Assignment 2

Date: / /

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Q4) Univariate dataset : $\begin{bmatrix} x_0 & y_0 \\ x_1 & y_1 \\ \vdots & \vdots \\ x_n & y_n \end{bmatrix}$

For univariate linear regression model :

$$\underbrace{\hat{y}}_{\text{predicted output}} = w_0 + w_1 \underbrace{X}_{\text{feature array}}$$

To get $w_0, w_1 \Rightarrow$ minimize sum of squared errors function $(J(w))$

$$\therefore J(w) = \frac{1}{2} \sum_{i=1}^{N} (\hat{y}_i - y_i)^2$$

$$= \frac{1}{2} \sum_{i=1}^{N} (w_0 + w_1 x_i - y_i)^2$$

$$= \frac{1}{2} (Xw - y)^2$$ [$\Leftarrow$ In matrix notation]

$$= \frac{1}{2} (Xw - y)^T (Xw - y)$$

$$\nabla_w J(w) = \nabla_w \frac{1}{2} (Xw - y)^T (Xw - y)$$

$$= \frac{1}{2} \nabla_w (w^T X^T X w - w^T X^T y - y^T X w + y^T y)$$

$$= \frac{1}{2} \nabla_w (w^T X^T X w - 2w^T X^T y + y^T y)$$

$$= \frac{1}{2} \times 2 (X^T X w - X^T y)$$

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$$= X^T X w - X^T y$$

As we are trying to minimize, set $\nabla_w J(w) = 0$

$$\therefore X^T X w - X^T y = 0$$

$$\therefore \boxed{w = (X^T X)^{-1} X^T y}$$

For univariate dataset :

$$w = \underset{2\times 1}{\begin{bmatrix} w_0 \\ w_1 \end{bmatrix}} \quad X = \underset{n\times 2}{\begin{bmatrix} 1 & x_0 \\ 1 & x_1 \\ \vdots & \vdots \\ 1 & x_{n-1} \end{bmatrix}} \quad y = \underset{n\times 1}{\begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_{n-1} \end{bmatrix}}$$

Plugging these values above we get :

$$\begin{bmatrix} w_0 \\ w_1 \end{bmatrix} = \left(\begin{bmatrix} 1 & 1 & \cdots & 1 \\ x_0 & x_1 & \cdots & x_{n-1} \end{bmatrix} \begin{bmatrix} 1 & x_0 \\ 1 & x_1 \\ \vdots & \vdots \\ 1 & x_{n-1} \end{bmatrix} \right)^{-1} \times \begin{bmatrix} 1 & 1 & \cdots & 1 \\ x_0 & x_1 & \cdots & x_{n-1} \end{bmatrix} \begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_{n-1} \end{bmatrix}$$

$$= \left(\begin{bmatrix} n & \sum_{i=0}^{n-1} x_i \\ \sum_{i=0}^{n-1} x_i & \sum_{i=0}^{n-1} x_i^2 \end{bmatrix} \right)^{-1} \times \begin{bmatrix} \sum_{i=0}^{n-1} y_i \\ \sum_{i=0}^{n-1} x_i y_i \end{bmatrix}$$

$$= \frac{1}{n \sum_{i=0}^{n-1} x_i^2 - \left(\sum_{i=0}^{n-1} x_i \right)}$$

$$= \frac{1}{n \sum_{i=0}^{n-1} x_i^2 - \left(\sum_{i=0}^{n-1} x_i \right)^2} \begin{bmatrix} \sum_{i=0}^{n-1} x_i^2 & -\sum_{i=0}^{n-1} x_i \\ -\sum_{i=0}^{n-1} x_i & n \end{bmatrix} \begin{bmatrix} \sum_{i=0}^{n-1} y_i \\ \sum_{i=0}^{n-1} x_i y_i \end{bmatrix}$$

$$= \frac{1}{n\sum_{i=0}^{n-1} x_i^2 - \left(\sum_{i=0}^{n-1} x_i\right)^2} \begin{bmatrix} \sum_{i=0}^{n-1} x_i^2 \cdot \sum_{i=0}^{n-1} y_i - \sum_{i=0}^{n-1} x_i \sum_{i=0}^{n-1} x_i y_i \\ n\sum_{i=0}^{n-1} x_i y_i - \sum_{i=0}^{n-1} x_i \sum_{i=0}^{n-1} y_i \end{bmatrix}$$

$$\therefore w_0 = \frac{1}{n\sum_{i=0}^{n-1} x_i^2 - \left(\sum_{i=0}^{n-1} x_i\right)^2} \times \left(\sum_{i=0}^{n-1} x_i^2 \sum_{i=0}^{n-1} y_i - \sum_{i=0}^{n-1} x_i \sum_{i=0}^{n-1} x_i y_i\right)$$

$$w_1 = \frac{1}{n\sum_{i=0}^{n-1} x_i^2 - \left(\sum_{i=0}^{n-1} x_i\right)^2} \left(n\sum_{i=0}^{n-1} x_i y_i - \sum_{i=0}^{n-1} x_i \sum_{i=0}^{n-1} y_i\right)$$

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