Subject Computational Numerical method (CNM)
subject code SEC-1/BCAS/T/107

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Module no 1

Introduction to numerical techniques:

Number theory, Division Algorithm, Greatest common Divisor, Mathematical Modelling, Characteristics, Error in numerical computation.

Solving questions: Bisection method, false position method, Newton Raphson method

Module no 2

Numerical solution of linear equation

Matrix notation, set of equations, gauss elimination method, gauss seidal method, matrix inversion method.

Interpolation and approximation

Introduction and polynomial interpolation, Newton Forward difference, Formula method, Newton backward difference formula method, langrage's interpolation.

Introduction to Numerical Method

==NUMERICAL METHOD==

Numerical method are techniques to approximation mathematical processes (example of mathematical processes is integral, differential equation, non- linear equation.)

==WHY WE NEED THE APPROXIMATION==

We can not solve the procedure analytically such as the standard normal cumulative distribution function The analytical method is intractable, such as solving a set of thousand simultaneous linear equation for a thousand unknown for finding forces in a truss So first we know about what is number and numeric as well as number theory.

==**WHY WE NEED NUMERICAL METHOD IN COMPUTER SCIENCE (APPLICATION OF NUMERICAL METHOD IN COMPUTER SCIENCE)**==

INTRODUCTION

Numerical method plays a crucial role in solving complex problem and making accurate prediction in various filed and their application in computer science and engineering is no exception. We will explore how numerical methods are utilized in computer science engineering to address challenges, optimize processes and enhance the performance of computational systems.

1) **SIMULATION AND MODELING:**

Numerical method is extensively used in computer science engineering for simulating and modelling real- world system. Whether its predicting weather patterns, simulating fluid dynamics, or modelling the behavior of financial markets, numerical techniques enables engineers to represents complex phenomena in a computationally feasible manner.

2) **Algorithm optimization**

Numerical method is employed to optimize algorithms for efficiency and speed when designing algorithm for tasks such as sorting, searching and data processing. Engineers use numerical techniques to analyzes and improve their performance.

3) **Machine   Learning and data analysis:**

In the realm of machine learning and data analysis. Numerical method are fundamental techniques like gradient descent, singular value decomposition and numerical optimization are applied to train machine learning models and process vast amount of data efficiently this method play a key role in enhancing the accuracy and speed of learning algorithms.

4) Finite element analysis (FEA):

FEA is a numerical technique widely used in computer aided design (CAD) and structural analysis. It enables engineering to simulate the behavior of complex structure under various, helping in the design and optimization of components ranging from bridges to microchips. Numerical methods ensure accurate and reliable predations of stress, strain and deformation.

5) Numerical solutions of differential equations:

Many engineering problems involve differential equational that cannot be solved analytically. Numerical methods such as finite difference method are employed to approximated solution to these equation. this is crucial in understanding dynamics systems are predicting their behaviors over time.

6) Error analysis and stability: -

Numerical methods help in analyzing the error introduced during computations and ensure the stability of algorithm engineers uses techniques like Richardson exploration and error propagation analysis to quantify and minimize error, ensuring the reliability of computational results.

7) Cryptographic Algorithms

In this field of cybersecurity, numerical method plays a vital role in the design and analysis of cryptographic algorithm techniques like modular arithmetic and prime number factorization are essential for ensuring the security of digital communication and data storage.

8) Optimization problems:

Numerical optimization method is applied to solve complex engineering problem involving resources allocation, network, optimization and logistics. These methods assistance in finding the optimal solution within a given set of constrain, enhancing the efficiency of various system.

9) Conclusion: in conclusion the application of numerical methods in computer science engineering is diverse and far reaching from simulation real – world systems to optimizing algorithms and solving complex equation, numerical techniques are indispensable in addressing the challenges faced by computer scientist and engineers.as technology continuous to advance the role of numerical method in shaping the future of computer science engineering is likely to become even more prominent.

Introduction to Numerical Method

Numerical method are techniques to approximate mathematical processes (example of mathematical processes is integral, differential equation, non – linear equation.) numerical method are mathematical tools that approximate solutions to equation and problems that are difficult or impossible to solve algebraically. They are used to compute numerical data by repeat tally generating a sequence of approximation that converge to exact solution. here is some example of numerical methods.

-Root finding

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-Solving Differential equation

Integration

Solving system of equation

- Finite difference method
- Interpolation

Numerical method is used in many filed of engineering science and are important for solving problems such as finding forces in a truss. When a numerical method is implemented in a programming language with a convergence checks it called a numerical algorithm. numerical analysis is a branch of mathematical in which we analyzed and solve the problem which require calculation.

Bisection method: - simple and rebuts; linear convergence.

Fixed point method:- newton method based on linear approximation around the current iterate quadratic convergence.

Secant method :- based on linear interpolation at least two iterate.

What is an example of numerical method

For example pie (π) = **3.14159 is** approximated as 3.141 for chopping (deleting all decimal after 3). or 3.142 for rounding up to 3 decimal point/ places .

What is numerical method in FEM.

The finite element method (FEM) is a popular method for numerically solving differential equation arising in engineering and mathematical method.

Typical problem areas of interest include the transitional field of structural analysis heat transfer fluid flow, mass transport, and electromagnetic potential.

What are the two types of Numerical?

The two major types of numerical data are discreate and continues operation can be performed on numerical data.

How do you know if data is categorial or numerical?

If the data uses numbers. It is numerical if the data does not have any numbers and has word / description it is categorial

What is the best definition of numerical data?

Numerical data is commonly called quantitative data. This data is in the form of number. and any data is in the form of number is numerical data. It can be number of object, number of sales mode, count of money. In daily life data exits everywhere it can be in the form that can be counted or read data is a set of values or information which is and   when analyzed together. Gives a interference this data can be in the form of number, word or even time items. numerical data as the names stated is the data in the form of number, word or even time item. These can be a count of object, cost revenue, sales data, percentage score or profits the list of example of numerical data is never ending numerical data is collected and analyzed on the needed parameters to arrive at the conclusion for given set of data numerical data is commonly called **QUENTITIVE DATA**

TYPES OF NUMERICAL DATA

Now that the definition of numerical data is understood it is time to get into the types of numerical data that exits. Based on the requirement or how the data needs to be analyzed. Numerical data is the sorted in the best possible way. These types are

-Discreate data

- continuous data

* discreate data

Discreate data is the type of numerical data, which specific or fixed data values. Discreate has some key function they are

-fixed set of values they can be integers or whole number. But are fixed.

- easily visualized using **pie chart or bar graph.**

Some example of discreate data are

Show shoes size available in a shoe store.

Number of blue cars sold in a showroom.

