

**CSCI971 Advance Computer Security:  
Homework #8**

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2019124044**

## Problem 1

For a RSA trapdoor function, if it is used directly as the encryption, then We assume the Challenger  $\mathcal{C}$  produce RSA params with  $G()$  and get  $pk = (N, e), sk = (N, d)$ , then Adversary produce two message  $m_1, m_2$  and  $|m_1| = |m_2|$ , Challenger select  $b$  randomly and perform encryption  $c_b = E(pk, m_b) = m_b^e$ , the decryption should be  $m_b = c_b^d$ . The Adversary get  $pk$  and generate  $b$ .

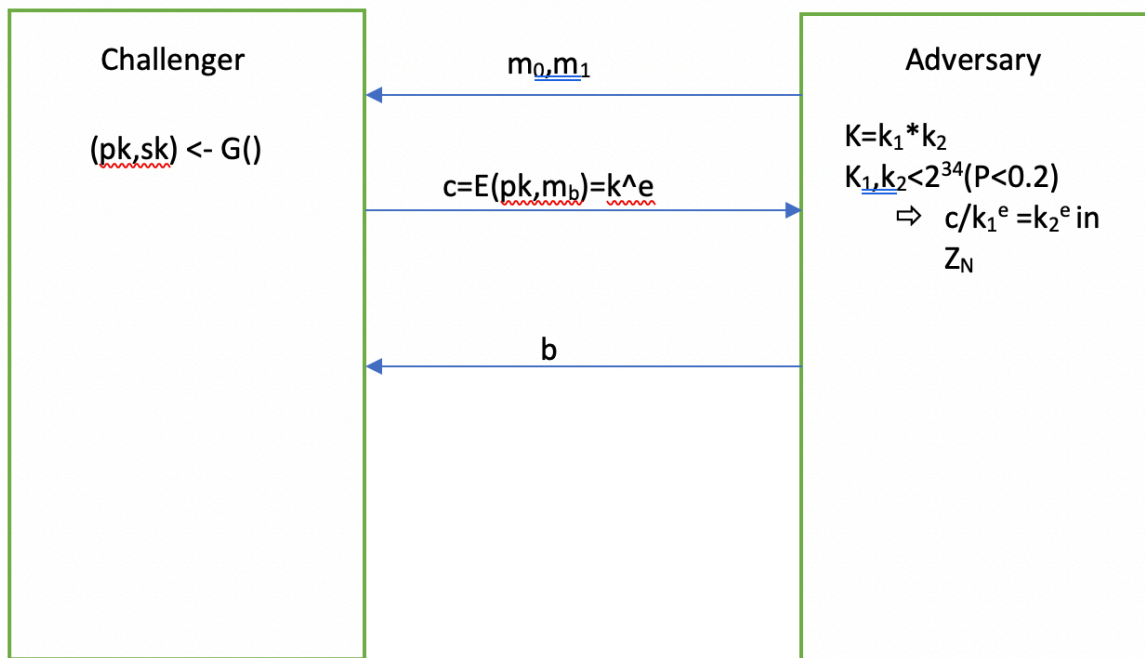


Figure 1: Attack Game

We suppose  $k$  is 64bits,  $k \in \{0, \dots, 2^{64}\}$ , it get  $c = k^e$  in  $Z_N$  if  $k = k_1 * k_2$  where  $k_1, k_2 < 2^{34}$  ( $Pr \approx 0.2$ ) then  $c/k_1^e = k_2^e$  in  $Z_N$ , firstly, Adversary  $\mathcal{A}$  can build table of  $k_2^e = c/1^e, c/2^e, \dots, c/2^{34e}$  then he can iteratively test if  $k_2^e$  is in table. he can output matching  $k_1, k_2$ .

