AI SMPS 2023 Week 3 Algorithms

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Node Order

MoveGen determines the order in which nodes are generated, and algorithms determine the order in which nodes are inspected. Some algorithms follow MoveGen order and some reorder the nodes.

For each algorithm discussed in this course, study the order in which nodes are added-to and removed-from OPEN/CLOSED lists and other data structures.

It is important to understand how list structures are constructed, accessed and printed. Study the 'cons' and 'append' operators and pay attention to the order in which items are cons-ed (added) and appended to lists.

WARNING: while answering auto-graded short-answer type questions, it is important to maintain the algorithm specific node order, otherwise, the autograder will mark the answer as wrong and there are no partial marks.

WARNING: often, participants understand the concepts correctly and they may follow a different variation of an algorithm or follow a different implementation of cons and append operators and arrive at a practically viable solution but with a different node order, in this case the auto-grader will mark the answer as wrong.

Tie Breaking a Pool of Best Nodes

MoveGen determines the order in which nodes are generated, and algorithms determine the order in which nodes are inspected. Some algorithms follow MoveGen order and some reorder the nodes.

Cost based (deterministic or heuristic) algorithms select the best node in each iteration. Often there will be a pool of best nodes with the same cost and we do not know which nodes lead to a solution.

In a real world scenario, an additional criteria (external to the algorithm) may be provided to filter the pool of best nodes, or else a random node may be selected from the pool. Each such selection may potentially lead to a different solution or no solution at all.

A pool of best nodes make it difficult to setup all possible answers for an auto-graded question, so we use node label to break ties.

When multiple nodes have the same (best) cost, sort those nodes in alphabetical order of node label and select nodes from the head of the sorted list.

This tie-breaker helps us to focus on the algorithm and not get beat up over finer details of (made up) toy problems in the assignments.

In the assignments and exams, when we apply a tie-breaker, we will accept the outcome without further debate/analysis.

On the other hand, outside the scope of auto-graded assignments and exams, we urge you to investigate all the problems/solutions/algorithms and find ways to improve it. Wish you the best.

Algorithms

```
BEST-FIRST-SEARCH(S)
    OPEN \leftarrow (S, null, h(S)) : []
 1
    CLOSED ← empty list
 2
    while OPEN is not empty
 3
 4
         nodePair ← head OPEN
         (N, \underline{\hspace{1em}}, \underline{\hspace{1em}}) \leftarrow nodePair
 5
         if GOALTEST(N) = TRUE
 6
              return RECONSTRUCTPATH(nodePair, CLOSED)
 7
         else CLOSED ← nodePair : CLOSED
 8
              neighbours \leftarrow MOVEGEN(N)
 9
              newNodes ← REMOVESEEN(neighbours, OPEN, CLOSED)
10
              newPairs ← MAKEPAIRS(newNodes, N)
11
              OPEN \leftarrow sort_h (newPairs ++ tail OPEN)
12
    return empty list
13
BEST-NEIGHBOUR-SEARCH(S)
1
   N \leftarrow S
   bestEver \leftarrow S
2
   until some termination condition
3
4
      N \leftarrow best Move-Gen(N)
        if N is better than bestEver
5
             bestEver \leftarrow N
6
   return bestEver
HILL-CLIMBING(S)
   N \leftarrow S
1
  do bestEver \leftarrow N
2
        N \leftarrow head sort_h MOVEGEN(bestEver)
3
   while h(N) is better than h(bestEver)
4
   return bestEver
5
Variable-Neighbourhood-Descent(S)
   MoveGenList \leftarrow MoveGen<sub>1</sub>: MoveGen<sub>2</sub>: \cdots: MoveGen<sub>n</sub>: []
1
   bestNode \leftarrow S
2
3
   while MoveGenList is not empty
        bestNode ← HILL-CLIMBING(bestNode, head MoveGenList)
4
        MoveGenList ← tail MoveGenList
5
   return bestNode
ITERATED-HILL-CLIMBING(N)
   bestNode ← random candidate solution
1
   repeat N times
3
        currentBest ← HILL-CLIMBING(new random candidate solution)
        if h(currentBest) is better than h(bestNode)
4
5
             bestNode ← currentBest
   return bestNode
   node ← random candidate solution or start node
1
2
   bestNode ← node
   repeat N times
3
        node ← random node from MoveGen(node)
4
        if h(node) is better than h(bestNode)
5
             bestNode \leftarrow node
6
   return bestNode
7
```

Beam Search

We will use the following version of Beam Search in assignments and exams because it is assignment friendly and prevents infinite loops.

