Assignment 2

Date: / /

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Q6) Given: $H = X(X^T X)^{-1} X^T$

(1) Prove: $H^T = H$

$H^T = [X.(X^T X)^{-1}.X^T]^T$

$= (X^T)^T.[X.(X^T X)^{-1}]^T$ [Using $(AB)^T = B^T A^T$]

$= X.[X.(X^T X)^{-1}]^T$ [Using $(A^T)^T = A$]

$= X.[(X^T X)^{-1}]^T.X^T$ [Using $(AB)^T = B^T A^T$]

$= X.[(X^T X)^T]^{-1} X^T$ [Using $(A^{-1})^T = (A^T)^{-1}$]

$= X.(X^T X)^{-1} X^T$ [Using $(AB)^T = B^T A^T$ and $(A^T)^T = A$]

$= H$ $\therefore$ Hence Proved!

(2) Prove: $HH = H$

$HH = [X(X^T X)^{-1}.X^T].[X(X^T X)^{-1} X^T]$

$= X.(X^T X)^{-1}.X^T X.(X^T X)^{-1}.X^T$

$= X.I.(X^T X)^{-1}.X^T$ [Using $A^{-1}.A = I$]

$= X.(X^T X)^{-1}.X^T$ [Using $A.I = A$]

$= H$ $\therefore$ Hence Proved!