Movie tickets available for a new release.

The above example show how discreate data is fixed value. A shoe size cannot be 7.00, number of cars sold cannot be a quarter. Hence all of them are fixed which gives them the name as discreate.

WHAT IS THE CONCEPT OF NUMBER THEORY

Number theory is branch of pure mathematics devoted to the study of natural number and the integer. So, the number theory also known as higher arithmetic is one of the oldest branches of mathematics and is used to study of the natural number and the integer.
        It is the study of the set of positive whole number usually called the set of the natural's numbers. This theory is experience/ experimental and theoretical.
Experimental number theory leads to question and suggest different way to answer them.
 Theoretical number system tries to provide a definite answer them.

WHY WE DO STUDY THE NUMBER SYSTEM/THEORY
Number theory is necessary for the study of the number because it shown the what number can do It helps to providing the valuable training in logical thinking and studying the relationship between the different kind of number. It is applied in cryptography device authentication, device authentication. Websites of the e – commerce, coding and security system.

Concept of NUMBER
 A number is an arithmetic value used to represent quantity. Hence a number is a mathematical concept used to count, measures and labels thus number from the basic of mathematics. Number are used for measurement, temperature, weigth, length, capacity and so on are measuring using number.

NUMBER REPRESENTATION

-digits
-symbols
-logical manner
-alphabetical form

- **DIGITS (0-9)**

  a number system is a writing system for denoting numbers using digits or symbols in a logical manner. We use the digits from 0 to 9 to form all other number. With the help of these number or digits we can create infinite number. For example (0-9) 123, 34, 987… m. this number system system using 10 digits is called as Decimal number system.

- **Alphabetical form of number**
  Number are written in word are the alphabetical form of number. As the name suggest these are number written in word form. For example 1-one,3-three and so on.

- **Symbolically using numerals**
  Numerical symbols are numerals such as Hindi, Arabic's and roman numerals. For example 112,15, and in roman (I,II,II and so on).

<mark>Cardinal and Ordinal number</mark>

Cardinal number are **counting** number. The numbers that we use for counting are called counting as well as cardinal numbers.
cardinal number tells us to how many thing, items, or object are there. For example 1,2,3,…m
Ordinal number gives us the **exact position of the things,** items or an object in the list. Ordinal number tells us position of an object rather than its quantity.
Exa. 1st, 2nd…..mnth

<mark>TYPES of NUMBER</mark>

a) **Natural number:** we start counting things from 1. The numbers 1,2,3,4… are called counting number or naturals number. 1 is the smallest naturals numbers 1,45,205…. m.

b) **Whole number:** To describe the numbers of object in a collection with no object, we use zero (0) to show nothing. the counting numbers or naturals number along with zero from whole numbers. 0,5,498,44555.

c) **Integer:** we use negative numbers to show number lesser than zero ("0") combining these with whole number, we obtained a new collection of numbers. Exa -3, -5, -1,0,1,2,3, 4, ...

d) **Rational: -** Rational number can be expressed as a fraction, where the denominator is not equal zero. every integer and every decimal numbers is a rational numbers.

e) **Irrational:** irrational numbers cannot be expressed as a fraction the square root, cube root, etc. of the natural number, are irrational number, if their extract values cannot be determined. Exa square root. $\sqrt{5}$, $\sqrt{9}$.

f) **Real:** real number includes all rational and irrational number. Apart from the above, there exits other number, namely even and odd number, prime number and composite number. These can be defined as follow

a) **Even Number**: the number that end with a 0 ,2,4,6,or 8 are called even number. Even number are evenly divisible by 2. Exa. 14,202,500,8186…

b) **Odd number:** number that end with 1,3,5,7, or 9 are called odd number. Number are not evenly divisible by 2 they are always leaving a remainder. Exa. 11,213,6005…n.

c) **Prime number:** number that are divisible by only 1 and themselves are called as prime number. A prime number has only 2 factor 1 and itself. 1,2,3,5,7,13 and etc.

d) **Composite number:** the number that are divisible by number other than 1 and themselves are called composite number. A composite number has more than 2 factor exa. 4,6,8,12,9,14,15…. etc.

Fraction and Decimal number

- Fraction number: 1) a fraction number is a quantity that expressed a part of whole. 2) the numerator and denominator are known as a term of fraction.
- **Fraction = numerator/ denominators.**
- In fraction 14/17 numerator=14 and denominator=17
- **Decimal number**
- **A** decimal number can be defined as a number whose whole numbers part and the fractional part is separated by a decimal numbers is called a decimal point.
- **T**he whole numbers part and decimal part are term of the decimal. Exa. 49.345 is 49+0.325 here 49 is the whole part and 0.345 is decimal part.

- HOW MANY TYPES OF NUMBER THEORY ARE THERE?
- The different types of modern number theory are classified into elementary, algebraic analytics, gematric and probabilities number theory.
- ALGEBRIC NUMBER
- Algebraic numbers include all of the natural numbers, all national number, so the irrational numbers and complex numbers of the pita where p and q are rational and I is a root of -1. For example I is a root of the polynomial $x^2 +1 =0$

- ANALYTICAL NUMBER THEORY
- Analytic number theory is a branch of number theory is uses techniques from analysis to solve problems about the integers.

- GEOMETRY
- The branch of mathematics that deals with the shapes, angles, dimensions and size of a variety we see in everyday life.

- PROBABILITY
- Probability is simply how likely something is happening whenever we are unsure about the out come how likely they are the analysis of events governed by probabilities is called statistics.
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Application of Number theory

Here are some of the most important number theory application. Number theory is used to find some of the important divisibility test, whether a given integer m. divided the integer n. number theory have countless application in mathematics.

- Security system like in banking securities.
- E- commerce theory.
- Coadding theory
- Bar coding
- Making management system.
- Making modular design.
- Memory management system.
- Authentication system.
- It is also defined in hash function, linear congruences and fast arithmetic operation.
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- ## MATHEMATICAL MODELLLING **IMP**

Mathematical modelling describes a process and an object by use of mathematical language. Mathematical models usually describe a system by a set of variables and set of e equation that establish relationship between the variables mathematical modelling is a mathematical form (i.e.) conversion of physical situation into mathematics with some situation/condition is known as mathematical modelling. Mathematical modelling. Mathematical modelling is nothing but a techniques and pedagogy taken from fine art and form the basic science.

the mathematical model definition is that a mathematical model is a quantitative description of a system. Mathematical modelling numerically describes the world. It is common for a math model to have numerical constants and variables that represent different aspect of the system.

A mathematical model is a representation of a system or event using mathematical equation, number, letter, symbol, algorithm, diagram and laws.

