West Virginia University

EE 224 Section 03

Electrical Circuits Lab
Spring 2023

Lab 4: Magnetically Coupled Circuits and Transformers

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The objective of this lab is to introduce the concept of Magnetic Coupling. This is when two coils of wire (Inductors) are placed close enough that through which a changing, time dependent current causes mutual inductance. These Circuits are implemented with transformers allowing induced voltage and current without a physical connection (not shorted).

Procedure

This lab has two parts. For part 1, the goal is to explore the impact of mutual inductance when the inductors are in a series aiding connection(dots on same side) compared to a series opposing connection(dots on opposite sides). Currents (phasor rms I) for both circuits are to be calculated and simulated on LTSpice. To calculate I, Ohm's Law is used ($I=V/Z_{total}$) and for the simulation, clicking on any element will give you the current. For the angle of current the Trigger sweep method is used.

For part 2, the ability to transform a magnetically coupled circuit (Linear Transformer) into its T equivalent is explored. The goal for part 2 is to show that ultimately one should see the same amount of current in both versions of the circuits to prove equivalence. For the circuit before any transformation, the current is calculated by writing two mesh equations implementing KVL. Then, the circuit is simulated for current I_2 with the trigger sweep method being used again for angle. Moving on, the calculation and Simulation for circuit after it has been replaced with its T-equivalent is the same. The only difference is that for the KVL equations, mutual inductance does not need to be specially accounted for as the T equivalent simply shorts the physically disconnected “transformer”.

Calculations

Circuit 1 Calculations for Current

%Given parameters
f=60;
V_s= 5; %rms voltage magnitude
R_L = 200;
M=3;
L_1=6;
L_2=12;

%calculations
w= 2 * pi * f;
L_{eq}= L_1 + L_2 + 2*M;
X_{eq} = w * L_{eq};
V_{rms} = Vs * (cos(-pi/2) + sin(-pi/2) * 1j);
Z_{tot}= R_L + X_{eq}*1j;
I=V_{rms} / Z_{tot}

I_{mag} = abs(I)
I_{angle} = angle(I) * 180/pi

I = -5.5235e-04 - 1.2210e-05i
I_{mag} = **5.5249e-04**
I_{angle} = -178.7337

Circuit 2 Calculations for Current

%Given parameters
f=60;
V_s= 5; %rms voltage magnitude
R_L = 200;
M=3;
L_1=6;
L_2=12;

%calculations
w= 2 * pi * f;
L_{eq}= L_1 + L_2 - 2*M;
X_{eq} = w * L_{eq};
V_{rms} = Vs * (cos(-pi/2) + sin(-pi/2) * 1j);
Z_{tot}= R_L + X_{eq}*1j;
I=V_{rms} / Z_{tot}

I_{mag} = abs(I)
I_{angle} = angle(I) * 180/pi

$I = -0.0011 - 0.0000i$
$I_{mag} = 0.0011$
$I_{angle} = -177.4686$

Circuit 3 Calculations for Currents (I_1 and I_2)

%given parameters
f=60;
V_s = 200;
R_1= 4000;
R_2 = 4000;
R_L= 8000;
M=2;
L_1=10;
L_2 = 4;

%Calculations
w= 2 *pi *f;
X_M = w *M;
X_{L1} = w * $L_{1;}$
X_{L2} = w * L_2;
V_{rms} = Vs * (cos(- pi/2) + sin(-pi/2) * 1j);

%I1 and I2 are symbols

syms I_1 I_2
eq1= I_1 *(R_1 + X_{L1} *1j) - X_M *1j * I_2 == V_rms;
eq2= I_2* (R_2 + R_L + X_{L2} *1j) - X_M *1j * I_1 ==0;

sol= solve([eq1,eq2], [I_1,I_2]);
I_1= vpa(sol.I_1)
I_2= vpa(sol.I_2)

I_{1mag} = abs(I_1)
I_{1angle} = vpa(angle(I_1) *180/pi)

I_2mag = abs(I_2)
I_2angle = vpa(angle(I_2) *180/pi)

I_1 = - 0.02464726 - 0.026497i
I_2 = 0.0014474334 - 0.00173052i

I_{1mag} =0.0361885
I_{1angle} = -132.927
I_{2mag} =0.00225605
I_{2angle} = -50.0904

Circuit 4 Calculations for Currents (I_1 and I_2)

%given parameters
f=60;
V_s = 200;
R_1= 4000;
R_2 = 4000;
R_L= 8000;
M=2;
L_1=10;
L_2 = 4;

%Calculations
w= 2 *pi *f;
X_{Lc} = w * L_c;
X_{La} = w * L_a;
X_{Lb} = w * L_b;

V_{rms} = Vs * (cos(- pi/2) + sin(-pi/2) * 1j);

%I1 and I2 are symbols

syms I_1 I_2
eq1= I_1 *(R_1 + X_{La} *1j + X_{Lc} *1j)- X_{Lc} * 1j * I_2== V_rms;
eq2= I_2* (R_2 + R_L+ X_{Lb} *1j + X_{Lc} *1j) - X_{Lc} *1j * I_1==0;

sol= solve([eq1,eq2], [I_1,I_2]);
I_1= vpa(sol.I_1)
I_2= vpa(sol.I_2)

