Q1. A curve passing through the point $(1,2)$ and satisfying the condition that slope of the normal at any point is equal to the ratio of ordinate and abscissa of that point, then the curve also passes through the point

(1) $(0,0)$

(2) $(2,2)$

(3) $(2,1)$

(4) $(3,2)$

Q2. Let α be the angle in radians between $\frac{x^2}{36}+\frac{y^2}{4}=1$ and the circle $x^2+y^2=12$ at their points of intersection. If $\tan^{-1}\frac{k}{2\sqrt{3}}$, then find the value of $\frac{k^2}{4}$.

Q3. The equation of the tangent line at the point $(4,2)$ to the curve with parametric equation given by $x=t^2$ and $y=t^3-3t$ where t is parameter is

(1) $y=15x-58$

(2) $y=2$

(3) $y=x-2$

(4) $y=\frac{9x}{4}-7$

Q4. If the line joining the points $(0,3)$ and $(5,-2)$ is a tangent to the curve $y=\frac{c}{x+1}$, then find the value of c

Q5. The shortest distance between the line $y=x$ and the curve $y^2=x-2$ is

(1) $\frac{7}{4\sqrt{2}}$

(2) $\frac{7}{8}$

(3) $\frac{11}{4\sqrt{2}}$

(4) 2

Q6. The number of values of a for which the curves $4x^2+a^2y^2=4a^2$ and $y^2=16x$ are orthogonal is

Q7. Find the least positive integer m for which function f defined as $f(x)=\sin x-mx+k$ is a decreasing function for all $x\in R$

Q8. Find the maximum value of the function $f(x)=3x^3-18x^2+27x-40$ in the set $S=\left\{x\in R: x^2+30-11x\leq 0\right\}$

(1) 122

(2) 222

(3) -122

(4) -222

Q9. Let $f(x)=ax^3+5x^2+cx+1$ be a polynomial function. If $f(x)$ has extreme at $x=\alpha$ and β such that $\alpha\beta<0$ and $f(\alpha)f(\beta)<0$, then the equation $f(x)=0$ has

(1) three distinct real roots.

(2) one positive root, if $f(\alpha)<0$ and $f(\beta)>0$.

(3) one negative root, if $f(\alpha)>0$ and $f(\beta)<0$.

(4) All of the above

Q10. Let a function $f:[0,5]\to R$ be continuous, $f(1)=3$ and F be defined as:

$F(x)=\int_1^x t^2 g(t)dt$, where $g(t)=\int_1^t f(u)du$.

Then for the function $F(x)$, the point $x=1$ is:

(1) a point of local minima

(2) not a critical point

(3) a point of local maxima

(4) a point of inflection

Q11. If $y=a\log|x|+bx^2+x$ has its extremum values at $x=-1$ and $x=2$ then find value of $a+b$.

Q12. The point on the curve $y=x^2$ which is closest to $\left(4,-\frac{1}{2}\right)$, is

(1) $(1,1)$

(2) $(2,4)$

(3) $\left(\frac{2}{3},\frac{4}{9}\right)$

(4) $\left(\frac{4}{3},\frac{16}{9}\right)$

Q13. If $x=1$ is a critical point of the function $f(x)=\left(3x^2+ax-2-a\right)e^x$, then

(1) $x=1$ and $x=-\frac{2}{3}$ are local minima of f

(2) $x=1$ and $x=-\frac{2}{3}$ is a local maxima of f

(3) $x=1$ is a local maxima and $x=-\frac{2}{2}$ is a local minima of f

(4) $x=1$ is a local minima and $x=-\frac{2}{3}$ are local maxima of f

Q14. On the interval $[0,1]$ the function $x^{25}(1-x)^{75}$ takes its maximum value at the point

(1) 0

(2) $1/4$

(3) $1/2$

(4) $1/3$

Q15. The value of k in order that $f(x)=\sin x-\cos x-kx+b$ decreases for all real values is given by:

(1) $k<1$

(2) $k>1$

(3) $k>\sqrt{2}$

(4) $k<\sqrt{2}$

Q16. $f(x)=x^3+4x^2+\lambda x+2$ is monotonically decreasing in the largest possible interval $\left(-2,\frac{-2}{3}\right)$. Then find greatest value of

Q17. The set of value(s) of a for which the function $f(x)=\frac{ax^3}{3}+(a+2)x^2+(a-1)x+2$ possess a negative point of inflection, is

(1) empty set.

(2) $\left\{-\frac{4}{5}\right\}$

(3) $(-2,0)$

(4) $(-\infty,-2)\cup(0,\infty)$

Q18. Consider the function $f: R\to R$ defined by

$f(x)=\begin{cases}\left(2-\sin\left(\frac{1}{x}\right)\right)|x|, x\neq 0\\ 0\end{cases}, x=0.$ Then f is :

(1) monotonic on $(-\infty,0)\cup(0,\infty)$

(2) not monotonic on $(-\infty,0)$ and $(0,\infty)$

(3) monotonic on $(0,\infty)$ only

(4) monotonic on $(-\infty,0)$ only

Q19. A spherical iron ball of 10 cm radius is coated with a layer of ice of uniform thickness that melts at a rate of $50\ \mathrm{cm}^3/\mathrm{min}$.

When the thickness of ice is 5 cm, then the rate (in cm/min.) at which of the thickness of ice decreases, is:

(1) $\frac{5}{6\pi}$

(2) $\frac{1}{3\pi}$

(3) $\frac{1}{36\pi}$

(4) $\frac{1}{18\pi}$

Q20. A spherical balloon is expanding. If the radius is increasing at the rate of $2\ \mathrm{cm}/\mathrm{min}$ the rate at which the volume increases

(in cubic centimeters per minute) when the radius is 5 cm, is

(1) 10π

(2) 100π

(3) 200π

(4) 50π

Q21. Suppose that f is differentiable for all x and that $\mathrm{f}'(\mathrm{x})\leq 2$ for all x. If $\mathrm{f}(1)=2$ and $\mathrm{f}(4)=8$, then $\mathrm{f}(2)$ has the value equal to

(1) 3

(2) 4

(3) 6

(4) 8

Q22. If Rolle's theorem holds for the function

$f(x)=2x^3+bx^2+cx, x\in[-1,1]$, at the point $x=\frac{1}{2}$, then 2 b + c equals:

(1) 2

(2) 1

(3) -1

(4) -3

Answer Key

Q1 (3)	**Q2** (4)	**Q3** (4)	**Q4** (4.00)
Q5 (1)	**Q6** (2)	**Q7** (2)	**Q8** (1)
Q9 (4)	**Q10** (1)	**Q11** (1.50)	**Q12** (1)
Q13 (4)	**Q14** (2)	**Q15** (3)	**Q16** (4)
Q17 (4)	**Q18** (2)	**Q19** (4)	**Q20** (3)
Q21 (2)	**Q22** (3)		

Q1. Given $-\frac{1}{\left(\frac{dy}{dx}\right)}=\frac{y}{x}$

$\frac{dy}{dx}=-\frac{x}{y}$

$ydy+xdx=0$

$y^2+x^2=C$

Since, it passes through (1,2)$\Rightarrow 4+1=C\Rightarrow C=5$

$x^2+y^2=5$

Also, passes through (2,1)

Q2.

Given ellipse equation $\frac{x^2}{36}+\frac{y^2}{4}=1\ \ldots(i)$

and circle equation $x^2+y^2=12\ \ldots(ii)$

After solving equation (i) & (ii) the points of intersection we get,

$y=\pm\sqrt{3}$ & $x=\pm 3$

consider the point $P\left(3,\sqrt{3}\right)$

Equation of the tangent at P to the circle is

$3x+\sqrt{3}y=12$

Equation of the tangent at P to the ellipse is $\frac{x}{12}+\frac{\sqrt{3}}{4}y=1$

if α is angle between these tangents, then

$\tan\alpha=\frac{2}{\sqrt{3}}=\frac{4}{2\sqrt{3}}$

$\tan^{-1}\left(\frac{4}{2\sqrt{3}}\right)=\alpha$

compare with $\alpha=\tan^{-1}\left(\frac{k}{2\sqrt{3}}\right)$ we get $k=4$,

so, $\frac{k^2}{4}=\frac{4^2}{4}=4$

Q3. Slope $M=\frac{dy}{dx}=\frac{dy/dt}{dx/dt}$

$\therefore\ M=\frac{3t^2-3}{2t}$ at $t=2\Rightarrow M=\frac{9}{4}$

Equation of tangent at $(4,2)$ is $M=\frac{9}{4}$

$\therefore\ (y-2)=\frac{9}{4}(x-4)\Rightarrow y=\frac{9}{4}x-7.$

Q4.

Given that

line joining $(0,\ 3)(5,-2)$

$y-3=\frac{-2-3}{5-0}(x-0)$

$y-3=-x$

$y=3-x$ is tangent to curve i.e

touches $y=\frac{c}{x+1}$

$(3-x)=\frac{c}{x+1}$

$3x+3-x^2-x=c$

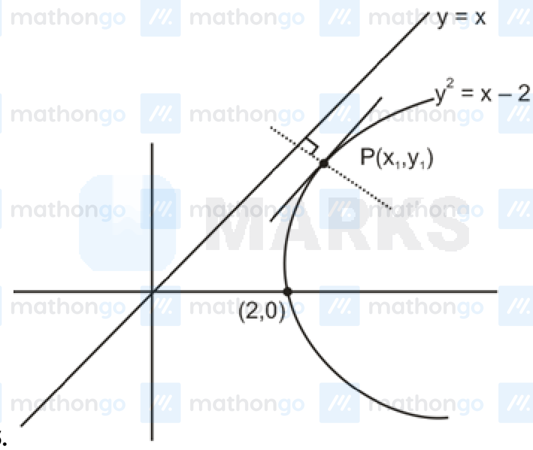
$x^2+2x+\left(c-3\right)=0$

$D=0$

$\left(-2\right)^2-4\times 1\left(c-3\right)=0$

$4-4\,c+12=0$

$c=4$



Q5.

$y^2=x-2$

Differentiating w.r.t. x we get,

$2yy'=1\Rightarrow y'=\frac{1}{2y}$

For the shortest distance, the tangent at point P will be parallel to the given line

$\Rightarrow y'|_p=\frac{1}{2y_1}=1\Rightarrow y_1=\frac{1}{2}$

$\Rightarrow x_1=2+\left(\frac{1}{2}\right)^2=\frac{9}{4}\ \left(\because y_1^2=x_1-2\right)$

The shortest distance between the given curve & the line

= The perpendicular distance of point P from the line

$=\left|\frac{\frac{9}{4}-\frac{1}{2}}{\sqrt{1^2+1^2}}\right|=\left|\frac{\frac{7}{4}}{\sqrt{2}}\right|=\frac{7}{4\sqrt{2}}$

Q6.

Given curves are

$4x^2+a^2y^2=4a^2$ ...(i)

and $y^2=16x$ ...(ii)

If the curves intersect at $P(\alpha,\beta)$, then

$\frac{\alpha^2}{a^2}+\frac{\beta^2}{4}=1$ and $\beta^2=16\alpha$.

On differentiating equation (i), we get,

$\frac{2x}{a^2}+\frac{2y}{4}y'=0$

$\Rightarrow y'=-\frac{4x}{a^2y}$

$\Rightarrow m_1=\frac{-4\alpha}{a^2\beta}$

On differentiating equation (ii), we get,

$2yy'=16$

$\Rightarrow m_2=\frac{8}{\beta}$

For curves to be orthogonal,

$m_1m_2=-1$

i.e. $\left(-\frac{4\alpha}{a^2\beta}\right)\left(\frac{8}{\beta}\right)=-1$

$\Rightarrow 32\alpha=a^2\beta^2$

$\Rightarrow 2\beta^2=a^2\beta^2$

$\Rightarrow a^2=2$

$\Rightarrow a=\pm\sqrt{2}$

Hence, two values of a

Q7.

$f(x)=\sin x-mx+k$

f is differentiable. $f'(x)=\cos x-m$

$f'(x)<0 \Leftrightarrow \cos x-m<0$

$\Leftrightarrow m>\cos x$

For function f to be decreasing for all x $m>1$

$\Rightarrow$The least positive integer $m=2$

Q8.

$f(x)=3x^3-18x^2+27x-40$

$x^2-11x+30\leq 0 \Rightarrow x\in[5,\ 6]$

$f^{'}(x)=9x^2-36x+27=9\left(x^2-4x+3\right)=9\ (x-1)\ (x-3)$

for $x\in[5,\ 6]$ $f(x)$ is increasing function

hence maximum value in this interval occurs at $x=6$

so $f(6)=648-648+162-40=122$

Q9.

Given,

$\alpha\beta<0$

$\Rightarrow \alpha$ and β are of opposite signs.

Let $\alpha<0$ and $\beta>0$.

It is given that $f(x)$ has extremum at $x=\alpha,\ \beta$.

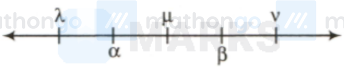
Therefore, α and β are two distinct real roots of $f'(x)=0$.

But we know that between two distinct real roots of a polynomial,

there is at least one real root of its derivative.

Therefore, $f(x)$ has three distinct real roots $\lambda,\ \mu$ and v (say) such

that $\lambda<\alpha<\mu<\beta<v$.



Thus, first option is correct.

If $f(x)=0$ has exactly one positive root, then it is evident from

the figure that $v>0$ and $\lambda,\ \mu<0$.

Therefore, $\alpha<\mu<0$

$\Rightarrow f(\alpha)f(0)<0$ [$\because \mu$ lies between α and 0]

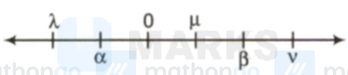
$\Rightarrow f(\alpha)<0$ [$\because f(0)=1>0$]

$\Rightarrow\ f(\beta)>0$ [$\because f(\alpha)f(\beta)<0$]

Thus, second option is also correct.

If $f(x)=0$ has exactly one negative real root, then from the

figure, we have $\lambda<0$ and $\mu,\ v>0$.



$\Rightarrow\ 0<\mu<\beta$

$\Rightarrow f(0)f(\beta)<0$ [$\because \mu$ lies between 0 and β]

$\Rightarrow f(\beta)<0$ [$\because f(0)=1>0$]

Thus, third option is also correct.

Q10.

Given, $F(x)=\int_1^x t^2g(t)dt$

By Leibnitz rule we get,

$F'(x)=x^2g(x)$

$\Rightarrow F'(1)=1.g(1)=0\ (\because g(1)=0)$

Now $F''(x)=2xg(x)+x^2g'(x)$

$\Rightarrow F''(x)=2xg(x)+x^2f(x)\ (\because g'(x)=f(x))$

$\Rightarrow F''(1)=0+1\times 3$

$\Rightarrow F''(1)=3$

$F(x)$ has a local minimum at $x=1$.

Q11.

$\frac{dy}{dx}=\frac{a}{x}+2bx+1$ or $\frac{dy}{dx}=0$ at $x=-1,\ 2$

$\frac{a}{-1}+2b(-1)+1=0$ or $-a-2b+1=0$

$\frac{a}{2}+4b+1=0$ or $a+8b+2=0$

solving we get ; $a=2,\ b=-\frac{1}{2}$

Q12.

$y=x^2$ Let any point on this parabola is $\left(t,t^2\right)$

Equation of normal at this point is

$x+2ty=t+2t^3$

It passes through $\left(4,-\frac{1}{2}\right)$

$4-t=t+2t^3$

$2t^3+2t-4=0$

$t^3+t-2=0$

$t=1$

So point is $(1,1)$

Q13.

$f(x)=\left(3x^2+ax-2-a\right)e^x$

$f'(x)=\left(3x^2+ax-2-a\right)e^x+e^x(6x+a)=e^x\left(3x^2+(a+6)x-2\right)$

$\because\ \ x=1$ is a critical point $\ \therefore\ \ f'(1)=0$

$\therefore 3+a+6-2=0$

$a=-7$

$\therefore f'(x)=e^x\left(3x^2-x-2\right)=e^x\left(3x^2-3x+2x-2\right)=e^x(3x+2)(x-1)$

$\therefore$ maxima at x $=\frac{-2}{3}$ $\therefore$ minima at x $=1$

Q14. Let f (x) = $x^{25}(1-x)^{75}$, x $\in$ [0,1]

$\therefore f'(x)=25x^{24}(1-x)^{75}-75x^{25}(1-x)^{74}$

$= 25\ x^{24}\ (1 - x)^{74}\ [(1 - x) - 3x]$

$= 25\ x^{24}\ (1 - x)^{74} (1 - 4x)$

For maximum value of f (x), put f'(x) = 0

$\therefore 25x^{24} (1 - x)^{74} (1 - 4x) = 0$

$\therefore \quad x = 0, 1, \frac{1}{4}$

Also, at x = 0 , y = 0

at x = 1, y = 0

and x = 1/4, y > 0

f (x) attains maximum at x = 1 / 4.

Q15.

Given:

$\Rightarrow f(x) = \sin x - \cos x - kx + b \Rightarrow f'(x) = \cos x + \sin x - k$

For decreasing function $f'(x) < 0$

So, $\cos x + \sin x - k < 0$

The maximum value of $\cos x + \sin x$ is $\sqrt{2}$.

$\Rightarrow \sqrt{2} - k < 0$

$\Rightarrow k > \sqrt{2}$

Q16. $f'(x) = 3\left(x^2 + \frac{8x}{3} + \frac{16}{9}\right) + \lambda - \frac{16}{3} = 3\left(x + \frac{4}{3}\right)^2 + \lambda - \frac{16}{3}$

now $-2 < x < -\frac{2}{3} \Leftrightarrow -2 + \frac{4}{3} < x + \frac{4}{3} < \frac{2}{3}$

$\Leftrightarrow -\frac{2}{3} < \left(x + \frac{4}{3}\right) < \frac{2}{3} \Leftrightarrow \left(x + \frac{4}{3}\right)^2 < \frac{4}{9}$

$\therefore f'(x) < 3 \cdot \frac{4}{9} + \lambda - \frac{16}{3}$

$\therefore \lambda \le 4$

$\therefore$ Greatest value of λ is 4

Q17.

Given $f(x) = \frac{ax^3}{3} + (a+2)x^2 + (a-1)x + 2$

Differentiating both sides with respect to x, we get

$f'(x) = \frac{3ax^2}{3} + 2(a+2)x + (a-1) = ax^2 + 2(a+2)x + (a-1)$

Again, differentiating both sides with respect to x, we get

$f''(x) = 2ax + 2(a+2)$

Since, $f(x)$ has a point of inflection.

So, $f''(x) = 0$

$\Rightarrow 2ax + 2(a+2) = 0$

$\Rightarrow x = -\frac{(a+2)}{a}$

Since, $f(x)$ has a negative point of inflection.

So, $x = -\frac{(a+2)}{a} < 0$

$\Rightarrow \frac{(a+2)}{a} > 0$

$\Rightarrow a \in (-\infty, -2) \cup (0, \infty)$

Q18.

$$f(x) = \begin{cases} -x\left(2 - \sin\left(\frac{1}{x}\right)\right) & x < 0 \\ 0 & x = 0 \end{cases}$$

$$f'(x) = \begin{cases} -\left(2 - \sin\frac{1}{x}\right) - x\left(-\cos\frac{1}{x} \cdot \left(-\frac{1}{x^2}\right)\right) & x < 0 \\ \left(2 - \sin\frac{1}{x}\right) + x\left(-\cos\frac{1}{x}\left(-\frac{1}{x^2}\right)\right) & x > 0 \end{cases}$$

$$f'(x) = \begin{cases} -2 + \sin\frac{1}{x} - \frac{1}{x}\cos\frac{1}{x}, & x < 0 \\ 2 - \sin\frac{1}{x} + \frac{1}{x}\cos\frac{1}{x}, & x > 0 \end{cases}$$

$f'(x)$ is an oscillating function which is non-monotonic in $(-\infty, 0) \cup (0, \infty)$.

Q19. Let thickness $= x$ cm

Total volume $V = \frac{4}{3}\pi(10 + x)^3$

$\frac{dV}{dt} = 4\pi(10 + x)^2 \frac{dx}{dt} \ldots\ldots (i)$

Given $\frac{dV}{dt} = 50\ \text{cm}^3 / \text{min}$

At $x = 5$ cm

$50 = 4\pi(10 + 5)^2 \frac{dx}{dt}$

$\frac{dx}{dt} = \frac{1}{18\pi}$ cm / min

Q20.

As, volume of sphere $V = \frac{4}{3}\pi r^3$

$\Rightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} = 4\pi r^2 . (2) \quad \left[\because \frac{dr}{dt} = 2\right]$

$\Rightarrow \frac{dV}{dt} = 4\pi . 25 . 2 = 200\pi$

Q21. Using Lagrange's mean value theorem for f in [1, 2]

for $c \in (1, 2), \frac{f(2) - f(1)}{2 - 1} = f'(c) \le 2$

$\Rightarrow \quad f(2) - f(1) \le 2$

$\Rightarrow \quad f(2) \le 4 \qquad (1)$

again using Lagrange's mean value theorem in [2, 4]

for $d \in (1, 2), \frac{f(4) - f(2)}{4 - 2} = f'(d) \le 2$

$\Rightarrow \quad f(4) - f(2) \le 4$

$\Rightarrow \quad 8 - f(2) \le 4$

$\Rightarrow \quad f(2) \ge 4 \qquad (2)$

from (1) and (2), f(2) = 4.