Mathematical model can be **used to**

- Understand how a system work.
- Identify important variables in a system and how they relate to each other.
- Predict what a system will do under different conditions.
- Highlights common behavior in seemingly unrelated systems.

What is mathematical **computer model?**

mathematical modelling describes a process and object by use of the math emetically language. The process or an object is presented in a "**pure form**" in mathematical modelling when external perturbations disturbing the study are absent. Computer simulation is a natural continuation of the mathematical modelling.

Function of mathematical model

A mathematical model can be used to gain understanding of a real- life situation by learning how to the system work. Which variables are important in system. And how they are related to each other. Models can also be used to predict what a system will do for different values of the independent variable.

Some <mark>types</mark> of mathematical models include

- **Mechanistic**: based on fundamental mechanism or underlying phenomena.
- **Empirical:** based on input -output data, trials or experimental results.
- **Stochastic:** contain model variables that are probabilistic in nature.
- **Deterministic:** based on cause -effect relationship.

what is the <mark>component</mark> of a mathematical modelling

- Identifying and defining the problems.
- Making assumption and identifying the variables.
- Applying mathematics to solve the problem.
- Verifying and interpreting solutions in the context of the problems.
- Refining the mathematical model.
- Reporting the findings.

<mark>CHARACTRISTICS</mark> OF GOOD MATHEMATICAL MODELLING:

A good mathematical modelling should be accurate, simple and applicable to real world situation.

A. **Accuracy:**
Accuracy is crucial characteristics of a good mathematical model. The model should be able to predict outcome with a high degree of precision. This means that the model should be based on reliable data and assumption that are as close to reliable as possible. The model should also be able to account for any uncertainties or error in the data.

B. **Simplicity:**

Simplicity is another important characteristic of a good mathematical model. The model should be easy to understand and use, even those who are not expert in the filled. This means that the model should be based on simple equation and concept that are easy to explain and interpreted. A simple model is also easier to modify and adapt to different situation.

C. **Applicability**
Applicability is the find characteristics of a good mathematical model. The model should be able to applied for real world. Situation and provide useful insight and prediction this means that the model should be able to account for all relevant all factor and variable that affected the situation being insight that are relevant and useful to decision makers. for more on how model is applied in practical situation.
    In summery a good mathematical model should be accurate, simple and applicable to real would situation by ensuring these characteristics, mathematical model can provide valuable insight and predictions that can help decision making make informed choice. Additionally, the process of analyzing model is a crucial to verify their accuracy and applicability for example demonstrating the real would utility of such model, explore the section of real would application.

Why we need to mathematical model? A simple answer to this technique that we need the mathematical modelling to understand the real world and how it works. It simply signifies the world in simple model and shapes. This model and shapes. This model should be work in with engineers and scientist. To solve real world problem using simple math. It will also involve equation.

Application of mathematical modelling

More ever using model, one can see the future of our climatic condition understanding how atoms and practical work. There is more application.

Common example of mathematical modelling is

- **Whether forecasting**
    1) First come the laws of physics
    2) **Now encodes** them as differential equation
    3) Equatially the naiver stoke equation.
    4) **Take** whether situation and Starlite data to demonstrate accurate situation whether for the day.
    5) Consider that as an initial condition now use a supercomputer to solve the 24-hours equation it will provides tomorrow whether.
    6) Update the forecast constantly.
    7) Finally present this result in an understandable form.

    In addition, people claim these methods is a failure, although it is not, despite rumors, this process work fine. It may not predict whether condition for a long time of period however it work well in the short term. It is a special case of method in mathematical modelling.
    It can be described as follow
    1) Talk to the people and identify the problem.
    2) Clarify the scientific method involved.
    3) Formulate mathematically
    4) With the help of computer try to solving the math.
    5) Start concluding
    6) Explain the result.
    Apart from whether forecasting there are several applications as well
    The following are some of them.
- Design and develop any vehicle model, starting from cars to spaceship.
- Develop new medicinal
- Controlling electricity supply network.
    o It helps in establishing the course of any naturals or manmade disaster.
    o Provide pressure and temperature condition in the workplaces etc.

CLASSIFICATION OF MATHEMATICAL MODELLING

There are different types of math models present some of them list is listed below.

-static vs. dynamic

- linear vs. non-linear

-Explicit vs. implicit.

- Deterministic vs. probabilistic

- discreate vs. continues

- floating, inductive or deductive.

1) static vs. dynamic:

Another name for the static model is the steady -state model. This model helps to calculate the system in equilibrium. Therefor it is time invariant in contrast, dynamic model is mainly used for time dependent changes in a system. hence their representation is by difference equation.

2) linear vs non -linear:

Linear mathematical modelling is obtained when all the operator exhibits linearity. On the contrary it is considered a nonlinear model when an object does not exhibit linearity. The descriptions of linearity are mainly based on context. A linear object might have some nonlinear expression it.

For instant take a linear statically model its relationship is linear in some of parameter however it is nonlinear in the predictor variables. Likewise, a differential equation. Contain nonlinear expression in it. But it trends to be linear while writing with linear differential operator.

A mathematical programming model is considered linear if the constrain and objective function are described fully by linear equation. This model is regarded as a nonlinear one if there are many constrain and objective function describe with a nonlinear equation.

3) Explicit vs. Implicit:

The model is considered explicit if the overall models input model are known as one can calculate output parameter in an explicit function by a finite computation series. In constrain is called an implicit model it outputs are already shown. Two examples of implicit function are buoyed method and newton method.

Let us take a example to understand these method, models properly considered the physical properties of a let engine. Nozzle throat and turbid areas aa explicitly. Calculated using a design theorem dynamic cycle, which uses area model math for derivative, it involves data regarding temperature pressure and fuel and airflow rates. It is only calculated at the specific power setting and fight condition. It may not be possible to calculate explicitly in such cases we use implicit model.

4) deterministic vs probabilistic

Parameter in deterministic model separately define all variables stated in involving a collection of previous stated of this variable. So, it is common for this model to perform the same. For initial condition. In probabilistic model, randomness is present variable state are not determined using unique values instead they are done by probabilities distribution other than name of these model are the statically model.

5) deductive, inductive or floating

Deductive models are usually logical structure they are used or based on a theory, inductive model. Comes from empirical finding or deductive model. Another hand floating model rest on neither observation nor theory. They are purely based on invocation of the expected structure.

6) **discreate vs. continuous**

in discreate model, object is treated as discreate, namely stated in statically models or practical in molecular model. In continuous model, object is represented continuously for example stresses and temperature.in solids and the application of electric filed over an entire model continuously are some of them.

