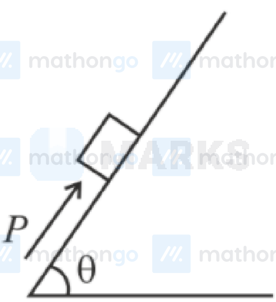
(1) 50 N

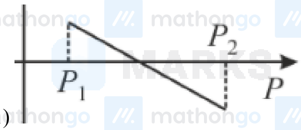
(2) 25 N

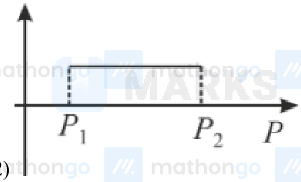
(3) 500 N

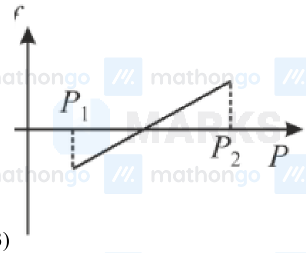
(4) 10 N

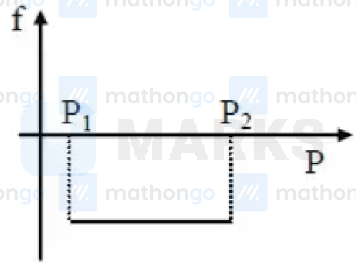
Q10. A block of mass m is on an inclined plane of angle θ. The coefficient of friction between the block and the plane is μ and $\tan\theta > \mu$. The block is held stationary by applying a force P parallel to the plane. The direction of force pointing up the plane is taken to be positive. As P is varied from $P_1 = mg(\sin\theta - \mu\cos\theta)$ to $P_2 = mg(\sin\theta + \mu\cos\theta)$, the frictional force f versus P graph will look like



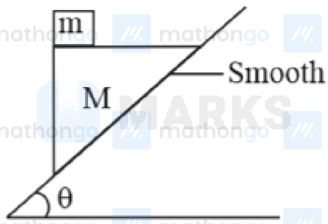
(1)


(2)


(3)


(4)


Q11. A triangular prism of mass M with a block of mass m placed on it, is released from rest on a smooth inclined plane of inclination θ. The block does not slip on the prism. Then



(1) The acceleration of the prism is $g\cos\theta$.

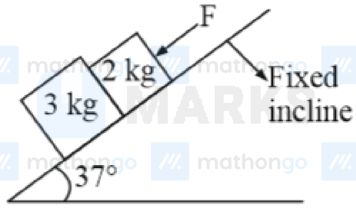
(2) The acceleration of the prism is $g\tan\theta$.

(3) The minimum coefficient of friction between the block and the prism is $\mu_{\min} = \cot\theta$.

(4) The minimum coefficient of friction between the block and the prism is $\mu_{\min} = \tan\theta$.

Q12. Two blocks of masses 3 kg and 2 kg are placed side by side on an incline as shown in figure. A force $F = 20$ N is acting on 2 kg block along the incline. The coefficient of friction between the block and the incline is same and equal to 0.1.

Find the normal contact force exerted by 2 kg block on 3 kg block.



(1) 18 N

(2) 30 N

(3) 12 N

(4) 27.6 N

Q13. An object takes n times as much time to slide down a 45° rough inclined plane as it takes to slide down a perfectly smooth 45° inclined plane. The coefficient of kinetic friction between the rough plane and the object is

(1) $n^2 - 1$

(2) $1 - \frac{1}{n^2}$

(3) $n^2 + 1$

(4) $1 + \frac{1}{n^2}$

Q14. A uniform rod of length L is fixed at one end. It is initially held vertically upward, then released. Find the vertical reaction

at hinge when it becomes horizontal.

(1) $\frac{mg}{2}$

(2) $\frac{mg}{3}$

(3) $\frac{3mg}{2}$

(4) $\frac{mg}{4}$

Q15. A circular racing car track has a radius of curvature of 500 m. The maximum speed of the car is 180 kmhr^{-1}. The angle of banking θ is $\left(g = 10 \text{ m s}^{-2}\right)$

(1) $\theta = \tan^{-1}(0)$

(2) $\theta = \tan^{-1}(0.5)$

(3) $\theta = \tan^{-1}(0.3)$

(4) $\theta = \tan^{-1}(0.1)$

Answer Key

Q1 (3)	**Q2** (1)	**Q3** (3)	**Q4** (1)
Q5 (1)	**Q6** (3)	**Q7** (4)	**Q8** (2)
Q9 (2)	**Q10** (1)	**Q11** (4)	**Q12** (3)
Q13 (2)	**Q14** (4)	**Q15** (2)	

Q1.

I	II
Fundamental forces in nature	**Relative strength**
(A) Strong nuclear force	(f) 1
(B) Weak nuclear force	(g) 10^{-13}
(C) Electromagnetic force	(e) 10^{-2}
(D) Gravitational force	(h) 10^{-39}

So, $\mathrm{A-f,\ B-g,\ C-e,\ D-h}$

Q2.

Here, mass of monkey, $m = 40$ kg

Maximum tension the rope can stand, $T = 500$ N

Tension in the rope will be equal to apparent weight of the monkey.

The rope will break when R exceeds T.

(A) When the monkey climbs up with an acceleration $a = 5\ \mathrm{ms}^{-2}$

$R = m\,(g + a) = 40\,(10 + 5) = 600\ \mathrm{N}\ \therefore R > T$

Hence, the rope will break

(B) When the monkey climbs down with an acceleration $a = 5\ \mathrm{ms}^{-2}$

$R = m\,(g - a) = 40\,(10 - 5) = 200\ \mathrm{N} \therefore R < T$

Hence, the rope will not break.

(C) When the monkey climbs up with a uniform speed $v = 5\ \mathrm{ms}^{-1}$,its acceleration $a = 0\ \mathrm{m/s}^2$

$\therefore R = mg = 40 \times 10 = 400\ \mathrm{N}\ \therefore R < T$

Hence, the rope will not break.

(D) When the monkey falls down the rope freely under gravity

$a = g \therefore R = m\,(g - a) = m\,(g - g) = 0$

Hence, the rope will not break.

Q3. Newton's first law is based on principle of inertia. Newton's second law gives relation between force and acceleration. Impulse gives us the effect of force.

Recoiling of gun is accounted for by Newton's 3^{rd} law.

Q4.

In centre of mass, frame acceleration of centre of mass is zero.

Hence, using Newton's second law of motion,

$F_{\text{net}} = ma_{\text{cm}} = m \times 0 = 0$

Also, $F_{\text{net}} = F_{\text{ext}} + P_{\text{S}} = 0$

and in ground frame, $F_{\text{ext}} = ma_{\text{cm}}$

From above two equations, we get,

$P_{\text{S}} = -F_{\text{ext}} = -ma_{\text{cm}}$

Hence, both Assertion and Reason are true and the Reason is correct explanation of the Assertion.

Q5.

We know that when vectors lie on the same plane, then they are called coplanar vectors.

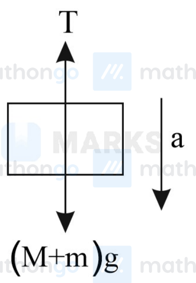
A concurrent vector system is a set of two or more vectors whose lines of action intersect at a point.

So, the forces, which meet at one point but their lines of action do not lie in one plane, are called non-coplanar and concurrent force.

Q6.

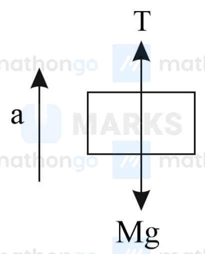
Suppose once the block starts moving tension in the string is T, acceleration of the blocks is a and tension in the spring balance is T_{S}. When the thread is burned, blocks start moving.

From FBD of the block of mass $(M + m)$ kg,



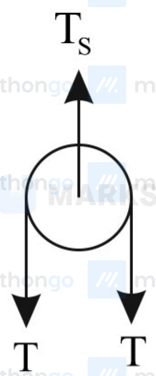
$(M + m)g - T = (M + m)a\quad \ldots\text{(i)}$

From FBD of the block of mass M kg,



$T - Mg = Ma\quad \ldots\text{(ii)}$

From FBD of the pulley,



Since the pulley is in equilibrium.

$\therefore T_{\text{S}} = 2T\quad \ldots\text{(iii)}$

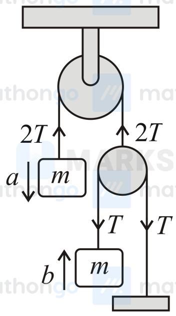
From the equation (i), (ii) and (iii),

$\Rightarrow T_{\text{S}} = \frac{4M(M+m)g}{2M+m} < (2M + m)g.$

Hence, the reading of spring balance will be less than the initial reading.

Q7.

Forces acting on different parts of the system is shown below.



For block A,
$mg - 2T = ma \quad ...(i).$

For block B,

$T - mg = mb \quad ...(ii).$

And by constraint motion, if A will move by x distance, then B will move by $2x$ distance.

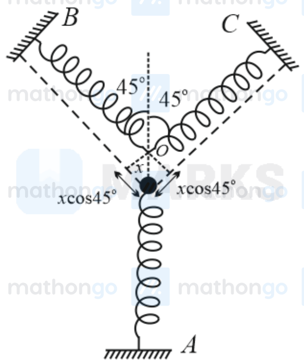
$(a - b) + a = 0 \quad ...(iii).$

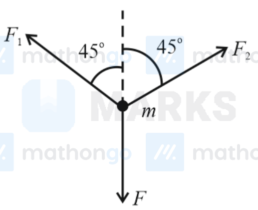
Solving (i), (ii) and (iii), we get,

$a = -\frac{g}{5}$ and

$b = -\frac{2g}{5}$.

Q8.





When particle is displaced by a distance x in the downward direction, lower spring is compressed by x distance in downward direction and both upper string is displaced by $\mathrm{xcos}(45^\circ)$ at an angle of 45° from vertical.Due to these compressions and expansions, net force start working on the particle which tries to bring the particle back to its original position.

Writing net spring force on the particle-

$\mathrm{F_{net}} = \mathrm{F} + \mathrm{F_1}\cos(45^\circ) + \mathrm{F_2}\cos(45^\circ)$

$\Rightarrow \mathrm{F_{net}} = \mathrm{kx} + [\mathrm{kxcos}(45^\circ)]\cos(45^\circ) + [\mathrm{kx}(\cos 45^\circ)]\cos(45^\circ)$

$\Rightarrow \mathrm{F_{net}} = 2\,\mathrm{kx}$

From newton's second law-

$\mathrm{ma} = -2\,\mathrm{kx}$

$\Rightarrow \mathrm{a} = -\left(\frac{2\mathrm{k}}{\mathrm{m}}\right)\mathrm{x}$

Above equation represents SHM.Comparing it with equation of SHM

$\mathrm{a} = -\omega^2 \mathrm{x}$

$\Rightarrow \omega = \sqrt{\frac{2\mathrm{k}}{\mathrm{m}}}$

Since time period $\mathrm{T} = \frac{2\pi}{\omega}$

$\therefore \mathrm{T} = 2\pi\sqrt{\frac{\mathrm{m}}{2\mathrm{k}}}$

Q9.

Acceleration of the body down the rough inclined plane= $g\sin\theta$.

$\therefore$ Force applied on the spring balance

$= mg\sin\theta = 5 \times 10 \times \sin(30^\circ)$

$= 5 \times 10 \times \frac{1}{2} = 25\ \mathrm{N}.$

Q10. When

$P = mg\,(\sin\theta - \mu\cos\theta)$

$f = \mu mg\cos\theta$ (upwards)

when $P = mg\sin\theta$

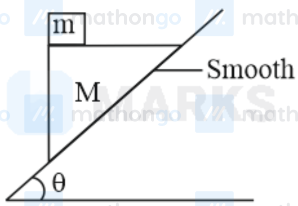
$f = 0$

and when $P = mg(\sin\theta + \mu\cos\theta)$

$f = \mu\, mg\cos\theta$ (downwards)

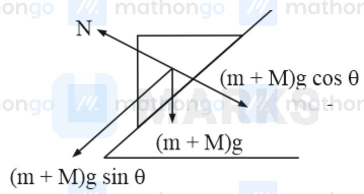
Hence, friction is first positive, then zero and then negative.

