Assignment 3

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Q2) Problem 1 :-

Beta Distribution $\Rightarrow p(x;\alpha,\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha).\Gamma(\beta)} * x^{\alpha-1} * (1-x)^{\beta-1}$

Likelihood $\Rightarrow L(\theta;D) = \prod_{i=1}^{N} \theta^{\alpha}(1-\theta)^{1-\alpha}$

Likelihood $P(\theta;\alpha,\beta) = \prod_{j=1}^{N} \prod_{k=1}^{c} \prod_{j=1}^{|H|} \left(\pi_k \theta_{jk}^{\alpha_{jk}} (1-\theta_{jk})^{1-\alpha_{jk}}\right)^{y_{jk}}$

$P(\theta/D;\alpha,\beta) \propto \left(\theta_{jk}^{x_{jj}} (1-\theta_{jk})^{1-x_{jj}}\right).\left(\theta^{\alpha-1}(1-\theta)^{\beta-1}\right)$

$\therefore P(\theta/D;\alpha,\beta) \propto \theta_{jk}^{x_{jj}+\alpha-1} . (1-\theta_{jk})^{\beta-x_{jj}+1-1}$

$\therefore P(\theta/D;\alpha,\beta) = \text{Beta}(x_{jj}+\alpha, \beta-x_{jj}+1)$

$\therefore \theta^{MAP} = \underset{\theta}{\text{argmax}}\left(P(\theta/D;\alpha,\beta)\right)$

$$\boxed{\theta^{MAP} = \frac{x_{jj}+\alpha-1}{\alpha+\beta-2}}$$

Q2) Problem 2 :-

Dirchlet Distribution $\Rightarrow P(x_1, x_2 \dots, x_{K-1}; \alpha_1, \alpha_2 \dots \alpha_K) = \frac{\Gamma(\sum_{i=1}^{K} \alpha_i)}{\prod_{i=1}^{K} \Gamma(\alpha_i)} \cdot \prod_{j=1}^{K} x_j^{\alpha_j - 1}$

Likelihood $\Rightarrow L(\pi, \theta / D) = \prod_{i=1}^{N} \prod_{k=1}^{C} \left(\pi_k \cdot \frac{L^{\circ}}{\prod_{i=1}^{|V|} f_{ij}!} \cdot \prod_{i=1}^{|V|} \theta_{jk}^{f_{ij}} \right)^{y_{ik}}$

$P(\theta / D ; \alpha_{1 \dots K}) \propto \left(\theta_j^{\alpha_j - 1}\right) \cdot \left(\theta_{jk}^{f_{ij}}\right)$

$\propto \theta_j^{\alpha_j + f_{ij} - 1}$

$P(\theta / D, \alpha_{1 \dots K}) = \text{Dirichlet}\left(f_{ij} + \alpha_j\right)$

$$\boxed{\theta^{MAP} = \alpha_j + f_{ij} - 1}$$

Q2) Problem 3 :-

Bernoulli MLE $\Rightarrow \theta_{jk} = \frac{\sum_{i=1}^{N} \mathbb{I}(x_{ij} = 1 \wedge y_{ik} = 1)}{\sum_{i=1}^{N} \mathbb{I}(y_{ik} = 1)}$

$\pi_k = \frac{\sum_{i=1}^{N} \mathbb{I}(y_{ik} = 1)}{N}$

MAP $\Rightarrow \theta^{MAP} = \frac{x_{ij} + \alpha - 1}{\alpha + \beta - 1}$

Multinomial

~~Dirichlet~~ MLE $\Rightarrow$ $\theta_{pq} = \dfrac{\sum_{i=1}^{N} \mathbb{I}(y_{iq} = 1) f_{ip}}{\sum_{i=1}^{N} \mathbb{I}(y_{iq} = 1) l_i}$

$$\pi_k = \frac{\sum_{i=1}^{N} \mathbb{I}(y_{ik} = 1)}{N}$$

MAP $\Rightarrow$ $\theta^{MAP} = \alpha_j + f_{ij} - 1$