UNIT 08

INTRODUCTION TO SOLID GEOMETRY

Session 28 Properties of Geometric objects

Session 29 Representation of 3D objects

Introduction

Geometry is a subject in mathematics that focuses on the study of shapes, sizes, relative configurations, and properties of geometrical bodies. The etymological meaning of the word Geometry, derived from the Greek word meaning " measurement of earth," is one of the ancient subject area of mathematics. Geometry was first formally structured by the famous Greek mathematician Euclid around 300 BC when he arranged 465 geometric propositions into 13 books, titled 'Elements'.

As a result of developments of Geometry by mathematicians, many types of geometry were created such as Euclidean geometry, non-Euclidean geometry, Riemannian geometry and algebraic geometry.

In this unit discussion primarily focuses on the properties of lines, points, and angles. We also describe on geometric measurements including lengths, areas, and volumes of various shapes.

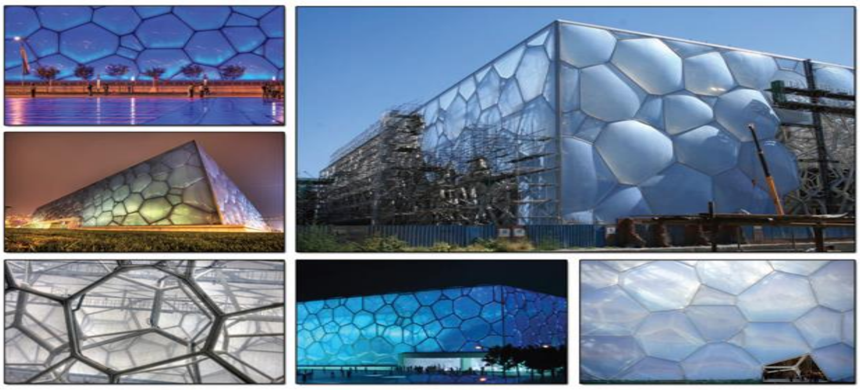
Session 28

Representation of 3-D Objects

Introduction

Buildings are solid geometrical shapes that are often constructed from basic geometrical shapes: cubes, pyramids, and spheres.

The Beijing National Aquatics Center, commonly called the Water Cube, is an amazing architectural and engineering feat. The building's design is based on how soap bubbles naturally connect to each other. Each "bubble" in this building is a large, inflated



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plastic pillow that uses solar energy to help regulate the heating inside the building. Even though the structure is not actually a cube, as the nickname implies, it is a marvel of modern design.

28.1 Two- Dimensional figures

In the plane geometry, each figure is restricted so that all its parts lie in the same plane. Such figures are called two dimensional figures.

A figure, all the parts of which lie in one straight line, is a one-dimensional figure, while a point is of zero dimensions.

28.2 Three Dimensional Figures

A figure not all the parts of which lie in the same plane is called a three-dimensional figure.

Thus, a figure consisting of a plane and a line not in the plane is a three-dimensional figure because the whole figure does not lie in a plane.

Solid geometry treats of the properties of three-dimensional figures.

28.2.1 Representation of a plane

While a plane is endless in extent in all its directions, it is represented by a parallelogram, some other limited plane figure.

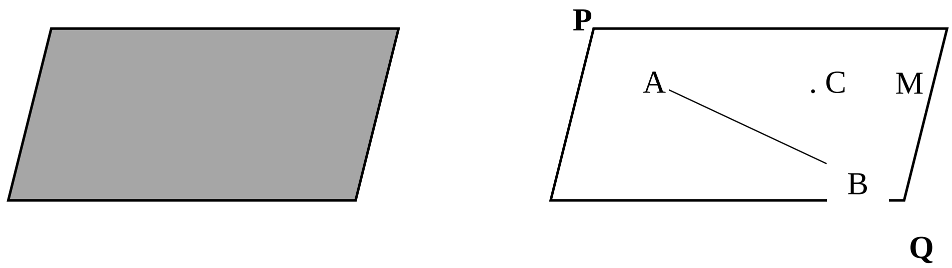


Figure 1

A plane is denoted by a single Roman alphabet or Greek alphabet letter on it, by two letters of opposite corners of the parallelogram representing it, or by any three letters on it but not the same straight line. Then, we can say ***plane M*** or the ***plane PQ*** or ***plane ABC.***

28.2.2 Figures in plane and solid geometry

In describing a figure in plane geometry, it is assumed that all parts of figure lie on the same plane.