Q11.



As the small block does not slip over the prism,

We can consider the mass of small block into prism.



Where, N is normal reaction

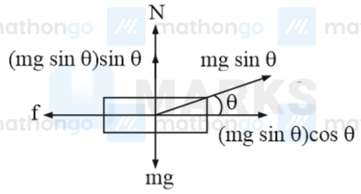
and here $(m + M)g\ \sin\ \theta$ is the only force in the line of inclination acting downwards

$F_{net} = ma$
$\Rightarrow (m+M) g \sin\theta = (m+M) a$
$\Rightarrow a = g \sin\theta$

$\therefore\ g\ \sin\ \theta$ will be its acceleration

$\therefore$ option 1 and 2 are wrong

Now FBD of block with respect to prism



Where,

$mg\ \sin\ \theta$ is the pseudo force

f is force of friction

N is normal reaction

Now writing equations of forces

$\Rightarrow\ N\ +\ mg\ \sin\ \theta\ \sin\ \theta\ =\ mg$

$\Rightarrow\ N\ =\ mg\ -\ mg\ \sin^2\theta\ \ \ldots(1)$

and $f\ =\ mg\ \sin\ \theta\ \cos\ \theta$

$f\ =\ \mu\text{N}$

$\Rightarrow \mu\left(mg\ -\ mg\ \sin^2\theta\right) = mg\ \sin\ \theta\ \cos\ \theta$

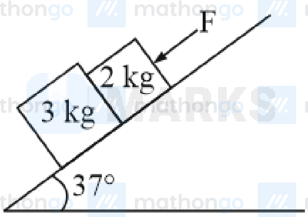
$\Rightarrow\ \mu\left(1\ -\ \sin^2\theta\right)\ =\ \sin\ \theta\ \cos\ \theta \Rightarrow\ \mu\ \cos^2\theta\ =\ \sin\ \theta\ \cos\ \theta$

$\Rightarrow\ \mu\ =\ \tan\ \theta$

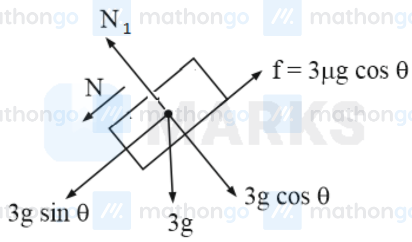
$\Rightarrow\ \mu_{min}\ =\ \tan\ \theta$

Q12.

According to question a F force act on the block



Now F.B.D. for block 3 kg



Where, N is normal provided by 2 kg block

f is force of friction

μ if coefficient of friction

According to newton's second law

$\sum F = ma$

Applied in the direction of incline

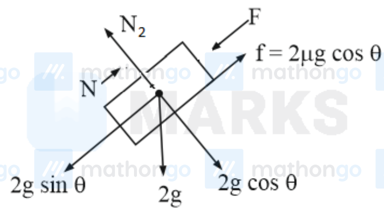
$3g\ \sin\ \theta + N - f = 3a$

$f = \mu N_1$

Here N_1 is normal force applied by the incline on 2 Kg block

$3g\ \sin\ \theta + N - 3\mu g\ \cos\ \theta = 3a\ \ \ \ldots(1)$

for 2 kg block



$\therefore\ F + 2g\ \sin\ \theta - N - 2\mu g\ \cos\ \theta = 2a\ \ \ \ldots(2)$

adding (1) and (2)

We get

$5g\ \sin\ \theta - 5\mu g\ \cos\ \theta + F = 5a$

given $\theta\ =\ 37^\circ$

$F = 20\ \text{N}$

$\mu = 0.1$

$\Rightarrow\ 5\ \times\ 10\ \times\ \left(\frac{3}{5}\right)\ -\ 50\left(0.1\right)\left(\frac{4}{5}\right)\ +\ 20\ =\ 5a$

$\Rightarrow\ a = \frac{46}{5}\ \text{m s}^{-2}$

Now putting the value of acceleration in (2)

We get

$20 + 20\left(\frac{3}{5}\right) - N - \left(2 \times \frac{4}{5}\right) = 2 \times \frac{46}{5}\ 20 + 12 - N - \left(\frac{8}{5}\right) = \frac{92}{5}$

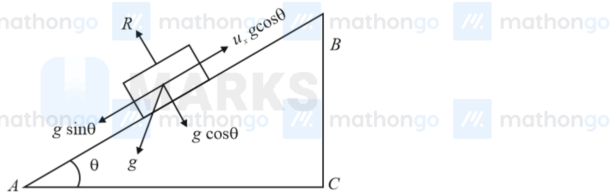
$\Rightarrow\ N = 12\ \text{N}$

Q13.

$s = ut + \frac{1}{2}at^2, a = g\sin\theta, u = 0$

Then, $\ s = 0 + \frac{1}{2}at^2$

or $\ t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2s}{g\sin\theta}}$ [for smooth plane]



For the rough plane. the effective value of acceleration along the incline is

$a' = g\sin\theta - \mu_k g\cos\theta$

and $t' = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2s}{g\sin\theta - \mu_k g\cos\theta}}$

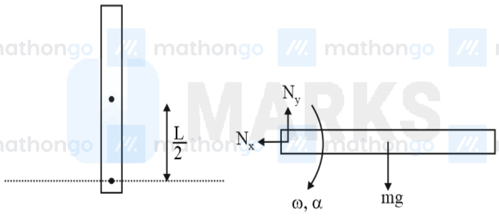
Now, $\ \frac{t'}{t} = n = \sqrt{\frac{g\sin\theta}{g\sin\theta - \mu_k g\cos\theta}}$

$= \sqrt{\frac{\sin 45^\circ}{\sin 45^\circ - \mu_k\cos 45^\circ}}$

$= \sqrt{\frac{1}{1-\mu_k}}$

$\Rightarrow\ \frac{1}{n^2} = 1 - \mu_k \Rightarrow \mu_k = 1 - \frac{1}{n^2}$

Q14.



$mg - N_y = ma_y$

$a_y =$ Acceleration of the center of mass in y-direction.

$= \frac{L}{2} \times \alpha$ (Since it is tangential acceleration)

Now,

$N_y = mg - \frac{mL\alpha}{2}$

But,

$\alpha = \frac{\tau_A}{I_A} = \frac{mg\left(\frac{L}{2}\right)}{\frac{mL^2}{3}} = \frac{3g}{2L}$

So, $N_y = mg - \frac{mL}{2} \times \frac{3g}{2L} = \frac{mg}{4}$

Q15.

Angle of banking θ is given by

$\tan\theta = \frac{v^2}{rg} \ldots(1)$

where $v =$speed of the vehicle

$r =$ radius of the road

$\therefore$ Given, $v = 180 \text{ km h}^{-1} = 180 \times \frac{5}{18} \text{ m s}^{-1} = 50 \text{ m s}^{-1}$

and $r = 500$ m.

Putting in equation (1) we get

$\tan\theta = \frac{(50)^2}{500\times10} = 0.5$

$\Rightarrow \theta = \tan^{-1}(0.5)$.

Q1. A particle moves along X-axis from $x=0$ to $x=5$ m under the influence of force given by $F=\left(7-2x+3x^2\right)$ N. The work done in the process is

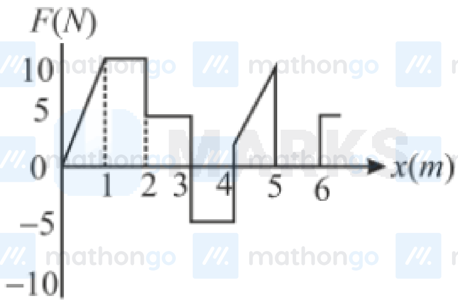
(1) 70 J

(2) 270 J

(3) 35 J

(4) 135 J

Q2. The relationship between the force F and position x of a body is as shown in figure. The work done in displacing the body form $x=1$ m to $x=5$ m will be



(1) 30 J

(2) 15 J

(3) 25 J

(4) 20 J

Q3. The work done on a body by a force is minimum when the force and the displacement of the body are in direction.

(1) Same

(2) Opposite

(3) Different

(4) Some

Q4. The only force acting on a block is along x-axis is given by $F=-\left(\frac{4}{x^2+2}\right) N$ When the block moves from $x=-2m$ to x = 4 mthe change in kinetic energy of block is -

(1) Positive

(2) Negative

(3) Zero

(4) May be positive or negative

Q5. The potential energy of a certain spring when stretched through a distance S is 10 joule. The amount of work (in joule) that must be done on this spring to stretch it through additional distance S will be

(1) 30

(2) 40

(3) 10

(4) 20

Q6. Two equal masses are attached to two each of a spring of spring constant k. The masses are pulled out symmetrically to stretch the spring by a length x over is natural length. The work done by the spring on each mass is

(1) $\frac{1}{2}kx^2$

(2) $-\frac{1}{2}kx^2$

(3) $\frac{1}{4}kx^2$

(4) $-\frac{1}{4}kx^2$

Q7. Assertion: Friction is a conservative force.

Reason: Friction does not depend upon mass of the body.

(1) Assertion is true, reason is true; reason is a correct explanation for assertion.

(2) Assertion is true, reason is true; reason is not a correct explanation for assertion.

(3) Assertion is true, reason is false.

(4) Assertion is false, reason is false.

Q8. A ball balanced on a vertical rod is an example of

(1) stable equilibrium

(2) unstable equilibrium

(3) neutral equilibrium

(4) perfect equilibrium

Q9. The potential energy of a particle of mass m is given by $U=\frac{1}{2}kx^2$ for $x<0$ and $U=0$ for $x\geq 0$. If total mechanical energy of the particle is E. Then its speed at $x=\sqrt{\frac{2E}{k}}$ is

(1) Zero

(2) $\sqrt{\frac{2\mathrm{E}}{\mathrm{m}}}$

(3) $\sqrt{\frac{\mathrm{E}}{\mathrm{m}}}$

(4) $\sqrt{\frac{E}{2m}}$

Q10. If the dissociation energy of the molecule is $D = \left[U_{(x=\infty)} - U_{\text{at equilibrium}}\right]$. The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$ where a and b are constants and x is the distance between the atoms. Find the dissociation energy.

(1) $\frac{b^2}{2a}$
(2) $\frac{b^2}{12a}$
(3) $\frac{b^2}{4a}$
(4) $\frac{b^2}{6a}$

Q11. A constant power p is applied to a car starting from rest. If v is the velocity of the car at time t, then

(1) $v \propto t$
(2) $v \propto \frac{1}{t}$
(3) $v \propto \sqrt{t}$
(4) $v \propto \frac{1}{\sqrt{t}}$

Q12. The power applied to a particle $P = \left[3t^2 - 2t + 1\right]$ W, where t is in seconds. Find the change in kinetic energy of particle from $t = 2$ s to 4 s.

(1) 46 J
(2) 52 J
(3) 92 J
(4) 104 J

Answer Key

Q1 (4)	**Q2** (2)	**Q3** (2)	**Q4** (2)
Q5 (1)	**Q6** (4)	**Q7** (4)	**Q8** (2)
Q9 (2)	**Q10** (3)	**Q11** (3)	**Q12** (1)

Q1. The given force varies with distance

Therefore, work done,

$W=\int_0^{x=5} F . d x$

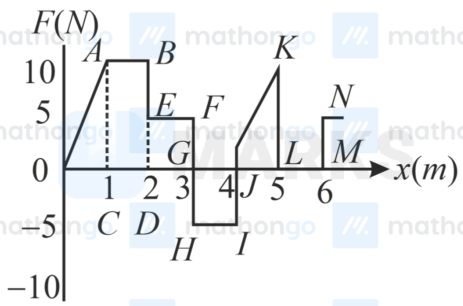
$=\int_0^5\left(7-2 x+3 x^2\right) d x$

$=\left[7 x-\frac{2 x^2}{2}+\frac{3 x^3}{3}\right]_0^5$

$=\left[7 \times 5-(5)^2+(5)^3-0\right]$

$=135$ J

Q2.



Work done $=$ area under F-x graph

$=$ area of rectangle $ABDC+$ area of rectangle $EFHD+$ area of rectangle $GJHI+$ area of triangle JKL.

$=(2-1)\left(10-0\right)+(3-2)(5-0)+(4-3)(-5-0)$

$+\frac{1}{2}(5-4)(10-0)=15$ J

.