7) **conclusion**

To conclude, mathematical model is useful in all domain, may application starting from furniture to spaceship can be done using mathematical modeling. It has a huge effect on our lives. Model with mathematical mainly contain equation. And theories which will help to solve real time problem they are used in engineering and science to perform various application. A mathematical model is used to maximize output and enhance the productivity. Apart from that, we have also discussed everything related to simulation in mathematics and how they differ from the model. And explained their subtopic. In detail all this information provided in this part will help to provide and obtained full knowledge.

DIVISION ALGORITHM

Division is an arithmetic operation that involve grouping object into equal part. It is also understood as the inverse operation of multiplication. For example, in multiplication 3 group of 6 make 18. Now if 18 is divided into 3 group is give 6 objects in each group here 18 is the dividend, 3 is the divisor and the quotient. Added to the remainder (if any) and this rule is known as division algorithm. The division algorithm applies to the division of polynomial as well.

The division of polynomial involves dividing one polynomial by a monomial, binomial, trinomial, or a polynomial of a lower degree. In a polynomial division the degree of the dividend is greater than or equal to the divisor polynomial and the questioned and add it to the remainder, if any i.e. we use the division algorithm to verify the result.

What is division algorithm.

The division algorithm says when a number 'a' is **divided** by a number "b" ang gives the quotient to be 'q' and remainder is 'r'. then **a=bq+r.** this is also known as **"Euclid division algorithm".**

The division algorithm can be represented in simple word as follow .

Dividend = Divisor * Quotient + Remainder

Let us just verify the division algorithm for some number. We know that when 59 is divided by 7 the quotient is 8 and the remainder is 3.

Here 59 /7 $\frac{59}{7}$ is **59 is divided by 7 is 56 then 59-56 = 03 is remainder and 7 *8 =56 here 8 is a quotient.**

Dividend = 59, **Divisor =7, Quotient=8, Remainder= 3**

the verification of division algorithm is **Dividend = Divisor * Quotient + Remainder.**

1) $59 = 7*8+3$

$59 = 56+03$

$59 = 59$

Division algorithm for polynomial

The division algorithm for polynomial says if p(x) and g(x) are two polynomial, where g(x) ≠ 0, we can write the division of polynomial as p(x) = g(x) + r(x) where degree of r(x) < degree of g(x) and

P(x) is the dividend.

g(x) is the divisor

q(x) is the quotient

r(x) is the remainder.

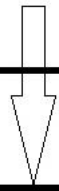If we compare this to the regular division of numbers we can easily understand this as

Dividend = Divisor * Quotient + Remainder.

We will verify the division algorithm for polynomial in the following example.

Example: find the quotient and the remainder when the polynomial $4x^3 + 5x^2 + 5x +8$ is divided by (4x +1) and verify the result by the division algorithm.

Solution: first we divided the given polynomial

$P(x) = 4x^3 + 5x^2 + 5x + 8$ by g(x) = (4x +1) using long division.

We found the quotient to be $q(x) = x^2 + x + 1$ and r(x) = 7 we will now verify the division algorithm.

P(x) = q(x) * g(x) + r(x)

$4x^3 + 5x^2 + 5x + 8 = x^2 + x + 1* 4x +1) +7$

$4x^3 + 5x^2 + 5x + 8 = 4x^3 + 4x^2 + 4x + x^2 + x + 1 + 7$

$4x^3 + 5x^2 + 5x + 8 = 4x^3 + 5x^2 + 5x + 8$

Thus, the division algorithm has proved.

Procedure to divide a polynomial by another polynomial

The step for the polynomial division is given follow

Step1: arrange and the divided and the divisor in descending order.

Step2: find the first term of the quotient by dividing the highest degree terms of the dividend by the highest degree term of the divisor.

Step3: then multiply the divisor by the current quotient and subtract the result from the current dividend. This will give a new dividend.

Step4: find the next term of the quotient by dividing the largest degree term of the new dividend obtained in the step 3 by the largest degree term of the divisor.

 Let us understand this process with an example.

Divide $2x^3 + 3x^2 + 4x + 3$ and g(x) =x+1 we will use above step to divide p(x) by g(x)

Step 1: the polynomial is already arranged in the descending order to their degree.

Step 2: the first term of the quotient is obtained by dividing the largest degree term of the dividend with the largest degree term of the divisor.

First term ($2x^2$)

Step 3: then the new dividend is $3x^2 + 4x$ which is obtained as follow. Which is obtained as follows

Step 4: the second term of the quotient is obtained by dividing the largest degree term of the dividend obtained in step 2 with the largest degree term of the divisor.

Second term = $(x^2) / x = x$.

Step 5 : repeat the step 3 and 4 again until the remainder degree is less than the divisor degree then we get the quotient to be $2x^2 + x + 3$

$$2x^2$$

divisor $\quad$ x + 1 $\quad | \quad 2x^3 + 3x^2 + 4x + 3 \qquad$ Divindend

$$2x^3 + 2x^2$$

$$0 + x^2 + 4x \qquad \text{new dividend}$$

$$- \quad x^2 + x$$

$$0 - 0 + 3x + 3$$

$$- 3x + 3$$

$$0 \qquad \text{Remainder}$$

First term of the quotient $= 2x^3 / x = 2x^2$

second term of the quotient $= x^2 / x = x.$

Third term of the quotient $= 2x^3 / x = 2x^2$

Here $p(x) = 2x^3 + 3x^2 + 4x + 3.$

$g(x) = x+1.$

$q(x) = 2x^2 +x+3.$

$r(x) = 0.$

<mark>Division Algorithm for linear divisors.</mark>

When a polynomial of degree $n \geq 1$ is divided by a divisor with degree 1, then we call it as a division by linear divisor. The division algorithm for linear divisor is same as that of the polynomial division algorithm. Discussed above except for the fact the divisor is of degree 1.

Let us look at an example below. **Let p(x) = x² +x+1. Be the dividend is g(x) = x-1.** Be the divisor. Here the degree of the divisor is 1.

Here g(x) is called as "linear divisor". To know more about this division algorithm.

Let us divided p(x) by g(x).

$$x+2$$

$$x-1. \quad | \quad x^2 +x+1$$

$$x^2 -x$$

$$2x+1$$

$$2x-2$$

$$3$$

Let us verify the division algorithm for polynomial here

$x^2 + x + 1 = (x-1)(x+2) + 3$

$x^2 + x + 1 = x^2 + 2x - 1x - 2 + 3$

$x^2 + x + 1 = x^2 + x + 1.$

Division algorithm for general divisors.