Q22. If Rolle's theorem is satisfied in the interval [-1, 1], then

$f(-1) = f(1)$

$-2 + b - c = 2 + b + c$

$c = -2$ also $f'(x) = 6x^2 + 2bx + c$

Also if $f'\left(\frac{1}{2}\right) = 0$ them

$6\frac{1}{4} + 2b\frac{1}{2} + c = 0$

$\frac{3}{2} + b + c = 0$

$\because \quad c = -2,$

$b = \frac{1}{2}$

$\therefore \quad 2b + c = 2\left(\frac{1}{2}\right) + (-2)$

$= 1 - 2$

$= -1.$

Q1. The Area of a region bounded by $y=\sin ax, y=0$ from $x=\frac{\pi}{3a}$ to $x=\frac{\pi}{a}$ is 3 square unit, find the integral value of $a(a>0)$

Given that graph of $y=\sin ax$ lies above the axis from $x=\frac{\pi}{a}$ to $\left.x=\frac{\pi}{3a}\right\}$

(1) 3

(2) $\frac{1}{3}$

(3) $\frac{1}{2}$

(4) $\frac{1}{1}$

Q2. The area of closed region bounded by the parabolas $y=4x^2, y=\frac{x^2}{9}$ and the line $y=2$ is

(1) $\frac{20\sqrt{2}}{3}$ sq. unit

(2) $\frac{10\sqrt{2}}{3}$ sq. unit

(3) $\frac{40\sqrt{2}}{3}$ sq. unit

(4) None of these

Q3. If the area bounded by $f(x)=\max(\sin x, \cos x); 0 \leq x \leq \frac{\pi}{2}, x=\frac{\pi}{2}$ and the coordinates axes is equal to k, then $[k+3]$ is equal to (where, [.] denotes the greatest integer function

(1) 2

(2) 8

(3) 4

(4) 6

Q4. The area enclosed between the curve $y=\log_e(x+e)$ and the coordinate-axes is

(1) 3

(2) 4

(3) 1

(4) 2

Q5. The area of the closed figure bounded by $x=-1, x=2$ and $y=\begin{cases}-x^2+2, & x \leq 1 \\ 2x-1, & x>1\end{cases}$ and the x-axis is

(1) $\frac{16}{3}$ sq. units

(2) $\frac{10}{3}$ sq. units

(3) $\frac{13}{3}$ sq. units

(4) $\frac{7}{3}$ sq. units

Q6. The area common to $x^2+y^2=64$ and $y^2=4x$ is

(1) $\frac{16}{3}(4\pi+\sqrt{3})$

(2) $\frac{16}{3}(8\pi-\sqrt{3})$

(3) $\frac{16}{3}(4\pi-\sqrt{3})$

(4) $\frac{16}{3}(8\pi+\sqrt{3})$

Q7. The area of the region bounded by $f(x)=\sin x, \mathrm{g}(x)=\cos x$ from $x=0$ to $x=\frac{\pi}{2}$ is

(1) $2(\sqrt{2}+1)$ sq. units

(2) $(\sqrt{3}-1)$ sq. units

(3) $2(\sqrt{3}-1)$ sq. units

(4) $2(\sqrt{2}-1)$ sq. units

Q8. The volume of the solid formed by rotating the area enclosed between the curve $y=x^2$ and the line $y=1$ about $y=1$ is (in cubic units)

(1) $\frac{9\pi}{5}$

(2) $\frac{4\pi}{3}$

(3) $\frac{8\pi}{3}$

(4) $\frac{7\pi}{5}$

Q9. The area of the region $\{(x, y): xy \leq 8, 1 \leq y \leq x^2\}$ is

(1) $16\log_e 2-\frac{14}{3}$

(2) $8\log_e 2-\frac{7}{3}$

(3) $8\log_e 2-\frac{14}{3}$

(4) $16\log_e 2-6$

Q10. Area enclosed between the curves $|y|=1-x^2$ and $x^2+y^2=1$ is

(1) $\frac{3\pi-8}{3}$ sq. units

(2) $\frac{\pi-8}{3}$ sq. units

(3) $\frac{2\pi-8}{3}$ sq. units

(4) None of these

Q11. The area of the region bounded by $x=0, y=0, x=2, y=2, y \leq e^x$ and $y \geq \ln x$ is

(1) $6-4\ln 2$ sq. units

(2) $4\ln 2-2$ sq. units

(3) $2\ln 2-4$ sq. units

(4) $6-2\ln 2$ sq. units

Q12. Let $f(x)=$ Maximum $\{x^2,(1-x)^2, 2x(1-x)\}$, where $0 \leq x \leq 1$. Determine the area of the region bounded by the curves $y=f(x)$, x-axis, $x=0 \& x=1$

(1) $\frac{17}{27}$

(2) $\frac{15}{25}$

(3) $\frac{13}{23}$

(4) $\frac{14}{29}$

Answer Key

Q1 (3) **Q2** (1) **Q3** (3) **Q4** (3)

Q5 (1) **Q6** (2) **Q7** (4) **Q8** (2)

Q9 (1) **Q10** (1) **Q11** (1) **Q12** (1)

Q1.

We have, $y = \sin ax$

Given that the given curve is above the axis from $x = \frac{\pi}{a}$ to $x = \frac{\pi}{3a}$.

Then, the area bounded by the curve from

$x = \frac{\pi}{a}$ to $x = \frac{\pi}{3a} = \int_{\pi/a}^{\pi/3a} y dx$

$\Rightarrow A = \int_{\frac{\pi}{3a}}^{\frac{\pi}{a}} \sin ax dx$

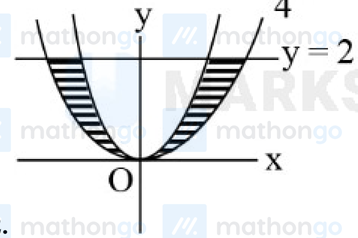
$\Rightarrow A = \frac{1}{a}(-\cos ax)_{\pi/a}^{\pi/3a}$

$\Rightarrow A = \frac{1}{a}\left[\frac{3}{2}\right]$

$\therefore\ A = \frac{3}{2a} = 3$ (Given)

$\Rightarrow a = \frac{1}{2}$

$x^2 = \frac{y}{4}, x^2 = 9y$



Q2.

$$\text{Area} = 2\int_0^2\left(\sqrt{9y} - \sqrt{\frac{y}{4}}\right) dy$$

$$= 2\left\{3.\frac{2}{3}(y)^{\frac{3}{2}} - \frac{1}{2}.\frac{2}{3}(y)^{\frac{3}{2}}\right\}_0^2$$

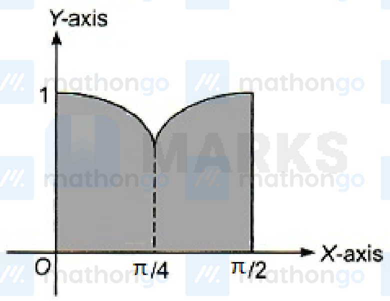
$$= 2\left\{\frac{5}{3}.y^{\frac{3}{2}}\right\}_0^2$$

$$= \frac{10}{3}.2\sqrt{2}$$

Q3. $f(x) = \begin{cases} \cos x \text{ for } 0 \leq x \leq \pi/4 \\ \sin x \text{ for } \pi/4 < x \leq \pi/2 \end{cases}$

$\therefore$ Required Area $= 2\int_0^{\pi/4} \cos x dx = 2[\sin x]_0^{\pi/4}$

$= \sqrt{2}$ sq units



$\Rightarrow \quad k = \sqrt{2}$

$\Rightarrow \quad [k+3] = \left[\sqrt{2}+3\right] = 4$

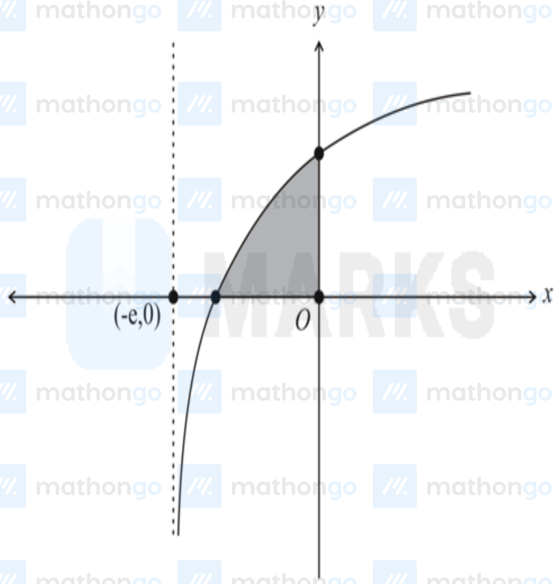
Q4.

Curve : $y = \log_e(x+e),\ x-\text{axis},\ y-\text{axis}$

at x-axis $y = 0 \Rightarrow \log_e(x+e) = 0 \Rightarrow x + e = 1$

$\Rightarrow x = 1 - e$

when $x \to -e^{+} \Rightarrow y \to -\infty$



Required area $= \int_{1-e}^{0} \log(x+e)\, dx$

$= x\log\ (x+e)\Big|_{1-e}^{0} - \int_{1-e}^{0} \frac{1}{x+e} x\, dx$

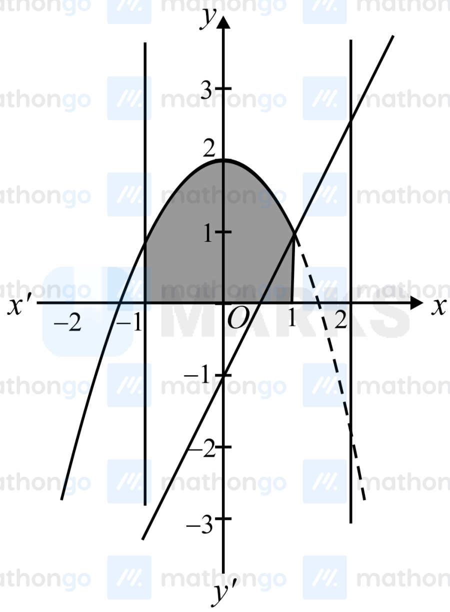
$= 0 - \int_{1-e}^{1} \frac{x+e-e}{x+e} dx$

$= 0 - (x - e\ \log(x+e))_{1-e}^{0}$

$= e + (1-e) - e\log(1)$

$= 1.$

Q5.



$A=\int_{-1}^{1}\left(-x^2+2\right)dx+\int_{1}^{2}(2x-1)dx$

$=\left(-\frac{x^3}{3}+2x\right)_{-1}^{1}+\left(x^2-x\right)^2$

$=\frac{16}{3}$ sq. units

Q6.

The given curves are $x^2+y^2=64\ldots$(i) and $y^2=4x\ldots$(ii)

Solving both the equations, we get

$x^2+4x-64=0$

$x=-2\pm2\sqrt{17}$

Since the intersection point is lying in the first quadrant, so

$x=-2+2\sqrt{17}$

So, the required area is

$=2\int_0^{-2+2\sqrt{17}}\sqrt{4x}dx+\int_0^{-2+2\sqrt{17}}\sqrt{64-x^2}dx$

On integrating, we get

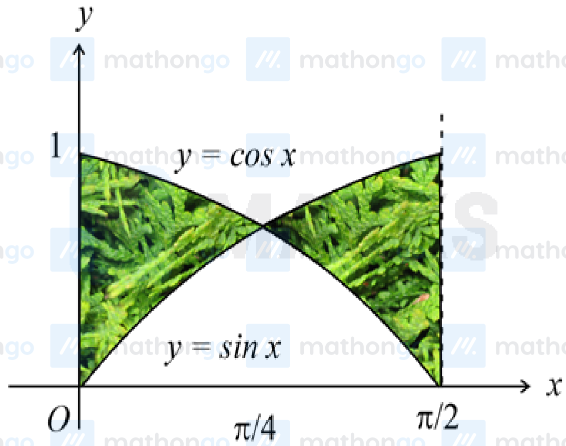
$=\frac{16}{3}\left(8\pi-\sqrt{3}\right).$

Q7.

We have, $\cos x\geq\sin x,\ 0\leq x\leq\frac{\pi}{4}$ and $\sin x\geq\cos x,\ \frac{\pi}{4}\leq x\leq\frac{\pi}{2}$

Area of the shaded region

$\Delta=\int_0^{\frac{\pi}{4}}(\cos x-\sin x)dx+\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}(\sin x-\cos x)dx$



$=[\sin x+\cos x]_0^{\frac{\pi}{4}}+[-\cos x-\sin x]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$

$=\left\{\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}-(0+1)\right\}-\left\{1-\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right)\right\}$

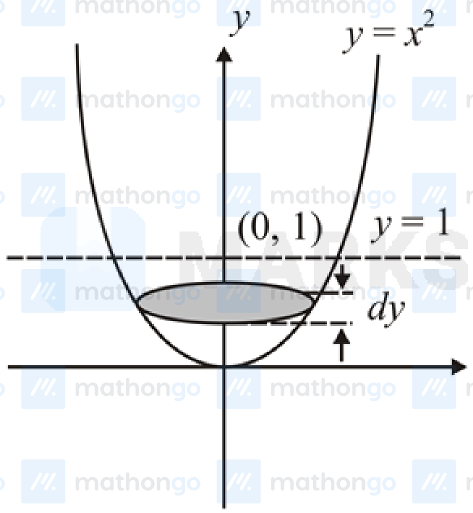
$=\frac{4}{\sqrt{2}}-2$

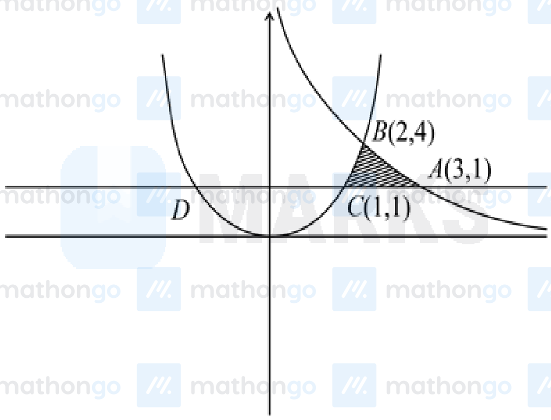
$=2\sqrt{2}-2$

$=2\left(\sqrt{2}-1\right)$ sq. units.

Q8. Volume of the solid formed by rotating the area enclosed between the curve $y=x^2$ and line $y=1$ will be

$V=\int_0^1 2\pi x dy=2\int_0^1\pi\sqrt{y}dy=\frac{4\pi}{3}\left[y^{\frac{3}{2}}\right]_0^1=\frac{4\pi}{3}$ cu. units.





Q9.

To draw the inequality, let us draw the equation

$xy=8$ and $y=1$ and $y=x^2$

For point of intersection

(i) $xy=8$ and $y=1 \Rightarrow A(8,1)$

(ii) $xy=8$ and $y=x^2 \Rightarrow x^3=8 \Rightarrow x=2 \Rightarrow B(2,\ 4)$

(iii) $y=x^2$ and $y=1 \Rightarrow C(1,\ 1)$ and $D(\ 1,\ 1)$

Now $xy \leq 8 \Rightarrow$ region which contains origin

$y \geq 1 \Rightarrow$ region above line $y=1$

$y \leq x^2 \Rightarrow$ region outside the parabola

Now required area

Method I:

Using x-axis:

$A=\int_1^2 (x^2-1) \cdot dx+\int_2^8\left(\frac{8}{x}-1\right) \cdot dx$

$A=\left[\frac{x^3}{3}-x\right]_1^2+[8 \ln x-x]_2^8$

$A=-\frac{14}{3}+16 \ln 2$

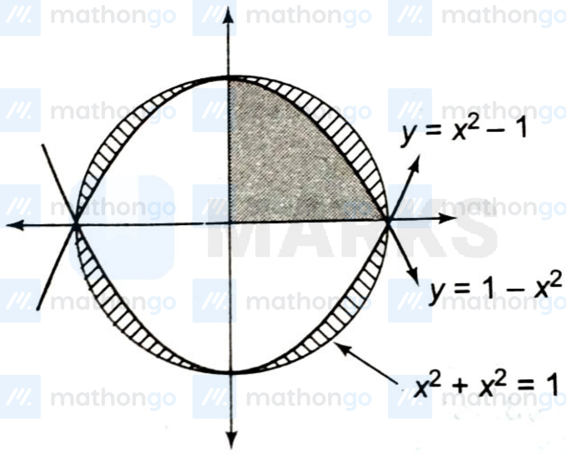
Method II:

Using y-axis:

$A=\int_1^4\left(\frac{8}{y}-\sqrt{y}\right) \cdot dy$

$A=\left[8 \ln y-\frac{2}{3} y^{3/2}\right]_1^4 \Rightarrow A=-\frac{14}{3}+16 \ln 2$

Note: The question should include bounded area term as in 2^{nd} quadrant there exist a area which satisfy the inequality and is unbounded.



Q10.

The dotted area is

$A=\int_0^1(1-x^2) dx=\left(x-\frac{x^3}{3}\right)_0^1=1-\frac{1}{3}=\frac{2}{3}$

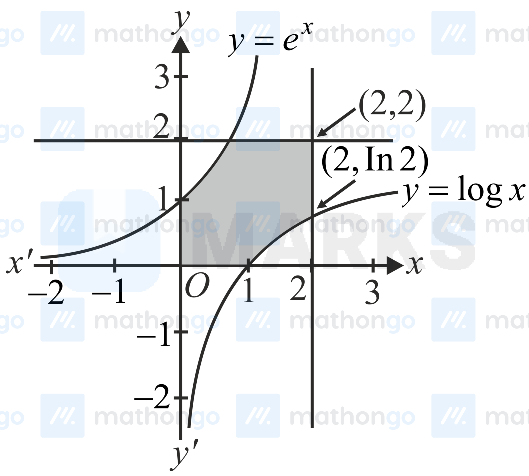
Hence, area bounded by circle $x^2+y^2=1$ and $|y|=1-x^2$

$=$ Lined area

$=$ Area of curcle $-$ Area bounded by $|y|=1-x^2$

$=\pi-4.\left(\frac{2}{3}\right)=\frac{3\pi-8}{3}$ sq. units

Q11.



$A=\int_1^2 \ln x\, dx$

$=[x \ln x-x]_1^2$

$=2 \ln 2-1$

$\Rightarrow$ Required area $=4-2(2 \ln 2-1)=6-4 \ln 2$ sq.units

Q12.

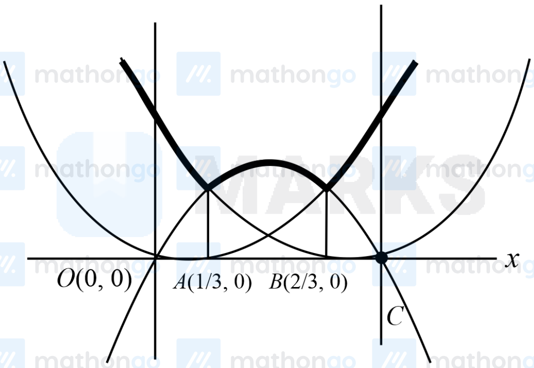
$$f(x)=\operatorname{Max}\left\{x^2,(1-x)^2, 2x(1-x)\right\}$$

coordinate of $A,(1-x)^2=2x(1-x)$

$(1-x)(1-x-2x)=0$

$x=\frac{1}{3}, A\left(\frac{1}{3}, 0\right)$ coordinate of B

$2x(1-x)=x^2, 2(1-x)=x, 2-3x=0, x=\frac{2}{3}$



$B\left(\frac{2}{3}, 0\right)$ Req. Area $=\int_0^{1/3}(1-x)^2 dx+\int_{1/3}^{2/3} 2x(1-x) dx+\int_{\frac{2}{3}}^{1} x^2 dx$

$=\frac{17}{27}$

Q1. If $f: R \rightarrow R$ is defined by

$$f(x)=\begin{cases}\frac{x+2}{x^2+3x+2}, & \text{if } x \in R-\{-1,-2\}\\ -1, & \text{if } \quad x=-2\\ 0, & \text{if } \quad x=-1\end{cases}$$ then f is continuous on the se

(1) R

(2) $R-\{-2\}$

(3) $R-\{-1\}$

(4) $R-\{-1,-2\}$

Q2. If

$$f(x)=\begin{cases}\frac{\sin(p+1)x+\sin x}{x} & , x<0\\ q & , x=0\\ \frac{\sqrt{x+x^2}-\sqrt{x}}{x^{3/2}} & , x>0\end{cases}$$ is continuous at $x=0$, then the ordered pair (p, q) is equal to

(1) $\left(-\frac{3}{2},-\frac{1}{2}\right)$

(2) $\left(-\frac{1}{2},\frac{3}{2}\right)$

(3) $\left(\frac{5}{2},\frac{1}{2}\right)$

(4) $\left(-\frac{3}{2},\frac{1}{2}\right)$

Q3. The number of points of discontinuity of $f(x)=\left[x^3+1\right]$ in $(1,2)$ is/are (where [.] denotes Greatest Integer Function)

(1) 1

(2) 6

(3) 5

(4) 4

Q4. Let $y=\frac{1}{u^2+u-2}$ where $u=\frac{1}{x-1}$ then y is discontinuous only at $x=$

(1) $1,2$

(2) $1,-2$

(3) $1,\frac{1}{2},2$

(4) None of these

Q5. Let $f(x)=\operatorname{sgn}(x)$ and $g(x)=x\left(x^2-5x+6\right)$. The function $f(g(x))$ is discontinuous at

(1) infinitely many points.

(2) exactly one point.

(3) exactly three points.

(4) no point.

Q6. Let $f(x)=\begin{cases}\frac{1+\cos x}{(\pi-x)^2}\cdot\frac{\sin^2 x}{\log\left(1+\pi^2-2\pi x+x^2\right)} & , \quad x\neq\pi\\ k & , \quad x=\pi\end{cases}$. If $f(x)$ is continuous functions at $x=\pi$, then k is equal to

(1) $\frac{1}{4}$

(2) $\frac{1}{2}$

(3) $\frac{-1}{2}$

(4) $-\frac{1}{4}$

Q7. If

$$f(x)=\begin{cases}\mathrm{x}+2, & \mathrm{x}>0\\ \mathrm{x}^2-2, & 0\leq\mathrm{x}<1\\ \mathrm{x}, & \mathrm{x}\geq 1\end{cases}$$ then the number of points of discontinuity of $|f(x)|$ is:

(1) 1

(2) 0

(3) 2

(4) none of these

Q8. Let f : $[0,3]\rightarrow$ R be defined by $f(x)=\min\{x-[x], 1+[x]-x\}$ where [x] is the greatest integer less than or equal to x.