Q3.

We know that the work done by the force is defined to be the product of a component of the force in the direction of the displacement and the magnitude of the displacement.

Thus the work done is a scalar product of force and displacement.

$\Rightarrow W=F \cdot d=\left(F \cos {}^{\circ}\right) \times d=F \times d \times \cos \theta$.

When the force and displacement are in opposite direction, $\theta=180^{\circ}$.

So,

$W=F \times d \times \cos 180^{\circ}=F \times d \times(-1)=-(F \times d)$.

This is the minimum value of work done which is equal to the negative of product of force and displacement.

Thus, the work done on a body by a force is minimum when the force and the displacement of the body are in opposite direction.

Q4. As W $=\Delta$K

Force is along negative x-axis and displacement is along + x-axis

$\therefore$ W = negative

Hence

ΔK = negative

Q5. $U=\frac{1}{2} K x^2=\frac{1}{2} K S^2=10$ J ...(i)

$U^{\prime}=\frac{1}{2} K(x+x)^2=\frac{1}{2} K(S+S)^2$

$=\frac{1}{2} K\left(4 S^2\right)=40$ J ...(ii)

$\therefore\ W=U'-U=40$ J -10 J $=30$ J

Q6. Work done by spring on both mass is given by, $W=U_1-U_2$, where U_1, U_2 represents initial potential energy and final potential energy.

$\Rightarrow W=0-\frac{1}{2} k x^2=-\frac{1}{2} k x^2$.

So work done by spring on each mass is given by, $W'=-\frac{1}{4} k x^2$.

Q7.

The work done by friction force depends upon the path, so it is a non-conservative force. The friction force is given by,

$f=\mu N$; where μ is coefficient of friction and N is normal reaction.

For a body kept of the plane horizontal surface of mass m, the normal reaction will be weight of the body, i.e., $N=m g$; where g is acceleration due to gravity,

The normal reaction depends upon the mass, so frictional force depends upon the mass. Hence, Assersion is false and reason is false.

Q8.

The body is said to be in the state of stable equilibrium when the centre of gravity of a body lies below the point of suspension or support. When the centre of gravity of a body lies above the point of suspension or support, it is said to be in the state of unstable equilibrium. Now, as the ball is at the top of the rod, is slightly disturbed, the centre of gravity is lowered down and line of action of its weight lies outside the base of the rod due to which a torque start acting on the ball. As a result the ball fell down. When the centre of gravity of a body lies on the point of suspension or support, it is said to be in state of neutral equilibrium.

Hence, the correct answer is "unstable equilibrium".

Q9. Potential energy of particle at $x = \sqrt{\frac{2E}{k}}$ is zero

$\therefore \quad KE = E$

or $\quad \frac{1}{2}mv^2 = E \qquad$ or $\qquad v = \sqrt{\frac{2E}{m}}$

Q10.

The potential energy function for the force between two atoms in a diatomic molecule is approximately given by

$U = \frac{a}{x^{12}} - \frac{b}{x^6}$

If Force is denoted by F, then

$$F = -\frac{dU}{dx} = -\frac{d}{dx}\left(\frac{a}{x^{12}} - \frac{b}{x^6}\right)$$

$$= -\left[\frac{-12a}{x^{13}} + \frac{6b}{x^7}\right] = \left[\frac{12a}{x^{13}} - \frac{6b}{x^7}\right]$$

At equilibrium, $F = 0$.

$$\therefore \ \frac{12a}{x^{13}} - \frac{6b}{x^7} = 0 \text{ or } x^6 = \frac{2a}{b}$$

By using the above equation we can write

$$U_{\text{at equilibrium}} = \frac{a}{\left(\frac{2a}{b}\right)^2} - \frac{b}{\left(\frac{2a}{b}\right)}$$

$$= \frac{ab^2}{4a^2} - \frac{b^2}{2a} = \frac{b^2}{4a} - \frac{b^2}{2a} = -\frac{b^2}{4a}$$

Since, $U_{(x=\infty)} = 0$

Therefore, th dissociation energy is given by

$$D = [U_{(x=\infty)} - U_{\text{at equilibrium}}$$

$$= \left[0 - \left(-\frac{b^2}{4a}\right)\right] = \frac{b^2}{4a}$$

Q11. Power,

$p = m \times a \times v$

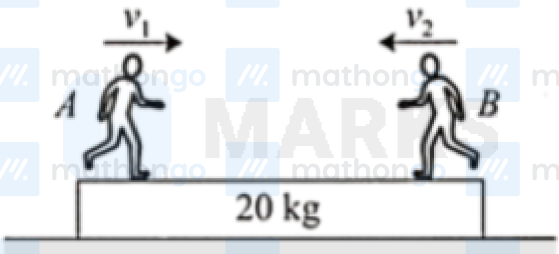
$p = m \times \frac{v^2}{t}$

If p is constant, then for a given body $v^2 \propto t$

Or $v \propto \sqrt{t}$

Q12. $\Delta K = \int_2^4 P dt = \left[t^3 - t^2 + t\right]_2^4 = 46J$

Q1. In the figure shown, the system is at rest, initially. Two persons, A and B, of masses 40 kg each, move with speeds v_1 and v_2, respectively, towards each other on a plank lying on a smooth horizontal surface as shown in the figure. The plank travels a distance of 20 m towards right direction in 5 s. (Here, v_1 and v_2 are given with respect to the plank). Then,



(i) $v_1 = 10\ \mathrm{m\ s^{-1}}, v_2 = 0\ \mathrm{m\ s^{-1}}$ (ii) $v_1 = 15\ \mathrm{m\ s^{-1}}, v_2 = 5\ \mathrm{m\ s^{-1}}$ (iii) $v_1 = 20\ \mathrm{m\ s^{-1}}, v_2 = 10\ \mathrm{m\ s^{-1}}$

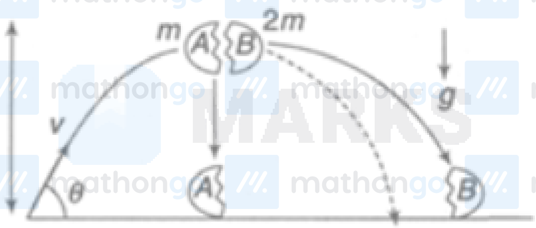
(1) only (i) and (iii) are possible.

(2) all (i), (ii), (iii) are not possible.

(3) only (ii) and (iii) are possible.

(4) All (i), (ii), (iii) are possible.

Q2. A projectile is launched from the origin with speed v at an angle θ from the horizon. At the highest point in the trajectory, the projectile breaks into two pieces, A and B of masses m and $2m$ respectively. Immediately after the breakup, piece A is at rest relative to the ground. Neglect air resistance. Which of the following sentences most accurately describes what happens next?



(1) Piece B will hit the gorund first, since it is more massive.

(2) Both pieces have zero vertical velocity immedialtely after th breakup, and therefore they hit the ground at the same time.

(3) Piece A will hit the ground first, because it will have a downward velocity immediatly after the breakup.

(4) There is no way of knowing which piece will hit the ground first, because not enough information is given about the breakup.

Q3. Two particles of masses m_1 and m_2 have equal kinetic energies. The ratio of their momentum is:

(1) $m_1 : m_2$

(2) $m_2 : m_1$

(3) $\sqrt{m_1} : \sqrt{m_2}$

(4) $m_1^2 : m_2^2$

Q4. The center of mass of a system of particles does not depend upon,

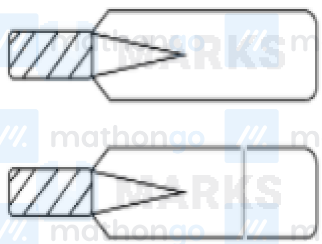
(1) Position of particles

(2) Relative distance between particles

(3) Masses of particles

(4) Force acting on particle.

Q5. A cricket bat is cut at the location of its centre of mass as shown. Then



(1) the two pieces will have the same mass.

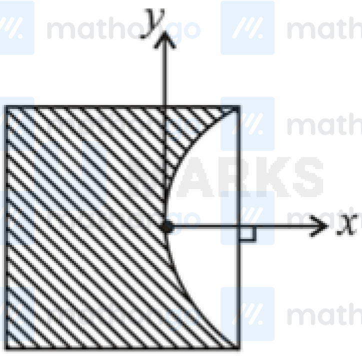
(2) the bottom piece will have larger mass.

(3) the handle piece will have larger mass.

(4) mass of handle piece is double the mass of bottom piece.

Q6. In the figure shown a semicircular area is removed from a uniform square plate of side l and mass (before removing) m.

The x-coordinate of centre of mass of remaining portion is (The origin is at the centre of square)



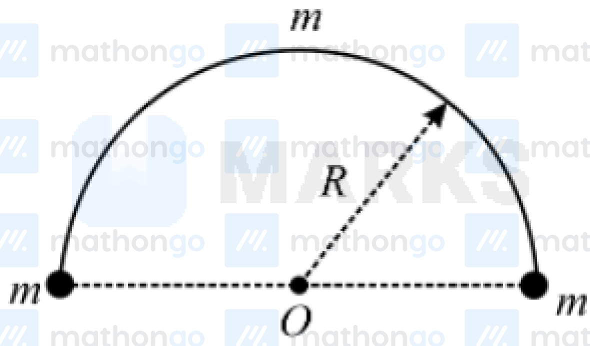
(1) $-\frac{\pi(\pi-2)\ell}{2(8-\pi)}$

(2) $\frac{\pi(\pi-2)\ell}{2(8-\pi)}$

(3) $-\frac{\pi(\pi-2)\ell}{(8-\pi)}$

(4) None of these

Q7. Two small particles each of mass m are attached at the two ends of a semicircular ring of same mass as shown. R is radius of ring and O is the centre of the ring. Moment of inertia about an axis passing through O and normal to its plane is



(1) $3mR^2$

(2) $\frac{3mR^2}{2}$

(3) $2mR^2$

(4) $\frac{mR^2}{2}$

Q8. Two particles A and B, initially at rest, move towards each other under a mutual force of attraction. At the instant when the speed of A is v and the speed of B is $2v$, the speed of the center of mass of the system is:

(1) $3v$

(2) $2v$

(3) $1.5v$

(4) zero

Q9. Two balls are thrown in air. The second ball is thrown 2 seconds after the first ball. The acceleration of the center of mass of the two balls while in air:

(1) Depends on the direction of the motion of the balls

(2) Depends on the masses of the balls

(3) Depends on the speed of two balls

(4) is equal to ' g '

Q10. A uniform rod of length l is pivoted at point A. It is struck by a horizontal force which delivers an impulse J at a distance x from point A as shown in the figure. The impulse delivered by pivot is zero if x is equal to,

(1) $\frac{l}{2}$

(2) $\frac{\frac{1}{3}}{3}$

(3) $\frac{2l}{3}$

(4) $\frac{3l}{4}$

Q11. In a head on elastic collision of a very heavy body moving at v with a light body at rest, velocity of heavy body after collision is

(1) v

(2) $2v$

(3) Zero

(4) $\frac{v}{2}$

Q12. A sping balance is adjusted at zero. Elastic collisions are brought about by dropping particles of one gram each on the pan of the balance. They recoil upwards without change is their speeds. If the height of fall of particles is 2 meter and the rate of particle dropping is 100 per seconds, then the reading of the balance is -

(1) 128gmwt.

(2) 1252gmwt.

(3) 625gmwt.

(4) 1.25gmwt.

Q13. A uniform rod of mass m and length $2a$ lies at rest on a smooth horizontal table. A perfectly elastic particle of same mass m, moving with speed v on the table in a direction perpendicular to the rod, strikes one end of the rod. The kinetic energy generated in the rod is

(1) $\frac{4}{13}\mathrm{mv}^2$

(2) $\frac{1}{4}\mathrm{mv}^2$

(3) $\frac{8}{25}\mathrm{mv}^2$

(4) None of these

Answer Key

Q1 (2)	**Q2** (2)	**Q3** (3)	**Q4** (4)
Q5 (2)	**Q6** (4)	**Q7** (1)	**Q8** (4)
Q9 (4)	**Q10** (3)	**Q11** (1)	**Q12** (1)
Q13 (3)			

Q1.