In solid geometry, it is assumed that all parts need not lie on any one plane.

In plane geometry "Through a fixed point on a line, one and only one perpendicular can be drawn to the line."

28.2.3 Loci in plane and solid geometry

- In plane geometry, the locus of the points at a constant distance from a given point is a circle.

  In solid geometry, the locus of the points at a constant distance from a given point is

  sphere.

- In plane geometry, the locus of the points at a constant distance from a given line is a parallel line to the given line.
  In solid geometry, the locus of the points at a constant distance from a given line is a cylindrical surface whose radius is the given distance.
- In plane geometry, two lines that do not meet are called parallel lines.
  In solid geometry, two lines that do not meet need not be parallel. i.e. they may not in the same plane. The lines that are not parallel and do not meet are called skew lines.

28.3 Fundamental Geometric Concepts

- A physical solid occupies a limited portion of space. The portion of space occupied by a physical solid is called ***a geometric solid***.
- A geometric solid has a length, a breadth, and a thickness. The boundary of a solid is called ***a surface***.
- A surface has a length and a breadth, but it does not have a thickness. The boundary of a surface is called ***a line***.
- A line has only a length. Boundaries or extremities of a line are ***points.***

  A point has neither length, nor breadth, nor thickness. It is ***only a position***.

Now let us consider these concepts in reverse order.

- As we have considered geometric solid independently of surface, line, and point, so we may consider point independently, and from it build up to the solid. A small dot made with a sharp pencil on a sheet of paper represents approximately ***a geometric point.***
- If a point is allowed to move in space, the path in which it moves will be a line. A piece of fine wire, or. a line drawn on paper with a sharp pencil, represents approximately a geometric line. This, however fine it may be, has some thickness and is not therefore an ideal, or geometric line.
- If a line is allowed to move in space, its path in general will be ***a surface***.
- If a surface is allowed to move in space, its path in general will be a ***geometric solid***.

Representation of three-dimensional figures

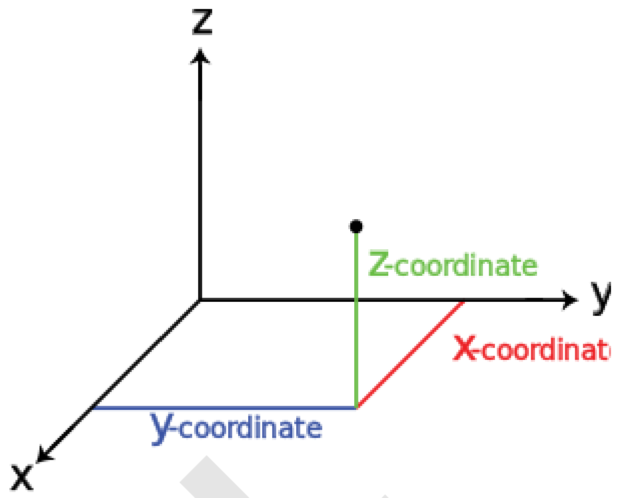


Figure 2

The student is already familiar with the physical objects about him, such as a ball or a block of wood. By a careful study of the following exercise, he may be led to see the relation of such physical solids to the geometric figures with which he must become familiar.

Activity 1

Look at a block of wood (or a chalk box).

- Has its weight? color? taste? shape? size? ***These are called properties of the solid***.
- What do we call such a solid? ***A physical solid***.
- Can you think of the properties of this solid apart from the block of wood?

  Imagine the block removed.
- Can you imagine the space which it occupied? What name would you give to this space?

  A geometric solid.
- What properties has it that the block possessed? ***Shape and size***.
- What is it that separates this geometric solid from surrounding space? How thick is this surface? How many surfaces has the block? Where do they

intersect? How many intersections are there? How wide are the intersections? how long? What is their name?

