CSCI971 Advance Computer Security: Homework #2

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Problem 1

Solution
We define the outputs as O_0, O_1
for 0^{64}, there is

$$R_0 = 0^{32},\ L_0 = 0^{32}$$

$$L_1 = R_0 = 0^{32},\ R_1 = F(k_1, R_0) \oplus L_0 = F(k_1, R_0)$$

$$L_2 = R_1 = F(k_1, R_0),\ R_2 = F(k_2, R_1) \oplus L_1 = F(k_2, F(k_1, 0^{32}))$$

Similarly, for $1^{32}0^{32}$, there is

$$L_2 = \bar{F}(k_1, 0^{32}),\ R_2 = F(k_2, \bar{F}(k_1, 0^{32}))$$

thus we can define, $m_0 = F(k_1, 0^{32})$, $c_0 = F(k_2, m_0)$, $m_1 = \bar{F}(k_1, 0^{32}) = \bar{m_0}$, $c_1 = F(k_2, m_1)$
if two outputs are from PRP, then the left 32 bits of $O_1 \oplus O_2$ is 1^{32}
we can easily find that 2) is from PRP, and the other 3 is from random permutation.

Problem 2

Solution
We can draw the whole process of the protocol

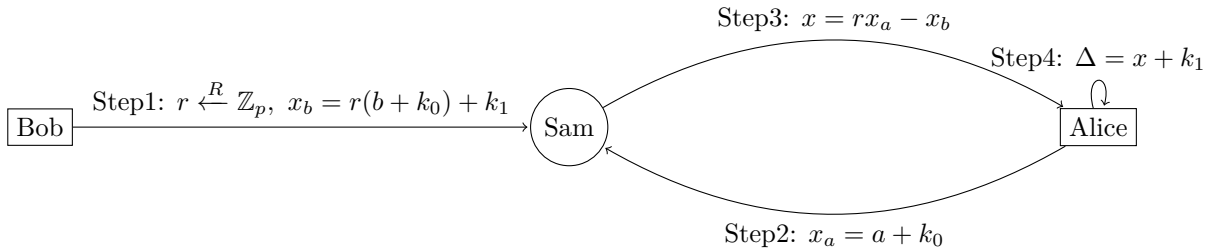

Figure 1: The protocol procedure

As $\Delta = x + k_1 = r(a - b)$ so we get the condition that $r \neq 0$
if (k_0, k_1) are used for more than once. We assume it was tested if $a = b$ and $a' = b'$
For Sam:

$$\begin{cases} x_a = a + k_0 \\ x_b = r(b + k_0) + k_1 \end{cases}$$

$$\begin{cases} x'_a = a' + k_0 \\ x'_b = r'(b' + k_0) + k_1 \end{cases}$$

So Sam learned that $a' - a = x'_a - x_a$.
For Alice:

$$\begin{cases} x = r(a - b) - k_1 \\ x' = r'(a' - b') - k_1 \end{cases}$$

Alice learned ration of $(a - b)/(a' - b')$ which reveal b/b'

Bugfix
the core of the issue was the reusing of the (k_0, k_1), so we should generate new indenpendent key from it by PRF. We define a secure PRF F defined at $\{K, \mathbb{Z}_b, \mathbb{Z}_p^2\}$ we can derive the key-pair (k_p, k_q) from F and

Problem 2 continued on next page...

initial seed key k as: $F(k, n) = (k_p, k_q)$ and n is value of counter.
We should let Alice and Bob get the value of counter synchronically for the syncronicity of key-pair. each time Alice and Bob finish the comparison, they increase the their counter by 1. so they get the syncronical key-pair.

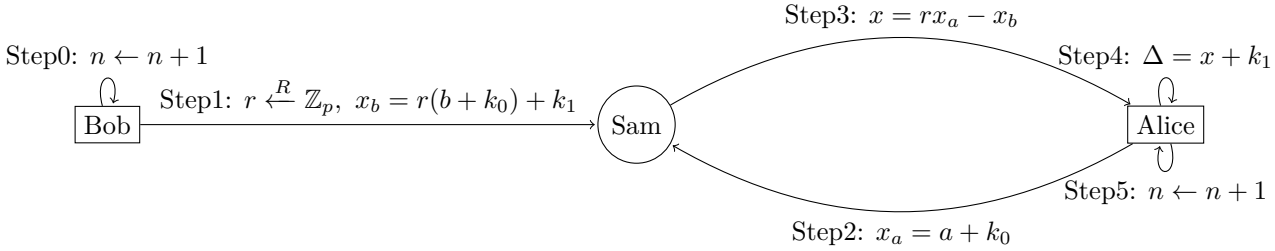

Figure 2: The fixed protocol procedure