Let the distance travelled by plank be s in time t.

So, the velocity of plank, $v_{\mathrm{P}}=\frac{s}{t}$.

$\Rightarrow v_{\mathrm{P}}=\frac{20}{5}$

$\Rightarrow v_{\mathrm{P}}=4\ \mathrm{m\ s^{-1}}$

$\Delta x_{\mathrm{CM}}=0$.

Velocity of A with respect to ground

$=v_1+4$

Velocity of B with respect to ground

$=v_2-4$

As the net external force in $x-$direction is zero, the linear momentum is conserved.

$40(v_1+4)-40(v_2-4)+20\times(4)=0$.

$40(v_1-v_2+10)=0$

$v_1-v_2=-10$.

If $v_1=10\ \mathrm{m\ s^{-1}}$ then $v_2=20\ \mathrm{m\ s^{-1}}$

If $v_1=20\ \mathrm{m\ s^{-1}}$ then $v_2=30\ \mathrm{m\ s^{-1}}$

If $v_1=15\ \mathrm{m\ s^{-1}}$ then $v_2=25\ \mathrm{m\ s^{-1}}$.

Q2.

At the highest point before explosion the vertical component of velocity is zero. So, as per law of conservation of momentum, after explosion also the pieces will have the same vertical component of velocity. So, in the vertical direction they will be under free fall with zero initial velocity. Hence, they will hit the ground at the same time.

Q3.

Kinetic energy, $K=\frac{P^2}{2m}$

$\therefore P^2=2mK$

$P=\sqrt{2mK}$

$\frac{P_1}{P2}=\sqrt{\frac{2m_1K}{2m_2K}}$

$\frac{P_1}{P2}=\sqrt{\frac{m_1}{m_2}}$

Q4.

The location of the centre of mass of a variety of bodies does not rely on the internal or external force acting on the system. Rather, the movement of the centre of mass like acceleration relies on the system's net external force.

The variables on which the centre of mass depends on the

- The mass of a rigid body.
- Position of mass from the axis.
- Distribution of mass.

Q5.

We know that the centre of mass is given by,

$x_{\mathrm{cm}}=\frac{m_1x_1+m_2x_2}{m_1+m_2}$

or for continuos body we have, $x_{\mathrm{cm}}=\frac{\int x\,\mathrm{d}m}{\int \mathrm{d}m}$

.

For a cricket bat we have the mass distributed over the greater distance on the left side of the bat than to the right side of the bat, so the r is greater for the left side, to compensate this larger value of distance the right side will have greater mass.

The centre of mass lies closest to the heavier mass.

Q6.

The centre of mass about $x-$axis is given as,

$x_{\text{cm}}=\frac{m_1 x_1+m_2 x_2}{m_1+m_2}$

Here, m_1 = mass of the square plate $=m$

x_1 =centre of mass of the square plate $=0$

(As the origin is at centre of the square)

m_2 = mass of the removed part=mass density of square$\times$area of the semicircular

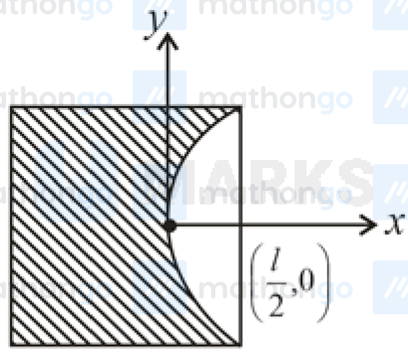
$=-\frac{m}{l^2}\left(\frac{\pi\left(\frac{l}{2}\right)^2}{2}\right)=-\frac{\pi}{8}m$

x_2 =centre of mass of the removed part

$=\frac{l}{2}-\frac{4}{3\pi}\left(\frac{l}{2}\right)=\frac{l}{2}\left(1-\frac{4}{3\pi}\right)$

Now, the centre of mass of the system will be, $x_{\text{cm}}=\frac{-\frac{\pi m}{8}\times\frac{l}{2}\left(1-\frac{4}{3\pi}\right)}{m-\frac{\pi}{8}m}$

Hence, the centre of mass of remaining portion $x_{\text{cm}}=-\frac{l\left(\pi-\frac{4}{3}\right)}{2(8-\pi)}$.



Q7. Here ring is half but its mass is m so its moment of inertia will me mR^2

$I=mR^2+mR^2+mR^2$

$I_{net}=mR^2+2\left(mR^2\right)$

$I=3mR^2$

Q8.

The two bodies are moving only because of a mutual force of attraction. Therefore, the net external force of the masses as a system is zero. Therefore, the acceleration of the center of mass is zero, so as the center of mass is initially at rest, it will continue to be at rest.

Q9. $a_{CM}=\frac{f_{\text{net}}}{m}=\frac{mg+mg}{2m}=g$

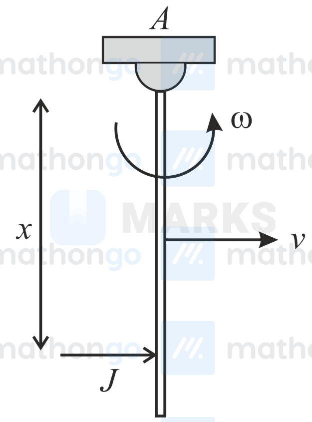
Q10.

From impulse-momentum theorem, we can write,

Impulse=change in momentum or, $J=\Delta p$.

Let velocity of the centre of mass is v, then, we can write,

$J=mv-0=mv\ldots(i)$



The rod starts rotating. Let the angular speed of rod about point A is ω. Then, from conservation of angular momentum, we have,

$J.x = I\omega = \frac{ml^2}{3}\omega \quad \ldots(ii)$

And from the relation between linear and angular velocity of the centre of mass, we have,

$v = \frac{l}{2}\omega \quad \ldots$(iii)

Solving these three equations, we get,

$x = \frac{2}{3}l.$

Q11. In a perfectly elastic collision the relative velocity remains unchanged in magnitude but reserved in direction. Therefore , velocity of heavy body after collision is v.

Q12. Velocity of the particle after it has fallen 2m,

$v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 2}\, m/s$

When a particles rebound from the pan, change in its momentum $= 2\, mv$

If n particles fall per second, then force exerted

$= n.2mv$

$= 100 \times 2 \times 1 \times 10^{-13} \times \sqrt{4 \times 9.8N}$

$= \frac{100 \times 2 \times 1 \times 10^{-3} \times \sqrt{4 \times 9.8}\, gm\, wt}{9.8 \times 10^{-3}}$

$= 128\, gm\, wt$

Q13. From conservation of linear momentum we have,

$$\mathrm{v} = \mathrm{v}_1 + \mathrm{v}_2 \qquad \ldots(1)$$

From conservation of angular momentum about centre of rod we have,

$$\mathrm{mv}a = \mathrm{mv}_2 a + \frac{\mathrm{m}a^2}{3} \cdot \omega$$

or $$\mathrm{v} = \mathrm{v}_2 + \frac{a\omega}{3} \qquad \ldots(2)$$

Further from the definition of coefficient of restitution (e = 1) at point of impact.

Relative speed of approach = relative speed of separation

$\therefore$ $$\mathrm{v} = \mathrm{v}_1 + a\omega - \mathrm{v}_2 \qquad \ldots(3)$$

solving these three Eqs. (1), (2) and (3) we get,

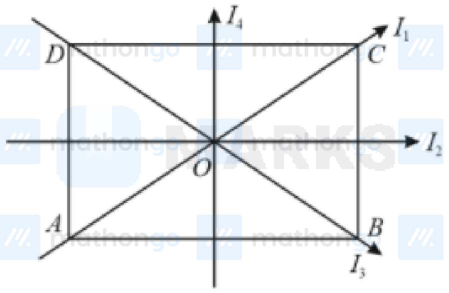
$$\mathrm{v}_1 = \frac{2}{5}\mathrm{v} \text{ and } \omega = \frac{6\mathrm{v}}{5a}$$

$\therefore$ Kinetic energy of rod,

$$\mathrm{K} = \frac{1}{2} \times \mathrm{m} \times \left(\frac{2}{5}\mathrm{v}\right)^2 + \frac{1}{2} \times \frac{\mathrm{m}a^2}{3} \times \left(\frac{6\mathrm{v}}{5a}\right)^2$$

$$= \frac{8}{25}\mathrm{mv}^2$$

Q1. The moment of inertia of a thin rectangular plate $ABCD$ of uniform thickness about an axis passing through the centre O and perpendicular to the plane of the plate is



(1) $I_1 + I_2$

(2) $I_2 + I_4$

(3) $I_1 + I_3$

(4) $I_1 + I_2 + I_3 + I_4$

Q2. Length, width and mass of a rectangular plate are ℓ, b and m respectively. The radius of gyration about the axis passing through centre and perpendicular to the plane is :

(1) $\sqrt{\frac{\ell^2+\mathrm{b}^2}{2}}$

(2) $\sqrt{\frac{\ell^2+\mathrm{b}^2}{8}}$

(3) $\sqrt{\frac{\ell^2+\mathrm{b}^2}{12}}$

(4) $\sqrt{\frac{\ell^2+b^3}{12}}$

Q3. The radius of gyration of a body about an axis at a distance 6 cm from its centre of mass is 10 cm. Then its radius of gyration about a parallel axis through its centre of mass will be

(1) 80 cm

(2) 8 cm

(3) 0.8 cm

(4) 8.0 cm

Q4. Consider two uniform discs of the same thickness and different radii $R_1 = R$ and $R_2 = \alpha R$ made of the same material. If the ratio of their moments of inertia I_1 and I_2, respectively, about their axes is $I_1 : I_2 = 1 : 16$ then the value of α is:

(1) $2\sqrt{2}$

(2) $\sqrt{2}$

(3) 2

(4) 4

Q5. Which of the following statements is true in case of the principle of perpendicular axes?

(1) It is applicable to only three dimensional objects.

(2) It is applicable to planar as well as three dimensional objects.

(3) It is applicable to only planar objects.

(4) It is applicable to only denser objects.

Q6. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: Moment of inertia of a circular disc of mass M and radius R about X, Y axes (passing through its plane) and Z-axis which is perpendicular to its plane were found to be I_x, I_y and I_z, respectively. The respective radii of gyration about all the three axes will be the same.

Reason R: A rigid body making rotational motion has fixed mass and shape. In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both A and R are correct but R is not the correct explanation of A.

(2) A is not correct but R is correct.

(3) A is correct but R is not correct.

(4) Both A and R are correct and R is the correct explanation of A.

Q7. A hoop of mass M and radius R is hung from a support fixed in a wall. Its moment of inertia about the support is

(1) $2MR^2$

(2) $3MR^2$

(3) $4MR^2$

(4) $6MR^2$

Q8. Three point sized bodies each of mass M are fixed at three corners of light triangular frame of side length L. About an axis perpendicular to the plane of frame and passing through centre of frame the moment of inertia of three bodies is ML^2. In above problem about an axis passing through any side of frame the moment of inertia of three bodies is

(1) ML^2

(2) $\frac{3ML^2}{2}$

(3) $\frac{3ML^2}{4}$

(4) $\frac{2ML^2}{3}$

Q9. If the force applied to a body produces rotational motion in an anticlockwise sense, the moment of the force is considered as,

(1) negative

(2) zero

(3) positive

(4) infinity

Q10. A solid disc of radius 20 cm and mass 10 kg is rotating with an angular velocity of 600rpm, about an axis normal to its circular plane and passing through its centre of mass. The retarding torque required to bring the disc at rest in 10 s is _____

(1) $4\pi \times 10^{-2}$

(2) 2π

(3) $4\pi \times 10^{-1}$

(4) $2\pi \times 10^{-2}$

Q11. A constant torque acting on a uniform circular wheel changes its angular momentum from A_0 to 4 A_0 in 4 s. The magnitude of this torque is

(1) $\frac{3\,\mathrm{A}_0}{4}$

(2) 4 A_0

(3) A_0

(4) 12 A_0

Q12. A door 1.6 m wide requires a minimum force of 1 N to be applied at the free end to open or close it. The minimum force that is required at a point 0.4 m away from the hinged edge for opening or closing the door is:

(1) 1.2 N

(2) 2.4 N

(3) 3.6 N

(4) 4 N

Q13. A disc of moment of inertia I_1 is rotating in horizontal plane about an axis passing through a centre and perpendicular to its plane with constant angular speed ω_1. Another disc of moment of inertia I_2 having zero angular speed is placed coaxially on a rotating disc. Now both the discs are rotating with constant angular speed ω_2. The energy lost by the initial rotating disc is

(1) $\frac{1}{2}\left[\frac{I_1+I_2}{1_1}\right]\omega_1^2$

(2) $\frac{1}{2}\left[\frac{\mathrm{II}_1}{\mathrm{I}_1-\mathrm{I}}\right]\omega_1^2$

(3) $\frac{1}{2}\left[\frac{I_1-I_2}{I_{I_2}}\right]\omega_1^2$

(4) $\frac{1}{2}\left[\frac{\mathrm{I},\mathrm{I}_2}{\mathrm{I}_1+\mathrm{I}_2}\right]\omega_1^2$

Q14. A uniform rod of length L is hinged at one end and initially held in horizontal position. After it is released, find its angular speed when the rod becomes vertical.