Let P denote the set containing all $x\in[0,3]$ where f is discontinuous, and Q denote the set containing all $x\in(0,3)$

where f is not differentiable. Then the sum of number of elements in P and Q is equal to

Q9. Let $f(x)=x|x|, g(x)=\sin x$ and $h(x)=(gof)(x)$. Then

(1) $h(x)$ is differentiable at $x=0$, but $h'(x)$ is not continuous at $x=0$

(2) $h(x)$ is not differentiable at $x=0$

(3) $h'(x)$ is differentiable at $x=0$

(4) $h'(x)$ is continuous at $x=0$ but is not differentiable at $x=0$

Q10. Let f : $(-1,1)\rightarrow$ R be a differentiable function with $\mathrm{f}(0)=-1$ and $\mathrm{f}'(0)=1$, $\mathrm{g}(\mathrm{x})=\{\mathrm{f}(2\mathrm{f}(\mathrm{x})+2)\}^2$. Then $\mathrm{g}'(0)=$

(1) 4

(2) -4

(3) 0

(4) -2

Q11. Let S be the set of points where the function, $f(x)=|2-|x-3||, x\in R$, is not differentiable. Then $\sum_{x\in S} f(f(x))$ is equal to

Q12. Let $f(x)=\begin{cases}3x-p & : 0\leq x\leq 2\\ 2x^2+qx & : 2<x\leq 3\end{cases}$

If $f(x)$ is differentiable at $x=2$, $(p,q)=$

(1) $(8,-5)$

(2) $\left(-\frac{5}{2},3\right)$

(3) $(-10,4)$

(4) None of these

Q13. If $f(x+y)=f(x)+f(y)+|x|y+x^2y^2, \forall x, y\in R$ and $f'(0)=0$, then

(1) not be differentiable at every non-zero x.

(2) differentiable for all $x\in R$.

(3) twice differentiable at $x=0$.

(4) none of the above.

Q14. If $f(x)=\begin{cases}3^x, & -1\leq x\leq 1\\ 4-x, & 1<x<4\end{cases}$, then at $x=1$, $f(x)$ will be

(1) Continuous but not differentiable

(2) Neither continuous nor differentiable

(3) Continuous and differentiable

(4) Differentiable but not continuous

Q15. Let $g(x)$ be a polynomial of degree one and $f(x)$ be a continuous and differentiable function defined by

$f(x)=\begin{cases} g(x), & x \leq 0 \\ \left(\frac{1+x}{2+x}\right)^{\frac{1}{x}}, & x>0 \end{cases}$. If $f'(1)=f'(-1)$, then

(1) $f'(-1)=-\frac{2}{3}\left(\frac{1}{6}+\ln \frac{3}{2}\right)$

(2) $f'(-1)=+\frac{3}{2}\left(\frac{1}{6}+\ln \frac{3}{2}\right)$

(3) $f'(-1)=-\frac{2}{3}\left(\frac{6}{1}+\ln \frac{2}{3}\right)$

(4) $f'(-1)=-\frac{2}{3}\left(\frac{1}{6}-\ln \frac{3}{2}\right)$

Answer Key

Q1 (3)	**Q2** (4)	**Q3** (2)	**Q4** (3)
Q5 (3)	**Q6** (2)	**Q7** (1)	**Q8** (5)
Q9 (4)	**Q10** (2)	**Q11** (3)	**Q12** (1)
Q13 (2)	**Q14** (1)	**Q15** (1)	

Q1.

Since, $f(x)$ is continuous for R, not sure about $\{-1,-2\}$.

Now, we have check continuity at these points.

At $x=-2$,

$\mathrm{LHL}=\lim_{n\to 0}\frac{(-2-n)+2}{(-2-n)^2+3(-2-n)+2}$

$=\lim_{n\to 0}\frac{-n}{n^2+n}=-1$

$\mathrm{RHL}=\lim_{n\to 0}\frac{(-2+n)+2}{(-2+n)^2+3(-2+n)+2}$

$=\lim_{n\to 0}\frac{n}{n^2-n}=-1$

$\mathrm{LHL}=\mathrm{RHL}=f(-2)$

It is continuous at $x=-2$

Now, check for $x=-1$

$\mathrm{LHL}=\lim_{n\to 0}\frac{(-1-n)+2}{(-1-n)^2+3(-1-n)+2}$

$=\lim_{n\to 0}\frac{1-n}{n^2-n}=\infty$

$\mathrm{RHL}=\lim_{n\to 0}\frac{(-1+n)+2}{(-1+n)^2+3(-1+n)+2}$

$=\lim_{n\to 0}\frac{1+n}{n^2+n}=\infty$

$\mathrm{LHL}=\mathrm{RHL}\neq f(-1)$

It is not continuous at $x=-1$

The required function is continuous in $R-\{-1\}$

Q2.

$$\because f(x)=\begin{cases}\frac{\sin(p+1)x+\sin x}{x} & :x<0\\ q & :x=0\\ \frac{\sqrt{x^2+x}-\sqrt{x}}{x^{3/2}} & :x>0\end{cases}$$

$\because f$ is continuous at $x=0$

$L.H.L=$

$\lim_{h\to 0}\frac{\sin(p+1)(-h)+\sin(-h)}{-h}=\lim_{h\to 0}\frac{\sin(p+1)h+\sin h}{h}$

$=\lim_{h\to 0}\frac{\sin(p+1)h}{h(p+1)}(p+1)+\lim_{h\to 0}\frac{\sin h}{h}=p+1+1$

$=p+2$

$R.H.L=\lim_{h\to 0}\frac{\sqrt{h^2+h}-\sqrt{h}}{h^{\frac{3}{2}}}\times\frac{\sqrt{h^2+h}+\sqrt{h}}{\sqrt{h^2+h}+\sqrt{h}}=\lim_{h\to 0}\frac{h^2+h-h}{h^{\frac{3}{2}}h^{\frac{1}{2}}(\sqrt{h+1}+1)}$

$\lim_{h\to 0}\frac{1}{\sqrt{1+h}+1}=\frac{1}{2}$

$\Rightarrow L.H.L.=R.H.L.=f(0)\Rightarrow p+2=\frac{1}{2}=q$

$=p=-\frac{3}{2}\quad \&\quad q=\frac{1}{2}$

Q3.

$f(x)=[x^3+1]$

By property of GIF, we know that $[x]$ is discontinuous at every integral point.

So we have to just check that for how many values of $x\in(1,2)$, (x^3+1) is taking integral values.

$1<x<2\Rightarrow\ 1<x^3<8\Rightarrow\ 2<x^3+1<9$

As $(x^3+1)\in(2,9)$, then integer lying in this range are

$[x^3+1]=3,4,5,6,7,8\longleftarrow$ 6 points are there at which $f(x)$ is becoming discontinuous.

Q4. $u=\emptyset(x)=\frac{1}{x-1}$ is discontinuous at $x=1$, $y=f(u)=\frac{1}{u^2+u-2}=\frac{1}{(u+2)(u-1)}$ is discontinuous at $u=-2,\ u=1$. If $u=-2$ then $-2=\frac{1}{x-1}\ \Rightarrow x=\frac{1}{2}$. If $u=1$ then $1=\frac{1}{x-1}\Rightarrow x=2$. Hence the composite function is discontinuous only at $x=1,\ \frac{1}{2},\ 2$.

Q5.

$$f(x)=\operatorname{sgn}(x)=\begin{Bmatrix}-1, & x<0\\ 0, & x=0\\ 1, & x>0\end{Bmatrix}$$

$g(x)=x(x^2-5x+6)=x(x-2)(x-3)$

$f(g(x))=\operatorname{sgn}(g(x))=\operatorname{sgn}(x(x-2)(x-3))$

$$=\begin{Bmatrix}-1, & x(x-2)(x-3)<0\\ 0, & x(x-2)(x-3)=0\\ 1, & x(x-2)(x-3)>0\end{Bmatrix}$$

Let $h(x)=f(g(x))$

$h(x)=-1$ if $x\in(-\infty,0)\cup(2,3)$

$h(x)=0$, if $x=0,2,3$

$h(x)=1$ if $x\in(0,2)\cup(3,\infty)$

Therefore, $f(g(x))$ is discontinuous at three points $0,\ 2,\ 3$.

Q6. The given function is

$$\mathrm{f(x)}=\begin{cases}\frac{1+\cos x}{(\pi-x)^2}\cdot\frac{\sin^2 x}{\log(1+\pi^2-2\pi x+x^2)}, & x\neq\pi\\ k, & x=\pi\end{cases}$$

Since, f(x) continuous at $x=\pi$.

$\therefore\qquad \lim_{x\to\pi}f(x)=f(\pi)$

$\Rightarrow\qquad \lim_{h\to 0}f(\pi+h)=k\Rightarrow\lim_{h\to 0}\frac{1+\cos(\pi+h)}{(\pi-\pi-h)^2}\cdot\frac{\sin^2(\pi+h)}{\log\left[1+\pi^2-2\pi(\pi+h)+(\pi+h)\right]^2}=k$

$\Rightarrow\qquad \lim_{h\to 0}\frac{1-\cos h}{\pi^2}\times\frac{\sin^2 h}{\log(1+h^2)}=k$

$\Rightarrow\qquad \lim_{h\to 0}\frac{1}{2}\left(\frac{\sin h/2}{h/2}\right)^2\cdot\frac{h^2}{\log(1+h^2)}\cdot\left(\frac{\sin h}{h}\right)^2=k$

$\Rightarrow\qquad \frac{1}{2}=k$

Q7.

$$\text{Given function } f\left(x\right)=\begin{cases}x+2, & x>0\\ x^2-2, & 0\le x<1\\ x, & x\ge 1\end{cases}.$$

$$\therefore|f(x)|=\begin{cases}|x+2|, & x<0\\ |x^2+2|, & 0\le x<1\\ |x|, & x\ge 1\end{cases}.$$

Using properties of modulus function,

$$\therefore|f(x)|=\begin{cases}-x-2, & x>-2\\ x+2, & -2\le x<0\\ x^2+2, & 0\le x<1\\ x, & x\ge 1\end{cases}$$

For continuityof $|f(x)|$ at $x=1$,

$\mathrm{L.H.L.}=\mathrm{f}(1^-)=1^2+2=3$

$\mathrm{R.H.L.}=\mathrm{f}(1^+)=1$

Hence, function is discontinuous at $x=1$.

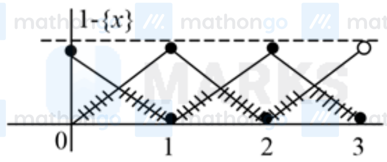
Therefore, number of discontinuity is 1.

Q8.

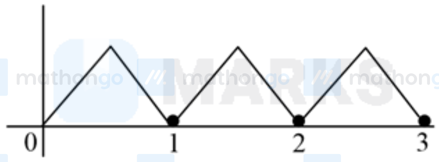
Given,

$f(x) = \min\{x - [x], 1 + [x] - x\}$

So, the graph of the function is



$1 - \{x\} = 1 - x; 0 \leq x < 1$



From the graph we can say that the function is continuous at all points in the interval $[0, 3]$.

So, $P = 0$

Again, from the diagram, we can say that at x $= \frac{1}{2},\ 1,\ \frac{3}{2},\ 2,\ \frac{5}{2}$, there are sharp edges.

So, the function is non differentiable at x $= \frac{1}{2},\ 1,\ \frac{3}{2},\ 2,\ \frac{5}{2}$

Hence, $Q = 5$

So, $P + Q = 5$

Q9. $h(x)=\begin{cases} \sin x^2 & x \geq 0 \\ -\sin x^2 & x < 0 \end{cases}$

$h'(0^+)= \lim\limits_{x \to 0^+} \frac{\sin x^2}{x} = 0$

$h'(0^-)= \lim\limits_{x \to 0^-} \frac{-\sin x^2}{x} = 0$

Hence, $h'(0)= 0$

So $h'(x)\begin{cases} 2x\cos x^2 & x > 0 \\ 0 & x = 0 \\ -2x\cos x^2 & x < 0 \end{cases}$

Now, $h''(0^+)= \lim\limits_{x \to 0^+} \frac{2x\cos x^2}{x} = 2$

$h''(0^-) = \lim\limits_{x \to 0^-} \frac{-2x\cos x^2}{x} = -2$

Hence, $h'(x)$ is non-derivable at $x = 0$.

Q10. $g(x)= \{ f(2 f(x)+2)\}^2$

We have on differentiation with respect to x,

$g'(x) = 2f(2f(x) + 2) f'(2f(x) + 2) 2f'(x)$

$g'(0) = 2f(2f(0) + 2) f'(2f(0) + 2 f'(0)$

$= -4.$

Q11.

Check non-differentiability of function, we get,

$\because f(x)$ is non differentiable at $x = 1, 3, 5$

$\sum f(f(x))= f(f(1))+f(f(3))+f(f(5))$

$= 1 + 1 + 1$

$= 3$

Q12. $\because$ diff. at $x = 2 \Rightarrow$ conti. at $x = 2$

$\therefore\ f(2)= f(2+)\Rightarrow 6- p = 8 + 2q \Rightarrow\ p + 2q =- 2 \ldots.(1)$

Now $f\,'(x)\ = \begin{bmatrix} 3 & , & 0 < x < 2 \\ 4x + q & , & 2 < x \leq 3 \end{bmatrix}$

$\therefore\ f\,'(2+)= f\,'(2-)\ \Rightarrow\ 8 + q = 3\ \Rightarrow\ q =- 5,\ p = 8$

Q13.

Given:

$f(x + y)= f(x) + f(y) + |x|y + x^2y^2$

and $f'(0) = 0$

Substitute, $x = y = 0$

$f(0 + 0) = f(0) + f(0) + 0 + 0$

$\Rightarrow f(0) = 0$

$f'\left(x\right) = \lim\limits_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$f'\left(x\right) = \lim\limits_{h \to 0} \frac{f(x)+f(h)+|x|h+x^2h^2-f(x)}{h}$

$= \lim\limits_{h \to 0} \frac{f(h)}{h} + |x| + x^2h$

$= f'(0) + |x|$

$f'(x) = 0 + |x| = |x|$

$f'\left(x\right) = \begin{cases} -x & x < 0 \\ x & x \geq 0 \end{cases}$

Clearly $f(x)$ is differentiable at all points.

Q14. Since $f(1-0)= \lim\limits_{x \to 1} 3^x = 3$

$f(1 + 0)= \lim\limits_{x \to 1}(4 - x)= 3$

and $f(1) = 3^1 = 3$

$\therefore\ f(1 - 0)= f(1 + 0)= f(1)$

$\Rightarrow$ f(x) is continuous at x = 1

Again $f'(1 + 0)= \lim\limits_{x \to 1^+} \frac{f(x)-f(1)}{x-1} = \lim\limits_{x \to 1} \frac{3^x-3}{x-1} = \lim\limits_{h \to 0} \frac{3^{1+h}-3}{h} = 3\lim\limits_{h \to 0} \frac{3^h-1}{h} = 3\log 3$

and $f'(1 + 0)= \lim\limits_{x \to 1} \frac{f(x)-f(1)}{x-1} = \lim\limits_{x \to 1} \frac{4-x-3}{x-1} = -1$

$\therefore\ f'(1 + 0) \neq f'(1 - 0)$

$\Rightarrow$ f(x) is not differentiable at x = 1

Q15.

Let $g(x)= ax + b$

LHL $= \lim\limits_{x \to 0^-} f(x)= \lim\limits_{x \to 0^-} g(x)= b$

and RHL $= \lim\limits_{x \to 0^+} f(x)= \left(\frac{1+0}{2+0}\right)^{\frac{1}{0}} = \left(\frac{1}{2}\right)^{\infty} = 0$

As $f(x)$ is continuous, LHL $=$ RHL $= f(0)\Rightarrow b = 0$

For $x > 0$, $f(x)= \left(\frac{1+x}{2+x}\right)^{1/x}$

$\ln(f(x))= \frac{1}{x}(\ln(1 + x)- \ln(2 + x))$

$\frac{1}{f(x)}f'(x)=\left[\frac{-1}{x^2}\left(\log\frac{(1+x)}{(2+x)}\right)+\frac{1}{x}\left(\frac{1}{1+x} - \frac{1}{2+x}\right)\right]$

$f'(1)= f(1)\left(-\ln\frac{2}{3} + 1\left(\frac{1}{2} - \frac{1}{3}\right)\right)$

$=\left(\frac{2}{3}\right)\left(-\ln\frac{2}{3} + \frac{1}{6}\right)$

$f'(-1)= a$

$\because\ f'(1)= f'(-1)\Rightarrow a = -\frac{2}{3}\left(\frac{1}{6} + \ln\frac{3}{2}\right)$

$f'(-1)= -\frac{2}{3}\left(\frac{1}{6} + \ln\frac{3}{2}\right)$

Q1. If $I_n=\int_0^{\frac{\pi}{2}} x^n \sin x d x$, where $n>1$, then

(1) $I_n+n(n-1) I_{n-2}=n\left(\frac{\pi}{2}\right)^n$

(2) $I_n+n(n-1) I_{n-2}=n\left(\frac{\pi}{2}\right)^{n-1}$

(3) $I_n-n(n-1) I_{n-2}=n\left(\frac{\pi}{2}\right)^n$

(4) $I_n-n(n-1) I_{n-2}=-n\left(\frac{\pi}{2}\right)^n$

Q2. The value of $x>1$ satisfying the equation $\int_1^x t \log t d t=\frac{1}{4}$, is

(1) $\sqrt{e}$

(2) $e^{\frac{3}{2}}$

(3) e^2

(4) $2 e-1$

Q3. The integral $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec ^{\frac{2}{3}} x \operatorname{cosec}^{\frac{4}{3}} x d x$ is equal to

(1) $3^{\frac{7}{6}}-3^{\frac{5}{6}}$

(2) $3^{\frac{4}{3}}-3^{\frac{1}{3}}$

(3) $3^{\frac{5}{6}}-3^{\frac{2}{3}}$

(4) $3^{\frac{5}{3}}-3^{\frac{1}{3}}$

Q4. Let $l_1=\int_0^{\infty} \frac{x^2 \sqrt{x}}{(1+x)^6} d x, l_2=\int_0^{\infty} \frac{x \sqrt{x}}{(1+x)^6} d x$, then

(1) $l_1=2 l_2$

(2) $l_2=2 l_1$

(3) $l_1=l_2^2$

(4) $l_1=l_2$

Q5. A value of α such that $\int_\alpha^{\alpha+1} \frac{d x}{(x+\alpha)(x+\alpha+1)}=\log _e\left(\frac{9}{8}\right)$ is

(1) $-\frac{1}{2}$

(2) $\frac{1}{2}$

(3) -2

(4) 2

Q6. The value of $\int_{-\pi / 2}^{\pi / 2} \frac{d x}{[x]+[\sin x]+4}$, where $[t]$ denotes the greatest integer less than or equal to t, is

(1) $\frac{3}{20}(4 \pi-3)$

(2) $\frac{3}{10}(4 \pi-3)$

(3) $\frac{1}{12}(7 \pi-5)$

(4) $\frac{1}{12}(7 \pi+5)$

Q7. Consider $\mathrm{I}(\alpha)=\int_\alpha^{\alpha^2} \frac{d x}{x}$ (where $\alpha>0$), then the value of $\sum_{r=2}^5 \mathrm{I}(r)+\sum_{k=2}^5 \mathrm{I}\left(\frac{1}{k}\right)$ is

(1) 0

(2) 1

(3) $\ln 2$

(4) $\ln 4$

Q8. If f is a real valued function defined by $\mathrm{f}(\mathrm{x}+\mathrm{y})+\mathrm{f}(\mathrm{x}-\mathrm{y})=\mathrm{f}(2 \mathrm{x})$, then the value of $\int_{\mathrm{f}(2)}^{\mathrm{f}(-2)} \mathrm{f}(\mathrm{x})$ is equal to

(1) 0

(2) 1

(3) 2

(4) 3

Q9. If $\int_0^{10 \pi+\alpha}|\sin x| d x=k-\cos \alpha$, where $0<\alpha<\pi$, then $k=$

(1) 101

(2) 100

(3) 201

(4) none of these

Q10. The value of $\int_0^{2 \pi}[\sin 2 x(1+\cos 3 x)] d x$, where $[t]$ denotes the greatest integer function is

(1) π

(2) 2π

(3) $-\pi$

(4) -2π

Q11. $\int_0^{2 n \pi}\left\{|\sin x|-\left|\frac{1}{2} \sin x\right|\right\} d x$ equals:

(1) n

(2) $2 n$

(3) $-2 n$

(4) None of these

Q12. If $I=\int_1^4(\{x\})^{[x]} d x$ then $\frac{24}{13} I$ is equal to

(1) 1

(2) 2

(3) 3

(4) 4

Q13. The integral value of $\int_{\frac{\pi}{4}}^{\frac{3 \pi}{4}} \frac{x}{1+\sin x} d x=$

(1) $\pi(\sqrt{2}+1)$

(2) $2 \pi(\sqrt{2}-1)$

(3) $2 \pi(\sqrt{2}+1)$

(4) $\frac{\pi}{\sqrt{2}+1}$

Q14. Statement I : If $\int_0^1 e^{\sin x} d x=\lambda$, then $\int_0^{200} e^{\sin x} d x=200 \lambda$

Statement II : $\int_0^{n a} f x d x=n \int_0^a f(x) d x, n \in I$ given $\mathrm{f}(\mathrm{a}+\mathrm{x})=\mathrm{f}(\mathrm{x})$

(1) Both Statement I and Statement II are true and the Statement II is the correct explanation of the Statement I

(2) Both Statement I and Statement II are true but the Statement II is not the correct explanation of the Statement I

(3) Statement I is true but Statement II is false

(4) Statement I is false but Statement II is true

Q15. $\int_0^4\{\sin\{x\}\}dx = N$, where $\{x\}$ represents fractional part of x. Then $\frac{N}{4}$ is

(1) $-\cos 1$

(2) $1-\cos 1$

(3) $1-\sin 1$

(4) None of these

Q16. Evaluate: $\int_{-\pi}^{199\pi}\sqrt{\left(\frac{1-\cos 2x}{2}\right)}dx$

(1) 200

(2) 400

(3) -200

(4) -400

Q17. The value of $Lt\left\{\frac{\int_0^{x^2}\sec^2 t dt}{x\sin x}\right\}$ is

(1) 0

(2) 3

(3) 2

(4) 1

Q18. $\int_{\sin x}^{1} t^2 f(t)dt = 1-\sin x \forall x\in(0,\pi/2)$, then $f\left(\frac{1}{\sqrt{3}}\right)$ is :

Q19. If $\int_0^{x^2(1+x)} f(t)\mathrm{d}t = x$, then the value of $f(2)$ is

(1) $1/2$

(2) $1/3$

(3) $1/4$

(4) $1/5$

Q20. $\lim_{n\to\infty}\frac{1^p+2^p+3^p+\ldots+n^p}{n^{p+1}}$ is $(p>-1)$

(1) $\frac{1}{p+1}$

(2) $\frac{1}{p-1}$

(3) $\frac{1}{p}-\frac{1}{p-1}$

(4) $\frac{1}{p+2}$

Q21. $\lim_{n\to\infty}\left(\frac{(n+1)^{1/3}}{n^{4/3}}+\frac{(n+2)^{1/3}}{n^{4/3}}+\ldots\ldots+\frac{(2n)^{1/3}}{n^{4/3}}\right)$ is equal to

(1) $\frac{3}{4}(2)^{4/3}-\frac{3}{4}$

(2) $\frac{4}{3}(2)^{3/4}$

(3) $\frac{4}{3}(2)^{4/3}$

(4) $\frac{3}{4}(2)^{4/3}-\frac{4}{3}$

Q22. The value of $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{k^{\frac{1}{a}}\left\{n^{a-\frac{1}{a}}+k^{a-\frac{1}{a}}\right\}}{n^{a+1}} =$

(1) 1

(2) 2

(3) 3

(4) None of these

Q23. If $f(x+y)=f(x)+3y^2+kxy$, $f(1)=1$ and $f(2)=6$, then the value of $\frac{1}{10}\int_{-1}^{2}f(x)dx$ is

Answer Key

Q1 (2)
Q2 (1)
Q3 (1)
Q4 (4)
Q5 (3)
Q6 (1)
Q7 (1)
Q8 (1)
Q9 (3)
Q10 (3)
Q11 (2)
Q12 (2)
Q13 (4)
Q14 (4)
Q15 (2)
Q16 (2)
Q17 (4)
Q18 (3)
Q19 (4)
Q20 (1)
Q21 (1)
Q22 (1)
Q23 (1.3)

Q1.

