

Assignment 2

Date: / /

Mayuri Kadam

Q8) Dataset: $(x) = \{x_1, x_2, \dots, x_n\}$

$$p(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$$

$$LL(\mu; x) = \ln \left(\frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2} \right)$$

$$= \ln \left(\frac{1}{\sqrt{2\pi\sigma}} \right) + \ln \left(e^{-\frac{1}{2\sigma^2}(x-\mu)^2} \right)$$

$$= \ln \left(\frac{1}{\sqrt{2\pi\sigma}} \right) - \frac{1}{2\sigma^2} (x-\mu)^2$$

$$\frac{\partial LL(\mu; x)}{\partial \mu} = 0 - \frac{1}{2\sigma^2} \times 2(x-\mu)$$

To maximize: equating $\frac{\partial LL(\mu; x)}{\partial \mu} = 0$

$$\therefore 0 = 0 - \frac{1}{2\sigma^2} \times 2(x-\mu)$$

$$\therefore \boxed{\mu = x}$$

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Q8.1) Dataset: $(x) = \{x_1, x_2, \dots, x_N\}$

$$p(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$$

$$LL(\sigma; x) = \ln\left(\frac{1}{\sqrt{2\pi}\sigma}\right) + \ln\left(e^{-\frac{1}{2\sigma^2}(x-\mu)^2}\right)$$

$$= \ln\left(\frac{1}{\sqrt{2\pi}}\right) + \ln\left(\frac{1}{\sigma}\right) + e^{-\frac{1}{2\sigma^2}(x-\mu)^2} - \frac{1}{2\sigma^2}(x-\mu)^2$$

$$\frac{\partial LL(\sigma; x)}{\partial \sigma} = 0 \Rightarrow -\frac{\partial}{\partial \sigma}(\ln(\sigma)) - \frac{1}{2}(x-\mu)^2 \sigma^{-3} - 2x\sigma^{-3}$$

$$= -\frac{1}{\sigma} + \frac{(x-\mu)^2}{\sigma^3}$$

To maximize: equating $\frac{\partial LL(\sigma; x)}{\partial \sigma} = 0$

$$\therefore 0 = -\frac{1}{\sigma} + \frac{(x-\mu)^2}{\sigma^3}$$

$$\therefore 0 = -\sigma^2 + (x-\mu)^2$$

$$\therefore \boxed{\sigma = x - \mu}$$