They are lines.

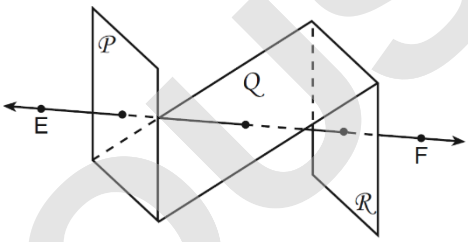
- Do these lines intersect? where? How wide are these intersections? how thick? how long? Can you say where this one is and so distinguish from where that one is? What is its name?

  It is a point.

- If you move the block through space, what will it generate as it moves? What will the surfaces of the block generate? all of them? Can you move a surface so that it will not generate a solid? ***Yes, by moving it along itself***.

Activity 2

1) As shown in the diagram below, $\overleftrightarrow{EF}$ intersects planes *P*, *Q*, and *R*.



If $\overleftrightarrow{EF}$ is perpendicular to planes *P* and *R*, which statement must be true?

1) Plane *P* is perpendicular to plane *Q.*
2) Plane *R* is perpendicular to plane *P.*
3) Plane *P* is parallel to plane *Q.*
4) Plane *R* is parallel to plane *P.*

2) If $\overleftrightarrow{AB}$ is contained in plane P, and $\overleftrightarrow{AB}$ is perpendicular to plane R, which statement is true?

1) $\overleftrightarrow{AB}$ is parallel to plane R.

2) Plane P is parallel to plane R.

3) $\overleftrightarrow{AB}$ is perpendicular to plane P.

4) Plane P is perpendicular to plane R.

Summary

In this session, we have introduced the concepts of two dimensions and three dimensions. We discussed the properties of geometric figures in two dimensions and three dimensions. We have described the concepts of point, line and plane. We have presented the method how to represent a point in three dimensions using coordinates.

Learning outcomes

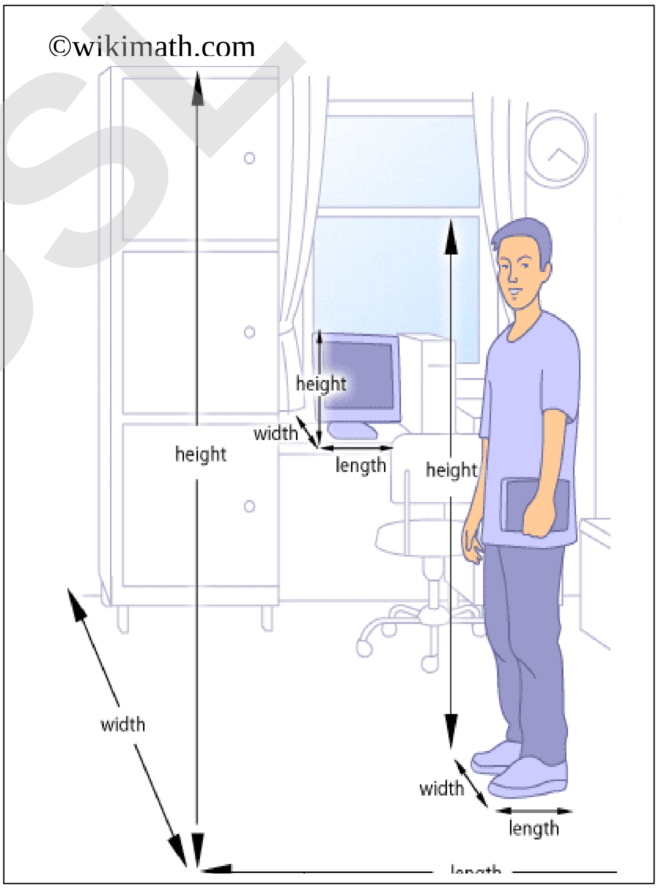
At the end of this session the student should be able to

1. describe the geometrical figures of 2-D and 3-D.
2. compare 2-D figures and 3-D figures.
3. represent 2-D figures and 3-D figures.

