MATRICES: -

A rectangular array of m*n numbers (real or complex) in the form of m horizontal lines (called rows) and n vertical lines (called column) is called a matrix order m by n written as m*n matrix such as enclosed by [] or ().

INTRODUCTION TO MATRIX

An introduction m*n matrix is usually written as

A= [a11 a12 …aln
A21 a22 a2n
Am1 am2 amn]

In brief the above matrix is represented by A= [aij] m*n. the number a11, a12,..etc, are known as the element of the matrix A , where aij belongs to the i^{th} row and j^{th} column and is called the $(i, j)^{the}$ element of the matrix A =[aij].

If A and B are square matrix of order n and In is a corresponding unit matrix then **(a) A (adj. A) = |A|/n = (adj A) A**

Terms Related to Matrix Inverse

The following terms below are helpful for more clear understanding and easy calculation of the inverse of matrix.

Minor: The minor is defined for every element of a matrix. The minor of a particular element is the determinant obtained after eliminating the row and column containing this element.

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a21 a22 a2n
am1 am2 amn]

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Determinant: The determinant of a matrix is the single unique value representation of a matrix. The determinant of the matrix can be calculated with reference to any row or column of the given matrix. The determinant of the matrix is equal to the summation of the product of the elements and its cofactors, of a particular row or column of the matrix.

Singular Matrix: A matrix having a determinant value of zero is referred to as a singular matrix. For a singular matrix A, |A| = 0. The inverse of a singular matrix does not exist.

Non-Singular Matrix: A matrix whose determinant value is not equal to zero is referred to as a non-singular matrix. For a non-singular matrix |A| ≠ 0 and hence its inverse exists.

Adjoint of Matrix: The adjoint of a matrix is the transpose of the cofactor element matrix of the given matrix.

Rules For Row and Column Operations of a Determinant:

The following rules are helpful to perform the row and column operations on determinants.

- The value of the determinant remains unchanged if the rows and columns are interchanged.
- The sign of the determinant changes, if any two rows or (two columns) are interchanged.
- If any two rows or columns of a matrix are equal, then the value of the determinant is zero.
- If every element of a particular row or column is multiplied by a constant, then the value of the determinant also gets multiplied by the constant.
- If the elements of a row or a column are expressed as a sum of elements, then the determinant can be expressed as a sum of determinants.
- If the elements of a row or column are added or subtracted with the corresponding multiples of elements of another row or column, then the value of the determinant remains unchanged.

TYPES of Matrices

[1] Symmetric matrix: - A square matrix A= [aij] is called as symmetric matrix

aij= aij, for all i, j

[2] Skew – Symmetric matrix: when aij = -aij

[3] Hermitian and skew -Hermitian matrix:

[4] orthogonal matrix: - if AA^T =|n =A^T A.

[5] Idempotent matrix: - if A^2 =A

[6] Involuntary Matrix: - a square matrix A is nilpotent, if A^p=0, p is an integer.

TRANSPOSE OF MATRIX: -

The matrix obtains from a given matrix A by changing its row into column or column into rows is called the transpose of matrix A and is denoted by A^T or A' from the definition, it is obvious that if the order of A is M*N then, the order of A^T becomes N*M for example, transpose of matrix.

[a1 a2 a3

B1 b2 b3] $_{2*3}$

Is

[a1 b1

A2 b2

A3 b3] $_{3*2}$

Properties of Transpose of matrix:

I. $(A^T)^T = A$
II. $(A + B)^T = A^T + B^T$
III. $(AB)^T = B^T A^T$
IV. $(KA)^T = K (A)^T$
V. $(A_1 A_2 A_3 \ldots\ldots A_{n-1} A_n)^T = A^T{}_n A^T{}_{n-1} \ldots\ldots A_3^T, A_2^T A_1^T$
VI. $I^T = I$
VII. $tr (A) = tr (A^T)$

so, the matrix refers to a rectangular array of number. A matrix consists of rows and column those rows and column define the size or dimension of a matrix. The various types of matrix are row matrix column matrix, null matrix, square matrix, diagonal matrix, upper triangular matrix, lower triangular matrix, symmetric matrix and asymmetric matrix.

PROGRAM FOR MATRIX TRANSPOSE IN C

The transpose of matrix is a new matrix that is obtained by exchanging the rows and column. In this program the user asked to enter the number of rows ® and number of columns © their values should be less then 10 in this program. Then the user asked to enter the element of matrix (of order (r*c)). The program below then computers the transpose of the matrix and print it on the screen.