$q(x) = x^2 - 2x - 2$

the division algorithm for general's divisor in the same as that the polynomial division algorithm discussed in this section of the division of the one polynomial by another polynomial. One important fact about the division is that the degree of the divisor can be any positive integer lesser than the divided. Let us take an **example** $p(x) = x^4 - 4x^3 + 3x^2 + 2x - 1$ **be the divided by**

$g(x) = x^2 - 2x + 1$ be the divisor is 2 . which is lesser than or equal to the dividend degree to knoe about the this division algorithm we will divided p(x) by g(x)

```
                        |  x² -2x-2
                        |
         _____ |
                        |
         x² -2x+1        |  x⁴ – 4x³ + 3x² +2x-1
                        |  x⁴ – 4x³ + 3x²
                          _____
                             - 2x³+ 2x² +2x
                            -2x³ +4x +2x
                          _____
                             - 2x² +4x-1
                            -2x² -4x-2
                          _____
                                1
```

$p(x) = x^4 - 4x^3 + 3x^2 + 2x - 1$

$q(x) = x^2 - 2x - 2$

$g(x) = x^2 - 2x + 1$

important notes on division algorithm

a polynomial can be divided by another polynomial of a lower degree only.

Arrange the dividend polynomial from the greatest to the lowest power before starting the division.

If the divisor polynomial is not a factor of the dividend obtained at any step of the polynomial division then it means that a remainder other than 0 will be left behind.

We can use the division algorithm to find one of the dividend, divisor, quotient and remainder when the other three of these

GREATEST COMMON DIVISOR (GCD)[IMP]

The greatest common divisor (GCD) of two or more number is the greatest common factor number that divides them, exactly. It is also called **Highest Common Factor (HCF).** The greatest common divisor for a given set of number. It is also known as greatest common factor (GCF) For example, the greatest common factor of 15 and 10 is 5. Since both the number can be divided by 5.

10/5 = 2

15/5 = 3

For a set of positive integers (a, b) then the greatest common divisor GCD of any two number is negative or 0 as the least positive integers are common of any two number is always **1.**

If a and b are two number then the greatest common divisor of both the number is denoted by gcd (a, b) to find the gcd of number, we need to list all the factor of number, we need to list all the factor of number and find the largest common factor.

Suppose 4,8 and are three number. Then the factors 4,8 and 16 are

4 --1,2,4

8—1,2,4,8

16—1,2,4,8,16 there for we can calculate that 4 is the highest common factor among all the three number. Before we move ahead on finding the greatest common factor let us understand what a factor and common factor are?

What is factor?

Factor of a number divides the original number evenly. For example if 8 is the factor of 64 then 8 can divide 64 into 8 equal part.

What is the common factor?

If factor of a number is a factor of another number then it is said to be a common factor for the two number. For example 2 is a factor of 4 and 8, hence 2 is a common factor.

GCD

GCD is the greatest common factor of two or more number. A factor that is the highest among the number.

How to find the greatest common divisor (GCD)

For a set of two positive integer (a, b) we use the following step to find the greatest common divisor.

Step1 - write a divisor of the number "a".

Step 2 - write a divisor of the number "b".

Step 3- list of the common divisor of the "a" and "b".

Step 4- now find the divisor which is the highest among the common divisor.

Example: find the greatest common divisor of 13 and 48.

Solution: we will use following step to find the greatest common divisor (13,48).

 Divisor of 13 = 1and 13.

Divisor of 48 = 1,2,3,4,6,8,12,16,24, and 48

The common divisor of 13 and 48 is **1. The** GCD of 13 and 48 is **1.**

Therefor GCD (13,48) = 1.

Finding the greatest common divisor by using (LCM) method

As per the LCM method for the greatest common divisor the GCD of two positive number (a, b) can be calculated by using the following formula.

GCD (a, b) = (a*b) / LCM (a, b)

As per the LCM let us see to calculated the gcd of (a, b)

Step 1: find the product of the a and b.

Step 2: find the least common multiple (LCM) of a and b.

Step 3: divide the product of the number by the lcm of the number.

Step 4: the obtained value after division is the greatest common divisor of (a, b).

Example find the greatest common divisor of **15 and 70 by using LCM method**.

Solution: the greatest common divisor of 15 and 70 using lcm method can be calculated as follow

The product of the 15 and 70 is 15 *70 = **210**

The LCM is (15, 70) = 210

We know the formula of GCD by using LCM of two number is **GCD (a, b) = (a*b) / LCM (a, b)**

Then the GCD is **(15 * 70) / 210 = 5.**

GCD (15,70) = 5.

Exercises:

1) Determine the greatest common divisor of 12 and 26.
Solution: the greatest common divisor of 12 and 26 can be calculated by
Divisor of 12 = 1,2,3,4,6, and 12
Divisor of 26 = 1,2,13, and 26.
The common factor of 12 and 26 are 1 and 2.
The greatest common divisor of (12, 26) are 2
The **GCD (12, 26) = 2.**

2) Using LCM method determine the value of the greatest common divisor of 20 and 65.

the greatest common divisor of 20 and 65 using lcm method can be calculated as follow

The product of the 20 and 65 is 20 *65 = **1300**

The LCM is (20, 65) = 210

How to find the LCM (20, 65)

- Multiple of 20: 20, 40,60,80,100,120,140,160,180,200,220,240,260,280…
- Multiple of 65: 65, 130, 195, 260, 325, …
- Find the smallest number that is on all of the lists is **(260)** so we have the LCM value is 260.

We know the formula of GCD by using LCM of two number is **GCD (a, b) = (a*b) / LCM (a, b)**

Then the GCD is **(20 * 65) / 260 = 5.**

GCD (20,65) = 5.

METHOD TO FIND THE GCD

There are several methods to find the greatest common divisor of the given number.

1) Prime Factorization method.
2) Long division method.
3) Euclid division method

Each of this method can be explain as follow

1) Prime Factorization method.

Every composite number I.e. a number with more than one factor can be written as a product of prime number. In the prime factorization method, each given number is written as the product of **prime numbers** and then find the product of the smallest power of each common factor. This method is applicable only for positive integers i.e. Natural Number.

Example: find the greatest common factor of 24,30 and 36.

Solution: prime factor of 24 is **2^3*3** = 8*3= 24.

prime factor of 30 is **2*3** *5= 6*5= 30.

prime factor of 36 is **2^2*3^2** = 4*9= 36

from the factorization we can see only 2*3 are common prime factor. Therefor the

GCD (24,30,36) = 2*3 = 6.

2) Long division method:

In this method the largest number among the given set of number, should be divided by the second largest numbers. And again, the second largest number should be divided by the remainder of the previous operation. This process will continue till the remainder is zero is called the greatest common divisor of the given number.

3) Euclid division algorithm:

This method is stated only for positive integer find the below step in order to get the HCF of two positive integers a and b here a > b.