Session 29

Properties of geometrical objects

Introduction



We live in a three-dimensional world. Every object you can see or touch has three dimensions that can be measured: ***length***, ***width***, and ***height***. The room you are sitting in can be described by these three dimensions. The monitor you're looking at has these three dimensions. Even you can be described by these three dimensions. In fact, the clothes you are wearing were made specifically for a person with your dimensions.

In the world around us, there are many three-dimensional geometric shapes. In these lessons, you'll learn about some of the shapes introduced in Ordinary Level Mathematics were plane figures and therefore two-dimensional. Basic shapes also form the base and ***sides*** of objects in three dimensions—length, width, and height— and are called geometric solids.

29.1 geometric solids.

A box is an example of a geometric solid.
Volume measures how much something “holds”. Volume is found by multiplying the area of the base by the height of the figure. Each of the length, width, and height measurements must be in the same unit (feet, meters, inches, etc.), so when multiplied together the result is in cubic units: ft^3, m^3, in^3

- **Volume:** Each ***face*** or surface of a geometric solid is a basic shape. Use the area formula for
- **Surface area:** each surface and find the sum of all the surfaces. Measure ***surface area*** in square units.
- **Unit requirements:** All measurements must be in the same units or converted to the same units. Use standard conversion ratios for changing between units, if necessary.

29.2 Types of Geometric Solids

Solids come in 2 types: **polyhedra** and **non-polyhedra**.

Non-polyhedra describes any geometric solid that has any surface that is not flat, like a sphere, cone, or cylinder.

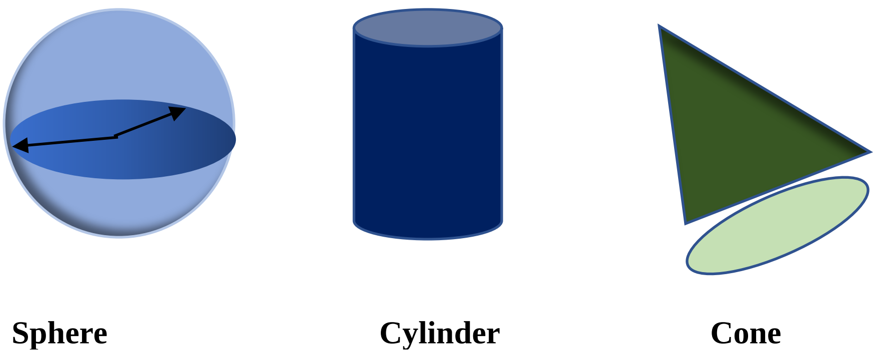


Figure 1

Polyhedra describes a geometric solid that has all flat faces, but the faces don't have to be the same size or shape. Polyhedra must have at least 4 faces but there is no limit to how many faces they can have. Some examples of polyhedra are pictured below:

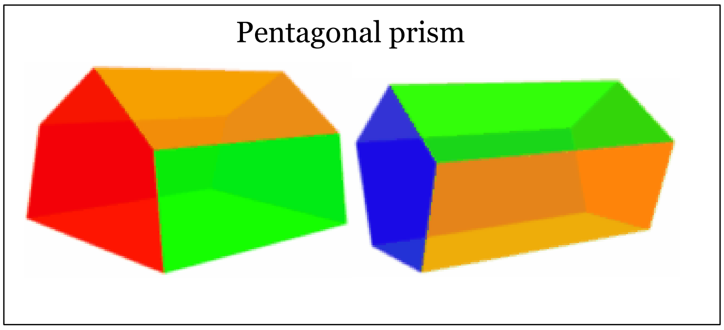


Figure 2

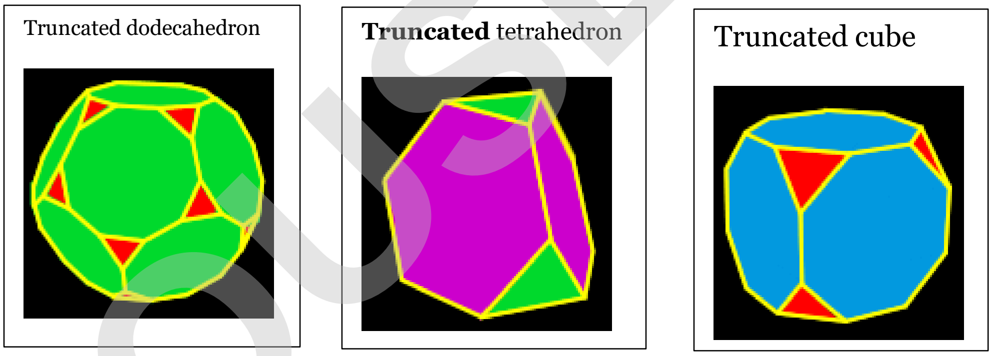


Figure 3

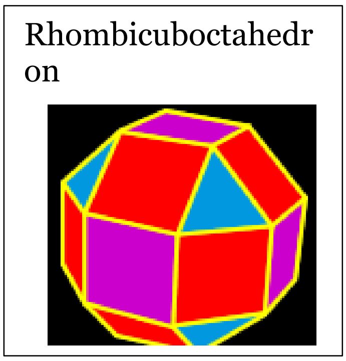


Figure 4

29.1.1 Cube

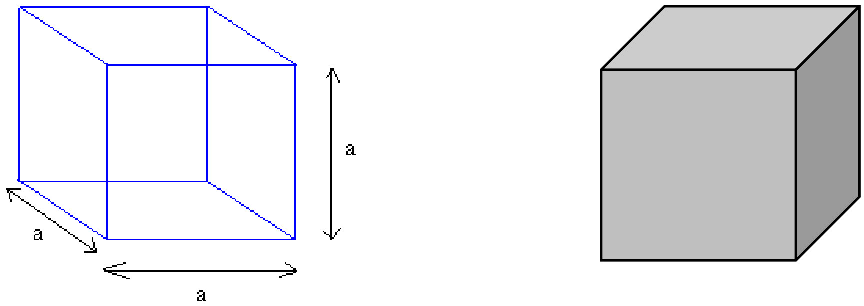


Figure 5

A **cube** is a three-dimensional solid object bounded by six square faces, facets or sides, with three meeting at each vertex.

- **Volume of a cube:**

  Volume is equal to the length of any one side raised to the third power.

$$\boldsymbol{V = a^3}$$

- **Surface area of a cube:**

  Each face (6 total) is an equivalent square.

$$\boldsymbol{S = 6a^2}$$

- **Observations:** Dice are cubes with dots indicating each side.

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29.1.2 Rectangular Parallelepiped

Also called a rectangular, ***prism*** or box. Each face is a rectangle and opposite
faces are equivalent. The dimensions are length, width, and height (or depth). The base is a rectangle.

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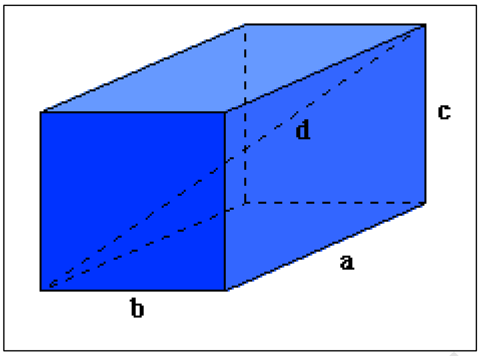


Figure 6

29.1.3 Sphere

A sphere is a perfectly round geometrical object in three- dimensional space that is the surface of a completely round ball.

Like a circle in a two-dimensional space, a sphere is defined mathematically as the set of points that are all at the same distance *a* from a given point, but in a three-dimensional space.

This distance *a* is the radius of the ball, which is made up from all points with a distance less than a from the given point, which is the ***center*** of the mathematical ball. These are also referred to as the radius and center of the sphere, respectively. The longest straight-line segment through the ball, connecting two points of the sphere, passes through the center and its length is thus twice the radius; it is a ***diameter*** of both the sphere and its ball.