We have,

$I_n=\int_0^{\frac{\pi}{2}} x^n \sin x dx$

$\Rightarrow I_n=\left[-x^n \cos x\right]_0^{\frac{\pi}{2}}+\int_0^{\frac{\pi}{2}} n x^{n-1} \cos x dx$

$\Rightarrow I_n=n \int_0^{\frac{\pi}{2}} x^{n-1} \cos x dx$

$\Rightarrow I_n=n\left[\left[x^{n-1} \sin x\right]_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}(n-1) x^{n-2} \sin x dx\right]$

$\Rightarrow I_n=n\left[\left(\frac{\pi}{2}\right)^{n-1}-(n-1) I_{n-2}\right]$

Hence,

$I_n+n(n-1) I_{n-2}=n\left(\frac{\pi}{2}\right)^{n-1}$

Q2. Consider that, $I=\int\limits_1^x t \log t\, dt$

$=\left[\log t \cdot \frac{t^2}{2}\right]_1^x-\int\limits_1^x \frac{1}{t} \cdot \frac{t^2}{2} dt$

$=\frac{x^2}{2} \log x-\frac{1}{2}\left[\frac{t^2}{2}\right]_1^x$

$=\frac{x^2}{2} \log x-\frac{1}{2}\left[\frac{x^2}{2}-\frac{1}{2}\right]$

$\Rightarrow \quad \frac{1}{4}=\frac{x^2}{2} \log x-\frac{1}{4}\left(x^2-1\right)$

$\Rightarrow \quad \frac{1}{2} x^2 \log x-\frac{1}{4} x^2=0$ (as $x>1$)

$\Rightarrow \quad x^2(2 \log x-1)=0$

$\Rightarrow \quad 2 \log x-1=0$

$\Rightarrow \quad \log x=\frac{1}{2}$

$\Rightarrow \quad x=e^{\frac{1}{2}}$

$\Rightarrow \quad x=\sqrt{e}$

Q3.

$I=\int\limits_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec ^{\frac{2}{3}} x \operatorname{cosec}^{\frac{4}{3}} x\, d x$

$=\int\limits_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sec ^2 x}{\tan ^{4 / 3} x} d x$

Let $\tan x=t,\ \sec ^2 x d x=d t$

$\Rightarrow I=\int\limits_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \frac{d t}{t^{\frac{4}{3}}}$

$=\left[\frac{t^{-\frac{4}{3}+1}}{-\frac{4}{3}+1}\right]_{\frac{1}{\sqrt{3}}}^{\sqrt{3}}$

$=-3\left(\frac{1}{3^{\frac{1}{6}}}-3^{\frac{1}{6}}\right)$

$=3^{\frac{7}{6}}-3^{\frac{5}{6}}$.

Q4. We have, $I_1=\int\limits_0^{\infty} \frac{x^2 \sqrt{x}}{(1+x)^6} d x$

Let $x=\frac{1}{t} \Rightarrow d x=-\frac{1}{t^2} d t$ then,

$\Rightarrow I_1=\int\limits_{\infty}^0 \frac{\frac{1}{t^2 \sqrt{t}}}{\left(1+\frac{1}{t}\right)^6}\left(-\frac{1}{t^2} d t\right)$

$\Rightarrow I_1=\int\limits_0^{\infty} \frac{t \sqrt{t}}{(1+t)^6} d t=I_2$

Q5. $\mathrm{I}=\int\limits_\alpha^{\alpha+1} \frac{d x}{(x+\alpha)(x+\alpha+1)}=\int\limits_\alpha^{\alpha+1}\left(\frac{1}{x+\alpha}-\frac{1}{x+\alpha+1}\right) d x$

Using partial fractions

$=[\ln |x+\alpha|-\ln |x+\alpha+1|]_\alpha^{\alpha+1}$

$=\left[\ln \left|\frac{x+\alpha}{x+\alpha+1}\right|\right]_\alpha^{\alpha+1}$

$=\ln \left|\frac{2 \alpha+1}{2 \alpha+2}\right|-\ln \left|\frac{2 \alpha}{2 \alpha+1}\right|$

$=\ln \left|\frac{(2 \alpha+1)^2}{(2 \alpha+1)^2-1}\right|=\ln \frac{9}{8}$

$\Rightarrow(2 \alpha+1)^2=9$

$\Rightarrow 2 \alpha+1=\pm 3$

$\Rightarrow \alpha=1$ or -2.

Q6.

We have $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{d x}{[x]+[\sin x]+4}$

$=\int_{-\frac{\pi}{2}}^0 \frac{d x}{[x]+3}+\int_0^{\frac{\pi}{2}} \frac{d x}{[x]+4}$

$=\int_{-\frac{\pi}{2}}^{-1} \frac{d x}{1}+\int_{-1}^0 \frac{d x}{2}+\int_0^1 \frac{d x}{4}+\int_1^{\frac{\pi}{2}} \frac{d x}{5}$

$=[x]_{-\frac{\pi}{2}}^{-1}+\left[\frac{x}{2}\right]_{-1}^0+\left[\frac{x}{4}\right]_0^1+\left[\frac{x}{5}\right]_1^{\frac{\pi}{2}}=\left(-1+\frac{\pi}{2}\right)+\left(0+\frac{1}{2}\right)+\frac{1}{4}+\frac{\pi}{10}-\frac{1}{5}$

$=\frac{-20+10 \pi+10+5+2 \pi-4}{20}=\frac{12 \pi-9}{20}=\frac{3}{20}(4 \pi-3)$

Q7.

$\mathrm{I}(\alpha)=[\ln x]_\alpha^{\alpha^2}=\ln \alpha^2-\ln \alpha$

$=\ln \left(\frac{\alpha^2}{\alpha}\right)$

$=\ln \alpha$

Thus, $\sum_{r=2}^5 \mathrm{I}(r)=\ln 2+\ln 3+\ln 4+\ln 5$ and

$\sum_{r=2}^5 \mathrm{I}\left(\frac{1}{k}\right)=\ln \left(\frac{1}{2}\right)+\ln \left(\frac{1}{3}\right)+\ln \left(\frac{1}{4}\right)+\ln \left(\frac{1}{5}\right)$

$=-(\ln 2+\ln 3+\ln 4+\ln 5)$

Hence, their sum equals to zero

Q8. $\mathrm{f}(\mathrm{x}+\mathrm{y})+\mathrm{f}(\mathrm{x}-\mathrm{y})=\mathrm{f}(2 \mathrm{x})=\mathrm{f}(\mathrm{x}+\mathrm{y}+\mathrm{x}-\mathrm{y})$

Put $\mathrm{x}=0,\ \mathrm{y}=-\mathrm{x}$

$\mathrm{f}(\mathrm{x})+\mathrm{f}(-\mathrm{x})=0 \Rightarrow \mathrm{f}(\mathrm{x})=-\mathrm{f}(\mathrm{x}) \Rightarrow \mathrm{f}(\mathrm{x})$ is an odd function.

$\Rightarrow \mathrm{f}(2)=-\mathrm{f}(-2)$

$\Rightarrow \int\limits_{\mathrm{f}(2)}^{\mathrm{f}(-2)} \mathrm{f}(\mathrm{x}) \mathrm{dx}=\int\limits_{\mathrm{f}(2)}^{-\mathrm{f}(2)} \mathrm{f}(\mathrm{x}) \mathrm{dx}=0$

Q9.

Let $I=\int_0^{100 \pi+\alpha}|\sin\ x|\ d x$

$=\int_0^{100 \pi}|\sin\ x|\ d x+\int_{100 \pi}^{100 \pi+\alpha}|\sin\ x|\ d x$

$=100 \int_0^\pi \sin\ x\ d x+\int_0^\alpha \sin\ x\ d x$

$\because$ Period of $|\sin\ x|$ is π & $0<\alpha<\pi$

$=100\,(-\cos x)_0^\pi+(-\cos x)_0^\alpha$

$= 100\ (1 + 1) + (-\cos\ \alpha + 1) = 201 - \cos\ \alpha$

$\therefore\ k = 201$

Q10.

$I = \int_0^{2\pi} [\sin 2x(1 + \cos 3x)]dx \quad \ldots(1)$

$I = \int_0^{2\pi} [\sin(2\pi - 2x)(1 + \cos(2\pi - 3x))]dx$

Applying $\left(\int_0^a f(x) = \int_0^a f(a - x)dx\right)$

$I = \int_0^{2\pi} [-\sin 2x(1 + \cos 3x)]dx \quad \ldots(2)$

Adding (1) and (2)

$2I = \int_0^{2\pi} ([\sin 2x(1 + \cos 3x)] + [-\sin 2x(1 + \cos 3x)])dx$

$\Rightarrow\ 2I = \int_0^{2\pi} -1dx \quad \left\{\because\ [x] + [-x] = \begin{cases} 0 & : x \in \mathrm{I} \\ -1 & : x \notin \mathrm{I} \end{cases}\right.$

$\Rightarrow\ 2I = (-x)_0^{2\pi}$

$\Rightarrow\ 2I = -2\pi$

$\Rightarrow\ I = \ -\pi$

Q11.

Let $I = \int_0^{2n\pi} \left\{|\sin\ x| - \left|\frac{1}{2}\sin\ x\right|\right\}dx$

$= \frac{1}{2}\int_0^{2n\pi} |\sin\ x|dx$

$= \frac{1}{2}\left(n \times \int_0^{2\pi} |\sin x|dx\right)$

Now, $I_1 = \int_0^{2\pi} |\sin\ x|dx$

$I_1 = \int_0^{\pi} \sin\ x\ dx - \int_{\pi}^{2\pi} \sin\ x\ dx$

$= [-\cos\ x]_0^{\pi} + [\cos\ x]_{\pi}^{2\pi} = -[-1 - 1] + [1 + 1]$

$= 2 + 2 = 4$

$\therefore\ I = \frac{1}{2}(4n) = 2n$

Q12. $I = \int_1^2 \{x\} + \int_2^3 \{x\}^2 + \int_2^3 \{x\}^3$

$= \int_0^1 x + x^2 + x^3 = \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{6+4+3}{12}$

$= \frac{13}{12}$

Q13. Let $I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{x}{1+\sin x}dx$ ....(i)

$I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{\left(\frac{3\pi}{4} + \frac{\pi}{4} - x\right)}{1 + \sin\left(\frac{3\pi}{4} + \frac{\pi}{4} - x\right)}$

$= \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{(\pi - x)dx}{1+\sin x}$ ....(ii)

$\left[\because\ \int_a^b f(x)dx = \int_a^b f(a + b - x)dx\right]$

By adding Equations (i) and (ii), we get

$2I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{\pi\ dx}{1+\sin x}$

$\Rightarrow\ 2I = \pi \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{1-\sin x}{(1+\sin x)\ (1-\sin x)}dx$

$\Rightarrow\ 2I = \pi \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{1-\sin x}{\cos^2 x}dx$

$\Rightarrow\ 2I = \pi \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \left[\sec^2 x - \sec x \tan x\right]dx$

$\Rightarrow\ 2I = \pi[\tan x - \sec x]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$

$\Rightarrow\ 2I = \pi\left[-1 - \left(-\sqrt{2}\right) - \left(1 - \sqrt{2}\right)\right]$

$\Rightarrow\ 2I = \pi\left[-1 + \sqrt{2} - 1 + \sqrt{2}\right]$

$\Rightarrow\ 2I = \pi\left[-2 + 2\sqrt{2}\right]$

$\therefore\ I = \pi\left[\sqrt{2} - 1\right]$

$= \frac{\pi\left(\sqrt{2}-1\right)}{\left(\sqrt{2}+1\right)}\left(\sqrt{2} + 1\right)$

$= \frac{\pi}{\sqrt{2}+1}$

Q14.

We have given that $\int_0^1 e^{\sin x}dx = \lambda$

$$\mathrm{f}(\mathrm{x}) = \mathrm{e}^{\mathrm{sinx}}$$

f(x) is a periodic function, its value will repeat as sinx repeat its value.

$\therefore$Period of $\mathrm{f}(\mathrm{x}) = 2\pi$

$$\int_0^{200} e^{\sin x}dx = \int_0^{\frac{100}{\pi}\cdot 2\pi} e^{\sin x}dx$$

But $\frac{100}{\pi}$ is not an integer

$\therefore$ Statement I is wrong.

and $\int_0^{na} fxdx = n\int_0^a f(x)dx,\ n \in I$

So Statement II is true.

Q15.

Let $I = \int_0^4 \left\{\sin\{x\}\right\}dx$

Using $\int_0^{nT} f(x)dx = n\int_0^T f(x)dx,\ n \in Z$ if $f(x)$ is a periodic function with period T.

We know that $\{x\}$ has period equals to 1.

$\Rightarrow\ I = 4\int_0^1 \{\sin\{x\}\}dx$

Also, for $x \in [0,\ 1],\ \{\sin\{x\}\} = \sin x$

$\Rightarrow\ I = 4\int_0^1 \sin x dx$

$\Rightarrow I = -4[\cos x]_0^1$

$\Rightarrow\ I = 4(1 - \cos 1).$

Q16. Let, $I = \int_{-\pi}^{199\pi} \sqrt{\left(\frac{1-\cos 2x}{2}\right)}\ dx$

$= \int_{-\pi}^{199\pi} |\sin x|dx$

$=(199-(-1))\int_0^{\pi}|\sin x|dx$

$(\because |\sin x|$ is periodic with period π and $\int_{mT}^{nT} f(x)dx = (n-m)\int_0^T f(x)dx$ if T is the period of the function $f(x)$).

$= 200\int_0^{\pi}\sin x dx$

$= 200|-\cos x|_0^{\pi}$

$= 200(1-(-1)) = 400.$

Q17.

$$\underset{x\to 0}{Lt}\left\{\frac{\int_0^{x^2}\sec^2 t\, dt}{x\sin x}\right\} \quad \left(\frac{0}{0}\text{ form}\right)$$

Apply L'Hospital's Rule, we get

$$= \underset{x\to 0}{Lt}\frac{(\sec^2 x^2)\,2x}{x\cos x+\sin x} \quad \left(\frac{0}{0}\text{ form}\right)$$

$$= \underset{x\to 0}{Lt}\frac{2\sec^2 x^2}{\cos x+\frac{\sin x}{x}} = \frac{2}{1+1} = 1$$

Q18.

$\int_{\sin x}^{1} t^2 f(t)\, dt = 1-\sin x$

Applying Leibnitz Rule,

$-\sin^2 x\, f(\sin x)\cos x = -\cos x$

$\Rightarrow f(\sin x) = \frac{1}{\sin^2 x}$

$\Rightarrow f(t) = \frac{1}{t^2}$

$f\left(\frac{1}{\sqrt{3}}\right) = \frac{1}{\left(\frac{1}{\sqrt{3}}\right)^2} = 3$

Q19.

Differentiating both sides w.r.t. x, we get

$f(x^2(1+x))\times(2x+3x^2) = 1$ (Use Newton-Leibniz Rule for differentiation)

At $x=1 \Rightarrow f(2) = \frac{1}{5}$

Q20.

$\underset{n\to\infty}{Lt}\frac{1^p+2^p+\ldots+n^p}{n^p}\times\frac{1}{n}$

$= \underset{n\to\infty}{Lt}\frac{1}{n}\left[\left(\frac{1}{n}\right)^p+\left(\frac{2}{n}\right)^p+\ldots+\left(\frac{n}{n}\right)^p\right]$

$= \underset{n\to\infty}{Lt}\sum_{r=1}^{r=n}\frac{1}{n}\left(\frac{r}{n}\right)^p$

$= \int_0^1 x^p dx \left[\frac{x^{p+1}}{p+1}\right]_0^1 = \frac{1}{p+1}$

Q21. The given limit can be written as

$\lim_{n\to\infty}\frac{1}{n}\left[\left(1+\frac{1}{n}\right)^{\frac{1}{3}}+\left(1+\frac{2}{n}\right)^{\frac{1}{3}}+\ldots.+\left(1+\frac{n}{n}\right)^{\frac{1}{3}}\right]$

$= \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\left(1+\frac{r}{n}\right)^{\frac{1}{3}}$

$= \int_0^1(1+x)^{1/3}dx$

$= \left.\frac{(1+x)^{\frac{4}{3}}}{\frac{4}{3}}\right|_0^1$

$= \frac{3}{4}\left(2^{4/3}-1\right)$

$= \frac{3}{4}.2^{4/3}-\frac{3}{4}$

Q22.

$\lim_{n\to\infty}\sum_{k=1}^{n}\frac{k^{\frac{1}{a}}\left\{n^{a-\frac{1}{a}}+k^{a-\frac{1}{a}}\right\}}{n^{a+1}}$

$= \lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n}\cdot\left\{\left(\frac{k}{n}\right)^{\frac{1}{a}}+\left(\frac{k}{n}\right)^{a}\right\}$

$= \int_0^1\left(x^{\frac{1}{a}}+x^a\right)dx$ (applying definite integral as limit of sum)

$= \left\{\frac{x^{\left(\frac{1}{a}\right)+1}}{\frac{1}{a}+1}+\frac{x^{a+1}}{a+1}\right]_0^1$

$= \frac{a}{a+1}+\frac{1}{a+1} = 1$

Q23.