(1) $(gL)^{1/2}$

(2) $\left(\frac{2g}{L}\right)^{1/2}$

(3) $\left(\frac{3g}{L}\right)^{1/2}$

(4) $\left(\frac{4g}{L}\right)^{1/2}$

Q15. The moment of inertia of the body about an axis is $1.2\ \mathrm{kg}-\mathrm{m}^2$. Initially the body is at rest. In order to produce a rotational kinetic energy of 1500 J, an angular acceleration of $25\mathrm{rad/s}^2$ must be applied about the axis for the duration of

(1) 2 s

(2) 4 s

(3) 8 s

(4) 10 s

Q16. A circular disc of radius R is free to rotate about an axis passing through its centre. An external tangential force F is applied on the disc along its edge. If the angular velocity of disc is increased from 0 to w in a time t then the work done by F during same time t is

(1) RFwt

(2) 2RFwt

(3) $\frac{\mathrm{RFwt}}{2}$

(4) $\mathrm{RF}(\mathrm{wt})^2$

Q17. A solid sphere is in rolling motion. In rolling motion a body possesses translational kinetic energy (K_t) as well as rotational kinetic energy (K_r) simultaneously. The ratio $K_t : (K_t + K_r)$ for the sphere is

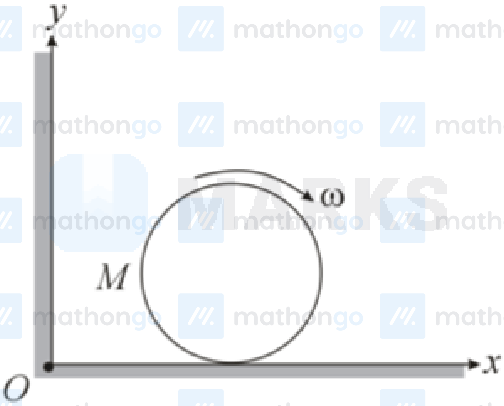
(1) 10 : 7

(2) 5 : 7

(3) 7 : 10

(4) 2 : 5

Q18. A disc of mass M and radius R is rolling with angular speed ω on a horizontal plane as shown. The magnitude of angular momentum of the disc about the origin O is



(1) $\left(\frac{1}{2}\right) MR^2\omega$

(2) $MR^2\omega$

(3) $\left(\frac{3}{2}\right) MR^2\omega$

(4) $2MR^2\omega$

Q19. A rod AB of mass M and length L is lying on a horizontal frictionless surface. A particle of mass m travelling along the surface hits the end A of the rod with a velocity v_0 in a direction perpendicular to AB. The collision is perfectly elastic and after the collision, the particle comes to rest. The ratio $\frac{m}{M}$ is

(1) $\frac{1}{4}$

(2) $\frac{1}{3}$

(3) $\frac{1}{2}$

(4) 1

Q20. The density of a rod gradually decreases from one end to the other. It is pivoted at an end, so that it can move about a vertical axis through the pivot. A horizontal force F is applied on the free end in a direction perpendicular to the rod. The quantities that do not depend on which end of the rod is pivoted are

(1) angular acceleration.

(2) angular velocity when the rod completes one rotation.

(3) angular momentum when the rod completes one rotation.

(4) torque of the applied force.

Q21. Which of the following statement is not correct?

(1) During rolling, the instantaneous speed of the point of contact is zero.

(2) During rolling, the instantaneous acceleration of the point of contact is zero.

(3) For perfect rolling motion, work done against friction is zero.

(4) A wheel moving down a perfectly frictionless inclined plane will slip but not roll on the plane.

Q22. A rod of length, l is given two velocities, v_1 and v_2 in opposite directions at its two ends at right angles to the length. The distance of the instantaneous axis of rotation from v_1 is

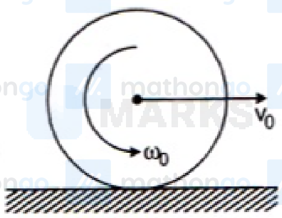
(1) 0

(2) $\frac{v_1}{v_1+v_2} l$

(3) $\frac{v_2}{v_1+v_2} l$

(4) $\frac{1}{2}$

Q23. A uniform sphere of radius R is placed on a rough horizontal surface and given a linear velocity v_0 and angular velocity ω_0 as shown. The sphere comes to rest after moving some distance to the right. It follows that



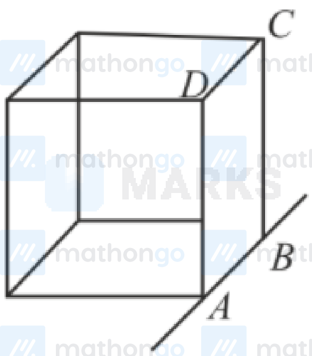
(1) $v_0 = \omega_0 R$

(2) $2v_0 = 5\omega_0 R$

(3) $5v_0 = 2\omega_0 R$

(4) $2v_0 = \omega_0 R$

Q24. A solid cube of wood of side $2a$ and mass M is resting on a horizontal surface as shown in the figure. The cube is free to rotate about a fixed axis AB. A bullet of mass $m(<< M)$ and speed v is shot horizontally at the face opposite to $ABCD$ at a height of $\frac{4a}{3}$ from the surface to impart the cube and angular speed ω. It strikes the face and embeds in the cube. Then ω is close to (note: the moment of inertia of the cube about an axis perpendicular to the face and passing through the center of mass is $\frac{2Ma^2}{3}$.)



(1) $\frac{Mv}{ma}$

(2) $\frac{Mv}{2ma}$

(3) $\frac{mv}{Ma}$

(4) $\frac{mv}{2Ma}$

Answer Key

Q1 (2)	**Q2** (3)	**Q3** (2)	**Q4** (3)
Q5 (3)	**Q6** (2)	**Q7** (1)	**Q8** (3)
Q9 (3)	**Q10** (1)	**Q11** (1)	**Q12** (4)
Q13 (4)	**Q14** (3)	**Q15** (1)	**Q16** (3)
Q17 (2)	**Q18** (3)	**Q19** (1)	**Q20** (4)
Q21 (2)	**Q22** (2)	**Q23** (3)	**Q24** (4)

Q1.

According to the theorem of perpendicular axis, the moment of inertia for any plane body for either of its axes direction normal to the plane is equivalent to the total of the moment of inertia along the two normal axis in the plane of the body that intersect the first axis in the plane.

From the figure moment of inertia about an axis passing through the centre O and perpendicular to the plane of the plate is $I = I_2 + I_4$.

Since the plate is rectangular of uniform thickness the $x-$axis and $y-$axis will not be symmetrical, i.e, AC and BD are not perpendicular to each other and this gives $I_1 \neq I_3$.

Q2.

Moment of inertia of a rectangular plate about an axis passing through its centre and perpendicular to the plane is given by

$I = \frac{m\left(l^2+b^2\right)}{12} \qquad \ldots.(1)$

As radius of gyration k is given by using the expression

$I = mk^2$

So, $k = \sqrt{\frac{I}{m}}$

$\Rightarrow k = \sqrt{\frac{l^2+b^2}{12}}$.

Q3. From the theorem of parallel axis, the moment of inertia I is equal to

$I = I_{CM} + Ma^2$

where I_{CM} moment of inertia is about centre of mass and a the distance of axis from centre.

$\therefore I = MK^2 + M \times (6)^2$

$MK_1^2 = MK^2 + 36M$

$\Rightarrow K_1^2 = K^2 + 36$

$\Rightarrow (10)^2 = K^2 + 36$

$\Rightarrow K^2 = 100 - 36 = 64$

$\Rightarrow K = 8$ cm

Q4.

Moment of inertia of disc is given by $\mathrm{I} = \frac{\mathrm{MR}^2}{2} = \frac{\left[\rho\left(\pi \mathrm{R}^2\right)\mathrm{t}\right]\mathrm{R}^2}{2}$

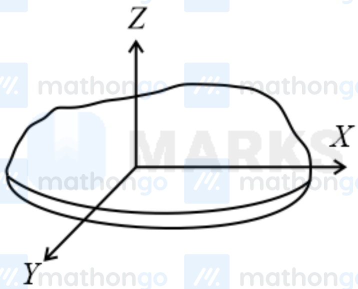
$\mathrm{I} \, \alpha \, \mathrm{R}^4$

$\frac{\mathrm{I}_2}{\mathrm{I}_1} = \left(\frac{\mathrm{R}_2}{\mathrm{R}_1}\right)^4$

$\frac{16}{1} = \alpha^4$

$\alpha = 2$

Q5.



Perpendicular axes theorem is applicable of two dimensional bodies or planer bodies having almost negligible thickness.

From figure the moment of inertia about rotational axis (Z-axis) which is normal to plane of body is given by

$I_{\mathrm{Z}} = I_{\mathrm{X}} + I_{\mathrm{Y}}$

Where I_x, I_y are moment of inertia of body about two mutually perpendicular axes in the plane of body.

Above theorem is not applicable for three-dimensional bodies.

Q6.

$I_z = I_x + I_y$ (using perpendicular axis theorem)

and $I = mk^2$ (K : radius of gyration)

so $mK_z^2 = mK_x^2 + mK_y^2$

$K_z^2 = K_x^2 + K_y^2$

So radius of gyration about axes x, y and z will not be same, hence assertion A is not correct reason R is correct statement (property of a rigid body).

Q7. A hoop is a circular ring $\therefore \; \mathrm{I}_0 = \mathrm{MR}^2$

By theorem of parallel axes,

$\mathrm{I} = \mathrm{I}_0 + \mathrm{MR}^2 = \mathrm{MR}^2 + \mathrm{MR}^2 = 2\mathrm{MR}^2$

Q8.

Moment of inertia of a body with discrete particles is given by

$I=\sum MR^2$

Here, M is the mass of each particle and R is the distance of the axis from the particle.

Given, the axis is passing through a side of the triangular frame.

So, the moment of inertia due to the two particles present on the axis and at the corners of the frame will be zero due to the zero distance.

Therefore, moment of inertia will be only due to one particle which is not on the axis.

$I=MR^2=M\left[L^2-\left(\frac{L}{2}\right)^2\right]$

$\Rightarrow\ I=\frac{3ML^2}{4}$

Q9.

Sign convention used for moment of couple.

When the forces applied to a body rotates the body in the anticlockwise manner, then the moment of force is assumed to be positive. On the other hand, if the rotation takes place in clockwise sense then the moment of force is assumed to be negative.

Q10.

$\tau=\frac{\Delta L}{\Delta t}=\frac{I\left(\omega_f-\omega_i\right)}{\Delta t}$

$\tau=\frac{\frac{mR^2}{2}\times\left[0-\omega\right]}{\Delta t}$

$=\frac{10\times\left(20\times10^{-2}\right)^2}{2}\times\frac{600\times\pi}{30\times10}$

$=0.4\pi=4\pi\times10^{-2}$

Q11.

The angular momentum is changing, means an angular impulse will arise in the system, which is given by,

angular impulse=change in angular momentum

$\tau\times t=L_f-L_i,$

where, τ is the torque and

t is the time taken.

$4\,\tau=4\,A_0-A_0$

$\Rightarrow\quad\tau=\frac{3\,A_0}{4}.$

Q12.