While outside mathematics the terms "sphere" and "ball" are sometimes used interchangeably

Enclosed volume

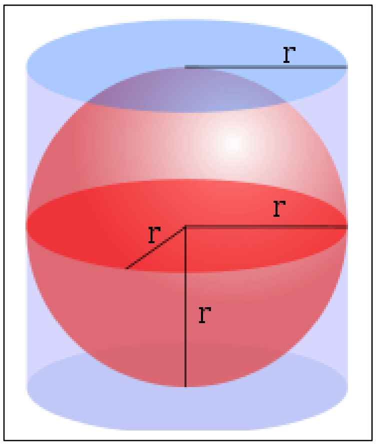


Figure 7

In three dimensions, the ***volume*** inside a sphere (that is, the volume of a ball, but classically referred to as the volume of a sphere) is

$$V = \frac{4}{3}\pi a^3$$

Archimedes first derived this formula by showing that the volume inside a sphere is twice the volume between the sphere and the ***circumscribed cylinder*** of that sphere of (having the height and diameter equal to the diameter of the sphere)

Surface area of a sphere of a radius a, $\boldsymbol{A = 4\pi a^2}$

29.1.4 Cylinder

- A ***cylindrical surface*** is a surface consisting of all the points on all the lines which are parallel to a given line and which pass through a fixed plane curve in a plane not parallel to the given line.
- Any line in this family of parallel lines is called an ***element*** of the cylindrical surface.
- A ***solid*** bounded by a cylindrical surface and two parallel planes is called a (solid) ***cylinder***.

- The line segments determined by an element of the cylindrical surface between the two parallel planes is called an ***element of the cylinder***.
- All the elements of a cylinder have equal lengths. The region bounded by the cylindrical surface in either of the parallel planes is called a ***base*** of the cylinder. The two bases of a cylinder are congruent figures.
- If the elements of the cylinder are perpendicular to the planes containing the bases, the cylinder is a ***right cylinder***, otherwise it is called an ***oblique cylinder***.
- If the bases are disks (regions whose boundary is a circle) the cylinder is called a ***circular cylinder***. In some elementary treatments, a cylinder always means a circular cylinder
- The ***height*** (or altitude) of a cylinder is the *perpendicular distance* between its bases.
- The cylinder obtained by rotating a *line segment* about a fixed line that it is parallel to is a ***cylinder of revolution***. A cylinder of revolution is a right circular cylinder.
  The height of a cylinder of revolution is the length of the generating line segment.
  The line that the segment is revolved about is called the ***axis*** of the cylinder and it passes through the centers of the two bases.

29.1.5 Cone

A **cone** is a three-dimensional geometric shape that tapers smoothly from a flat base to a point called the apex or ***vertex.***

- A cone is formed by a set of line segments, half lines or lines connecting a common point, the apex, to all of the points on a base that is in a plane that does not contain the apex.
- A Cone is a Rotated Triangle
  A cone can be made by rotating a triangle!

The triangle is a right- angled triangle, and it gets rotated around one of its two short sides.

The side it rotates around is the **axis** of the cone.

- When the apex is aligned on the center of the base it is a Right Cone otherwise it is an Oblique Cone:

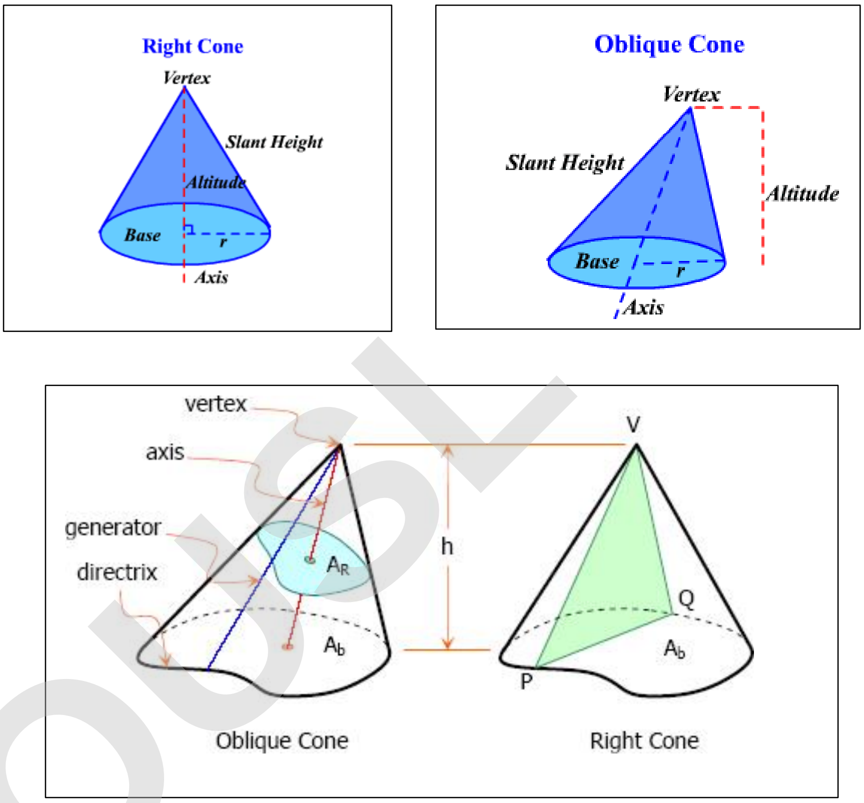
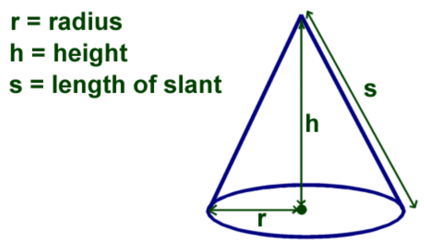


Figure 8

Surface Area of a Cone

Surface area has two parts

Base area = $\boldsymbol{\pi r^2}$ Side area $= \boldsymbol{\pi r s}$



Then, **Surface area** = $\boldsymbol{\pi r}(\boldsymbol{r} + \boldsymbol{s})$

- **Volume of a cone** = $\boldsymbol{\pi r^2 h}$

29.1.6 Prism

A **prism** is a Polyhedrane comprising an n-sided polygonal base, a second base which is a translated copy (rigidly moved without rotation) of the first, and n other faces (necessarily all Parallelogram) joining corresponding sides of the two bases. All cross sections parallel to the bases are translations of the bases. Prisms are named for their bases, so a prism with a pentagonal base is called a pentagonal prism. The prisms are a subclass of the prismoid.

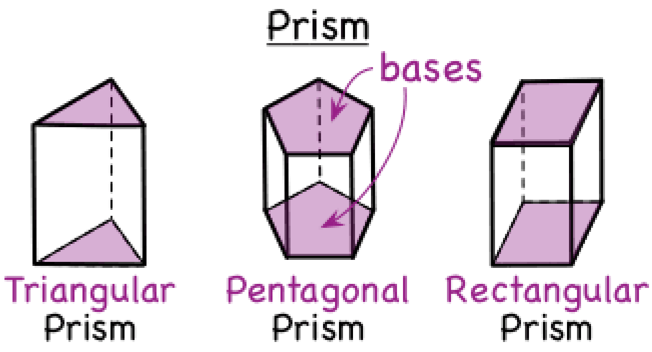
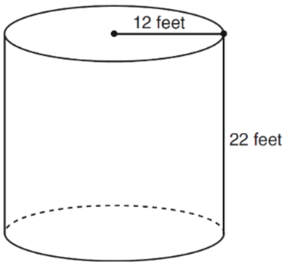


Figure 9

Activity 1

1) The cylindrical tank shown in the diagram below is to be painted. The tank is open at the top, and the bottom does *not* need to be painted. Only the outside needs to be painted. Each can of paint covers 600 square feet. How many cans of paint must be purchased to complete the job?



2) If the surface area of a sphere is represented by 144π, what is the volume in terms of π?

1) 36π

2) 48π

3) 216π

4) 288π

Summary

In this session, we have discussed the properties of geometric solids. We have introduced the volume and surface area of some geometric solids. We have described polyhedral and non-polyhedral solids. We have presented the formulae to calculate the volume and surface area of some geometric solid such as a cube, a sphere and a cone.

Learning outcomes

At the end of this session the student should be able to

1. Recognize some geometric solids such as sphere, cone and cylinder.
2. Describe some properties of geometric solids.
3. Calculate the volume and surface area of some geometric solids.