$f(x+y) = f(x)+3y^2+kxy$..(i)

In (i), put $x=1$, $f(1+y) = f(1)+3y^2+ky$

$\Rightarrow f(y+1) = 3y^2+ky+1$.......(ii)

In (ii), put $y=1$, $f(2) = 3(1)+k+1 = 6$

$\Rightarrow k=2$

Using (ii), $f(y+1) = 3y^2+2y+1$

Replace $y\to x-1$

$\Rightarrow f(x) = 3(x-1)^2+2(x-1)+1 = 3(x-1)^2+2x-1$

Now $\int_{-1}^{2} f(x)dx = \int_{-1}^{2}\left(3(x-1)^2+2x-1\right)dx$

$= \left[(x-1)^3+x^2-x\right]_{-1}^{2} = [(8+1)+(4-1)-(2+1)] = 13$

Q1. Given a differential equation $x\left(\frac{dy}{dx}\right)^{\frac{3}{2}}-1=2\left(\frac{d^2y}{dx^2}\right)+\sin x$, whose order and degree are $p\&q$ respectively, then

(1) $-q=3$

(2) $p-q=1$

(3) $p^q+q^p=8$

(4) $p^q-q^p=1$

Q2. If m and n are order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^5+4\cdot\frac{\left(\frac{d^2y}{dx^2}\right)^3}{\frac{d^3y}{dx^3}}+\frac{d^3y}{dx^3}=x^2-1$, then the value of $m+n$ is

Q3. The degree of the differential equation, of which $y^2=4a(x+a)$ is a solution, is

(1) 1

(2) 2

(3) 3

(4) None of these

Q4. Solution of the differential equation $\frac{x+\frac{x^3}{3!}+\frac{x^5}{5!}\cdots}{1+\frac{x^2}{2}+\frac{x^4}{4}\cdots}=\frac{dx-dy}{dx+dy}$ is

(1) $2ye^{2x}=ce^{2x}+1$

(2) $2ye^{2x}=2ce^{2x}-1$

(3) $ye^{2x}=ce^{2x}+2$

(4) None of these

Q5. If $f(x)$ satisfies the differential equation $\frac{dy}{dx}=(x-y)^2$ and given that $y(1)=1$, then

(1) $-\ln\left|\frac{1-x+y}{1+x-y}\right|=2(x-1)$

(2) $\ln\left|\frac{2-y}{2-x}\right|=x+y-1$

(3) $\ln\left|\frac{1-x+y}{1+x-y}\right|=2(x-1)$

(4) $\frac{1}{2}\ln\left|\frac{1-x+y}{1+x-y}\right|+\ln|x|=0$

Q6. The slope at any point of a curve $y=f(x)$ is given by $\frac{dy}{dx}=3x^2$ and it passes through $(-1,1)$ The equation of the curve is

(1) $y=x^3+2$

(2) $y=-x^3-2$

(3) $y=3x^3+4$

(4) $y=-x^3+2$

Q7. The general solution of the differential equation $\frac{dy}{dx}+\sin\left(\frac{x+y}{2}\right)=\sin\left(\frac{x-y}{2}\right)$ is (where c is an arbitrary constant)

(1) $\ln\tan\left(\frac{y}{2}\right)=c-2\sin x$

(2) $\ln\tan\left(\frac{y}{4}\right)=c-2\sin\left(\frac{x}{2}\right)$

(3) $\ln\tan\left(\frac{y}{2}+\frac{\pi}{4}\right)=c-2\sin x$

(4) $\ln\tan\left(\frac{y}{4}+\frac{\pi}{4}\right)=c-2\sin\left(\frac{x}{2}\right)$

Q8. If $x^3dy+xy\cdot dx=x^2dy+2ydx; y(2)=e$ and $x>1$, then $y(4)$ is equal to :

(1) $\frac{\sqrt{e}}{2}$

(2) $\frac{1}{2}+\sqrt{e}$

(3) $\frac{3}{2}\sqrt{e}$

(4) $\frac{3}{2}+\sqrt{e}$

Q9. The equation of the curve satisfying the equation $\left(xy-x^2\right)\frac{dy}{dx}=y^2$ and passing through the point $(-1,1)$ is

(1) $y=(\log \mathrm{y}-1)x$

(2) $y=(\log \mathrm{y}+1)x$

(3) $x=(\log \mathrm{x}-1)y$

(4) $x=(\log \mathrm{x}+1)y$

Q10. The solution of $\left(x^2+xy\right)dy=\left(x^2+y^2\right)dx$ is

(1) $\log x=\log(x-y)+\frac{y}{x}+C$

(2) $\log x=2\log(x-y)+\frac{y}{x}+C$

(3) $\log x=\log(x-y)+\frac{x}{y}+C$

(4) none of these

Q11. Let $y=y(x)$ be the solution of the differential equation $\sin x\frac{dy}{dx}+y\cos x=4x, x\in(0,\pi)$. If $y\left(\frac{\pi}{2}\right)=0$, then $y\left(\frac{\pi}{6}\right)$ is equal to

(1) $-\frac{4}{9}\pi^2$

(2) $\frac{4}{9\sqrt{3}}\pi^2$

(3) $\frac{-8}{9\sqrt{3}}\pi^2$

(4) $-\frac{8}{9}\pi^2$

Q12. The solution of $\frac{dz}{dx}+\frac{z}{x}\log z=\frac{z}{x^2}(\log z)^2$ is

(1) $\left(\frac{1}{\log z}\right)x=2-x^2c$

(2) $\left(\frac{1}{\log z}\right)x=2+x^2c$

(3) $\left(\frac{1}{\log z}\right)x=x^2c$

(4) $\left(\frac{1}{\log z}\right)x=\frac{1}{2}+cx^2$

Q13. A curve passes through the point $\left(1, \frac{\pi}{6}\right)$. Let the slope of the curve at each point (x, y) be $\frac{y}{x} + \sec\left(\frac{y}{x}\right), x > 0$. Then the equation of the curve is

(1) $\sin\left(\frac{y}{x}\right) = \log x + \frac{1}{2}$

(2) $\operatorname{cosec}\left(\frac{y}{x}\right) = \log x + 2$

(3) $\sec\left(\frac{2y}{x}\right) = \log x + 2$

(4) $\cos\left(\frac{2y}{x}\right) = \log x + \frac{1}{2}$

Q14. The integrating factor of the differential equation $\frac{dy}{dx} + \frac{y}{(1-x)\sqrt{x}} = 1 - \sqrt{x}$ may be

(1) $\frac{1-\sqrt{x}}{1+\sqrt{x}}$

(2) $\frac{1+\sqrt{x}}{1-\sqrt{x}}$

(3) $\frac{1-x}{1+x}$

(4) $\frac{\sqrt{x}}{1-\sqrt{x}}$

Q15. The solution of differential equation $\left(1 + y^2\right) + \left(x - e^{\tan^{-1} y}\right) \frac{dy}{dx} = 0$ is

(1) $2xe^{\tan^{-1} y} = e^{2\tan^{-1} y} + k$

(2) $2xe^{\tan^{-1} y} = e^{\tan^{-1} y} + k$

(3) $xe^{\tan^{-1} y} = e^{\tan^{-1} y} + k$

(4) $xe^{\tan^{-1} y} = e^{\tan^{-1} y} - k$

Q16. The solution of the initial value problem $(2\ln x)\frac{dy}{dx} + \frac{y}{x} = \frac{1}{y}\cos x, y > 0, x > 1$ and $y\left(\frac{3\pi}{2}\right) = 0$ is given by which of the following options?

(1) $y = a\sqrt{\frac{1-\sin x}{\ln x}}$

(2) $y = a\sqrt{\frac{1+\sin x}{\ln x}}$

(3) $y = a\sqrt{\frac{1-\cos x}{\ln x}}$

(4) $y = a\sqrt{\frac{1+\cos x}{\ln x}}$

Q17. The solution of the differential equation $y\left(\sin^2 x\right) dy + (\sin x \cos x)y^2 dx = x dx$ is (where C is the constant of integration)

(1) $\sin^2 x \cdot y = x^2 + C$

(2) $\sin^2 x \cdot y^2 = x^2 + C$

(3) $\sin x \cdot y^2 = x^2 + C$

(4) $\sin^2 x \cdot y^2 = x + C$

Q18. If $xdy = ydx + y^2 dy$ and $y(1) = 1$ then find the value of $y(-3)$

Q19. If a curve $y = f(x)$ passing through $(1, 2)$ satisfies the differential equation $y(1 + xy)dx - xdy = 0$, then which of the following option is correct?

(1) $f(x) = \frac{2x}{2-x^2}$

(2) $f(x) = \frac{x+1}{x^2+1}$

(3) $f(x) = \frac{x-1}{4-x^2}$

(4) $f(x) = \frac{4x}{1-2x^2}$

Q20. The solution of differential equation $\frac{dy}{dx} = \frac{yf'(x)-y^2}{f(x)}$ is

(1) $f(x) = y(x - c)$

(2) $f(x) = y(c - x)$

(3) $f(x) = y(x + c)$

(4) None of these

Q21. The orthogonal trajectory of $x^2 - y^2 = a^2$, where a is an arbitrary constant, is

(1) A parabola

(2) A circle

(3) An ellipse

(4) A hyperbola

Q22. The population $p(t)$ at a time t of a certain mouse species satisfies the differential equation $\frac{dp(t)}{dt} = 0.5p(t) - 450$. If $p(0) = 850$, then the time at which the population becomes zero is

(1) $\frac{1}{2}\ln 18$

(2) $\ln 18$

(3) $2\ln 18$

(4) $\ln 9$

Answer Key

Q1 (3) **Q2** (5.00) **Q3** (2) **Q4** (2)

Q5 (1) **Q6** (1) **Q7** (2) **Q8** (3)

Q9 (1) **Q10** (2) **Q11** (4) **Q12** (4)

Q13 (1) **Q14** (2) **Q15** (1) **Q16** (2)

Q17 (2) **Q18** (3.00) **Q19** (1) **Q20** (3)

Q21 (4) **Q22** (3)

Q1.

We have, $x\left(\frac{dy}{dx}\right)^{\frac{3}{2}}=2\left(\frac{d^2y}{dx^2}\right)+\sin x+1$

Square both the sides we get,

$x^2\left(\frac{dy}{dx}\right)^3=\left(2\left(\frac{d^2y}{dx}\right)+\sin x+1\right)^2$

Here, Order$=p=2$

Degree$=q=2$

Then, $p^q+q^p=2^2+2^2=8$

Q2.

The given differential equation can be written as

$\left(\frac{d^2y}{dx^2}\right)^5\frac{d^3y}{dx^3}+4\left(\frac{d^2y}{dx^2}\right)^3+\left(\frac{d^3y}{dx^3}\right)^2$

$=\left(x^2-1\right)\frac{d^3y}{dx^3}$

$\Rightarrow m=3,\ n=2$

Q3.

We have, $\mathrm{y}^2=4\mathrm{a}\ (\mathrm{x}+\mathrm{a})\ldots\ (1)$

On differentiating w.r.t. x , we get

$2\mathrm{y}\ \frac{\mathrm{dy}}{\mathrm{dx}}=4\mathrm{a}\ \Rightarrow\ \mathrm{a}=\frac{\mathrm{y}}{2}\frac{\mathrm{dy}}{\mathrm{dx}}.$

On substituting the value of a in equation (1), we get

$\mathrm{y}^2=2\mathrm{y}\frac{\mathrm{dy}}{\mathrm{dx}}\left[\mathrm{x}+\frac{\mathrm{y}}{2}\frac{\mathrm{dy}}{\mathrm{dx}}\right]$

$\Rightarrow\mathrm{y}=2\mathrm{x}\frac{\mathrm{dy}}{\mathrm{dx}}+\mathrm{y}\left(\frac{\mathrm{dy}}{\mathrm{dx}}\right)^2$

$\Rightarrow\mathrm{y}\left[1-\left(\frac{\mathrm{dy}}{\mathrm{dx}}\right)^2\right]=2\mathrm{x}\ .\ \frac{\mathrm{dy}}{\mathrm{dx}}$, which is the required differential equation.

The degree of the differential equation is 2

Hence, (B) is the correct answer.

Q4.

$e^x=1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots\ldots\ldots\ldots\ldots\infty$

$e^{-x}=1-\frac{x}{1!}+\frac{x^2}{2!}-\frac{x^3}{3!}+\ldots\ldots\ldots\ldots\ldots\infty$

$\frac{x+\frac{x^3}{3!}+\frac{x^5}{5!}\cdots}{1+\frac{x^2}{2!}+\frac{x^4}{4!}\cdots}=\frac{dx-dy}{dx+dy}\ \frac{\frac{e^x-e^{-x}}{2}}{\frac{e^x+e^{-x}}{2}}=\frac{dx-dy}{dx+dy}.$

Apply componendo and dividendo

$\frac{dy}{dx}=e^{-2x}$

$\int\frac{dy}{dx}=\int e^{-2x}\Rightarrow y=-\frac{1}{2}e^{-2x}+c$

$2ye^{2x}=2ce^{2x}-1.$

Q5. $x-y=t$

$1-\frac{dy}{dx}=\frac{dt}{dx}$

Hence $1-\frac{dt}{dx}=t^2$

$\Rightarrow 1-t^2=\frac{dt}{dx}$

$\Rightarrow\int\frac{dt}{1-t^2}=\int dx$

$\Rightarrow\frac{1}{2}\log\left|\frac{1+t}{t-1}\right|=x+c$

$\frac{1}{2}\log\left|\frac{x-y+1}{x-y-1}\right|=x+c$

At $x=1$ and $y=1$, we get $c=-1$

Hence $\frac{1}{2}\log\left|\frac{x-y+1}{x-y-1}\right|=x-1$

Hence $-\ln\left|\frac{1-x+y}{1+x-y}\right|=2(x-1)$

Q6.

We have,

$\frac{dy}{dx}=3x^2$

$\Rightarrow dy=3x^2dx$

Integrating both sides, we have

$\int dy=3\int x^2dx$

$\Rightarrow y=3\left(\frac{x^3}{3}\right)+c$

$\Rightarrow y=x^3+c$

It is passing through $(-1,1)$. Therefore,

$\Rightarrow 1=(-1)^3+c\Rightarrow c=2$

Hence, the required curve is

$y=x^3+2$

Q7.

Given equation

$\frac{dy}{dx}+\sin\left(\frac{x+y}{2}\right)=\sin\left(\frac{x-y}{2}\right)$

$\Rightarrow\frac{dy}{dx}=\sin\left(\frac{x-y}{2}\right)-\sin\left(\frac{x+y}{2}\right)$

$\Rightarrow\frac{dy}{dx}=-2\sin\left(\frac{y}{2}\right)\cos\left(\frac{x}{2}\right)$

$\Rightarrow\operatorname{cosec}\left(\frac{y}{2}\right)dy=-2\cos\left(\frac{x}{2}\right)dx$

On integrating both sides, we get

$\int\operatorname{cosec}\left(\frac{y}{2}\right)dy=-\int 2\cos\left(\frac{x}{2}\right)dx$

$\Rightarrow\frac{\ln\left(\tan\frac{y}{4}\right)}{\frac{1}{2}}=-\frac{2\sin\left(\frac{x}{2}\right)}{\frac{1}{2}}+c$

$\Rightarrow\ln\left(\tan\frac{y}{4}\right)=c-2\sin\left(\frac{x}{2}\right)$

Q8.

Given $x^3dy+xy\cdot dx=x^2dy+2ydx$

$\Rightarrow (x^3 - x^2)dy = (2-x)ydx$

$\Rightarrow \frac{dy}{y} = \frac{(2-x)}{(x^3-x^2)}dx$

Integrating both sides with respect to x, we get

$\int \frac{dy}{y} = \int \frac{(2-x)}{x^2(x-1)}dx + k \ldots\ldots\ldots\ldots(i)$

Let $\frac{(2-x)}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x-1)}$

$\Rightarrow (2-x) = Ax(x-1) + B(x-1) + Cx^2$

Putting $x = 0 \Rightarrow 2 = -B \Rightarrow B = -2$

Putting $x = 1 \Rightarrow 2 - 1 = C \Rightarrow C = 1$

Putting

$x = 2 \Rightarrow 2 - 2 = A(2)(1) + B(1) + C(2^2) \Rightarrow 2A + 2 = 0 \Rightarrow A = -1$

From equation (i), we get

$\int \frac{dy}{y} = \int \left(\frac{-1}{x} + \frac{-2}{x^2} + \frac{1}{x-1}\right)dx + k$

$\Rightarrow \ln y = -\ln x + \frac{2}{x} + \ln|x-1| + k \ldots\ldots\ldots(ii)$

Given $y(2) = e$ i.e. at $x = 2,\ y = e$

$\Rightarrow \ln e = -\ln 2 + \frac{2}{2} + \ln|2-1| + k$

$\Rightarrow 1 = -\ln 2 + 1 + 0 + k$

$\Rightarrow k = \ln 2$

Putting in equation (ii), we get

$\ln y = -\ln x + \frac{2}{x} + \ln|x-1| + \ln 2$

Now putting $x = 4$, we get

$\ln y = -\ln 4 + \frac{2}{4} + \ln|4-1| + \ln 2$

$\Rightarrow \ln y = -2\ln 2 + \frac{1}{2} + \ln 3 + \ln 2$

$\Rightarrow \ln y = -\ln 2 + \frac{1}{2} + \ln 3$

$\Rightarrow \ln y = \frac{1}{2} + \ln \frac{3}{2}$

$\Rightarrow \ln \frac{2y}{3} = \frac{1}{2}$

$\Rightarrow \frac{2y}{3} = e^{\frac{1}{2}}$

$\Rightarrow y = \frac{3}{2}\sqrt{e}$

$\Rightarrow y(4) = \frac{3}{2}\sqrt{e}$

Q9. $\frac{dy}{dx} = \frac{y^2}{xy - x^2}$

Put, $y = vx$

$\Rightarrow \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow \quad v + x\frac{dv}{dx} = \frac{v^2}{v-1}$

$\Rightarrow \quad x\frac{dv}{dx} = \frac{v^2}{v-1} - v = \frac{v}{v-1}$

$\Rightarrow \quad \int\left(\frac{v-1}{v}\right)dv = \int \frac{dx}{x}$

$\Rightarrow \quad \int\left(1 - \frac{1}{v}\right)dv = \int \frac{dx}{x}$

$\Rightarrow \quad v - \ell n\, v = \ell n\, x + c$

$\Rightarrow \quad \frac{y}{x} = \ell n\, y + c$

Alternate Solution :

We have, $\quad (xy - x^2)\frac{dy}{dx} = y^2$

$\Rightarrow \quad y^2\frac{dx}{dy} = xy - x^2$

$\Rightarrow \quad \frac{1}{x^2}\frac{dx}{dy} - \frac{1}{x}\cdot\frac{1}{y} = -\frac{1}{y^2}$

Put $\frac{1}{x} = v \Rightarrow -\frac{1}{x^2}\frac{dx}{dy} = \frac{dv}{dy}$

$\therefore \quad \frac{dv}{dy} + \frac{v}{y} = \frac{1}{y^2}$, which is linear

$\therefore \quad \text{IF} = e^{\int \frac{1}{y}dy} = e^{\log y} = y$

$\therefore$ The solution is $vy = \int \frac{1}{y^2}\cdot y\ \ dy + c$

$\Rightarrow \quad \frac{y}{x} = \log|y| + c$

$\Rightarrow \quad y = x(\log|y| + c)$

This posses through the point (–1, 1).

$\therefore \quad 1 = -1(\log 1 + c)$

ie, $\quad c = -1$

Thus, the equation of the curve is

y = x (log |y| – 1)

Q10.

$x^2dy + xydy = x^2dx + y^2dx$

dividing both sides by x^2

$dy + \frac{y}{x}dy = dx + \left(\frac{y}{x}\right)^2 dx$

$\left(1 + \frac{y}{x}\right)\frac{dy}{dx} = 1 + \left(\frac{y}{x}\right)^2 \qquad \ldots(i)$

As it is of type $\frac{dy}{dx} = f\left(\frac{y}{x}\right)$

Put $\frac{y}{x} = t \Rightarrow \frac{dy}{dx} = t + x\frac{dt}{dx} \qquad \ldots(ii)$

from equations (i) and (ii), we get

$\left(1 + t\right)\left(t + x\frac{dt}{dx}\right) = 1 + t^2$

$x\frac{dt}{dx} = \frac{1-t}{1+t} \Rightarrow -\left(\frac{1+t}{t-1}\right)dt = \frac{dx}{x} \qquad \ldots(iii)$

integrating both sides,

$-\int\left(\frac{2}{t-1} + \frac{t-1}{t-1}\right)dt = \int \frac{dx}{x}$

$-2\ln|t-1| - t = \ln x + C$

$-2\ln\left|\frac{y}{x} - 1\right| - \frac{y}{x} = \ln x + C$

$\ln x = 2\ln|y - x| + \frac{y}{x} + C$

or, $\ln x = 2\ln|x - y| + \frac{y}{x} + C.$

Q11. $\frac{dy}{dx} + y\cot x = \frac{4x}{\sin x},\ x \in (0,\ \pi)$

I.F. $= e^{\int \cot x} = e^{\ln \sin x} = \sin x$

$\therefore y \sin x = \int \frac{4x}{\sin x}.\sin x\, dx$

$y \sin x = 2x^2 + c$

$\because y\left(\frac{\pi}{2}\right)=0$

$\therefore 0\times 1=2\times\left(\frac{\pi}{2}\right)^2+c$

$\Rightarrow c=-\frac{\pi^2}{2}$

$y\left(\frac{\pi}{6}\right)=\frac{2\times\left(\frac{\pi}{6}\right)^2-\left(\frac{\pi^2}{2}\right)}{\frac{1}{2}}=\left[\left(\frac{\pi^2}{18}\right)-\left(\frac{\pi^2}{2}\right)\right]\times 2$

$=\frac{\pi^2}{9}-\pi^2$

$=-\frac{8\pi^2}{9}$

Q12.

Given, $\frac{dz}{dx}+\frac{z}{x}\log z=\frac{z}{x^2}(\log z)^2$

Dividing given equation by $z(\log z)^2$,

$\Rightarrow \frac{1}{z(\log z)^2}\frac{dz}{dx}+\frac{1}{x}\frac{1}{\log z}=\frac{1}{x^2}$

Put $\frac{1}{\log z}=t\Rightarrow -\frac{1}{(\log z)^2}\cdot\frac{1}{z}\cdot\frac{dz}{dx}=\frac{dt}{dx}$

$\therefore -\frac{dt}{dx}+\frac{t}{x}=\frac{1}{x^2}$

$\Rightarrow \frac{dt}{dx}-\frac{t}{x}=-\frac{1}{x^2}\ldots(i)$

Now, integrating factor $I.F=e^{\int-\frac{dx}{x}}=e^{-\ln x}=e^{\ln\frac{1}{x}}=\frac{1}{x}$

$\Rightarrow \frac{t}{x}=-\int\frac{1}{x^3}dx=\frac{1}{2x^2}+c$, where, c is the constant of integration.

$\Rightarrow \frac{1}{x\log z}=\frac{1}{2x^2}+c$

$\Rightarrow\left(\frac{1}{\log z}\right)x=\left(\frac{1}{2}\right)+cx^2.$

Q13.

Given slope at (x, y) is

$\frac{dy}{dx}=\frac{y}{x}+\sec\left(\frac{y}{x}\right)\ldots\ldots(i)$

let $\frac{y}{x}=t\Rightarrow y=xt$

Differentiating this wrt x

$\Rightarrow\quad \frac{dy}{dx}=t+x\frac{dt}{dx}\ldots\ldots(ii)$

Now using equation (i) and (ii), we have

$t+x\frac{dt}{dx}=t+\sec(t)$

$\Rightarrow\frac{dt}{\sec(t)}=\frac{dx}{x}$

Integrating both sides.

$\int\cos(t)dt=\int\frac{1}{x}dx$

$\sin(t)=\ln(x)+c$

Now Put value of t in above equation.

$\sin\left(\frac{y}{x}\right)=\ln(x)+c\ldots..(iii)$

Given that curve passes through $\left(1,\frac{\pi}{6}\right)$ so put in above equation.