Step 1: applying Euclid's division lemma to a and b we get two number a and r such that a = bq+r;

Step 2: if r=0 then b is the HCF of a and b if r ≠ 0 then apply Euclid division a to b and r.

Step 3: continue the above process until the remainder is zero. (0)

Step 4: when the remainder is zero, the divisor at this stage is the HCF of a given number.

Application of greatest common divisor

The concept of the greatest common divisor or the highest common factor is used many real – life incident is followed

A shopkeeper has 420 balls and 130 bats to pack in a day. He wants to pack them in such a way that each set has some number in box. And they take up the least area of the box. What is the number that can be placed in each set for this packing purpose. In this above problem the greatest common divisor of 420 and 130 will be the required number.

Other application like arranging student in rows and column in equal number, dividing the group if people into smaller section etc.

Can the greatest common divisor be NEGATIVE?

No the greatest common divisor cannot be negative as it.

Represent the greatest common divisor of two positive integers.

The least values of gcd can be 1 and not lesser than 1

GCD CAN COT HOLD THE NEGATIVE VALUE.

Are GCD and HCF the same?

Yes

Gcd and HCF are the same the values can be calculated by checking the common divisor of factor and finding the greatest divisor of both the numbers.

ERROR IN NUMERICAL ANALYSIS imp

Error in term is a residual variable produced by a statically or mathematical model, which is created when the model does not fully represent the actual relationship between the independent variable and the dependent variable. An error term in statistics is a value which represents how observed data differs from actual population data. It can also be a variable which represents how a given statistical model differs from reality.

There are three main sources of errors in numerical computation: rounding, data uncertainty, and truncation. Rounding errors, also called arithmetic errors, are an unavoidable consequence of working in finite precision arithmetic.

In numerical computation, an error is the difference between the true value and an approximation of that value. There are several types of errors that can occur, including:

Rounding error

- Round-off error, also known as rounding error, is the difference between the result of an algorithm using exact arithmetic and the result of the same algorithm using rounded arithmetic. It occurs when a number is altered to an integer or one with fewer decimals.
- Round-off error can be caused by **chopping** or rounding. For example, if a number like 0.3378 is written up to three significant digits, it would be written as 0.333. If rounding is used, it would be written as 0.334.
- Round-off error can be inconsequential in most cases, but it can have a cumulative effect in computerized financial environments. It can also impact the performance of numerical methods like finite differences. Caused by the finite precision of computations involving floating-point values. For example, computers can only store a limited number of digits of π.

- **Symmetric round-off error** is a type of rounding error that occurs when the last significant digit of a number is rounded up by one if the first discarded digit is greater than or equal to five. For example, 42.7893 would be rounded to 42.79, and 46.5432 would be rounded to 46.54.
- Round-off errors occur when computers represent numbers using a fixed number of bits and binary digits. The errors can be introduced at every stage of a numerical computation, and while each individual error may be small, the cumulative effect can be significant

Truncation error

The difference between the exact mathematical solution and the approximate solution. This can occur when simplifications are made to mathematical equations, such as truncating an infinite series expansion. Truncation error is the difference between the actual value of a function and the approximated value of that function. It occurs when a finite number of steps are used to approximate an infinite process, which is known as discretization. This is often done to make calculations easier.

Here are some examples of truncation error:

- **Numerical integration**

   Truncation error can be local or global. Local truncation error is the error caused by one iteration, while global truncation error is the cumulative error from many iterations.

- **Derivatives**

   Truncation error is the difference between the true derivative of a function and the derivative obtained by numerical approximation.

- **Infinite series**

   Truncation error occurs when an infinite series is approximated by a finite sum. For example, the infinite Taylor series can be approximated by a quadratic polynomial.

   The main difference between truncation and round-off error is the source of the error:

- **Truncation error**

   Occurs when an infinite process is replaced by a finite one. For example, when calculating the speed of light in a vacuum, truncating the value to two decimal places results in a truncation error.

- **Round-off error**

   Occurs when numbers are represented on a computer and rounded to a certain number of digits. For example, rounding 9.945309 to two decimal places (9.95) and then rounding again to one decimal place (10.0) results in a round-off error.

   The sum of truncation and round-off error is called the total numerical error. Reducing the step size can reduce the truncation error, but it can also increase the round-off error

- Some other categories of error

- **Environmental error**

   Caused by external changes in the environment of the instrument, such as temperature, humidity, or pressure.

   Other types of errors include:

   Relative error: The numerical difference divided by the true value

Percentage error: The relative error expressed as a percent

Random error: Distinguishes the effects of inherent imprecision from systematic error
The total error of an approximation is the summation of roundoff error and truncation error.

INTRODUCTION TO MATRIX

An introduction m*n matrix is usually written as

$$A= \begin{bmatrix} a11 & a12 & \dots aln \\ A21 & a22 & a2n \\ Am1 & am2 & amn \end{bmatrix}$$

In brief the above matrix is represented by A= [aij] m*n. the number a11, a12,..etc, are known as the element of the matrix A , where aij belongs to the ith row and j^{th} column and is called the $(i, j)^{the}$ element of the matrix A =[aij].

If A and B are square matrix of order n and ln is a corresponding unit matrix then **(a) A (adj. A) = |A|/n = (adj A) A**

MATRICES: -

A rectangular array of m*n numbers (real or complex) in the form of m horizontal lines (called rows) and n vertical lines (called column) is called a matrix order m by n written as m*n matrix such as enclosed by [] or ().

INTRODUCTION TO MATRIX

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If A and B are square matrix of order n and ln is a corresponding unit matrix then **(a) A (adj. A) = |A|/n = (adj A) A**

Terms Related to Matrix Inverse

The following terms below are helpful for more clear understanding and easy calculation of the inverse of matrix.

Minor: The minor is defined for every <u>element of a matrix</u>. The minor of a particular element is the determinant obtained after eliminating the row and column containing this element.

A= [a11   a12   …aln
      a21   a22   a2n
      am1   am2   amn]

In brief the above matrix is represented by A= [aij] m*n. the number a11, a12,..etc, are known as the element of the matrix A , where aij belongs to the ith row and j^{th} column and is called the $(i, j)^{the}$ element of the matrix A =[aij].

.

Determinant: The determinant of a matrix is the single unique value representation of a matrix. The determinant of the matrix can be calculated with reference to any row or column of the given matrix. The determinant of the matrix is equal to the summation of the <u>product</u> of the elements and its cofactors, of a particular row or column of the matrix.

Singular Matrix: A matrix having a determinant value of zero is referred to as a <u>singular matrix</u>. For a singular matrix A, |A| = 0. The inverse of a singular matrix does not exist.

