Week 1 Problem No. 3 :

Given : $\alpha_p = 3$ dB ;

$\omega_c = \omega_p = 2\pi \times 1000 = 2000\pi$ rad/sec

$\alpha_s = 10$ dB

$\omega_s = 2\pi \times 350 = 700\pi$ rad/sec

the digital frequency we have, prewraping it,

$$\Omega_p = \frac{2}{T}\tan\frac{\omega_p T}{2} = \frac{2}{2\times10^{-4}}\tan\frac{(2000\pi \times 2\times10^{-4})}{2}$$

$$\Omega_p = 10^4 \tan(0.2\pi) = 7265 \text{ rad/sec}$$

$$\Omega_s = \frac{2}{T}\tan\frac{\omega_s T}{2} = \frac{2}{2\times10-4}\tan\frac{(700\pi \times 2\times10^{-4})}{2}$$

$$\Omega_s = 10^4 \tan(0.07)\pi = 2235 \text{ rad/sec}$$

Order of filter : $N = \dfrac{\log\sqrt{\dfrac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\log\dfrac{\Omega_s}{\Omega_p}} = \dfrac{\log\sqrt{\dfrac{10^{0.1(10)}-}{10^{0.1(3)}-}}}{\log\dfrac{7265}{2235}}$

$$= \frac{\log 3}{\log 3.25} = \frac{0.4771}{0.5118} \doteq 0.932$$

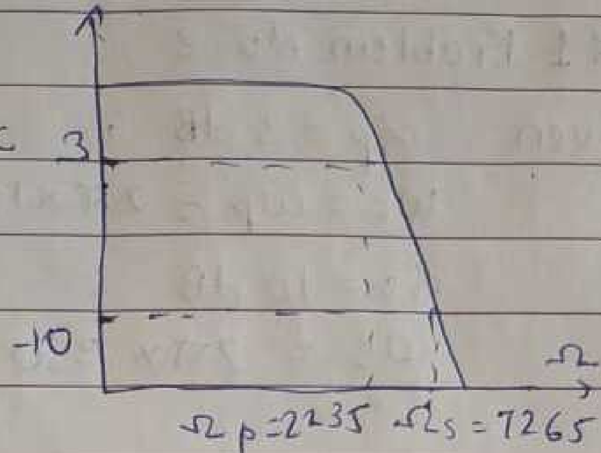
$\therefore$ taking $N \approx 1$

Applying 1st order butterworth filter,

$\Omega_c = 1$ rad/sec is $H(s) = \dfrac{1}{1+s}$

As, $\Omega_c = \Omega_p$

$\therefore \Omega_c = \Omega_p = 7265$ rad/sec



$$s \rightarrow \frac{\Omega_c}{s}$$

i.e. $s \rightarrow \frac{(7265)}{s}$

transfer function of high-pass filter:

$$H(s) = \left.\frac{1}{s+1}\right|_{s = 7265/s} = \frac{s}{s + 7265}$$

Using bilinear transformation,

$$H(z) = H(s)\Big|_{s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \left.\frac{s}{s + 7265}\right|_{s = \frac{2}{2\times10^{-4}}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$

$$= \frac{10000\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}{10000\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 7265}$$

$$H(z) = \frac{0.5792(1-z^{-1})}{1 - 0.1584 z^{-1}}$$

$\therefore H(z) + 0.1584 z^{-1} H(z) = 0.5792 (1 - z^{-1})$

$\therefore H(z) + 0.1584 z^{-1} H(z) = 0.5792 - 0.5792 z^{-1}$.