

**CSCI971 Advance Computer Security:
Homework #7**

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Problem 1

AE-secure \Leftrightarrow semantically secure under CPA and CI.

For the first cipher, assume an attacker who can perform CPA. He intercept the ciphertext $c = E_1(k, m) = (E(k, m), H_1(m))$. He can perform as many as CPA. We assume in CPA attack game. Adversary \mathcal{A} first send m_0, m_0 to challenger \mathcal{C} , he get the ciphertext $c = (E(k_0, m_0), H_1(m_0))$. Then \mathcal{A} send m_0, m_1 to \mathcal{C} , as E is CPA secure, so key has to be changed. \mathcal{A} get the ciphertext $c = (E(k_1, m_0), H_1(m_0))$ or $c = (E(k_1, m_1), H_1(m_1))$ based on b . Then if $b = 1$, \mathcal{A} can easily differ the plaintext from the tag $H_1(m_b)$. So $Adv_{CPA}(\mathcal{A}, \mathcal{E}) = 1/2$ is not negligible. Cipher1 is not CPA-secure, so it's not AE-secure.

For the second cipher, attacker can intercept the ciphertext $(c, H_2(c))$, so he can learn the mapping model of H_2 function. So in CI attack game, Adversary \mathcal{A} can easily generate an valid ciphertext-tag pair $(c_{atk}, H_2(c_{atk}))$. Then Decryptor $D_2(k, (c_{ack}, H_2(c_{ack}))) \neq \perp$. So $Adv_{CI}(\mathcal{A}, \mathcal{E})$ is not negligible. Cipher1 does not satisfy CI, so it's not AE-secure.

Problem 2

Addition \mathbb{Z}_6^* is a cyclic group.

$$\mathbb{Z}_6^* = \{0, 1, 2, 3, 4, 5, 6\}$$

1 generate $\{0, 1, 2, 3, 4, 5\}$

2 generate $\{0, 2, 4\}$

3 generate $\{0, 3\}$

4 generate $\{0, 2, 4\}$

5 generate $\{0, 1, 2, 3, 4, 5\}$

So the generators of \mathbb{Z}_6^* are 1, 5, the subgroups are $\{0, 1, 2, 3, 4, 5\}$, $\{0, 2, 4\}$, $\{0, 3\}$

Problem 3

Group under multiplication \mathbb{Z}_{13}^* is a cyclic group.

$$\mathbb{Z}_{13}^* = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$

$$\langle 1 \rangle = \{1\}$$

$$\langle 2 \rangle = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$

$$\langle 3 \rangle = \{1, 3, 9\}$$

$$\langle 4 \rangle = \{1, 3, 4, 9, 10, 12\}$$

$$\langle 5 \rangle = \{1, 5, 8, 12\}$$

$$\langle 6 \rangle = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$

$$\langle 7 \rangle = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$

$$\langle 8 \rangle = \{1, 5, 8, 12\}$$

$$\langle 9 \rangle = \{1, 3, 9\}$$

$$\langle 10 \rangle = \{1, 3, 4, 9, 10, 12\}$$

$$\langle 11 \rangle = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$

$$\langle 12 \rangle = \{1, 12\}$$

So subgroups are $\{1\}$, $\{1, 12\}$, $\{1, 3, 9\}$, $\{1, 5, 8, 12\}$, $\{1, 3, 4, 9, 10, 12\}$, $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

python codes:

```
for i in range(2,13):
    num=1
    cset = set()
    for j in range(1,20):
        num*=i
        cset.add(num % 13)
    print(i, cset)
```