I_{1mag} = abs(I_1)
I_{1angle} = vpa(angle(I_1) *180/pi)

I_2mag = abs(I_2)
I_2angle = vpa(angle(I_2) *180/pi)

I_1= - 0.024647265 - 0.0264976651331i
I_2 =0.001447433 - 0.0017305231i
I_{1mag} = 0.0361885
I_{1angle} =-132.9279
I_2mag = 0.0022560
I_2angle = -50.09042

Result Tables

Current I	Calculated Value (rectangular form)	Simulated Value (Magnitude)
Case 1	5.5235e-04 - j1.221e-05	533.87 µA
Case 2	-0.0011 - j0.0000	1.067 mA

Table 1: Series aiding vs Series Opposing Results

Current I_2	Calculated Value	Simulated Value (Magnitude)
Case 1	0.0014474334 - j0.00173052	2.2556 mA
Case 2	0.001447433 - j0.0017305231	2.2214mA

Table 2: Magnetically coupled circuit vs T equivalent Circuit

Figures and Circuit Diagram

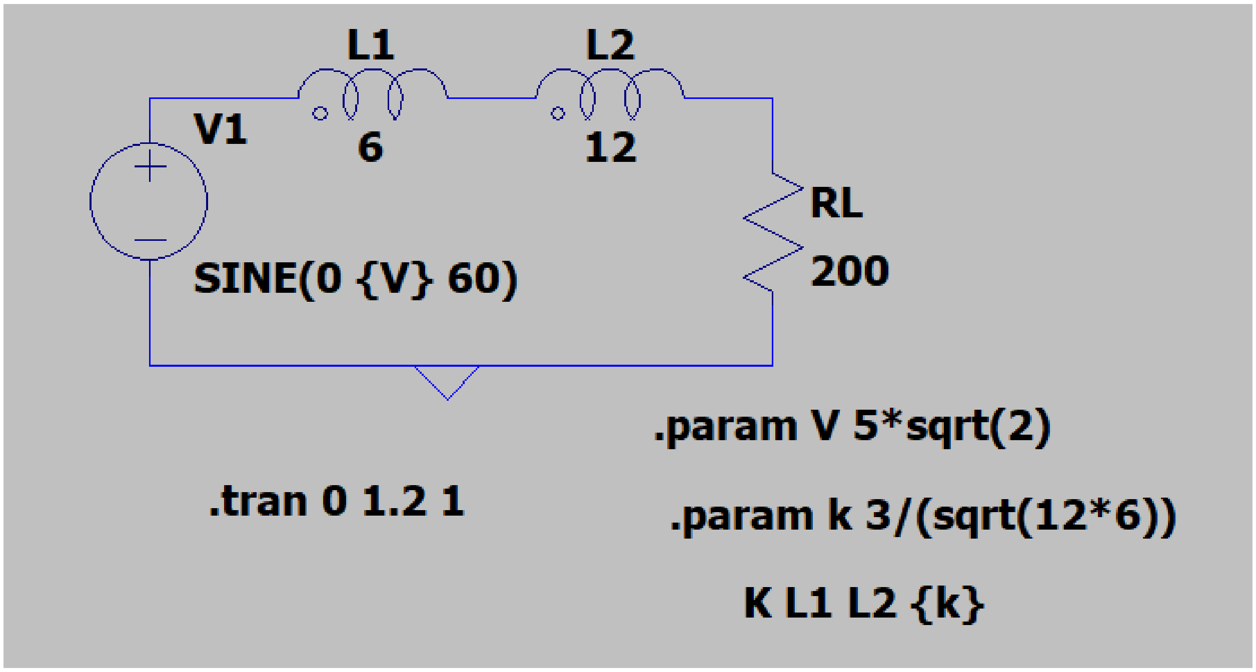

Figure 1a: Circuit 1 (series aiding mutual inductance)

