Assignment 3

Q1) 1-4-1

$$\sigma(a) = \frac{1}{1+e^{-a}}$$

given $a = w^T x$

$$\therefore \sigma(a) = \sigma(w^T x) = \frac{1}{1+e^{-(w^T x)}}$$

$$\frac{d\sigma(x)}{da} = \frac{d\sigma(w^T x)}{dx}$$

$$= \frac{d}{dx}\left(\frac{1}{1+e^{-w^T x}}\right)$$

$$= \frac{1}{(1+e^{-w^T x})^2} \cdot e^{-w^T x}$$

$$= \frac{1}{1+e^{-w^T x}} * \frac{(1+e^{-w^T x} - 1)}{(1+e^{-w^T x})}$$

$$= \frac{1}{1+e^{-w^T x}} * \left(1 - \frac{1}{1+e^{-w^T x}}\right)$$

$$= \sigma(w^T x) \cdot (1 - \sigma(w^T x))$$

Q1) 1.4.2

$$P(y=1|x,w) = \sigma(w^T x) = \frac{1}{1+e^{-w^T x}} = \frac{e^{w^T x}}{e^{w^T x}+1} \quad \text{①}$$

$$P(y=-1|x,w) = 1 - \frac{e^{w^T x}}{e^{w^T x}+1} = \frac{1}{e^{w^T x}+1} \quad \text{②}$$

Combine above eq^n ① and ②

$$\boxed{P(y=\pm 1|x,w) = \frac{1}{1+e^{-y w^T x}}}$$

Q1) 1.4.3

$$L = \prod_{i=1}^{N} \sigma(y_i w^T x_i)$$

~~Taking -ve log~~

-ve $$\log L = -\sum_{i=1}^{N} \log(\sigma(y_i w^T x_i))$$

$$= -\sum_{i=1}^{N} \log\left(\frac{1}{1+e^{-y_i w^T x_i}}\right)$$

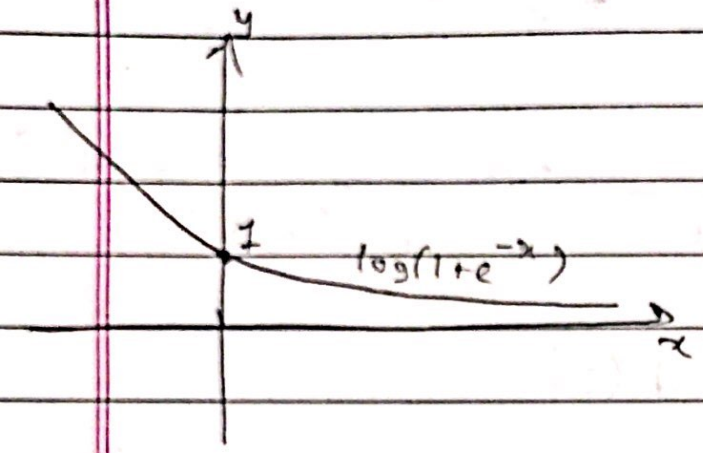
$$= -\sum_{i=1}^{N} \log(1+e^{-y_i w^T x})^{-1}$$

$$= \sum_{i=1}^{N} \log(1+e^{-y_i w^T x})$$

$$\therefore -ve\ LL = \sum_{i=1}^{N} \log(1+e^{-y_i w^T x})$$

If training label & dot product have same sign:

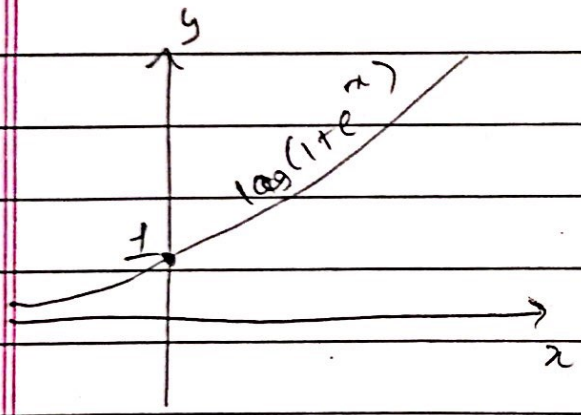
$$L_{log} = \sum_{i=1}^{N} \log(1 + e^{-y_i w^T x})$$



As can be seen if training label and dot product have same sign, then loss decreases.

If training label & dot product don't have same sign:

$$L_{log} = \sum_{i=1}^{N} \log(1 + e^{y_i w^T x})$$



As can be seen, loss increases if sign of dot product and training label is not same.