Non-Singular Matrix: A matrix whose determinant value is not equal to zero is referred to as a <u>non-singular matrix</u>. For a non-singular matrix |A| ≠ 0 and hence its inverse exists.

Adjoint of Matrix: The <u>adjoint of a matrix</u> is the <u>transpose</u> of the cofactor element matrix of the given matrix.

Rules For Row and Column Operations of a Determinant:

The following rules are helpful to perform the row and column <u>operations on determinants</u>.

- The value of the determinant remains unchanged if the rows and columns are interchanged.
- The sign of the determinant changes, if any two rows or (two columns) are interchanged.
- If any two rows or columns of a matrix are equal, then the value of the determinant is zero.
- If every element of a particular row or column is multiplied by a <u>constant</u>, then the value of the determinant also gets multiplied by the constant.
- If the elements of a row or a column are expressed as a sum of elements, then the determinant can be expressed as a sum of determinants.
- If the elements of a row or column are added or subtracted with the corresponding <u>multiples</u> of elements of another row or column, then the value of the determinant remains unchanged.

TYPES of Matrices

[1] Symmetric matrix: - A square matrix A= [aij] is called as symmetric matrix

aij= aij, for all i, j

[2] Skew – Symmetric matrix: when aij = -aij

[3] Hermitian and skew -Hermitian matrix:

[4] orthogonal matrix: - if $AA^{T} = |n = A^{T} A$.

[5] Idempotent matrix: - if $A^2 = A$

[6] Involuntary Matrix: - a square matrix A is nilpotent, if $A^p = 0$, p is an integer.

TRANSPOSE OF MATRIX: -

The matrix obtains from a given matrix A by changing its row into column or column into rows is called the transpose of matrix A and is denoted by A^T or A' from the definition, it is obvious that if the order of A is M*N then, the order of A^T becomes N*M for example, transpose of matrix.

$$[a1 \quad a2 \quad a3$$
$$B1 \quad b2 \quad b3]_{2*3}$$

Is

$$[a1 \quad b1$$
$$A2 \quad b2$$
$$A3 \quad b3]_{3*2}$$

Properties of Transpose of matrix:

I. $(A^T)^T = A$
II. $(A + B)^T = A^T + B^T$
III. $(AB)^T = B^T A^T$
IV. $(KA)^T = K (A)^T$
V. $(A_1 A_2 A_3 .... A_{n-1} A_n)^T = A^T_n \ A^T_{n-1} .... A_3^T, A_2^T A_1^T$
VI. $I^T = I$
VII. $tr (A) = tr (A^T)$

so, the matrix refers to a rectangular array of number. A matrix consists of rows and column those rows and column define the size or dimension of a matrix. The various types of matrix are row matrix column matrix, null matrix, square matrix, diagonal matrix, upper triangular matrix, lower triangular matrix, symmetric matrix and asymmetric matrix.

PROGRAM FOR MATRIX TRANSPOSE IN C

The transpose of matrix is a new matrix that is obtained by exchanging the rows and column. In this program the user asked to enter the number of rows ® and number of columns © their values should be less then 10 in this program. Then the user asked to enter the element of matrix (of order (r*c)). The program below then computers the transpose of the matrix and print it on the screen.

PROGRAM:

```
#include<stdio.h>

int main () {

int a [10] [10], transpose [10] [10], r, c;

printf ("enter the rows and column which you want in matrix");

scanf ("%d %d", &r, &c);
```

```
// assign the element in matrix

Printf ("\n enter the element of the matrix :");

For (int i=0; i<r; i++)

{

For (int j=0; j<c; j++)

{

Printf ("%d ", a[i][j]);

If (j==c-1)

Printf ("\n");

}

// computing the transpose of the matrix

For (int i=0; i<r; ++i)

{

For (int j=0; j<c; ++j)

{

Transpose [j][i] = a[i][j]);

}

Printf ("\n");

// printing the transpose

Printf ("\n transpose of the matrix:");

For (int i=0; i<c; ++i)

{

for (int j=0; j<r; ++j)

{

Printf ("%d", transpose[i][j]);

If (j == r-1)

Printf ("\n");

}

Return 0;

}
```

Output :

Enter rows and column

2   3

Enter the element a11: 1

Enter the element a12: 4

Enter the element a13: 0

Enter the element a21: -5

Enter the element a22: 2

Enter the element a23: 7

Enter matrix   1 4 0

                    -5 2 7

Transpose of the matrix

                    1   -5

                    4   2

                    0   7

Definition. The determinant of a matrix is a single numerical value which is used when calculating the inverse or when solving systems of linear equations. The determinant of a matrix A is denoted |A|, or sometimes det(A).

The value of the determinant of a matrix can be calculated by the following procedure:

- For each element of the first row or first column get the cofactor of those elements.
- Then multiply the element with the determinant of the corresponding cofactor.
- Finally, add them with alternate signs.

$$A = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

$$|A| = a(ei - fh) - b(di - gf) + c(dh - eg)$$

In terms of Cofactor:

Finding
$$\begin{bmatrix} a \times \begin{vmatrix} e & f \\ h & i \end{vmatrix} \end{bmatrix} - \begin{bmatrix} b \times \begin{vmatrix} d & f \\ g & i \end{vmatrix} \end{bmatrix} + \begin{bmatrix} c \times \begin{vmatrix} d & e \\ g & h \end{vmatrix} \end{bmatrix}$$

Let us consider three matrices X, A and B such that X = AB. To determine the inverse of a matrix using elementary transformation, we convert the given matrix into an identity matrix. Learn more about how to do elementary transformations of matrices here.

If the inverse of matrix A, A^{-1} exists then to determine A^{-1} using elementary row operations

- Write A = IA, where I is the identity matrix of the same order as A.
- Apply a sequence of row operations till we get an identity matrix on the LHS and use the same elementary operations on the RHS to get I = BA. The matrix B on the RHS is the inverse of matrix A.
- To find the inverse of A using column operations, write A = IA and apply column operations sequentially till I = AB is obtained, where B is the inverse matrix of A.
- be the 3 x 3 matrix. The inverse matrix is:

$$A^{-1} = \frac{1}{|A|} \begin{bmatrix} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} & \begin{vmatrix} a_{13} & a_{12} \\ a_{33} & a_{32} \end{vmatrix} & \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} \\ \begin{vmatrix} a_{23} & a_{21} \\ a_{33} & a_{31} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} & \begin{vmatrix} a_{13} & a_{11} \\ a_{23} & a_{21} \end{vmatrix} \\ \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} & \begin{vmatrix} a_{12} & a_{11} \\ a_{32} & a_{31} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \end{bmatrix}$$

-

Example: matrix inversion method

This method can be applied Inverse of Matrix

The inverse of Matrix for a matrix A is denoted by A^{-1}. The inverse of a 2×2 matrix can be calculated using a simple formula. Further, to find the inverse of a matrix of order 3 or higher, we need to know about the determinant and adjoint of the matrix. The inverse of a matrix is another matrix, which by multiplying with the given matrix gives the identity matrix.