Torque required in initial case is,

$\tau=1.6\times1=1.6\ \mathrm{N\,m}$

Now when the point of application of force is changed to, $d=0.4\ \mathrm{m}$

$F=\frac{\tau}{d}=\frac{1.6}{0.4}=4\ \mathrm{N}$

Q13.

Here, angular momentum is conserved, so the initial momentum is equal to final momentum.

$J_{\mathrm{i}}=J_{\mathrm{f}}$

$I_1\omega_1=(I_1+I_2)\omega_2$

$\Rightarrow\omega_2=\frac{I_1\omega_1}{(I_1+I_2)}$

Here, initial kinetic energy, $(K.E.)_{\mathrm{i}}=\frac{1}{2}I_1(\omega_1)^2$

Thus, final kinetic energy, $(K.E.)_{\mathrm{f}}=\frac{1}{2}(I_1+I_2)(\omega_2)^2$,

now put the value, $\frac{1}{2}(I_1+I_2)\left(\frac{I_1\omega_1}{(I_1+I_2)}\right)^2=\frac{1}{2}\frac{(I_1\omega_1)^2}{(I_1+I_2)}$

$\Delta K.E.=(K.E.)_{\mathrm{i}}-(K.E.)_{\mathrm{f}}=\frac{1}{2}I_1(\omega_1)^2-\frac{1}{2}\frac{(I_1\omega_1)^2}{(I_1+I_2)}$

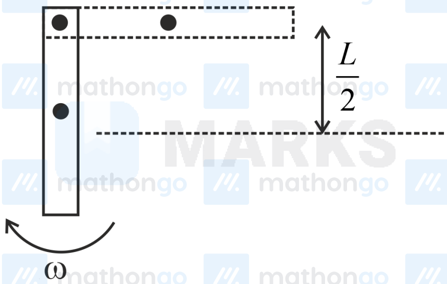
$\Delta K.E.=\frac{1}{2}I_1(\omega_1)^2\left(1-\frac{I_1}{I_1+I_2}\right)=\frac{1}{2}(\omega_1)^2\left(\frac{I_2I_1}{I_1+I_2}\right)$

Q14.

The gain in kinetic energy, $\Delta KE=\frac{1}{2}\left(\frac{ml^2}{3}\right)\omega^2$

Loss in potential energy, $-\Delta U=mgh_{\mathrm{loss}}$, where

$h_{\mathrm{loss}}=\frac{L}{2}$ is the loss of height of the center of mass.



By conservation of mechanical energy, $\Delta KE=-\Delta U$

$mg\frac{\mathrm{L}}{2}=\frac{1}{2}\left(\frac{mL^2}{3}\right)\omega^2\Rightarrow\omega=\left(\frac{3g}{L}\right)^{1/2}$

Q15. Rotational kinetic energy $=\frac{1}{2}I\,\omega^2$

According to question

$\frac{1}{2}I\,\omega^2=1500$

$\frac{1}{2}I\,(\alpha t)^2=1500$

$(1.2)\times(25)^2\times\mathrm{t}^2=3000$

$1.2\times625\times\mathrm{t}^2=3000$

$\mathrm{t}^2=\frac{3000}{1.2\times625}=4$

$\mathrm{t}=2\ \mathrm{s}$

Q16. Work done $W = \frac{1}{2} RF(\omega t)$

Q17. Transnational $KE = \frac{1}{2}mv^2$

Translational $KE +$

Rotational KE

$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

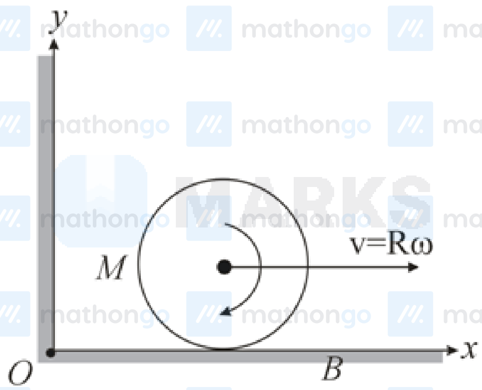
$= \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\omega^2$

$\because$ for rolling motion $v = r\omega$

$= \frac{1}{2}mv^2 + \frac{1}{2} \times \frac{2}{5}mr^2\left(\frac{v}{r}\right)^2 = \frac{7}{10}mv^2$

So, $\frac{K_t}{K_t + K_r} = \frac{\frac{1}{2}mv^2}{\frac{7}{10}mv^2} = \frac{5}{7}$

Q18. From the theorem



$\overrightarrow{L} = \overrightarrow{L_{cm}} + M\left(\overrightarrow{r} \times \overrightarrow{v}\right)$

Angular momentum about O = Angular momentum about CM +

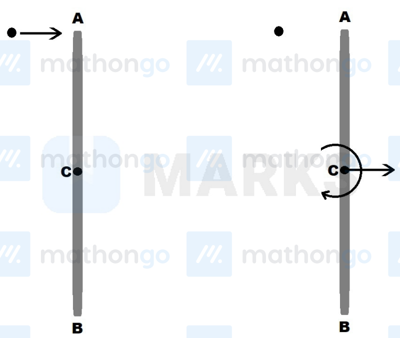
Angular momentum of CM about origin

$L_0 = I\omega + MRv$

$= \frac{MR^2\omega}{2} + MR^2\omega$

$= \left(\frac{3}{2}\right)MR^2\omega$

Q19.



Using the principle of conservation of linear momentum

$mv_0 = Mv_{cm}$

Using the principle of conservation of angular momentum

$mv_0 \frac{L}{2} = \frac{ML^2}{12}\omega$

Since the collision is completely elastic, the velocity of approach is equal to the velocity of separation

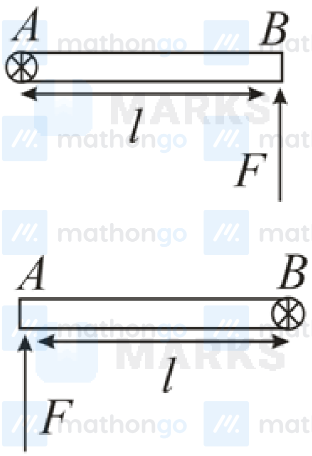
$v_0 = v_{cm} + \omega\frac{L}{2}$

$\Rightarrow v_0 = \frac{m}{M}v_0 + 3\frac{m}{M}v_0$

$\Rightarrow \frac{m}{M} = \frac{1}{4}$

Q20.

Density of the rod is increasing from one end to other, lets say it increases from end A to end B.



Therefore, the rod element near end B is heavier than end A.

So, the moment of inertia of rod about the axis passing through the end A is greater than about the axis passing through the end B,

i.e., $I_A > I_B$

Torque about an end, $\tau = (r_\perp)F$

where, $r_\perp = l$.

So, torque about both the ends is same.

Angular acceleration, $\alpha = \frac{\tau}{I}$

$\therefore \frac{\alpha_A}{\alpha_B} = \frac{I_B}{I_A} < 1$

All quantities are independent of time,

$\therefore \alpha$ is constant.

$\frac{\omega_A}{\omega_B} = \sqrt{\frac{\alpha_A}{\alpha_B}} = \sqrt{\frac{I_B}{I_A}} < 1$

Angular momentum, $L = I\omega$

$\therefore \frac{L_A}{L_B} = \frac{I_A}{I_B}\frac{\omega_A}{\omega_B} = \sqrt{\frac{I_A}{I_B}}$

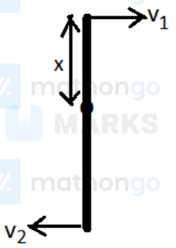
So, only torque is same about both ends.

Q21.

A rolling boy can be imagined to be rotating about an axis passing through the point of contact of the body with the ground. Hence, the instantaneous sped of the point of contact is zero. Thus, statement (a) is correct. As the body is rotating, its instantaneous acceleration is not zero. Hence, Statement (b) is incorrect. Once perfect the rolling begins, force of friction becomes zero. Hence, work done against friction is zero. Thus, statement (c) is correct.

Rolling cannot take place in the absence of friction because it is the frictional force that provides the necessary torque which makes the body roll on a surface. When the inclined plane is perfectly smooth, the wheel will simply slip under the effect of its weight. Hence, statement (d) is correct.

Q22.



Let, the distance of the instantaneous axis from $v_1 = x$.

About the instantaneous axis of rotation, angular velocity of each point will be the same.

So, $\omega_{v_1} = \omega_{v_2}$

$\frac{v_1}{x} = \frac{v_2}{l-x}$

$x = \frac{v_1}{v_1+v_2} l$.

Q23.

friction between the sphere and the horizontal surface,f=μN,

$\Rightarrow \mathrm{f} = \mu \mathrm{mg}$,

$acceleration,\ \ a = \mu g$

$where, \mu - coefficient\ of\ kinetic\ friction,$

$N - normal\ reaction = mg$

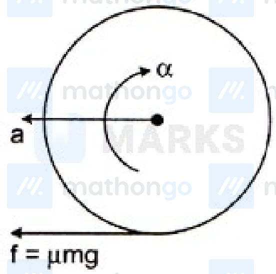
$m - mass\ of\ the\ sphere$

$we\ know\ that, torque \mathrm{T} = \mathrm{I}\alpha, \Rightarrow \alpha = \frac{\mathrm{T}}{\mathrm{I}}$

$where,\ \alpha - angular\ acceleration,$

$\mathrm{I} - moment\ of\ inertia\ of\ the\ sphere$

$\alpha = \frac{\mu \mathrm{mgR}}{\frac{2}{5}\mathrm{mR}^2} = \frac{5}{2}\frac{\mathrm{g}\mu}{\mathrm{R}}$



Now,

$u\sin g\ equation\ of\ motion,\ \mathrm{v} = \mathrm{u} + \mathrm{at},$

$\Rightarrow 0 = \mathrm{v}_0 - \mathrm{at}$

$\Rightarrow \mathrm{t} = \frac{\mathrm{v}_0}{\mathrm{a}}$

$t = \frac{\mathrm{v}_0}{a} = \frac{\omega_0}{\alpha},$

$where\ \mathrm{v}_0 = \mathrm{R}\omega,\ and\ \mathrm{a} = \mathrm{R}\alpha,\ \mathrm{R} - radius\ of\ the\ sphere$

or $\quad \frac{a}{\alpha} = \frac{\mathrm{v}_0}{\omega_0}$

$\therefore \quad \frac{2\mathrm{R}}{5} = \frac{\mathrm{v}_0}{\omega_0}$

$\therefore \quad 5\mathrm{v}_0 = 2\omega_0\mathrm{R}$

Q24.

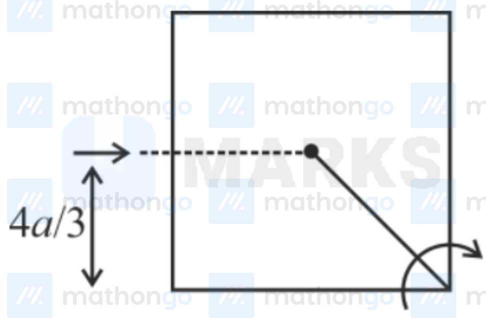
Applying the parallel axis theorem, $I = I_{\mathrm{cm}} + Ma^2$,

here I_{cm} is the moment of inertia passing through centre of mass, a is the distance between two axes.

The moment of inertia of solid cube about AB is

$I = \frac{2}{3}Ma^2 + M\left(a\sqrt{2}\right)^2$

$I = \frac{8}{3}Ma^2$



The cube is free to rotate about AB axis. Thus, the angular momentum should be conserved.

Angular momentum $mvr = I\omega$, here $r = \frac{4a}{3}$.

So, $mv \times \frac{4a}{3} = I\omega$

$\Rightarrow \omega = \frac{mv\left(\frac{4a}{3}\right)}{\frac{8}{3}Ma^2} = \frac{mv}{2Ma}$.

Q1. Reason of weightlessness in a satellite is

(1) zero gravity.

(2) no atmosphere.

(3) zero reaction force by satellite surface

(4) None of the above.

Q2. Which of the following statements about the gravitational constant is true?

(1) It is a force.

(2) It has same value in all systems of units.

(3) It depends on the value of the masses.

(4) It does not depend on the nature of the medium in which the bodies are kept.