$\sin\left(\frac{\pi}{6}\right)=\ln(1)+c\Rightarrow c=\frac{1}{2}$

Now, put the value of c in equation (iii)

$\sin\left(\frac{y}{x}\right)=\ln(x)+\frac{1}{2}$

Q14.

Given, $\frac{dy}{dx}+\frac{y}{(1-x)\sqrt{x}}=1-\sqrt{x}$

$\therefore$ If $=e^{\int\frac{1}{(1-x)\sqrt{x}}dx}$

Put $\sqrt{x}=t$

$\Rightarrow\frac{1}{2\sqrt{x}}dx=dt$

$\therefore$ If $=e^{\int\frac{2}{1-t^2}dt}$

$=e^{\frac{2}{2}\log\left|\frac{1+t}{1-t}\right|}=\frac{1+t}{1-t}=\frac{1+\sqrt{x}}{1-\sqrt{x}}$

Q15.

Given equation can be rewritten as

$\frac{dx}{dy}+\frac{1}{(1+y^2)}x=\frac{e^{\tan^{-1}y}}{(1+y^2)}$

$\therefore$ If $=e^{\int\frac{1}{1+y^2}dy}=e^{\tan^1 y}$

$\therefore$ Required solution is

$xe^{\tan^1 y}=\int\frac{e^{\tan^{-1}y}e^{\tan^{-1}y}}{1+y^2}dy$

Put $e^{\tan^{-1}y}=t\Rightarrow e^{\tan^{-1}y}\frac{1}{1+y^2}dy=dt$

$\therefore xe^{\tan^{-1}y}=\int t\,dt=\frac{t^2}{2}+c$

$\Rightarrow 2xe^{\tan^{-1}y}=e^{2\tan^{-1}y}+k$

Q16.

Given differential equation is

$2y\frac{dy}{dx}+y^2\left(\frac{1}{x\ln x}\right)=\frac{\cos x}{\ln x}$

$\Rightarrow\frac{dz}{dx}+\frac{z}{x\ln x}=\frac{\cos x}{\ln x}$

On substituting $y^2=z$ and $2y\frac{dy}{dx}=\frac{dz}{dx}$, we get

$\Rightarrow\frac{dz}{dx}+\frac{z}{x\ln x}=\frac{\cos x}{\ln x}$

Which is a LDE,

$\therefore y^2\ln x=z\ln x=\sin x+C$

Now, $y\left(\frac{3\pi}{2}\right)=0\Rightarrow C=1$

$\therefore y=\pm\sqrt{\frac{1+\sin x}{\ln x}}$

$y=\sqrt{\frac{1+\sin x}{\ln x}}\qquad(\because y>0)$

Required solution $y=a\sqrt{\frac{1+\sin x}{\ln x}}$ where a is any constant.

Q17. The given equation is

$(\sin^2x)(2ydy)+(2\sin x\cos x dx)y^2=2xdx$

or $d(\sin^2x\cdot y^2)=2xdx$

On integrating, we get

$\sin^2x\cdot y^2=x^2+C$

Q18.

Given that

$x\,dy = y\,dx + y^2 dy$

$\int \frac{ydx - xdy}{y^2} = \int -dy$

We know that $d\left(\frac{x}{y}\right) = \frac{ydx - xdy}{y^2}$

$\frac{x}{y} = -y + c$

As, $y(1) = 1$

$1 = -1 + c$

$\Rightarrow c = 2$

$\Rightarrow \frac{x}{y} = -y + 2$

If $x = -3$

$-3 = -y^2 + 2y$

$\Rightarrow y^2 - 2y - 3 = 0$

$\Rightarrow y = 3$

Q19.

The given differential equation is,

$y(1 + xy)dx - xdy = 0$

$\Rightarrow (ydx - xdy) + xy^2 dx = 0$

$\Rightarrow \frac{y\,dx - x\,dy}{y^2} + x\,dx = 0$

$\Rightarrow d\left(\frac{x}{y}\right) + x\,dx = 0$

On integrating both sides, we get

$\int d\left(\frac{x}{y}\right) + \int x dx = 0$

$\Rightarrow \frac{x}{y} + \frac{x^2}{2} = C$

If the above curve passes through the point $(1,\ 2)$, then,

$\frac{1}{2} + \frac{1}{2} = C \Rightarrow C = 1$

The given curve is $\frac{x}{y} + \frac{x^2}{2} = 1$

$\therefore\ y = \frac{2x}{2 - x^2}$

$\Rightarrow f(x) = \frac{2x}{2 - x^2}$

Q20. The given equation is

$\frac{dy}{dx} = \frac{yf'(x) - y^2}{f(x)}$

$\Rightarrow yf'(x)dx - f(x)dy = y^2 dx$

$\Rightarrow \frac{yf'(x)dx - f(x)dy}{y^2} = dx \Rightarrow d\left[\frac{f(x)}{y}\right] = dx$

On integration, we get

$\frac{f(x)}{y} = x + c \Rightarrow f(x) = y(x + c)$

Q21. $x^2 - y^2 = a^2$

$\Rightarrow 2x - 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{x}{y}$

Product of slope of $x^2 - y^2 = a^2$ and slope of orthogonal trajectory of $x^2 - y^2 = a^2$ is -1

$\therefore$ Slope of orthogonal trajectory of $x^2 - y^2 = a^2$ is

$\frac{dy}{dx} = -\frac{y}{x}$

$\int \frac{dx}{x} = \int -\frac{dy}{y}$

$\Rightarrow \log x = -\log y + \log c$

$\log xy = \log c$

$\Rightarrow xy = c$

which is a rectangular hyperbola.

Q22.

Let, $p = p(t)$

$\frac{dp}{dt} = 0.5p - 450$

$= \frac{p - 900}{2}$

$\Rightarrow \int_{850}^{p} \frac{2}{p - 900} dp = \int_0^t dt$

$\Rightarrow 2|\log|p - 900||_{850}^{p} = t$

$\Rightarrow 2|\log|p - 900| - \log|-50|| = t$

$\Rightarrow 2\log\left|\frac{p - 900}{-50}\right| = t \quad \ldots(1)$

$\Rightarrow \left|\frac{p - 900}{-50}\right| = e^{t/2}$

$\Rightarrow p = 900 - 50\, e^{t/2}$

Let, when $t = T,\ p = 0$ (using (1))

$\Rightarrow T = 2\ \ln 18$

Q1. Let $f(x), g(x)$ be two continuous differentiable functions satisfying the relationships $f'(x)=g(x)$ and $f''(x)=-f(x)$. Let $h(x)=[f(x)]^2+[g(x)]^2$. If $h(0)=5$, then $h(10)=$

(1) 10

(2) 5

(3) 15

(4) None of these

Q2. If $\log(x+y)-2xy=0$. Then $y'(0)=$

(1) 1

(2) -1

(3) 2

(4) 0

Q3. If $x=e^t\sin t, y=e^t\cos t, t$ is a parameter, then $\frac{d^2y}{dx^2}$ at $(1,1)$ is equal to

(1) $-\frac{1}{2}$

(2) $-\frac{1}{4}$

(3) 0

(4) $\frac{1}{2}$

Q4. If the derivative of $f(\tan x)$ w.r.t. $g(\sec x)$ at $x=\frac{\pi}{4}$, where $f'(1)=2$ and $g'(\sqrt{2})=4$ is $\frac{\sqrt{2}}{k}$ then find k ?

Q5. $\frac{d}{dx}\left(\tan^{-1}\frac{\cos x}{1+\sin x}\right)=$

(1) $-\frac{1}{2}$

(2) $\frac{1}{2}$

(3) -1

(4) 1

Q6. If $y=\sin^{-1}\left(\frac{5x+12\sqrt{1-x^2}}{13}\right)$, then $\frac{dy}{dx}$ is equal to

(1) $\frac{1}{\sqrt{1-x^2}}$

(2) $-\frac{1}{\sqrt{1-x^2}}$

(3) $\frac{3}{\sqrt{1-x^2}}$

(4) $\frac{-3}{\sqrt{1-x^2}}$

Q7. Let $\mathrm{f}(x)=x+\frac{1}{2x+\frac{1}{2x+\frac{1}{2x+\ldots\ldots\infty}}}$. Then the value of $f(100).f'(100)$ is

(1) 100

(2) $\frac{1}{100}$

(3) -100

(4) $-\frac{1}{100}$

Answer Key

Q1 (2) **Q2** (1) **Q3** (1) **Q4** (2.00)

Q5 (1) **Q6** (1) **Q7** (1)

Q1.

Given f(x), g(x) are two continuous differentiable functions satisfying $f'(x) = g(x)$, $f''(x) = g'(x)$ and $f''(x) = -f(x)$

Hence $g'(x) = -f(x)$

Given $h(x) = [f(x)]^2 + [g(x)]^2$

Differentiating the above equation,

$h'(x) = 2f(x)\, f'(x) + 2g(x)\, g'(x)$

$h'(x) = 2[f(x)\, g(x) + g(x)\, [-f(x)]]$

$h'(x) = 2[f(x)\, g(x) - f(x)\, g(x)] = 0$

$\therefore$ h(x) = C , a constant

$\therefore$ h(0) = C i.e. C = 5

h(x) = 5 for all x .

Hence h(10) = 5 .

Q2.

Given, $\log(x+y) - 2xy = 0 \quad \ldots\ldots\ldots(1)$

Differentiate equation (1) w.r.t x

$$\frac{1}{x+y}\left(1+\frac{dy}{dx}\right) - 2\left(x\frac{dy}{dx}+y\right) = 0$$
$$\left(\frac{1}{x+y}-2x\right)\frac{dy}{dx} + \left(\frac{1}{x+y}-2y\right) = 0$$
$$\Rightarrow \frac{dy}{dx} = -\frac{1-2xy-2y^2}{1-2xy-2x^2}$$

Hence, $y'(0) = -\dfrac{1-0-2y_{(0)}{}^2}{1-0-0} \quad \ldots\ldots\ldots(2)$

Now put $x=0$ in equation (1)

$\log(0+y) = 0 \Rightarrow y_{(0)} = 1$

from equation (2)

$y'(0) = 1$

Q3.

Given that, $x = e^t \sin t,\ y = e^t \cos t \quad \ldots(1)$

At point $(1,1)$, $1 = e^t \sin t,\ 1 = e^t \cos t$

$\tan t = 1 \ \Rightarrow t = \frac{\pi}{4}$

On differentiating Equation (1) w.r.t. x, we get

$\frac{dy}{dt} = e^t(\cos t - \sin t)$

and $\frac{dx}{dt} = e^t(\sin t + \cos t)$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\cos t - \sin t}{\cos t + \sin t}$$

Again differentiating w.r.t. x, we get

$$\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{\cos t-\sin t}{\cos t-\sin t}\right)\frac{dt}{dx}$$
$$= \left[\frac{[(\cos t+\sin t)(-\sin t-\cos t)-(\cos t-\sin t)(-\sin t+\cos t)]}{(\cos t+\sin t)^2}\right]\frac{dt}{dx}$$
$$= \frac{-2}{(\cos t+\sin t)^2}\cdot\frac{1}{e^t(\sin t+\cos t)}$$
$$= \frac{-2}{(e^t\cos t+e^t\sin t)}\cdot\frac{1}{(\cos t+\sin t)^2}$$
$$= \frac{-2}{x+y}\cdot\frac{1}{(\cos t+\sin t)^2} \quad [\text{from Equation}(1)]$$

At $t=\frac{\pi}{4}, x=1,\ y=1$

$$\therefore \frac{d^2y}{dx^2} = \frac{-2}{1+1}\cdot\frac{1}{\left(\cos\frac{\pi}{4}+\sin\frac{\pi}{4}\right)^2}$$
$$= \frac{-1}{\left[\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right]} = -\frac{1}{2}$$

Q4.

Let $u = f(\tan x)$ and $v = g(\sec x)$

$\Rightarrow \frac{du}{dx} = f'(\tan x)\sec^2 x$

and $\frac{dv}{dx} = g'(\sec x)\sec\ x\ \tan\ x$

$$\Rightarrow \frac{du}{dv} = \frac{du}{dx}\Big/\frac{dv}{dx} = \frac{f'(\tan\ x)\sec^2 x}{g'(\sec\ x)\sec\ x\ \tan\ x}$$
$$\Rightarrow \left[\frac{du}{dv}\right]_{x=\frac{\pi}{4}} = \frac{f'\left(\tan\frac{\pi}{4}\right)}{g'\left(\sec\frac{\pi}{4}\right)\sin\frac{\pi}{4}} = \frac{f'(1)\sqrt{2}}{g'\left(\sqrt{2}\right)}$$
$$= \frac{2\sqrt{2}}{4} = \frac{1}{\sqrt{2}}$$
$$= \frac{1}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$$
$$= \frac{\sqrt{2}}{2}$$
$$= \frac{\sqrt{2}}{k}$$

$\Rightarrow k = 2.$

Q5.

$$\frac{d}{dx}\left[\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)\right]$$
$$= \frac{d}{dx}\left[\tan^{-1}\left(\frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}\right)\right]$$
$$= \frac{d}{dx}\left[\tan^{-1}\left(\frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}\right)\right]$$
$$= \frac{d}{dx}\left[\tan^{-1}\left(\frac{1-\tan\left(\frac{x}{2}\right)}{1+\tan\left(\frac{x}{2}\right)}\right)\right]$$
$$= \frac{d}{dx}\left[\tan^{-1}\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right]$$
$$= -\frac{1}{2}$$

Q6.

Given,

$y=\sin^{-1}\left[\frac{5x+12\sqrt{1-x^2}}{13}\right]$

On putting $x=\sin\theta, 5=r\cos\alpha$ and

$12=r\sin\alpha$, we get

$y=\sin^{-1}\left(\frac{r\cos\alpha\sin\theta+r\sin\alpha\cos\theta}{13}\right)$

$=\sin^{-1}\left(\frac{13\sin(\theta+\alpha)}{13}\right);\left[\because\ r=\sqrt{25+144}=13\right]$

$=\theta+\alpha$

$=\sin^{-1}x+\tan^{-1}\left(\frac{12}{5}\right);\left[\because\ \tan\alpha=\frac{12}{5}\right]$

Differentiating both sides w.r.t. x, we get

$\frac{dy}{dx}=\frac{1}{\sqrt{1-x^2}}$

Q7. Let $y=$ f(x)= $x+\cfrac{1}{2x+\cfrac{1}{2x+\cfrac{1}{2x+\ldots\ldots\infty}}}$

$\Rightarrow y=x+\frac{1}{x+y}=\frac{x^2+xy+1}{x+y}$

$\Rightarrow xy+y^2=xy+x^2+1$

$\Rightarrow y^2-x^2=1$

Undefined y 2 = 1 + x 2

On differentiating

$\Rightarrow 2yy'=2x$

$\Rightarrow yy'=x$

$\Rightarrow f(100).\,f'(100)=100$

Q1. If the domain of $f(x)$ is $[1,3]$, then the domain of $f\left(\log_2\left(x^2+3x-2\right)\right)$ is

(1) $[-5,-4]\cup[1,2]$

(2) $[-13,-2]\cup\left[\frac{3}{5},5\right]$

(3) $[4,1]\cup[2,7]$

(4) $[-3,2]$

Q2. The domain of the function $f(x)=e^{\sin(x-[x])}+[x]\cos\left(\frac{\pi}{[x+1]}\right)$, (where $[\cdot]$ represents greatest integer function), is:

(1) R

(2) $R-[-1,0]$

(3) $R-[0,1]$

(4) $R-[-1,0)$

Q3. The range of values of m for which the line $y=mx$ and the curve $y=\frac{x}{x^2+1}$ enclose a region, is

(1) $(-1,1)$

(2) $(0,1)$

(3) 1]

(4) $(1,\infty)$

Q4. For $p>2$ and $x\in R$, if the number of natural numbers in the range of $f(x)=\frac{x^2+2x+p}{x^2+2x+2}$ is 3, then the value of p is equal to

(1) 3

(2) 4

(3) 5

(4) 6

Q5. If the graph of the function $f(x)=\frac{a^x-1}{x^n(a^x+1)}$ is symmetrical about Y-axis, then n equals

(1) 2

(2) $\frac{2}{3}$

(3) $\frac{1}{4}$

(4) $-\frac{1}{3}$

Q6. The function $f(x)=\frac{x}{e^x-1}+\frac{x}{2}+1$ is

(1) an odd function

(2) an even function

(3) neither an odd nor an even function

(4) a periodic function

Q7. The period of the function $f(\theta)=\sin\frac{\theta}{3}+\cos\frac{\theta}{2}$ is

(1) 3π

(2) 6π

(3) 9π

(4) 12π

Q8. If $f(x)=\frac{x}{\sqrt{1+x^2}}$, then $(fofof)(x)$ is

(1) $\frac{3x}{\sqrt{1+x^2}}$

(2) $\frac{x}{\sqrt{1+3x^2}}$

(3) $\frac{3x}{\sqrt{1-x^2}}$

(4) none of these

Q9. For $x\epsilon\left(0,\frac{3}{2}\right)$, let $f(x)=\sqrt{x}, g(x)=\tan x$ and $h(x)=\frac{1-x^2}{1+x^2}$. If $\phi(x)=(\text{hof})\,\text{og})\,(x)$, then $\phi\left(\frac{\pi}{3}\right)$ is equal to:

(1) $\tan\frac{\pi}{12}$

(2) $\tan\frac{5\pi}{12}$

(3) $\tan\frac{7\pi}{12}$

(4) $\tan\frac{11\pi}{12}$

Q10. The function $f: R\to R$ is defined as $f(x)=3^{-x}$. From the following statements,

I. f is one-one

II. f is onto

III. f is a decreasing function

the true statements are

(1) only I, II

(2) only II, III

(3) only I, III

(4) I, II, III

Q11. If $f: R\to A$ defined as $f(x)=\tan^{-1}\left(\sqrt{4\left(x^2+x+1\right)}\right)$ is surjective, then A is equal to

(1) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

(2) $\left[0,\frac{\pi}{2}\right)$

(3) $\left[\frac{\pi}{3},\frac{\pi}{2}\right)$

(4) $\left(0,\frac{\pi}{3}\right]$

Q12. If $A=\left\{x: -\frac{2}{5}\leq x\leq\frac{\pi-2}{5}\right\}, B=\{y: -1\leq y\leq 1\}$ and $f(x)=\cos(5x+2)$, then the mapping $f: A\to B$ is

(1) one-one but not onto

(2) onto but not one-one

(3) both one-one and onto

(4) neither one-one nor onto

Q13. Which of the following functions is inverse of itself?

(1) $f(t)=\frac{(1-t)}{(1+t)}$

(2) $f(t)=\frac{(1-t^2)}{(1+t^2)}$

(3) $f(t)=4^{\log t}$

(4) $f(t)=2^{t}$

Q14. The inverse of $f(x)=\frac{2}{3}\frac{10^{x}-10^{-x}}{10^{x}+10^{-x}}$ is

(1) $\frac{1}{3}\log_{10}\frac{1+x}{1-x}$

(2) $\frac{1}{2}\log_{10}\frac{2+3x}{2-3x}$

(3) $\frac{1}{3}\log_{10}\frac{2+3x}{2-3x}$

(4) $\frac{1}{6}\log_{10}\frac{2-3x}{2+3x}$

Q15. If $5f(x)+3f\left(\frac{1}{x}\right)=x+2$ and $y=xf(x)$ then $\left(\frac{dy}{dx}\right)_{x=1}$ is equal to

(1) $\frac{1}{4}$

(2) $\frac{7}{8}$

(3) 1

(4) None of these

Q16. If f is a function such that $2f(x)+f(2-x)=x^{2}$ then find value of $f(4)$.

Answer Key

Q1 (1)	**Q2** (4)	**Q3** (2)	**Q4** (3)
Q5 (4)	**Q6** (2)	**Q7** (4)	**Q8** (2)
Q9 (4)	**Q10** (3)	**Q11** (3)	**Q12** (3)
Q13 (1)	**Q14** (2)	**Q15** (2)	**Q16** (9.33)

Q1.

Domain of $f(x)$ is $[1,3]$

$1 \leq \log_2(x^2+3x-2) \leq 3$

$\Rightarrow 2 \leq x^2+3x-2 \leq 8$

It is the same as solving two inequalities.

Case 1 : $x^2+3x-2 \geq 2$

or, $x^2+3x-4 \geq 0$

or, $(x+4)(x-1) \geq 0$

$\therefore\ x \in (-\infty,-4] \cup [1,\infty)$

Case 2 : $x^2+3x-2 \leq 8$

or, $x^2+3x-10 \leq 0$

or, $(x+5)(x-2) \leq 0$

$\therefore\ x \in [-5,2]$

Considering both Case 1 and Case 2 we have,

$x \in [-5,-4] \cup [1,2]$

Q2.

We have,

$f(x)= e^{\sin\,(x-[x])}+[x]\cos\left(\frac{\pi}{[x+1]}\right)$

For $f(x)$ to be defined,

$[x+1] \neq 0$

$\Rightarrow [x]+1 \neq 0$

$\Rightarrow [x] \neq -1$

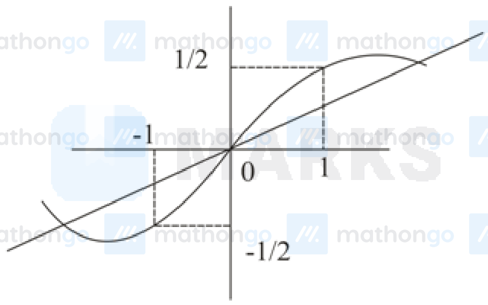
$\Rightarrow x \notin [-1,\ 0)$

Hence, domain of $f(x)$ is $x \in R-[-1,\ 0)$.

Q3.

Given:

$y=mx$ and $y=\frac{x}{x^2+1}$



Hence points of intersection are:

$x=0$

and

$x^2+1=\frac{1}{m}$

$x^2=\frac{1}{m}-1$

$\frac{1}{m}-1>0$

$\frac{1-m}{m}>0$

$\frac{m-1}{m}<0$

$m \in (0,1)$

Q4. Here, $f(x)=1+\frac{p-2}{(x+1)^2+1} \in (1,p-1]$

If there are 3 natural numbers in the range $\Rightarrow p-1=4 \Rightarrow p=5$

Q5.