PROGRAM:

```
#include<stdio.h>

int main () {

int a [10] [10], transpose [10] [10], r, c;

printf (“enter the rows and column which you want in matrix”);

scanf (“%d %d”, &r, &c);

// assign the element in matrix

Printf (“\n enter the element of the matrix :”);

For (int i=0; i<r; i++)

{

For (int j=0; j<c; j++)

{
```

```
Printf (“%d “, a[i][j]);
If (j==c-1)
Printf (“\n”);
}
// computing the transpose of the matrix
For (int i=0; i<r; ++i)
{
For (int j=0; j<c; ++j)
{
Transpose [j][i] = a[i][j]);
}
Printf (“\n”);
// printing the transpose
Printf (“\n transpose of the matrix:”);
For (int i=0; i<c; ++i)
{
for (int j=0; j<r; ++j)
{
Printf (“%d”, transpose[i][j]);
If (j = = r-1)
Printf (“\n”);
}
Return 0;
}
```

Output :

```
Enter rows and column
2   3
Enter the element a11: 1
```

Enter the element a12: 4

Enter the element a13: 0

Enter the element a21: -5

Enter the element a22: 2

Enter the element a23: 7

Enter matrix 1 4 0

-5 2 7

Transpose of the matrix

1 -5

4 2

0 7

Definition. The determinant of a matrix is a single numerical value which is used when calculating the inverse or when solving systems of linear equations. The determinant of a matrix A is denoted |A|, or sometimes det(A).

The value of the determinant of a matrix can be calculated by the following procedure:

- For each element of the first row or first column get the cofactor of those elements.
- Then multiply the element with the determinant of the corresponding cofactor.
- Finally, add them with alternate signs.

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$|A| = a(ei - fh) - b(di - gf) + c(dh - eg)$$

In terms of Cofactor:

$$\left[a \times \begin{vmatrix} e & f \\ h & i \end{vmatrix} \right] - \left[b \times \begin{vmatrix} d & f \\ g & i \end{vmatrix} \right] + \left[c \times \begin{vmatrix} d & e \\ g & h \end{vmatrix} \right]$$

Finding an **Inverse Matrix by Elementary Transformation**

Let us consider three matrices X, A and B such that X = AB. To determine the inverse of a matrix using elementary transformation, we convert the given matrix into an identity matrix. Learn more about how to do elementary transformations of matrices here.

If the inverse of matrix A, A^{-1} exists then to determine A^{-1} using elementary row operations

- Write A = IA, where I is the identity matrix of the same order as A.
- Apply a sequence of row operations till we get an identity matrix on the LHS and use the same elementary operations on the RHS to get I = BA. The matrix B on the RHS is the inverse of matrix A.
- To find the inverse of A using column operations, write A = IA and apply column operations sequentially till I = AB is obtained, where B is the inverse matrix of A.
- be the 3 x 3 matrix. The inverse matrix is:

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} & \begin{vmatrix} a_{13} & a_{12} \\ a_{33} & a_{32} \end{vmatrix} & \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} \\ \begin{vmatrix} a_{23} & a_{21} \\ a_{33} & a_{31} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} & \begin{vmatrix} a_{13} & a_{11} \\ a_{23} & a_{21} \end{vmatrix} \\ \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} & \begin{vmatrix} a_{12} & a_{11} \\ a_{32} & a_{31} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \end{bmatrix}$$

Example: matrix inversion method

This method can be applied Inverse of Matrix

The inverse of Matrix for a matrix A is denoted by A^{-1}. The inverse of a 2×2 matrix can be calculated using a simple formula. Further, to find the inverse of a matrix of order 3 or higher, we need to know about the determinant and adjoint of the matrix. The inverse of a matrix is another matrix, which by multiplying with the given matrix gives the identity matrix.

The inverse of matrix is used of find the solution of linear equations through the matrix inversion method. Here, let us learn about the formula, methods, and terms related to the inverse of matrix.

What is Inverse of Matrix?

The **inverse of matrix** is a matrix, which on multiplication with the given matrix gives the multiplicative identity. For a square matrix A, its inverse is A^{-1}, and $A \cdot A^{-1} = A^{-1} \cdot A = I$, where I is the identity matrix. The matrix whose determinant is non-zero and for which the inverse matrix can be calculated is called an invertible matrix.

Inverse Matrix Formula

In the case of real numbers, the inverse of any real number *a* was the number a^{-1}, such that *a* times a^{-1} equals 1. We knew that for a real number, the inverse of the number was the reciprocal of the number, as long as the number wasn't zero. The inverse of a square matrix A, denoted by A^{-1}, is the matrix so that the product of A and A^{-1} is the identity matrix. The identity matrix that results will be the same size as matrix A.

The formula to find the inverse of a matrix is: $A^{-1} = 1/|A| \cdot \text{Adj } A$, where

- |A| is the determinant of A and
- Adj A is the adjoint of A

$$\textit{inverse of matrix } A^{-1} = \frac{1}{|A|} . Adj\ A$$

Since |A| is in the denominator of the above formula, the inverse of a matrix exists only if the determinant of the matrix is a non-zero value. i.e., $|A| \neq 0$.

How to Find Matrix Inverse?

