

Lab 04:

8-2 *Sorting in place in linear time*

Suppose that we have an array of n data records to sort and that the key of each record has the value 0 or 1. An algorithm for sorting such a set of records might possess some subset of the following three desirable characteristics:

1. The algorithm runs in $O(n)$ time.
 2. The algorithm is stable.
 3. The algorithm sorts in place, using no more than a constant amount of storage space in addition to the original array.
- a. Give an algorithm that satisfies criteria 1 and 2 above.
 - b. Give an algorithm that satisfies criteria 1 and 3 above.
 - c. Give an algorithm that satisfies criteria 2 and 3 above.
 - d. Can you use any of your sorting algorithms from parts (a)–(c) as the sorting method used in line 2 of RADIX-SORT, so that RADIX-SORT sorts n records with b -bit keys in $O(bn)$ time? Explain how or why not.
 - e. Suppose that the n records have keys in the range from 1 to k . Show how to modify counting sort so that it sorts the records in place in $O(n + k)$ time. You may use $O(k)$ storage outside the input array. Is your algorithm stable? (*Hint*: How would you do it for $k = 3$?)

9-2 *Weighted median*

For n distinct elements x_1, x_2, \dots, x_n with positive weights w_1, w_2, \dots, w_n such that $\sum_{i=1}^n w_i = 1$, the **weighted (lower) median** is the element x_k satisfying

$$\sum_{x_i < x_k} w_i < \frac{1}{2}$$

and

$$\sum_{x_i > x_k} w_i \leq \frac{1}{2}.$$

For example, if the elements are 0.1, 0.35, 0.05, 0.1, 0.15, 0.05, 0.2 and each element equals its weight (that is, $w_i = x_i$ for $i = 1, 2, \dots, 7$), then the median is 0.1, but the weighted median is 0.2.

- a. Argue that the median of x_1, x_2, \dots, x_n is the weighted median of the x_i with weights $w_i = 1/n$ for $i = 1, 2, \dots, n$.
- b. Show how to compute the weighted median of n elements in $O(n \lg n)$ worst-case time using sorting.
- c. Show how to compute the weighted median in $\Theta(n)$ worst-case time using a linear-time median algorithm such as SELECT from Section 9.3.