## Problem 2

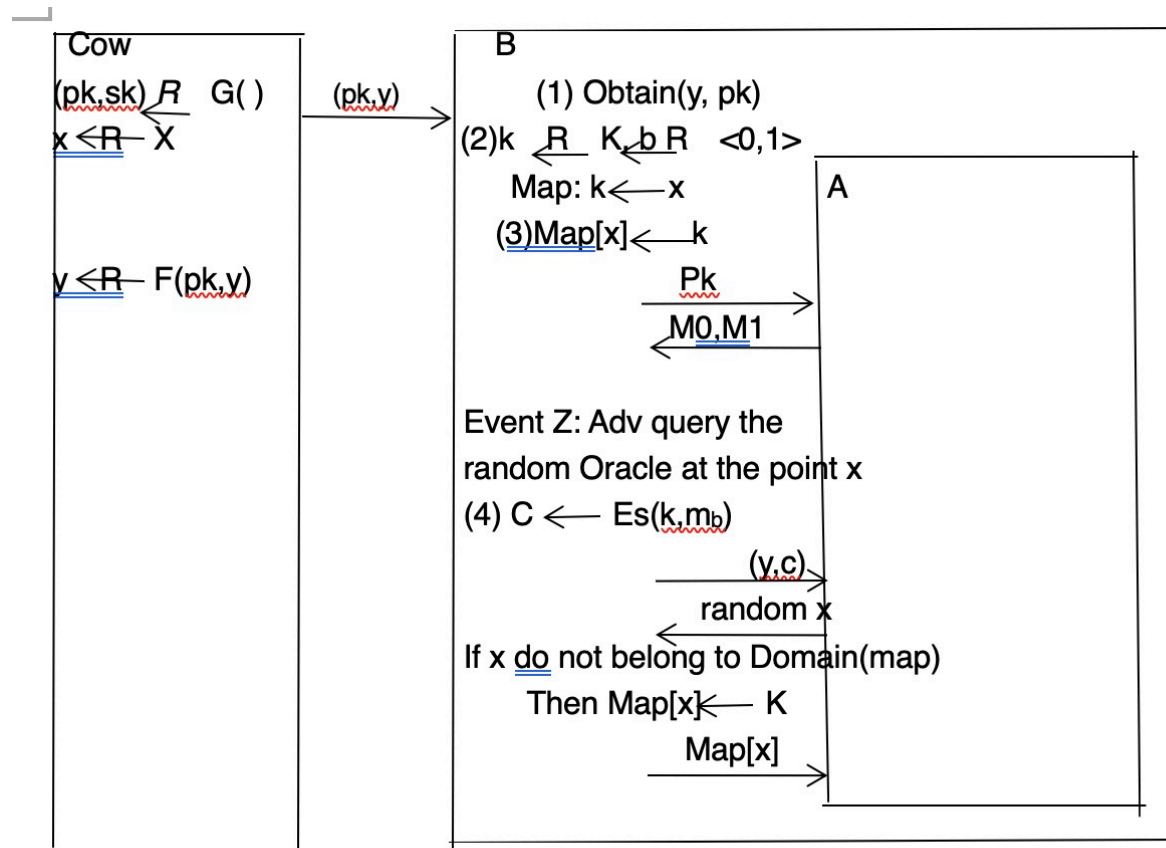


Figure 2:

As  $SSadv[A, \epsilon_{TDF}] = 2SS^{roadv*}[A, \epsilon_{TDF}]$ . We need to prove:

$$SS^*adv[A, \epsilon_{TDF}] \leq OWadv[B_{ow}, T] + SSadv^*[B_s, \epsilon_s]$$

We can define Game0 and Game1 like this: Set  $W_j$  as  $\hat{b} = b$  in Game  $j$ , ( $j=0,1$ ).

Then get  $|Pr[W_1] - Pr[W_0]|$  is negligible and  $|Pr[W1]| \approx 1/2$ .

Then  $SS^{roadv^*}[A, \epsilon_{TDF}] = |\Pr[W0] - 1/2|$  is negligible.

Game0: Adversary can make any number of random oracle queries but at most one encryption query.

Game1: The  $(PK, y)$  is obtained by Cow .

Set  $Z$ : Adversary queries the random oracle at the point  $x$  in Game1. At this time, Game0 and Game1 proceed identically unless  $Z$  occurs, So We can have:

$$|Pr[W1] - Pr[W0]| \leq Pr[Z]$$

If event Z happens, then one of the adv's random queries is the inverse of  $y$  under  $F(pk, \cdot)$ . In Game1, the value of  $x$  is only used to define  $y$ .

Then use that breaks the OW for TDF with advantage equal to  $\Pr[Z]$ .

Lets view Game1 and the game between Bow and Cow. By the definition above  $Z$  occurs if and only if  $x \in \text{Domain}(\text{Map})$  when Bow finishes its game. So we can indicate:

$$Pr[Z] = OWadv[B_{ow}, T]$$

Observe that in Game1, the key  $k$  is only used to encrypt the challenge plaintext. As such, the adversary is attacking the bit-guessing version as Attack Game 2.1 from which we can know that:

$$|Pr[W = 1] - 1/2| = SSadv^*[B_s], \epsilon_s$$

Then delete the process of (2),(3) change (4) to forward  $(m_0, m_1)$  to  $C_s$ , obtaining  $c$ . Additionally”

When  $A$  outputs  $\hat{b}$ , then output  $\hat{b}$

Then we can conclude

$$SS^{ro}adv[A, \epsilon_{TDF}] \leq 2OWadv[B_{ow}, T] + SSadv[B_s, E_s]$$