$f(x)=\frac{a^x-1}{x^n(a^x+1)}$

since, $f(x)$ is symmetrical about Y-axis.

(i.e., it is a even function.)

$\therefore\ f(x)=f(-x)$

$\Rightarrow \frac{a^x-1}{x^n(a^x+1)}=\frac{a^{-x}-1}{(-x)^n(a^{-x}+1)}$

$\Rightarrow \frac{a^x-1}{x^n(a^x+1)}=\frac{1-a^x}{(-x)^n(1+a^x)}$

$\Rightarrow x^n=-(-x)^n$

Hence, the value of n which satisfy this relation, is $-\frac{1}{3}$

Q6.

Given, $f(x)=\frac{x}{e^x-1}+\frac{x}{2}+1$

$=\frac{2x+xe^x-x}{2(e^x-1)}+1=\frac{x+xe^x}{2(e^x-1)}+1$

$f(-x)=\frac{-x-xe^{-x}}{2(e^{-x}-1)}+1=\frac{x+xe^x}{2(e^x-1)}+1$

$\therefore f(-x)=f(x)$ for all x.

$\therefore f(x)$ is an even function.

Q7. Period of $\sin\frac{\theta}{3}=6\pi$

And period of $\cos\frac{\theta}{2}=4\pi$

$\therefore$ Period of $f(x)=\text{LCM}(6\pi,\ 4\pi)=12\pi$

Q8.

Given:

$f(x)=\frac{x}{\sqrt{1+x^2}}$

Then,

$(fof)(x)=f[f(x)]=f\left(\frac{x}{\sqrt{x^2+1}}\right)=\frac{\frac{x}{\sqrt{1+x^2}}}{\sqrt{1+\frac{x^2}{1+x^2}}}=\frac{x}{\sqrt{2x^2+1}}$

$(fofof)(x)=f[f\{f(x)\}]=f\left(\frac{x}{\sqrt{2x^2+1}}\right)=\frac{\frac{x}{\sqrt{2x^2+1}}}{\sqrt{1+\frac{x^2}{2x^2+1}}}=\frac{x}{\sqrt{1+3x^2}}$

Q9.

$f(x)=\sqrt{x}$

$g(x)=\tan x$

$h(x)=\frac{1-x^2}{1+x^2}$

$fog(x)=\sqrt{\tan x}$

$hofog(x)=h(\sqrt{\tan x})$

$= \frac{1-\tan x}{1+\tan x} = \tan\left(\frac{\pi}{4} - x\right)$

$\phi(x) = \tan\left(\frac{\pi}{4} - x\right)$

$\phi\left(\frac{\pi}{3}\right) = \tan\left(\frac{\pi}{4} - \frac{\pi}{3}\right)$

$= \tan\left(-\frac{\pi}{12}\right) = -\tan\frac{\pi}{12}$

$= \tan\left(\pi - \frac{\pi}{12}\right) = \tan\frac{11\pi}{12}$

Q10.

Since, $f : R \rightarrow R$ such that $f(x) = 3^{-x}$

Let y_1 and y_2 be two elements of $f(x)$ such that $y_1 = y_2$

$\Rightarrow\ 3^{-x_1} = 3^{-x_2}\ \Rightarrow\ x_1 = x_2$

Since, if two images are equal, then their elements are equal, therefore it is one-one function.

Since, $f(x)$ is positive for every value of x, therefore $f(x)$ is into.

On differentiating w.r.t. x, we get $\frac{dy}{dx} = -3^{-x}\log 3 < 0$ for every value of x.

$\therefore$ It is decreasing function.

$\therefore$ Statement I and III are true.

Q11.

$\because\ x^2 + x + 1 = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$

$\therefore\ 4\left(x^2 + x + 1\right) \geq 3$

$\Rightarrow\ \sqrt{4(x^2 + x + 1)} \geq \sqrt{3}$

$\Rightarrow \tan^{-1}\left(\sqrt{4(x^2 + x + 1)}\right) \geq \tan^{-1}\left(\sqrt{3}\right)$

$\Rightarrow f\left(x\right) \geq \frac{\pi}{3}$

$\therefore$ Range of $f(x)$ *is* $\left[\frac{\pi}{3}, \frac{\pi}{2}\right)$

Q12.

Let $t = 5x + 2$, then $A = (t : 0 \leq t \leq \pi)$
A bijective function is both one-one and onto.

$\therefore\ f(t) = \cos t$, which is bijective in $[0, \pi]$ since $\cos t$ is decreasing in $[0, \pi]$ and Range of $\cos t$ is $[-1, 1]$.

Hence, $f(x)$ is bijective.

Q13. Let $y = f(t)\ \ \therefore t = f^{-1}(y)$

Now, $y = f(t) = \frac{1-t}{1+t} \Rightarrow y + ty = 1 - t$

$\Rightarrow t + ty = 1 - y \Rightarrow t = \frac{1-y}{1+y}$

i.e., $f^{-1}(y) = \frac{1-y}{1+y}$ or $f^{-1}(t) = \frac{1-t}{1+t}$

Thus, this function is inverse of itself.

Q14.

Given,

$y = \frac{2}{3}\left[\frac{10^x - 10^{-x}}{10^x + 10^{-x}}\right]$

$y = \frac{2}{3}\left[\frac{10^{2x}-1}{10^{2x}+1}\right]$

$\Rightarrow 10^{2x} = \frac{3y+2}{2-3y}$

Taking log both sides, we get

$2x\log_{10}(10) = \log_{10}\left(\frac{3y+2}{2-3y}\right)$

$\Rightarrow x = \frac{1}{2}\log_{10}\left(\frac{2+3y}{2-3y}\right)$

$\therefore\ f^{-1}(x) = \frac{1}{2}\log_{10}\left(\frac{2+3x}{2-3x}\right)$.

Q15. $5f(x) + 3f\left(\frac{1}{x}\right) = x + 2\ \ \ldots(1)$

Replacing x by $\frac{1}{x}$

$\therefore\ 5f\left(\frac{1}{x}\right) + 3f(x) = \frac{1}{x} + 2\ \ \ldots(2)$

From (1)

$25f(x) + 15f\left(\frac{1}{x}\right) = 5x + 10\ \ \ \ldots(3)$

and from (2)

$9f(x) + 15f\left(\frac{1}{x}\right) = \frac{3}{x} + 6\ \ \ \ldots(4)$

Subtracting (4) from (3)

$\therefore 16f(x) = 5x - \frac{3}{x} + 4$

$\therefore xf(x) = \frac{5x^2 - 3 + 4x}{16} = y$

$\therefore \frac{dy}{dx} = \frac{10x+4}{16}$

$\left.\frac{dy}{dx}\right|_{x=1} = \frac{10+4}{16} = \frac{7}{8}$.

Q16.

Given, $2f(x) + f(2-x) = x^2\ \ \ldots(i)$

Substitute x by $(2-x)$ in equation (i) we get,

$2f(2-x) + f(x) = (2-x)^2\ \ \ldots(ii)$

Now, $(i) \times 2 - (ii)$ we get,

$3f(x) = 2x^2 - (2-x)^2$

$\Rightarrow f(x) = \frac{1}{3}\left[2x^2 - (2-x)^2\right]$

Then, $f((4)) = \frac{1}{3}\left[2(4)^2 - (2-4)^2\right]$

$= \frac{1}{3}[32 - 4] = \frac{28}{3} = 9.33$

Q1. If $f(x)$ is a differentiable function such that $\int f(x)dx = 2[f(x)]^2 + C$, (where, C is the constant of integration) and $f(1) = 1/4$, then $f(\pi)$ equals

(1) $\frac{3}{4}$

(2) $\frac{\pi}{4}$

(3) $\frac{2}{3}$

(4) $\frac{7}{6}$

Q2. $\int \sqrt{\frac{1-x}{1+x}} \mathrm{dx} =$

(1) $\sin^{-1} \mathrm{x} - \frac{1}{2}\sqrt{1-\mathrm{x}^2} + \mathrm{c}$

(2) $\sin^{-1} \mathrm{x} + \frac{1}{2}\sqrt{1-\mathrm{x}^2} + \mathrm{c}$

(3) $\sin^{-1} x - \sqrt{1-x^2} + c$

(4) $\sin^{-1} \mathrm{x} + \sqrt{1-\mathrm{x}^2} + \mathrm{c}$

Q3.

$\int e^{-x} \tan^{-1}(e^x)dx = f(x) - \frac{1}{2}\log(1+e^{2x}) + c \Rightarrow f(x) =$

(1) $e^x - e^{-x} \tan^{-1}(e^x)$

(2) $x^2 + e^{-x} \tan^{-1}(e^x)$

(3) $-e^x \tan^{-1}(e^x)$

(4) $x - e^{-x} \tan^{-1}(e^x)$

Q4. $\int \frac{\cos x - \sin x}{7 - 9\sin 2x} dx =$

(1) $\frac{1}{24}\log\left|\frac{4+3(\sin x+\cos x)}{4-3(\sin x+\cos x)}\right| + c$

(2) $\frac{1}{24}\log\left|\frac{4-3(\sin x+\cos x)}{4+3(\sin x+\cos x)}\right| + c$

(3) $\frac{1}{24}\log\left|\frac{4-(\sin x-\cos x)}{4+(\sin x-\cos x)}\right| + c$

(4) $\frac{1}{24}\log\left|\frac{4+(\sin x-\cos x)}{4-(\sin x-\cos x)}\right| + c$

Q5. The value of $\int \frac{f(x)\phi'(x)+\phi(x)f'(x)}{(f(x)\cdot\phi(x)+1)\sqrt{\phi(x)\cdot f(x)-1}} dx$ is (Where C is the constant of integration)

(1) $\cos^{-1} \sqrt{f(x)^2 - \phi(x)^2}$

(2) $\tan^{-1}[f(x)\phi(x)]$

(3) $\sin^{-1} \sqrt{\frac{f(x)}{\phi(x)}}$

(4) None of these

Q6. $\int \left(\cos^{-3/7} x\right) \left(\sin^{-11/7} x\right) dx$ is equal to

(1) $\log\left|\sin^{4/7} x\right| + c$

(2) $\frac{4}{7}\tan^{4/7} x + c$

(3) $-\frac{7}{4}\tan^{-4/7} x + c$

(4) $\log\left|\cos^{3/7} x\right| + c$

Q7. If a function $f: R \rightarrow R$ is defined as $f(x) = \int \frac{x^8+4}{x^4-2x^2+2} dx$ and $f(0) = 1$, then which of the following is correct?

(1) $f(x)$ is an even function

(2) $f(x)$ is an onto function

(3) $f(x)$ is an odd function

(4) $f(x)$ is many one function

Q8. The value of indefinite integral: $\int \sqrt{\frac{\sin x - \sin^3 x}{1 - \sin^3 x}} dx$ equals: (where c is constant of integration)

(1) $\frac{3}{2}\cos^{-1}\left(\sin^{\frac{3}{2}} x\right) + c$

(2) $\frac{2}{3}\sin^{-1}\left(\sin^{\frac{3}{2}} x\right) + c$

(3) $\frac{3}{2}\sin^{-1}\left(\sin^{\frac{3}{2}} x\right) + c$

(4) $\frac{2}{3}\tan^{-1}\left(\sin^{\frac{3}{2}} x\right) + c$

Q9. If $\mathrm{I_n} = \int (\ln \mathrm{x})^{\mathrm{n}} \mathrm{dx}$, then $\mathrm{I_n} + \mathrm{nI_{n-1}} =$

(1) $\frac{(\ln \mathrm{x})^{\mathrm{n}}}{\mathrm{x}} + \mathrm{C}$

(2) $\mathrm{x}(\ln \mathrm{x})^{\mathrm{n}-1} + \mathrm{C}$

(3) $\mathrm{x}(\ln \mathrm{x})^{\mathrm{n}} + \mathrm{C}$

(4) None of these

Q10. $\int e^x \left[\frac{2+\sin 2x}{1+\cos 2x}\right] dx =$

(1) $e^x \tan x + C$

(2) $e^x + \tan x + C$

(3) $2e^x \tan x + C$

(4) $e^x \tan 2x + C$

Q11. If $\int \frac{3x+1}{(x-3)(x-5)} dx = \int \frac{-5}{(x-3)} dx + \int \frac{B}{(x-5)} dx$, then the value of B is

(1) 3

(2) 4

(3) 6

(4) 8

Q12. Find the ordered triplet (A, B, λ), If

$\int \frac{2\cos x - \sin x + \lambda}{\cos x + \sin x - 2} dx = A \log_e(|\cos x + \sin x - 2|) + Bx + C$

(1) $\left(\frac{1}{2}, \frac{3}{2}, -1\right)$

(2) $\left(\frac{3}{2}, \frac{1}{2}, -1\right)$

(3) $\left(\frac{1}{2}, -1, -\frac{3}{2}\right)$

(4) $\left(\frac{3}{2}, -1, -\frac{1}{2}\right)$

Q13. Evaluate: $\int \left\{\frac{(\log x - 1)}{1 + (\log x)^2}\right\}^2 dx$

(1) $\frac{x}{x^2+2} + C$

(2) $\frac{\log x}{(\log x)^2+1} + C$

(3) $\frac{x}{(\log x)^2+1} + C$

(4) $\frac{xe^x}{1+x^2} + C$

Q14. $\int e^{x \sin x}\left(x^2 \cos x + x \sin x + 1\right) dx$, equals (where C is constant of integration)

(1) $xe^{x^2 \sin x} + C$

(2) $x^2 e^{x \sin x} + C$

(3) $xe^{x \sin x} + C$

(4) $2x^2 e^{x \sin x} + C$

Answer Key

Q1 (2) **Q2** (4) **Q3** (4) **Q4** (1)

Q5 (4) **Q6** (3) **Q7** (2) **Q8** (2)

Q9 (3) **Q10** (1) **Q11** (4) **Q12** (2)

Q13 (3) **Q14** (3)

Q1. Differentiating w.r.t. 'x' we get $f(x)=4f(x)\cdot f'(x)$

$f'(x)=0$ or $\frac{1}{4}=f(x)$

Integrating, we have

$\int \frac{1}{4}dx=\int f'(x)dx$

$\Rightarrow \frac{x}{4}=f(x)+C$

As $f(1)=\frac{1}{4}\rightarrow C=0$

$\Rightarrow f(x)=\frac{x}{4}$

$\Rightarrow f(\pi)=\frac{\pi}{4}$

Q2.

$\int \sqrt{\frac{1-\mathrm{x}}{1+\mathrm{x}}}\mathrm{dx}$

Multiply and divide by $\sqrt{1-\mathrm{x}}$

$=\int \frac{1-\mathrm{x}}{\sqrt{1-\mathrm{x}^2}}\mathrm{dx}=\int \frac{\mathrm{dx}}{\sqrt{1-\mathrm{x}^2}}-\int \frac{\mathrm{x}}{\sqrt{1-\mathrm{x}^2}}\mathrm{dx}$

$=\sin^{-1}\mathrm{x}+\frac{1}{2}\int \frac{(-2\mathrm{x})}{\sqrt{1-\mathrm{x}^2}}\mathrm{dx}$

$=\sin^{-1}\mathrm{x}+\sqrt{1-\mathrm{x}^2}+\mathrm{c}$

Q3. Put $e^x=t,\ e^x dx=dt,\ dx=\frac{1}{t}\ dt$

$=\int \frac{\tan^{-1}t}{t^2}dt=\tan^{-1}t\left(\frac{-1}{t}\right)-\int\left(\frac{-1}{t}\right)\frac{1}{1+t^2}dt$

$=\int \frac{\tan^{-1}t}{t^2}+\frac{1}{2}\int \frac{2t}{t^2(1+t^2)}dt\ t^2=z,\ 2tdt=dz$

$=\frac{-\tan^{-1}t}{t}+\frac{1}{2}\int \frac{(z+1)-z}{z(1+z)}dz=\frac{-\tan^{-1}t}{t}+\frac{1}{2}\left[\int \frac{1}{z+1}dz\right]$

$=\frac{-\tan^{-1}t}{t}+\frac{1}{2}log\left|\frac{t^2}{1+t^2}\right|+c=\frac{-\tan^{-1}t}{t}+\frac{1}{2}\log\left|\frac{e^{2x}}{1+e^{2x}}\right|+c$

$=-e^{-x}\tan^{-1}(e^x)-\frac{1}{2}\log|1+e^{2x}|+\frac{1}{2}\ 2x+c$

$=x-e^{-x}\tan^{-1}(e^x)-\frac{1}{2}\log|1+e^{2x}|+c$

Q4.

$I=\int \frac{\cos x-\sin x}{7-9\sin 2x}dx$

$\Rightarrow\ I=\int \frac{\cos x-\sin x}{7-9[(1+\sin 2x)-1]}dx$

$\Rightarrow\ I=\int \frac{\cos x-\sin x}{7-9\left[\left(\sin^2 x+\cos^2 x+2\sin x\cos x\right)-1\right]}$

$\Rightarrow\ I=\int \frac{\cos x-\sin x}{7-9[(\sin x+\cos x)^2-1]}dx$

Let $\sin x+\cos x=t \Rightarrow(\cos x-\sin x)dx=dt$

$\Rightarrow\ I=\int \frac{dt}{7-9(t^2-1)}$

$\Rightarrow\ I=\int \frac{dt}{4^2-(3t)^2}$

$\Rightarrow\ I=\frac{1}{2\cdot 4}\cdot\frac{1}{3}\log\left|\frac{4+3t}{4-3t}\right|+c$

$\Rightarrow\ I=\frac{1}{24}\log\left|\frac{4+3(\sin x+\cos x)}{4-3(\sin x+\cos x)}\right|+c$

Q5.

Let $I=\int \frac{f(x)\phi'(x)+\phi(x)f'(x)}{(f(x)\cdot\phi(x)+1)\sqrt{\phi(x)\cdot f(x)-1}}dx$

Let $\phi(x)\cdot f(x)-1=t^2$

$\Rightarrow(\phi(x)\cdot f'(x)+f(x)\phi'(x))dx=2tdt$

$\therefore\ I=\int \frac{2tdt}{(t^2+2)\sqrt{t^2}}=\int \frac{2dt}{t^2+\left(\sqrt{2}\right)^2}$

$=2\cdot\frac{1}{\sqrt{2}}\tan^{-1}\frac{t}{\sqrt{2}}+C$

$=\sqrt{2}\tan^{-1}\left(\frac{\sqrt{\phi(x)\ f(x)-1}}{\sqrt{2}}\right)+C$

Q6. $\int\left(\cos^{-3/7}x\right)\left(\sin^{-11/7}x\right)dx=\int \frac{\sin^{-11/7}x}{\cos^{-11/7}x}\cdot\sec^2 x\ dx$

$=\int \tan^{-11/7}x\sec^2 x\ dx$

Put $\tan x=t\Rightarrow\ \sec^2 x\ dx=dt$

$\therefore\ I=\int \mathrm{t}^{-11/7}dt=-\frac{7}{4}\tan^{-4/7}x+c$

Q7.

We have, $f(x)=\int \frac{x^8+4}{x^4-2x^2+2}dx$

Now, $f(x)=\int \frac{(x^8+4+4x^4)-(4x^4)}{x^4-2x^2+2}dx$

$=\int\left(\frac{(x^4+2)^2-(2x^2)^2}{x^4-2x^2+2}\right)dx$

$=\int \frac{(x^4+2x^2+2)\ (x^4-2x^2+2)}{(x^4-2x^2+2)}dx$

Therefore, $f(x)=\frac{x^5}{5}+\frac{2x^3}{3}+2x+C$

$\Rightarrow f(0)=0+0+0+C=1$

$\Rightarrow C=1$

$\Rightarrow f(x)=\frac{x^5}{5}+\frac{2x^3}{3}+2x+1$

Range of $f(x)$ is R,

So, $f(x)$ is an onto function

$f(-x)\neq f(x)$,

So, $f(x)$ is not even

$f(-x)\neq -f(x)$,

So, $f(x)$ is not odd

$f'(x)>0\forall x\in R$,

So, $f(x)$ is one-one

Q8.

Let

$I=\int \sqrt{\frac{\sin x-\sin^3 x}{1-\sin^3 x}}\ dx$

$\Rightarrow I=\int\left\{\frac{(\sqrt{\sin x})\sqrt{1-\sin^2 x}}{\sqrt{1-\sin^3 x}}\right\} dx$

$\Rightarrow I=\int\left\{\frac{(\sqrt{\sin x})\cos x}{\sqrt{1-\sin^3 x}}\right\} dx$

Put $\sin^{\frac{3}{2}} x=t \Rightarrow \frac{3}{2}\sqrt{\sin x}\cos x dx=dt$

$\Rightarrow I=\frac{2}{3}\int\frac{dt}{\sqrt{1-t^2}}$

$\Rightarrow I=\frac{2}{3}\sin^{-1}t+c$

$\Rightarrow I=\frac{2}{3}\sin^{-1}\left(\sin^{\frac{3}{2}} x\right)+c$

Q9. Integrate by parts by taking $\int(\ln x)^n dx$ as

$I_n=\int(\ln x)^n .1 . dx$ $I_n=x(\ln x)^n-\int\frac{x(n)(\ln x)^{n-1}}{x}dx$

$=x(\ln x)^n-n I_{(n-1)}$

$\Rightarrow I_n+n I_{n-1}=x(\ln x)^n$

Q10.

Let, $I=\int e^x\left[\frac{2+\sin 2x}{1+\cos 2x}\right] dx$

We know that

$\sin 2x=2\sin x\cos x$ & $\cos 2x=2\cos^2 x-1$

Using the above formulas we can write

$I=\int e^x\left(\frac{2+2\sin x\cos x}{2\cos^2 x}\right)dx$

$I=\int e^x\left(\frac{2(1+\sin x\cos x)}{2\cos^2 x}\right)dx$

$I=\int e^x\left(\frac{1}{\cos^2 x}+\frac{\sin x\cos x}{\cos^2 x}\right)dx$

$I=\int e^x\left(\tan x+\sec^2 x\right)dx$

We know that $\int e^x(f(x)+f'(x))dx=e^x\cdot f+C$

Using the above formula we can write

$I=e^x\tan x+C$

Q11.