Q3. What is acceleration due to gravity at the surface of mars if its diameter is 6760 km and mass one tength that of earth. The diameter of earth is 12742 km and acceleration due to gravity on earth is $9.8 \mathrm{~m/sec^2}$

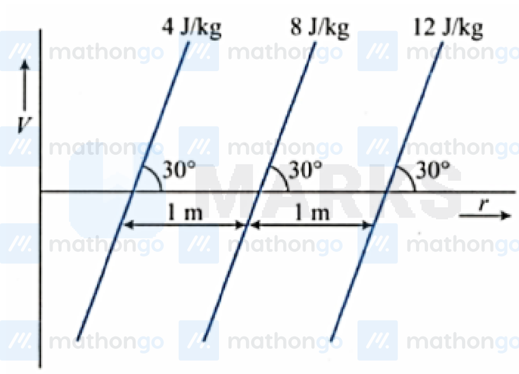
(1) $4.18 \mathrm{~m/sec^2}$

(2) $3.03 \mathrm{~m/sec^2}$

(3) $4.58 \mathrm{~m/sec^2}$

(4) $3.48 \mathrm{~m/sec^2}$

Q4. Gravitational potential V versus distance r graph is represented in figure. The magnitude of gravitational field intensity is equal to:



(1) $8 \mathrm{~N/Kg}$

(2) $4 \mathrm{~N/Kg}$

(3) $6 \mathrm{~N/Kg}$

(4) $2 \mathrm{~N/Kg}$

Q5. The mass density of a planet of radius R varies with the distance r from its centre as $\rho(r)=\rho_0\left(1-\frac{r^2}{R^2}\right)$ Then the gravitational field is maximum at:

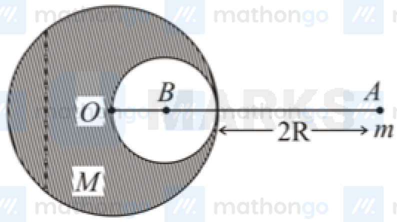
(1) $r=\sqrt{\frac{3}{4}} R$

(2) $r=R$

(3) $r=\frac{1}{\sqrt{3}} R$

(4) $r=\sqrt{\frac{5}{9}} R$

Q6. A solid sphere of radius R gravitationally attracts a particle placed at 3R from its centre with a force F_1. Now a spherical cavity of radius $\left(\frac{R}{2}\right)$ is made in the sphere (as shown in figure) and the force becomes F_2. The value of $F_1: F_2$ is:



(1) 41 : 50

(2) 50 : 41

(3) 25 : 36

(4) 36 : 25

Q7. The value of acceleration due to gravity at a height of 10 km from the surface of earth is x. At what depth inside the earth is the value of the acceleration due to gravity has the same value x?

(1) 10 km

(2) 15 km

(3) 5 km

(4) 20 km

Q8. The speed of the earth's rotation about its axis is ω. Its speed is increased to x times to make the effective acceleration due to gravity equal to zero at the equator, then x is around $\left(g=10 \mathrm{~m~s^{-2}}, R=6400 \mathrm{~km}\right)$

(1) 1

(2) 8.5

(3) 17

(4) 34

Q9. A body is projected from the Earth's surface with a velocity equal to half the escape velocity from that point at an angle 45° with horizontal. The maximum height attained by the body above the Earth's surface is (nearly)

(1) 0.125R

(2) 0.19R

(3) $\frac{\sqrt{3}}{\frac{\pi}{3}}$

(4) 0.9R

Q10. The mass of the earth is 6.0×10^{24} kg. The potential energy of a body of mass 50 kg at a distance of 6.3×10^9 m from the centre of the earth is

(1) 3.23×10^9 J

(2) 3.19×10^6 J

(3) 2.5×10^6 J

(4) 4.0×10^{11} J

Q11. Escape velocity from the earth's surface is

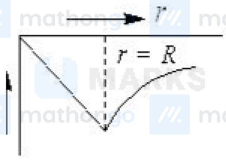
(1) equal to that from the moon's surface.

(2) greater than that from the moon's surface

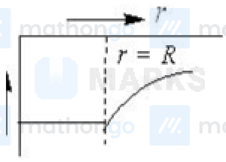
(3) less than that from the moon's surface.

(4) equal to that from the sun's surface.

Q12. P is point at a distance r from the centre of a solid sphere of radius R. The variation of gravitational potential at $P(i.e, V)$ and distance r from the centre of sphere is represented by the curve.



(1)



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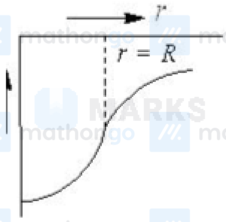
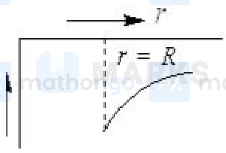
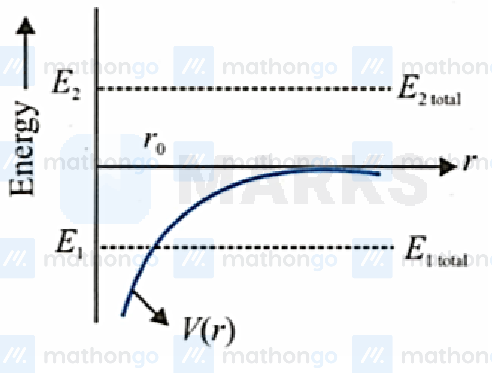


(3)



(4)

Q13. In the graph shown, the potential energy of the earth-satellite system is shown by a solid line as a function of distance r (the separation between the earth's centre and satellite). The total energy of the two objects which may or may not be bounded to the earth is shown in the figure by dotted lines. Mark the correct statement(s).



(1) e object having the total energy E_1 is bounded one.

(2) e object having the total energy E_2 is bounded one.

(3) th the objects are bounded.

(4) th the objects are unbounded.

Q14. A satellite is revolving in circular orbit of radius r around the earth of mass M. Time of revolution of satellite is

(1) $T \propto \frac{r^3}{GM}$

(2) $T \propto \sqrt{\frac{r^3}{GM}}$

(3) $T \propto \sqrt{\frac{r^3}{\frac{GM}{3}}}$

(4) $T \propto \sqrt{\frac{r^3}{\frac{GM}{4}}}$

Q15. The ratio of the energy required to raise a satellite upto a height h above the earth of radius R to that the kinetic energy of the satellite into that orbit is

(1) R : h

(2) $h : R$

(3) R : 2 h

(4) 2 h : R

Q16. In a satellite, if the time of revolution is T, then kinetic energy is proportional to

(1) $\frac{1}{T}$

(2) $\frac{1}{T^2}$

(3) $\frac{1}{T^3}$

(4) $T^{-\frac{2}{3}}$

Q17. A satellite of mass M is launched vertically upwards with an initial speed u from the surface of the earth. After it reaches height R ($R =$ radius of the earth), it ejects a rocket of mass $\frac{M}{10}$ so that subsequently the satellite moves in a circular orbit. The kinetic energy of the rocket is (G is the gravitational constant; M_e is the mass of the earth):

(1) $\frac{M}{20}\left(u^2 + \frac{113}{200}\frac{GM_e}{R}\right)$

(2) $5M\left(u^2 - \frac{119}{200}\frac{GM_e}{R}\right)$

(3) $\frac{3M}{8}\left(u + \sqrt{\frac{5GM_e}{6R}}\right)^2$

(4) $\frac{M}{20}\left(u - \sqrt{\frac{2GM_e}{3R}}\right)^2$

Q18. According to Kepler's law, the areal velocity of the radius vector drawn from the Sun to any planet always

(1) decreases.

(2) first increases and then decreases.

(3) remains constant.

(4) increases.

Q19. A body is projected horizontal on the top of the Mount Everest with speed much greater than $\sqrt{2gr}$, where r is the distance of the projection point from the centre of the earth and g is the acceleration due to gravity there. The trajectory of the body will be a

(1) straight line

(2) parabola

(3) hyperbola

(4) ellipse

Q20. In the case of a satellite moving along a circular orbit, a larger orbit corresponds to

(1) longer period and smaller velocity.

(2) larger velocity and longer period.

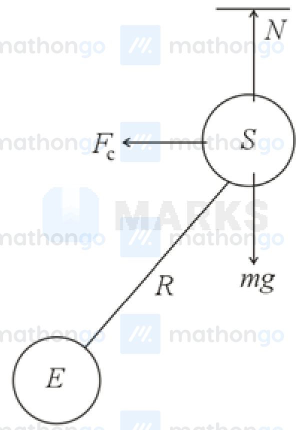
(3) smaller period and smaller velocity.

(4) smaller period and larger velocity.

Answer Key

Q1 (3)	**Q2** (4)	**Q3** (4)	**Q4** (1)
Q5 (4)	**Q6** (2)	**Q7** (4)	**Q8** (3)
Q9 (2)	**Q10** (2)	**Q11** (2)	**Q12** (3)
Q13 (1)	**Q14** (2)	**Q15** (4)	**Q16** (4)
Q17 (2)	**Q18** (3)	**Q19** (3)	**Q20** (1)

Q1.



Look the free body diagram here, $F_c = \frac{mv^2}{R}$, where v is orbital velocity.
It means,

$F - N = F_c$

$\Rightarrow G\frac{Mm}{R^2} - N = \frac{mv^2}{R}$.

The formula of orbital velocity,

$v = \sqrt{\frac{GM}{R}}$, it means,

$\Rightarrow G\frac{Mm}{R^2} - N = \frac{m\left(\sqrt{\frac{GM}{R}}\right)^2}{R}$

$\Rightarrow N = 0$, so zero reaction force by satellite surface.

Q2.

The expression of gravitational constant from Newton's law of gravitation is,

$F = \frac{GMm}{R^2} \Rightarrow G = \frac{FR^2}{Mm}$

Value of the universal gas constant is,

$G = 6.67 \times 10^{-11}\ \mathrm{N\ m^2\ kg^{-2}}$ or

$G = 6.67 \times 10^{-8}\ \mathrm{dyne\ \ cm^2\ g^{-2}}$.

That is, the value of G is not same for all system of units.

G is named as an Universal Gravitational Constant as it is constant at any location of the universe and as well as at any medium in the universe.

Hence, it does not depend on the nature of the medium in which the bodies are kept.

Q3.

From the universal law of gravitation, the gravitational force between two masses is, $F = G\frac{Mm}{r^2}$ Using newton's second law the acceleration due to gravity due to mass M is $mg = G\frac{Mm}{r^2} \Rightarrow g_m = G\frac{M}{r^2}$

Acceleration due to gravity at earth is, $g_e = G\frac{M_e}{R^2}$

Acceleration due to gravity at mars is, $g_m = G\frac{M_m}{r^2}$

Using given data, from the above equations,

$$\frac{g_e}{g_m} = \frac{G\frac{M_e}{R^2}}{G\frac{M_m}{r^2}} = \frac{M_e\left(\frac{6760}{2}\right)^2}{\frac{M_e}{10}\left(\frac{12742}{2}\right)^2}$$

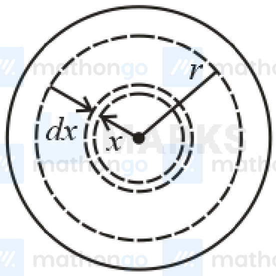
$g_m = 3.48\ \mathrm{m\ s^{-2}}$

Q4.

The gravitational field intensity is given by the Gradient of Gravitational potential means the Gravitational potential change in per unit length, hence the Gravitational field intensity will be,

$\left|\vec{E}\right| = \frac{\Delta V}{\Delta r} = \frac{8-4}{(1)\sin 30^\circ} = 8\mathrm{Nkg^{-1}}$; will be the correct answer.

Q5.