To find the inverse of a square matrix A, we use the following formula: $A^{-1} = \text{adj}(A) / |A|$**;** $|A| \neq 0$

where

- A is a square matrix.
- adj(A) is the adjoint matrix of A.
- |A| is the determinant of A.

Methods to Find Inverse of Matrix

The inverse of a matrix can be found using two methods. The inverse of a matrix can be calculated through elementary operations and through the use of an adjoint of a matrix. The elementary operations on a matrix can be performed through row or column transformations. Also, the inverse of a matrix can be calculated by applying the inverse of matrix formula through the use of the determinant and the adjoint of the matrix. For performing the inverse of the matrix through elementary column operations we use the matrix X and the second matrix B on the right-hand side of the equation.

- Elementary row or column operations

- Inverse of matrix formula (using the adjoint and determinant of matrix)

Let us check each of the methods described below.

Elementary Row Operations

To calculate the inverse of matrix A using elementary row transformations, we first take the augmented matrix [A | I], where I is the identity matrix whose order is the same as A. Then we apply the row operations to convert the left side A into I. Then the matrix gets converted into $[I \mid A^{-1}]$. For a more detailed process, click here.

Elementary Column Operations

We can apply the column operations as well just like how the process was explained for row operations to find the inverse of matrix.

Inverse of Matrix Formula

The inverse of matrix A can be computed using the inverse of matrix formula, $A^{-1} = (\text{adj } A)/(\det A)$. i.e., by dividing the adjoint of a matrix by the determinant of the matrix. The inverse of a matrix can be calculated by following the given steps:

- **Step 1:** Calculate the minors of all elements of A.
- **Step 2:** Then compute the cofactors of all elements and write the cofactor matrix by replacing the elements of A by their corresponding cofactors.
- **Step 3:** Find the adjoint of A (written as adj A) by taking the transpose of the cofactor matrix of A.
- **Step 4:** Multiply adj A by the reciprocal of the determinant

Important Points on Inverse of a Matrix:

- The inverse of a square matrix (if exists) is unique.
- If A and B are two invertible matrices of the same order then $(AB)^{-1} = B^{-1}A^{-1}$.
- The inverse of a square matrix A exists, only if its determinant is a non-zero value, $|A| \neq 0$.
- The determinant of matrix inverse is equal to the reciprocal of the determinant of the original matrix.
- The determinant of the product of two matrices is equal to the product of the determinants of the two individual matrices. $|AB| = |A|.|B|$

  FOR EXAMPLE

  Solve the following system of equation using matrix inversion method

  $2x_1 + 3x_2 + 3x_3 = 5;$

  $x_1 - 2x_2 + x_3 = -4;$

  $3x_1 - x_2 - 2x_3 = 3;$

  Soluation

First we can written the given system of eq. in matrix form

$2x_1 + 3x_2 + 3x_3 = 5;$

$x_1 - 2x_2 + x_3 = -4;$

$3x_1 - x_2 - 2x_3 = 3;$

Where consider the matrix equation. AX=B.

Then A $\begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}$; X= $\begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix}$; B= $\begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$ then find |A|

$|A| = \begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}$ det |A| =(a-b+c) find the cofactor of a,b, c which is the first row element in in matrix A i.e. 2, 3, 3;

|A|= |2*| (-2*-2) - (-1*-1) | - 3*| (1*-2) – (3*1) | + 3*| (1*-1) – (3*-2) |

|A|= 2 (4+1) – 3 (-2-3) + 3 (-1+6)

|A|= 10 +15 +15

|A|= 40

If $|A| \neq 0$ then we can find the inverse of matrix hence |A|= 40 then A^{-1} exists.

$$\textit{inverse of matrix } A^{-1} = \frac{1}{|A|} . Adj\ A$$

A^{-1} = A $\begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}$

= 1/40 * A $\begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}$ find adjoint of

Adj A $= \begin{vmatrix} +(4+1) & -(-2-3) & +(-1-6) \\ -(-6+3) & +(-4-9) & -(-2-9) \\ +(3+6) & -(-2-3) & +(-4-3) \end{vmatrix} = \begin{bmatrix} 5 & 5 & 5 \\ 3 & -13 & 11 \\ 9 & 1 & -7 \end{bmatrix}$

AdjA = transpose of adjA $= \begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$

Then applying x= $A^{-1*}B$

$$1/40 * = \begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix} * \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix} = 1/40 \begin{bmatrix} 25 & -12 & 27 \\ 25 & +52 & +3 \\ 25 & -44 & -21 \end{bmatrix}$$

$$= 1/40 \begin{bmatrix} 40 \\ 80 \\ -40 \end{bmatrix} = \begin{bmatrix} 40/40 \\ 80/40 \\ -40/40 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$ **then $x_1 = 1$; $x_2 = 2$; $x_3 = -1$**