Given,

$\int\frac{3x+1}{(x-3)(x-5)}dx=\int\frac{-5}{(x-3)}dx+\int\frac{B}{(x-5)}dx$

So, let

$\frac{3x+1}{(x-3)(x-5)}=\frac{-5}{x-3}+\frac{B}{x-5}$

$\Rightarrow 3x+1=-5(x-5)+B(x-3)$

Putting $x=5$ in the above equation, we get

$3(5)+1=0+B(5-3)$

$\Rightarrow 16=2B$

or $B=8$.

Q12.

We have,

$\int\frac{2\cos x-\sin x+\lambda}{\cos x+\sin x-2}dx=A\log_e(|\cos x+\sin x-2|)+Bx+C$

Differentiating R. H. S. w.r.t. x, we get

$\frac{d}{dx}[A\log_e(|\cos x+\sin x-2|)+Bx+C]$

$=A\frac{\cos x-\sin x}{\cos x+\sin x-2}+B$

$=\frac{A\cos x-A\sin x+B\cos x+B\sin x-2B}{\cos x+\sin x-2}$

$\therefore \frac{2\cos x-\sin x+\lambda}{\cos x+\sin x-2}=\frac{A\cos x-A\sin x+B\cos x+B\sin x-2B}{\cos x+\sin x-2}$

So, $A+B=2,\ B-A=-1,\ \lambda=-2B$

$\Rightarrow A=\frac{3}{2},\ B=\frac{1}{2},\ \lambda=-1$

Q13.

Method1: by cross checking the options

Consider $f(x)=\frac{x}{(\log x)^2+1}$

$\therefore\ f'(x)=\frac{1+(\log x)^2-\frac{2x\log x}{x}}{\left(1+(\log x)^2\right)^2}$

$\therefore\ f'(x)=\frac{1+(\log x)^2-2\log x}{\left(1+\log^2 x\right)^2}=\left(\frac{(\log x-1)}{(1+\log x)^2}\right)^2$

$\therefore\ \int\left(\frac{(\log x-1)^2}{1+(\log x)^2}\right)dx=\int f'(x)dx=f(x)+C$

$\therefore\ \int\left(\frac{\log x-1}{1+(\log x)^2}\right)^2 dx=\frac{x}{1+(\log x)^2}+C$

Hence option 3 is the correct answer and we can check the other choices by the similar argument.

Alternate solution

$\int\left\{\frac{\log(x)-1}{1+(\log x)^2}\right\}^2 dx$

Put $\log(x)=t \Rightarrow x=e^t \Rightarrow dx=e^t dt$

$=\int e^t\left\{\frac{(t-1)^2}{(t^2+1)^2}\right\}dt$

$\int e^t\left\{\frac{t^2+1-2t}{(t^2+1)^2}\right\}dt$

$=\int e^t\left\{\frac{1}{t^2+1}+\left(\frac{-2t}{(t^2+1)^2}\right)\right\}dt$

$=\frac{e^t}{t^2+1}+C \quad \left[\because \int e^x(f(x)+f'(x))dx=e^x f(x)+C\right]$

$=\frac{x}{(\log x)^2+1}$

Q14.

Let $I=\int e^{x\sin x}\left(x^2\cos x+x\sin x+1\right)dx$

$\Rightarrow I=\int\left(x.(x\cos x+\sin x)e^{x\sin x}+1.e^{x\sin x}\right)dx$

Let $e^{x\sin x}=t \Rightarrow e^{x\sin x}(\sin x+x\cos x)dx=dt$

Using integration by parts,

$I=\int\left(x\frac{dt}{dx}+t\right)dx=x\,t+C=x\,e^{x\sin x}+C$, where C is the constant of integration.

Q1. At $t=0$, the function $f(t)=\frac{\sin t}{t}$ has

(1) A minimum

(2) A discontinuity

(3) A point of inflexion

(4) A maximum

Q2. The value of $\lim_{x \rightarrow 1} \frac{\sqrt[5]{x^2}-2 \sqrt[5]{x}+1}{4(x-1)^2}$ is equal to

Q3. If $\lim_{x \rightarrow 1} \frac{a x^2+b x+c}{(x-1)^2}=2$, then (a, b, c)

(1) $(2,-4,2)$

(2) $(2,4,2)$

(3) $(2,4,-2)$

(4) $(2,-4,-2)$

Q4. $\operatorname{Lim}_{x \rightarrow \infty} \frac{x(\log x)^3}{1+x+x^2}$ equals

(1) 0

(2) -1

(3) 1

(4) Does not exist

Q5. The value of $\lim_{\mathrm{x} \rightarrow 0}\left(\left[\frac{100 \mathrm{x}}{\sin \mathrm{x}}\right]+\left[\frac{99 \sin \mathrm{x}}{\mathrm{x}}\right]\right)$, (where [x] represents greatest integral function less than or equal to x) is 99λ. Then the value of λ is

Q6. $\lim_{x \rightarrow 0} \frac{1}{x} \sin^{-1}\left(\frac{2 x}{1+x^2}\right)$ is equal to

(1) -2

(2) 0

(3) 2

(4) ∞

Q7. If $f(x)=[\tan x]^2, x \in\left(0, \frac{\pi}{3}\right)$, then $f^{\prime}\left(\frac{\pi}{4}\right)$ (where, [.] is the greatest integer function)

(1) is equal to 1

(2) is equal to 0

(3) does not exist

(4) None of these

Q8. $\lim_{x \rightarrow 2} \frac{3^x+3^{3-x}-12}{3^{-\frac{x}{2}}-3^{1-x}}$ is equal to

Q9. The value of $\lim_{x \rightarrow \infty} \frac{e^{x+1} \log\left(x^3 e^{-x}+1\right)}{10 x^3}$ is equal to (Use $e=2.7$)

Q10. The value of $\lim_{x \rightarrow 0}(\cos x+\sin x)^{\frac{1}{x}}$ is equal to (take $e=2.71$)

Q11. $\lim_{x \rightarrow 0}\left\{(1+x)^{\frac{2}{x}}\right\}$ (where $\{.\}$ denotes the fractional part of x) is equal to

(1) e^2-7

(2) e^2-8

(3) e^2-6

(4) None of these

Q12. The value of $\lim_{x \rightarrow 0} \frac{\int_0^{x^2} \sin \sqrt{t}}{x^3} d t$ is

(1) 0

(2) $2 / 9$

(3) $1 / 3$

(4) $2 / 3$

Q13. Let $f: R \rightarrow R$ be a positive increasing function with $\lim_{x \rightarrow \infty} \frac{f(3 x)}{f(x)}=1$. Then $\lim_{x \rightarrow \infty} \frac{f(2 x)}{f(x)}=$

(1) 1

(2) $\frac{2}{3}$

(3) $\frac{3}{2}$

(4) 3

Answer Key

Q1 (4) **Q2** (0.01) **Q3** (1) **Q4** (1)

Q5 (2) **Q6** (3) **Q7** (3) **Q8** (36.00)

Q9 (0.27) **Q10** (2.71) **Q11** (1) **Q12** (4)

Q13 (1)

Q1. Given, $f(t) = \frac{\sin t}{t}$

At t = 0, first we will check continuity of the function.

Now, $LHL = f(0-h)$

$= \lim_{h\to 0} \frac{\sin(0-h)}{(0-h)}$

$= \lim_{h\to 0} \frac{-\sin h}{-h}$

$= 1$

$RHL = f(0+h)$

$= \lim_{h\to 0} \frac{\sin(0+h)}{(0+h)}$

$= \lim_{h\to 0} \frac{\sin h}{h} = 1$

And $f(0) = 1$

Since, $LHL = RHL = f(0)$

So, the function is continuous at t = 0

Now, we check the function is maximum or minimum

$f'(t) = \frac{1}{t}\cos t - \frac{1}{t^2}\sin t$

and $f''(t) = \frac{-1}{t}\sin t - \frac{1}{t^2}\cos t - \frac{1}{t^2}\cos t + \frac{2}{t^3}\sin t$

$= \frac{-\sin t}{t} - \frac{2\cos t}{t^2} + \frac{2\sin t}{t^2}$

For maximum or minimum value of $f(x)$, put

$f'(x) = 0$

$\Rightarrow \frac{\cos t}{t} - \frac{\sin t}{t^2} = 0$

$\Rightarrow \frac{\tan t}{t} = 1$

Now, $\lim_{t\to 0} f''(t)$

$= -\lim_{t\to 0}\left(\frac{\sin t}{t}\right) - 2\lim_{t\to 0}\left(\frac{t\cos t - \sin t}{t^3}\right) \quad \left[\frac{0}{0} \text{ from}\right]$

$= -1 - 2\lim_{t\to 0}\left(\frac{\cos t - \sin t - \cos t}{3t^2}\right)$ [using L' Hospital rule]

$= -1 + \frac{2}{3}\lim_{t\to 0}\frac{\sin t}{t}$

$= -1 + \frac{2}{3}\times 1 = \frac{-1}{3} < 0$

So, function f(t) is maximum at t = 0

Q2. $\lim_{x\to 1} \frac{\sqrt[5]{x^2} - 2\sqrt[5]{x} + 1}{4(x-1)^2} = \lim_{y\to 1} \frac{y^2 - 2y + 1}{4(y^5-1)^2}$

(Putting $\sqrt[5]{x} = y;\ \text{as } x \to 1,\ y \to 1$)

$\lim_{y\to 1} \frac{(y-1)^2}{4(y-1)^2(y^4+y^3+y^2+y+1)^2}$

$= \frac{1}{25\times 4} = 0.01$

Q3. Given, $\lim_{x\to 1} \frac{ax^2+bx+c}{(x-1)^2} = 2$

This limit will exist, if

$ax^2 + bx + c = 2(x-1)^2$

$\Rightarrow ax^2 + bx + c = 2x^2 - 4x + 2$

$\Rightarrow \quad a = 2,\ \ b = -4, \quad\quad c = 2$

Q4. $\text{Lim}_{x\to\infty} \frac{(\log x)^3 + x.3(\log x)^2 \times \frac{1}{x}}{1+2x}$

(By D.L. Hospital rule)

$\Rightarrow \text{Lim}_{x\to\infty} \frac{3(\log x)^2 \times \frac{1}{x} + 6(\log x)\times\frac{1}{x}}{2}$

(By D.L. Hospital rule)

$\Rightarrow \text{Lim}_{x\to\infty} \frac{3(\log x)^2 + 6\log x}{2x}$

(By D.L. Hospital rule)

$\Rightarrow \text{Lim}_{x\to\infty} \frac{6\log x \times \frac{1}{x} + \frac{6}{x}}{2}$

(By D.L. Hospital rule)

$\Rightarrow \text{Lim}_{x\to\infty} \frac{6\log x + 6}{2x}$

(By D.L. Hospital rule)

$\Rightarrow \text{Lim}_{x\to\infty} \frac{6\left(\frac{1}{x}\right) + 0}{2}$

(By D.L. Hospital rule)

$= \frac{\frac{6}{\infty}}{2} = 0$

Q5.

We know that,

$\lim_{x\to 0} \frac{\sin x}{x} \to 1^-$

then,

$\lim_{x\to 0}\left(\left[\frac{100x}{\sin x}\right]\right) \Rightarrow \lim_{x\to 0}\left(\left[\frac{100}{\frac{\sin x}{x}}\right]\right) = 100$

$\lim_{x\to 0}\left(\left[\frac{99\sin x}{x}\right]\right) \Rightarrow \lim_{x\to 0}\left(\left[99\frac{\sin x}{x}\right]\right) = 98$

Hence $\Rightarrow \lim_{x\to 0}\left[\frac{100x}{\sin x}\right] + \left[\frac{99\sin x}{x}\right] = 100 + 98 = 198$

$99\lambda = 198$

Hence, $\lambda = 2$

Q6. Put $x=\tan\theta \Rightarrow \quad \theta=\tan^{-1}x$

As $x\to 0 \Rightarrow \theta\to 0$

$\therefore \lim\limits_{\theta\to 0}\frac{1}{\tan\theta}\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$=\lim\limits_{\theta\to 0}\frac{1}{\tan\theta}\sin^{-1}(\sin 2\theta)$

$=\lim\limits_{\theta\to 0}\frac{2\theta}{\tan\theta}=2$

Q7. LHD : $f'\left(\frac{\pi}{4}\right)=\lim\limits_{h\to 0^+}\frac{f\left(\frac{\pi}{4}-h\right)-f\left(\frac{\pi}{4}\right)}{-h}$

$=\lim\limits_{h\to 0^+}\frac{\left[\tan\left(\frac{\pi}{4}-h\right)\right]^2-\left[\tan\frac{\pi}{4}\right]^2}{-h}$

$=\lim\limits_{h\to 0^+}\frac{0-1}{-h}\to\infty$

$\therefore f'\left(\frac{\pi}{4}\right)$ does not exist.

Q8.

Let, $3^{\frac{x}{2}}=t,\ x\to 2\Rightarrow t\to 3$

$\lim\limits_{t\to 3}\frac{t^2\frac{27}{t^2}-12}{\frac{1}{t}-\frac{3}{t^2}}=\lim\limits_{t\to 3}\frac{t^4+27-12t^2}{t-3}$

$\lim\limits_{t\to 3}\frac{(t^2-3)(t+3)(t-3)}{(t-3)}=6\times 6=36$

Q9.

Given, $\lim\limits_{x\to\infty}\frac{e\cdot e^x\log\left(\frac{x^3}{e^x}+1\right)}{10(x)^3}$

$=\lim\limits_{x\to\infty}\frac{e\ \log\left(\frac{x^3}{e^x}+1\right)}{\left(\frac{x^3}{e^x}\right)}\times\frac{1}{10}$

$=e\times 1\times\frac{1}{10}=0.27\left(\because\frac{x^3}{e^x}\to 0 \text{ if } x\to\infty\right)$

Q10.

$\because \lim_{x\to a}[f(x)]^{\phi(x)}=e^{\lim\limits_{x\to a}\phi(x)[f(x)-1]}$

as $f(x)\to 1$ & $\phi(x)\to\infty$ as $x\to a$

So, $\lim\limits_{x\to 0}(\cos x+\sin x)^{\frac{1}{x}}=e^{\lim\limits_{x\to 0}\frac{1}{x}(\cos x+\sin x-1)}$

$=e^{\lim\limits_{x\to 0}\frac{(-\sin x+\cos x)}{1}}=e$

Q11.

We know, $\left\{(1+x)^{2/x}\right\}=(1+x)^{2/x}-\left[(1+x)^{2/x}\right]$

Also, $\lim\limits_{x\to 0}(1+x)^{2/x}=e^2$

$\Rightarrow\lim\limits_{x\to 0}\left\{(1+x)^{2/x}\right\}=\lim\limits_{x\to 0}\left((1+x)^{2/x}\right)-\lim\limits_{x\to 0}\left((1+x)^{2/x}\right)$

$=e^2-\left[e^2\right]=e^2-7$

Q12.

Let $\mathrm{L}=\lim\limits_{x\to 0}\frac{\int_0^{x^2}\sin\sqrt{t}}{x^3}dt$

$=\lim\limits_{x\to 0}\frac{(\sin x)\cdot 2x}{3x^2}$ (by Leibnitz rule)

$=\frac{2}{3}\lim\limits_{x\to 0}\frac{\sin x}{x}$

$=\frac{2}{3}(1)$

$=\frac{2}{3}$

Hence, $\mathrm{L}=\frac{2}{3}$.

Q13. As f is a positive increasing function, we have

$f(x)<f(2x)<f(3x)$

Dividing by $f(x)$ leads to $1<\frac{f(2x)}{f(x)}<\frac{f(3x)}{f(x)}$

As $\lim\limits_{x\to\infty}\frac{f(3x)}{f(x)}=1$, we have by squeeze theorem

or sandwich theorem, $\lim\limits_{x\to\infty}\frac{f(2x)}{f(x)}=1$

Q1. If R be a relation defined as aRb iff $|a-b|>0$, then the relation is

(1) Reflexive

(2) Symmetric

(3) Transitive

(4) Symmetric and transitive

Q2. A relation R is defined as $(x, y) \in R \Rightarrow x^y = y^x$ for $x, y \in I - \{0\}$, where I is the set of all integers. Then the relation R is:

(1) reflexive but not symmetric

(2) svmmetric but not reflexive

(3) reflexive and symmetric both

(4) equivalence relation

Q3. Given

$n(A)=11, n(B)=13, n(C)=16, n(A \cap B)=3, n(B \cap C)=6, n(A \cap C)=5$ and $n(A \cap B \cap C)=2$, then the value of $n\left[A^c \cap (B \Delta C)\right]=$

(1) 4

(2) 7

(3) 13

(4) 23

Q4. Out of 64 students, the number of students taking Mathematics is 45 and the number of students taking both Mathematics and Biology is 10. Then, the number of students taking the only Biology is

(1) 18

(2) 19

(3) 20

(4) 17

Q5. In a class of 60 students, 25 students play cricket and 20 students play tennis and 10 students play both the games, then the number of students who play neither is

(1) 45

(2) 0

(3) 25

(4) 35

Q6. Let $P=\{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$ and $Q=\{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$ be two sets. Then

(1) $P \subset Q$ and $P \neq \varnothing$

(2) $Q \not\subset P$

(3) $P \not\subset Q$

(4) $P = Q$

Answer Key

Q1 (2)
Q2 (3)
Q3 (3)
Q4 (2)
Q5 (3)
Q6 (4)

Q1. Since, R is defined as aRb iff $|a-b|>0$.

Reflexive : aRa iff $|a-a|>0$

Which is not true. So, R is not reflexive.

Symmetric : aRb iff $|a-b|>0$

Now, bRa iff $|b-a|>0$

$\Rightarrow\ |a-b|>0\ \Rightarrow\ aRb$

Thus, R is symmetric.

Transitive : aRb iff $|a-b|>0$

bRc iff $|b-c|>0$

aRb & $bRa \Rightarrow\ aRa$ which is not correct, hence relation is not transitive.

Q2. $\because (x,\ y) \in R \Rightarrow x^y = y^x$

$\therefore (x,\ x) \in R$ as $x^x = x^x\ \forall\ x \in I-\{0\}$

$\therefore R$ is reflexive

Now $(x,\ y) \in R \Rightarrow x^y = y^x \Rightarrow y^x = x^y$

$\Rightarrow (y,\ x) \in R\ \ \therefore R$ is symmetric

Now $\ (x,\ y) \in R \Rightarrow x^y = y^x$ and $(y,\ z) \in R \Rightarrow y^z = z^y$

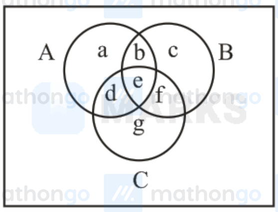
$\because x^y = y^x \Rightarrow y = x^{\frac{y}{x}}$

$\Rightarrow y^z = z^x \Rightarrow \left(x^{\frac{y}{x}}\right)^z = z^x \Rightarrow x^{\frac{yz}{x}} = z^x$

$\Rightarrow x^{yz} = z^{x^2} \Rightarrow \left(x,\ z\right) \notin R$

R is not transitive.

Q3.



Given

$n(A) = 11 \Rightarrow a+b+d+e = 11$

$n(B) = 13 \Rightarrow b+c+e+f = 13$

$n(C) = 16 \Rightarrow d+e+f+g = 16$

$n(A \cap B) = 3 \Rightarrow b+e = 3$

$n(B \cap C) = 6 \Rightarrow e+f = 6$

$n(A \cap C) = 5 \Rightarrow d+e = 5$

$n(A \cap B \cap C) = 2 \Rightarrow e = 2$

Solving all equations, we get

$e=2,\ d=3,\ f=4,\ b=1,\ a=5,\ c=6,\ g=7$

To find: $n[A^c \cap (B\Delta C)]$

$B\Delta C = b+c+d+g$

$A^c \cap (B\Delta C) = c+g$

$A^c \cap (B\Delta C) = c+g = 6+7 = 13$

Q4.

Let M & B denote the sets of students taking Mathematics and Biology respectively.

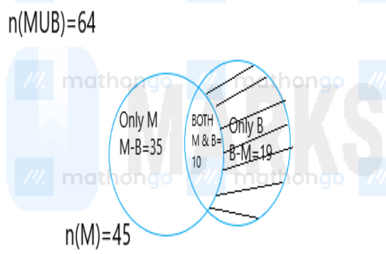
Given $n(M) = 45,\ n(M\cap B) = 10,\ n(M\cup B) = 64$

$\therefore n(B) = n(M\cup B) - n(M) + n(M\cap B)$

$\Rightarrow n(B) = 64-45+10 = 29$

$\therefore n(\text{only }\ B) = n(B-A) = n(B) - n(M\cap B) = 29-10 = 19$

Drawing the Venn diagram :



Therefore, the number of students taking only Biology $= 19$.

Q5. Let student play cricket $= C$

Student play tennis $= T$

and total number of students $= S$

$\therefore n(S) = 60,\ n(C) = 25,\ n(T) = 20$

and $n(C\cap T) = 10$

Now, $n(C\cup T) = n(C) + n(T) - n(C\cap T)$

$= 25+20-10 = 35$

$\therefore$ The number of students who play neither game

$= n(C\cap T)' = n(S) - n(C\cup T)$

$= 60-35 = 25$

Q6. In set P,

$\sin\theta = \left(\sqrt{2}+1\right)\cos\theta\ \Rightarrow\ \tan\theta =\ \sqrt{2}+1$

In set $Q,\ \left(\sqrt{2}-1\right)\sin\theta = \cos\theta$

$\Rightarrow \tan\theta = \frac{1}{\sqrt{2}-1} =\ \sqrt{2}+1 \Rightarrow\ \ P = Q$