The inverse of matrix is used of find the solution of linear equations through the matrix inversion method. Here, let us learn about the formula, methods, and terms related to the inverse of matrix.

What is Inverse of Matrix?

The **inverse of matrix** is a matrix, which on <u>multiplication</u> with the given matrix gives the <u>multiplicative identity</u>. For a square matrix A, its inverse is A^{-1}, and $A \cdot A^{-1} = A^{-1} \cdot A = I$, where I is the identity matrix. The matrix whose determinant is non-zero and for which the inverse matrix can be calculated is called an invertible matrix.

Inverse Matrix Formula

In the case of <u>real numbers</u>, the inverse of any real number a was the number a^{-1}, such that a times a^{-1} equals 1. We knew that for a real number, the inverse of the number was the <u>reciprocal</u> of the number, as long as the number wasn't zero. The inverse of a square matrix A, denoted by A^{-1}, is the matrix so that the product of A and A^{-1} is the <u>identity matrix</u>. The identity matrix that results will be the same size as matrix A.

The formula to find the inverse of a matrix is: $A^{-1} = 1/|A| \cdot Adj\ A$, where

- |A| is the determinant of A and
- Adj A is the adjoint of A

$$inverse\ of\ matrix\ A^{-1} = \frac{1}{|A|} . Adj$$

Since |A| is in the <u>denominator</u> of the above formula, the inverse of a matrix exists only if the <u>determinant of the matrix</u> is a non-zero value. i.e., |A| ≠ 0.

<mark>How to Find Matrix Inverse?</mark>

To find the inverse of a square matrix A, we use the following formula: $A^{-1} = adj(A) / |A|$; **|A| ≠ 0**

where

- A is a square matrix.
- adj(A) is the adjoint matrix of A.
- |A| is the determinant of A.

Methods to Find Inverse of Matrix

The inverse of a matrix can be found using two methods. The inverse of a matrix can be calculated through elementary operations and through the use of an adjoint of a matrix. The elementary operations on a matrix can be performed through row or column transformations. Also, the inverse of a matrix can be calculated by applying the inverse of matrix formula through the use of the determinant and the adjoint of the matrix. For performing the inverse of the matrix through elementary column operations we use the matrix X and the second matrix B on the right-hand side of the equation.

- Elementary row or column operations
- Inverse of matrix formula (using the adjoint and determinant of matrix)

Let us check each of the methods described below.

Elementary Row Operations

To calculate the inverse of matrix A using elementary row transformations, we first take the <u>augmented matrix</u> [A | I], where I is the identity matrix whose order is the same as A. Then we apply the row operations to convert the left side A into I. Then the matrix gets converted into [I | A^{-1}]. For a more detailed process, click <u>here</u>.

Elementary Column Operations

We can apply the column operations as well just like how the process was explained for row operations to find the inverse of matrix.

Inverse of Matrix Formula

The inverse of matrix A can be computed using the inverse of matrix formula, A^{-1} = (adj A)/(det A). i.e., by dividing the adjoint of a matrix by the determinant of the matrix. The inverse of a matrix can be calculated by following the given steps:

- **Step 1:** Calculate the <u>minors</u> of all elements of A.
- **Step 2:** Then compute the cofactors of all elements and write the cofactor matrix by replacing the elements of A by their corresponding cofactors.
- **Step 3:** Find the adjoint of A (written as adj A) by taking the transpose of the cofactor matrix of A.
- **Step 4:** Multiply adj A by the reciprocal of the determinant

Important Points on Inverse of a Matrix:

- The inverse of a square matrix (if exists) is unique.
- If A and B are two invertible matrices of the same order then $(AB)^{-1} = B^{-1}A^{-1}$.
- The inverse of a square matrix A exists, only if its determinant is a non-zero value, $|A| \neq 0$.
- The <u>determinant</u> of matrix inverse is equal to the reciprocal of the determinant of the original matrix.
- The determinant of the product of two matrices is equal to the product of the determinants of the two individual matrices. $|AB| = |A|.|B|$

<mark>FOR EXAMPLE</mark>
 Solve the following system of equation using matrix inversion method
$2x_1+ 3x_2+3x_3 =5$;
$x_1-2x_2+x_3 =-4$;
$3x_1-x_2-2x_3 =3$;
<mark>Soluation</mark>
First we can written the given system of eq. in matrix form
$2x_1+ 3x_2+3x_3 =5$;
$x_1-2x_2+x_3 =-4$;
$3x_1-x_2-2x_3 =3$;
 Where consider the matrix equation. <mark>AX=B.</mark>

Then A $\begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}$; $X = \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix}$; $B = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$ then find $|A|$

$|A| = \begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}$ det $|A| = (a-b+c)$ find the cofactor of a,b, c which is the first row element in in matrix

A i.e. 2, 3, 3;
$|A| = |2*| (-2*-2) - (-1*-1) | - 3*| (1*-2) – (3*1) | + 3*| (1*-1) – (3*-2) |$
$|A| = 2 (4+1) – 3 (-2-3) + 3 (-1+6)$
$|A| = 10 +15 +15$
<mark>$|A| = 40$</mark>

If <mark>$|A| \neq 0$</mark> then we can find the inverse of matrix hence $|A| = 40$ then A^{-1} exists.
$$\textit{\textbf{inverse of matrix }} A^{-1} = \frac{1}{|A|} . Adj\ A$$

$A^{-1} = A$ $\begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}$

$$= 1/40 * A \begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix} \quad \text{find adjoint of}$$

Adj A =| +(4+1)   -(-2-3)   +(-1-6)   [5   5   5

         -(-6+3)   +(-4-9)   -(-2-9)   =   3   -13   11

         +(3+6)   -(-2-3)   +(-4-3) | 9   1   -7]

AdjA = transpose of adjA =[5   3   9

                                          5   -13   1

                                          5   11   -7]

Then applying x= $A^{-1}*B$

1/40 * =[5   3   9   [5   [25   − 12   27

              5   -13   1   *   -4   = 1/40   25   +52   +3

              5   11   -7]   3]   25   -44   -21]

           [40   [40/40   [1

= 1/40   80   =   80/40   =   2   **then x₁ =1; x₂ =2; x₃ = -1**

             -40]   -40/40]   -1]