Here, sorth sorts from best to worst h-values.

```
BEAM-SEARCH(S, w)
    OPEN ← S:[7
 1
 2
    N \leftarrow S
 3
    do bestEver \leftarrow N
         if OPEN contains goal node
 4
 5
               return that goal node
         else neighbours \leftarrow MOVE-GEN(OPEN)
 6
               OPEN \leftarrow take w (sort_h neighbours)
 7
               N \leftarrow head OPEN
 8

    best in new layer

 9
    while h(N) is better than h(bestEver)
10
    return bestEver
Move-Gen(OPEN)
   neighbours \leftarrow []
1
   for each X in OPEN
3
        neighbours \leftarrow neighbours ++ MOVE-GEN(X)
   return neighbours ▷ the list preserves duplicates
(take n LIST) returns at most n values from the beginning of LIST.
   [o, u, t] = take 3 [o, u, t, r, u, n]
   [a, t] = take [a, t]
   \lceil a \rceil = take 3 \lceil a \rceil
   [] = take 3 []
```

```
BEST-NEIGHBOUR-SEARCH(S)
       N \leftarrow S
1
       bestEver \leftarrow S
2
3
       until some termination condition
                 N \leftarrow best Move-Gen(N)
4
5
                 if N is better than bestEver
6
                          bestEver \leftarrow N
       return bestEver
TABU-SEARCH(tt)
         F \leftarrow array of N zeros

    ▶ use frequency to compute penalty

  2
         M \leftarrow array of N zeros
                                                                                                   ▶ use memory to track tenure
        currentNode ← choose a node randomly
  3
                                                                                                                                      bestEver ← currentNode
  4
         while some termination criteria
  5
                   GENERATE-NEIGHBOURS
  6
  7
                   FIND-BEST-NEIGHBOUR
  8
                   if bestAllowedValue is not worse than eval(currentNode)
  9
                             Move-To(bestAllowedIndex)
                                                                                                                                 else if bestValue is better than eval(bestEver)
10
                             Move-To(bestIndex)
11

    b the aspiration criterion
    contact the co
12
                   else FIND-BEST-ALLOWED-WITH-PENALITY

    □ diversify search

                             MOVE-To(bestAllowedIndex)
13
         return bestEver
14
         GENERATE-NEIGHBOURS
15
16
                   for i \leftarrow 1 to N
17
                             neighbour(i) ← CHANGE(currentNode, i)
                             value(i) \leftarrow eval(neighbour(i))
18
                             tabu(i) \leftarrow if M(i) > 0 then YES else NO
19
20
         FIND-BEST-NEIGHBOUR
21
                   (bestValue, bestIndex) ← (worst value, null)
                   (bestAllowedValue, bestAllowedIndex) ← (worst value, null)
22
                   for i \leftarrow 1 to N
23
                             if value(i) is better than bestValue
24
                                        (bestValue, bestIndex) \leftarrow (value(i), i)
25
                             if tabu(i) = NO and value(i) is better than bestAllowedValue
26
                                        (bestAllowedValue, bestAllowedIndex) \leftarrow (value(i), i)
27
28
         FIND-BEST-ALLOWED-WITH-PENALITY

    □ use frequency to diversify search

29
                   (bestAllowedValue, bestAllowedIndex) ← (worst value, null)
                   for i \leftarrow 1 to N, if tabu(i) = NO
30
                             value(i) \leftarrow value(i) - Penalty(F(i))
31
                                                                                                                            32
                             if value(i) is better than bestAllowedValue
                                       (bestAllowedValue, bestAllowedIndex) \leftarrow (value(i), i)
33
         MOVE-To(index)
34
35
                   currentNode ← neighbour(index)
                   if value(index) is better than eval(bestEver)
36
                             bestEver ← currentNode
37
                   for i \leftarrow 1 to N, if M(i) > 0
38
                             M(i) \leftarrow M(i) - 1
39
40
                   M(index) \leftarrow tt
                   F(index) \leftarrow F(index) + 1
41
```

```
RANDOM-WALK(N)
   node ← random candidate solution or start node
2
   bestNode ← node
3
   repeat N times
4
        node \leftarrow random node from MoveGen(node)
5
        if h(node) is better than h(bestNode)
6
             bestNode ← node
   return bestNode
SIMULATED-ANNEALING
    node ← random candidate solution or start node
 1
    bestNode ← node
 3
    T \leftarrow some large value
    for time \leftarrow 1 to number-of-epochs
 4
          while some termination criteria
 5
                neighbour ← RANDOM-NEIGHBOUR(node)
 6
               \Delta E \leftarrow eval(neighbour) - eval(node)
 7
               if random(0, 1) < 1/(1 + e^{-\Delta E/T})
 8
 9
                     node ← neighbour
                     if eval(node) is better than eval(bestNode)
10
                           bestNode ← node
11
          T \leftarrow \text{COOLING-FUNCTION}(T, \text{time})
12
    return bestNode
13
```

TSP Algorithms

NEAREST-NEIGHBOUR-HEURISTIC

- 1 Start at some city
- 2 Move to the nearest neighbour as long as it does not close the loop prematurely

Greedy-Heuristic

- 1 Sort the edges by edge-cost
- 2 Add shortest available edge to the tour as long as it does not close the loop prematurely and as long as it does not form branches/forks

SAVINGS-HEURISTIC

```
1
    n \leftarrow select \ a \ base \ city \ from \ N \ cities
                                                    ▶ base city
 2
    construct (N-1) tours, each length 2 anchored at n
 3
    savingsList ← empty list
    for each non base city a
 4
 5
         for each non base city b, if b \neq a
               savings \leftarrow \cos((n, a) + \cos((n, b) - \cos((a, b)))
 6
               savingsList \leftarrow (a, b, savings): savingsList
 7
    sort savingsList in descending order of savings
 8
 9
    for each tuple (a, b, _ ) in savingsList
         if edges (n, a) and (n, b) are in different tours
10
11
               merge those tours into a single tour:
                    remove the edges (n, a) and (n, b)
                    and insert the edge (a, b)
12
    return the final tour
```