$\mathrm{dm} = \rho \times 4\pi x^2 \mathrm{dx} = \rho_0\left(1 - \frac{x^2}{r^2}\right) \times 4\pi x^2\ \mathrm{dx}$

$\mathrm{m} = 4\pi\rho\int_0^r\left(x^2 - \frac{x^4}{R^2}\right)\ \mathrm{dx\ \ s}$

$\mathrm{m} = 4\pi\rho_0\left|\frac{r^3}{3} - \frac{r^5}{5R^2}\right|$

$\mathrm{E} = \frac{\mathrm{Gm}}{r^2} = \frac{G}{r^2} \times 4\pi\rho_0\left(\frac{r^3}{3} - \frac{r^5}{5R^2}\right)$

$\mathrm{E} = 4\pi G\rho_0\left(\frac{r}{3} - \frac{r^3}{5R^2}\right)$

E is maximum when $\frac{dE}{dt} = 0 \Rightarrow \frac{dE}{dr} = 4\pi G\rho_0\left(\frac{1}{3} - \frac{3r^2}{5R^2}\right) = 0$

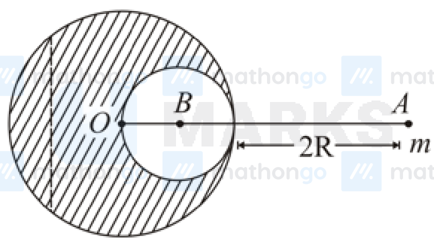
$\Rightarrow\ \ r = \frac{\sqrt{5}}{3}R$

$E_{max} = 4\pi G\rho_0 \times \frac{\sqrt{5}R}{3}\left[\frac{1}{3} - \frac{1}{5} \times \frac{5}{9}\right]$

$E_{max} = \frac{8\sqrt{5}}{27}\pi G\rho_0 R$

Q6.

Let the initial mass of the sphere is m'. Hence, mass of a removed portion will be $m'/8, F_1 = m.E. = \frac{m.Gm'}{9R^2}$



$F_2 = m\left[\frac{G\cdot m'}{(3R)^2} - \frac{G\cdot m'/8}{(5R/2)^2}\right] = \frac{Gm'}{9R^2} - \frac{Gm'\times 4}{8\times 25} = \left(\frac{1}{9} - \frac{1}{50}\right)\frac{Gm'}{R^2}$

$F_2 = \frac{41}{50\times 9}\cdot\frac{Gm'}{R^2} \Rightarrow \frac{F_1}{F_2} = \frac{1}{9} \times \frac{50\times 9}{41} = \frac{50}{41}$

Q7.

$g_h = g\left(1 - \frac{2h}{R}\right)$

$g_d = g\left(1 - \frac{d}{R}\right) \Rightarrow g_h = g_d$

$g\left(1 - \frac{2h}{R}\right) = g\left(1 - \frac{d}{R}\right) \Rightarrow d = 2h = 2 \times 10 = 20\ \mathrm{km}$

Q8.

At the equator, $g' = g - R\omega^2$

When angular velocity be $(\omega' = x\omega)$

then, $0 = g - R\omega'^2$ or

$\omega = \sqrt{\frac{g}{R}}$

present angular velocity of the earth $\omega_0 = \frac{2\pi}{24\times60\times60}$

$\frac{\omega}{\omega_0} = 17$

Q9.

Let h be the maximum height attained and v be the velocity at maximum height. From conservation of angular momentum

$\mathrm{m}\frac{\mathrm{V_e}}{2}\cos\ 45 \times \mathrm{R} = \mathrm{mvr}$ (where $r = R + h$)

$\frac{\mathrm{V_e}}{2} \times \frac{1}{\sqrt{2}}\mathrm{R} = \mathrm{vr} \Rightarrow \mathrm{v} = \frac{\mathrm{V_e}}{2\sqrt{2}}\frac{\mathrm{R}}{\mathrm{r}} = \frac{1}{2\sqrt{2}}\sqrt{\frac{2\,\mathrm{GM}}{\mathrm{R}}} \times \frac{\mathrm{R}}{\mathrm{r}} \Rightarrow \mathrm{v} = \sqrt{\frac{\mathrm{GMR}}{4\mathrm{r}^2}}$

From conservation of energy

$\frac{-\mathrm{GMm}}{\mathrm{R}} + \frac{1}{2}\mathrm{m}\frac{\mathrm{V_0^2}}{4} = -\frac{\mathrm{GMm}}{\mathrm{r}} + \frac{1}{2}\mathrm{mv}^2 \Rightarrow \frac{-\mathrm{GM}}{\mathrm{R}} + \frac{1}{2} \times \frac{1}{4} \times 2\frac{\mathrm{GM}}{\mathrm{R}} = -\frac{\mathrm{GM}}{\mathrm{r}} + \frac{1}{2} \times \frac{\mathrm{GMR}}{4\mathrm{r}^2}$

$-\frac{3}{4\mathrm{R}} = \frac{-8\mathrm{r}+\mathrm{R}}{8\mathrm{r}^2} \Rightarrow -6\mathrm{r}^2 = -8\mathrm{rR} + \mathrm{R}^2 \Rightarrow 6\mathrm{r}^2 - 8\mathrm{rR} + \mathrm{R}^2 = 0$

$\mathrm{r} = \frac{8\mathrm{R}\pm\sqrt{64\mathrm{R}^2-4\times6\times\mathrm{R}^2}}{12} \Rightarrow \mathrm{r} = \frac{8\mathrm{R}\pm2\sqrt{10}\mathrm{R}}{12} \Rightarrow \mathrm{R}+\mathrm{h} = \frac{4\mathrm{R}+\sqrt{10}\mathrm{R}}{6} \Rightarrow \mathrm{h} = 0.19\mathrm{R}$

Q10. Mass of the earth (M) = 6.0 x 10^{24} kg;

Mass of the body (m) = 50 kg and distance (r) = 6.3 x 10^9 m Potential energy (P.E)= $\frac{\mathrm{GMm}}{\mathrm{r}} = \frac{\left(6.7\times10^{-11}\right)\times\left(6.0\times10^{24}\right)\times50}{6.3\times10^9}$

= 3.19 x 10^6 J

Q11.

Escape velocity from the surface of earth is given as

$v_{\text{escape}} = \sqrt{2gR}$, where g is gravitational acceleration on the surface of earth and R is radius of earth. As we know that, radius of moon, $R_{\text{moon}} = \frac{R_{\text{earth}}}{4}$ and gravitational acceleration of moon, $g_{\text{moon}} = \frac{g_{\text{earth}}}{6}$.

$\Rightarrow (v_{\text{escape}})_{\text{moon}} = \sqrt{2 \times \frac{g_{\text{earth}}}{6} \times \frac{R_{\text{earth}}}{4}} = \frac{(v_{\text{escape}})_{\text{earth}}}{\sqrt{10}}$

Escape velocity from surface of earth is more than that from surface of moon.

Q12.

Let M be the mass of the sphere of radius R.

Gravitational potential at any point P due to the solid sphere can be written as,

(1) At all points lies inside $(r < R)$ the sphere is,

$V_P = -\frac{GM}{2R^3}\left(3R^2 - r^2\right)$

(2) On the sphere $(r = R)$ is, $V_P = -\frac{GM}{R}$

(3) At any point outside the sphere $(r > R)$ is,

$V_P = -\frac{GM}{r}$

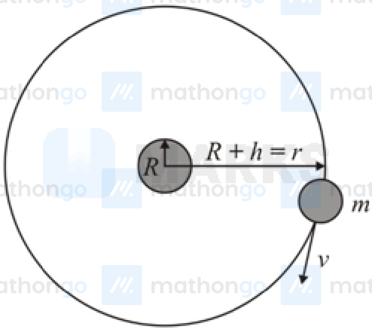
Q13.

$\mathrm{TE} = \mathrm{PE} + \mathrm{KE}$

For the system to be bounded one, the total energy of the system must be negative.

Therefore, the object having the negative total energy is bounded and if the total energy is positive is then object is unbounded.

Q14.



When the satellite is moving in circular orbit after launching the satellite, then centripetal force should be equal to the gravitational force,

$F_c = F_G$

$\frac{1}{2}\mathrm{m}(v_0)^2 = G\frac{Mm}{(r)^2}$

where $G,\ M,\ m,\ h$ are universal gravitational constant, the mass of the planet, the mass of satellite and height where the satellite is projected,

$\Rightarrow \frac{m(v_0)^2}{r} = G\frac{Mm}{(R+h)^2}$

$\Rightarrow (v_0)^2 = G\frac{M}{r}$.

Now the time period of the satellite,

$T = \frac{2\pi r}{v_0} = \frac{2\pi r}{\left(G\frac{M}{r}\right)^{\frac{1}{2}}}$

$\Rightarrow T^2 = \left(\frac{2\pi r}{\left(G\frac{M}{r}\right)^{\frac{1}{2}}}\right)^2$

$\Rightarrow T \propto \sqrt{\frac{r^3}{GM}}$.

Q15. Energy required to raise the satellite to a height h from surface of earth is given by,

$\mathrm{U} = -\frac{\mathrm{GmM}}{(\mathrm{R}+\mathrm{h})} - \left(-\frac{\mathrm{GmM}}{\mathrm{R}}\right) = \frac{\mathrm{GMmh}}{\mathrm{R}(\mathrm{R}+\mathrm{h})}$

Kinetic energy of satellite is given by,

$\mathrm{K} = \frac{1}{2}\mathrm{mv_0^2} = \frac{1}{2}\mathrm{m}\frac{\mathrm{GM}}{(\mathrm{R}+\mathrm{h})}$

$\therefore\ \frac{\mathrm{U}}{\mathrm{K}} = \frac{2\mathrm{h}}{\mathrm{R}}$

Q16.

Suppose mass of satellite is M and radius of satellite is r.

Velocity of satellite, $v = \sqrt{\frac{GM}{r}},\ KE \propto v^2 \propto \frac{1}{r}$

and time period is

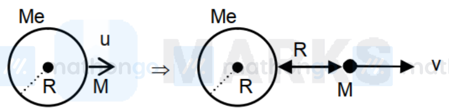
$T = 2\pi\sqrt{\frac{r^3}{GM}}$

$\therefore T^2 \propto r^3$,

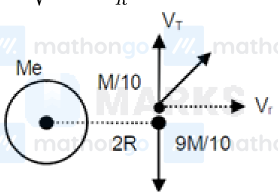
So, $KE \propto T^{-\frac{2}{3}}$.

Q17.

$\frac{-GM_eM}{R} + \frac{1}{2}Mu^2 = \frac{-GM_eM}{2R} + \frac{1}{2}Mv^2$



$v = \sqrt{u^2 - \frac{GM_e}{R}}$



$V_\tau \rightarrow$ Transverse velocity of rocket

$V_R \rightarrow$ Radial velocity of rocket

$V = \sqrt{\frac{GM_e}{2R}}$

$\frac{M}{10}V_T = \frac{9M}{10}\sqrt{\frac{GM_e}{2R}}$

$\frac{M}{10}V_r = M\sqrt{u^2 - \frac{GM_e}{R}}$

Kinetic energy $= \frac{1}{2}\frac{M}{10}\left(V_T^2 + V_r^2\right) = \frac{M}{20}\left(81\frac{GM_e}{2R} + 100u^2 - 100\frac{GM_e}{R}\right)$

$= \frac{M}{20}\left(100u^2 - \frac{119GM_e}{2R}\right)$

$= 5M\left(u^2 - \frac{119\,GM_e}{200R}\right)$

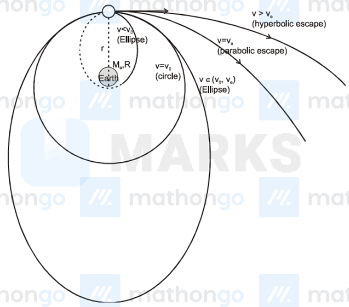
Q18.

Kepler's law of planetary motion

1. All planets move around the sun in elliptical path, having the sun at one of its foci
2. Radial vector joining the sun and the planet is sweeps equal area in equal interval of time.
3. The square of time period of revolution of planet is directly proportional to mean distance of planet from the Sun.

Q19.

Escape velocity if given by $v_e = \sqrt{2gr}$



In astrodynamics or celestial mechanics, a hyperbolic trajectory is the trajectory of any object around a central body with more than enough speed to escape the central object's gravitational pull. The name derives from the fact that according to Newtonian theory such an orbit has the shape of a hyperbola

According to question projection speed is much greater than $\sqrt{2gr}$. So, body escape from earth along hyperbola.

Q20.

From Kepler's third law, the time period of satellite is

$T^2 \propto r^3$

And larger the orbit then larger is the value of r, larger will be the time period.

Now the velocity of satellite is

$v = \sqrt{\frac{GM}{r}} \Rightarrow v \propto \frac{1}{r}$.

Hence, larger value of r Corresponds to smaller value of v.