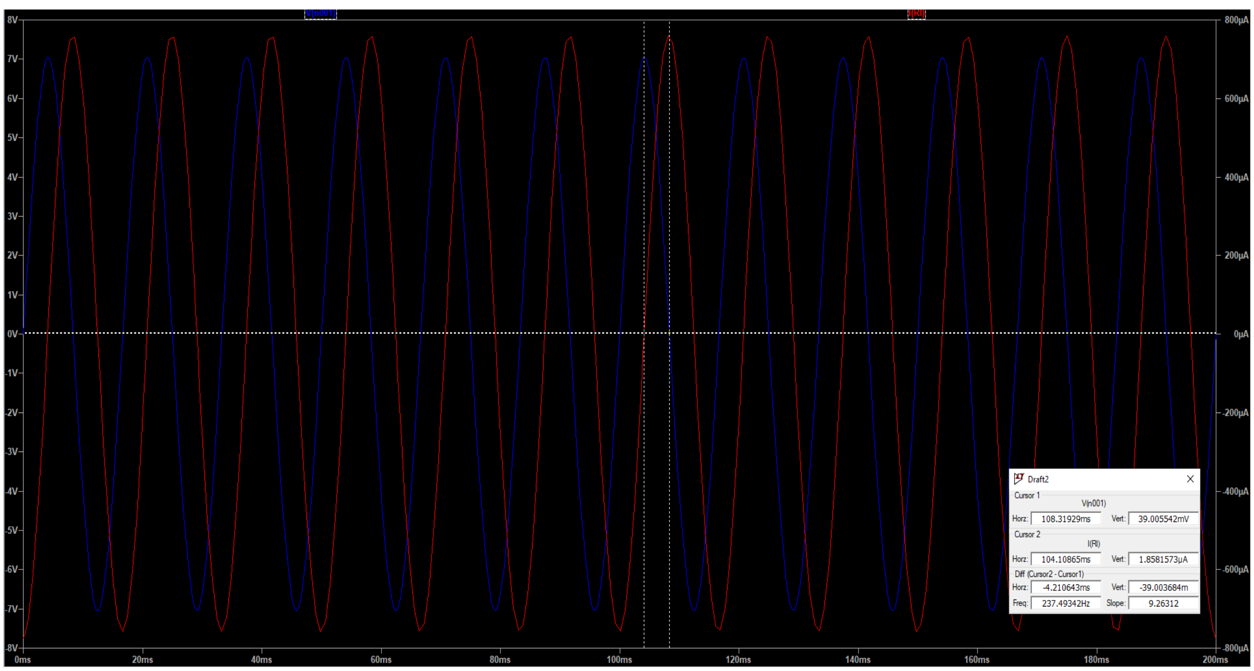

Figure 1b: Plot of Source Voltage and Circuit Current (cursors used for Δt to get I_{angle})

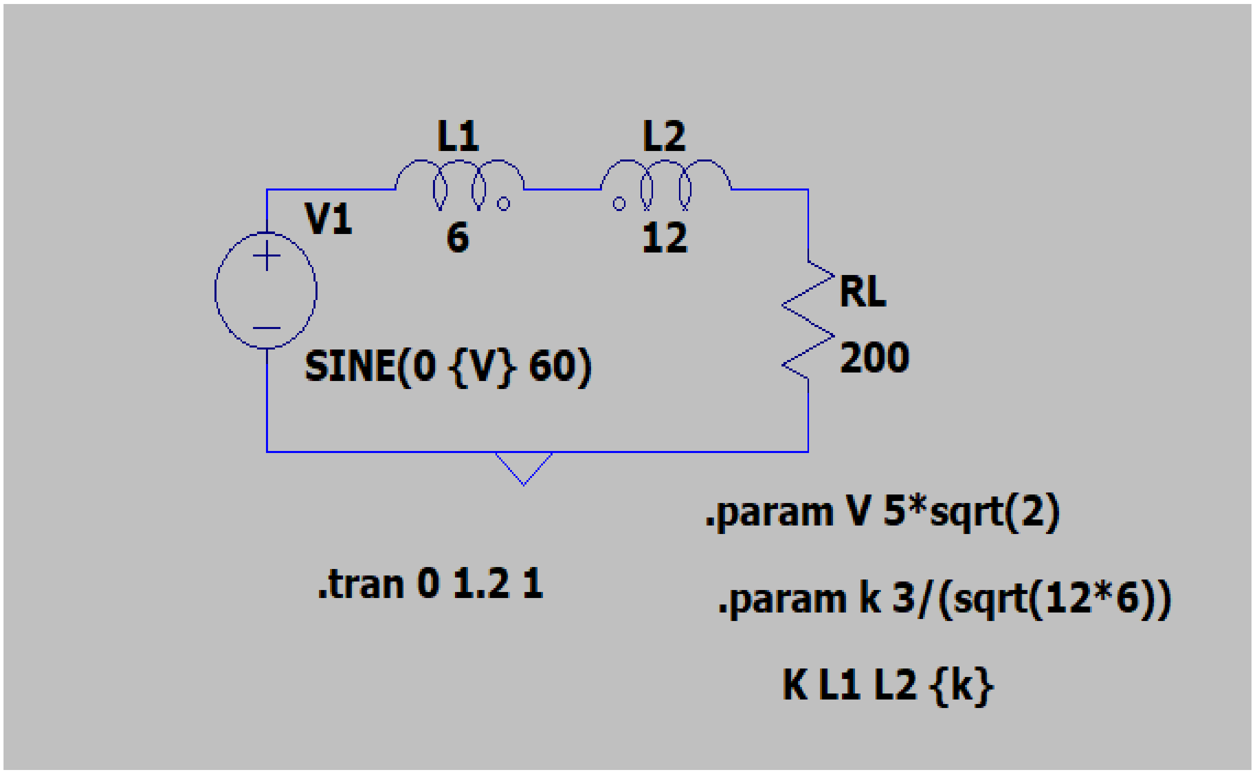

Figure 2a: Circuit 2 (series opposing mutual inductance)

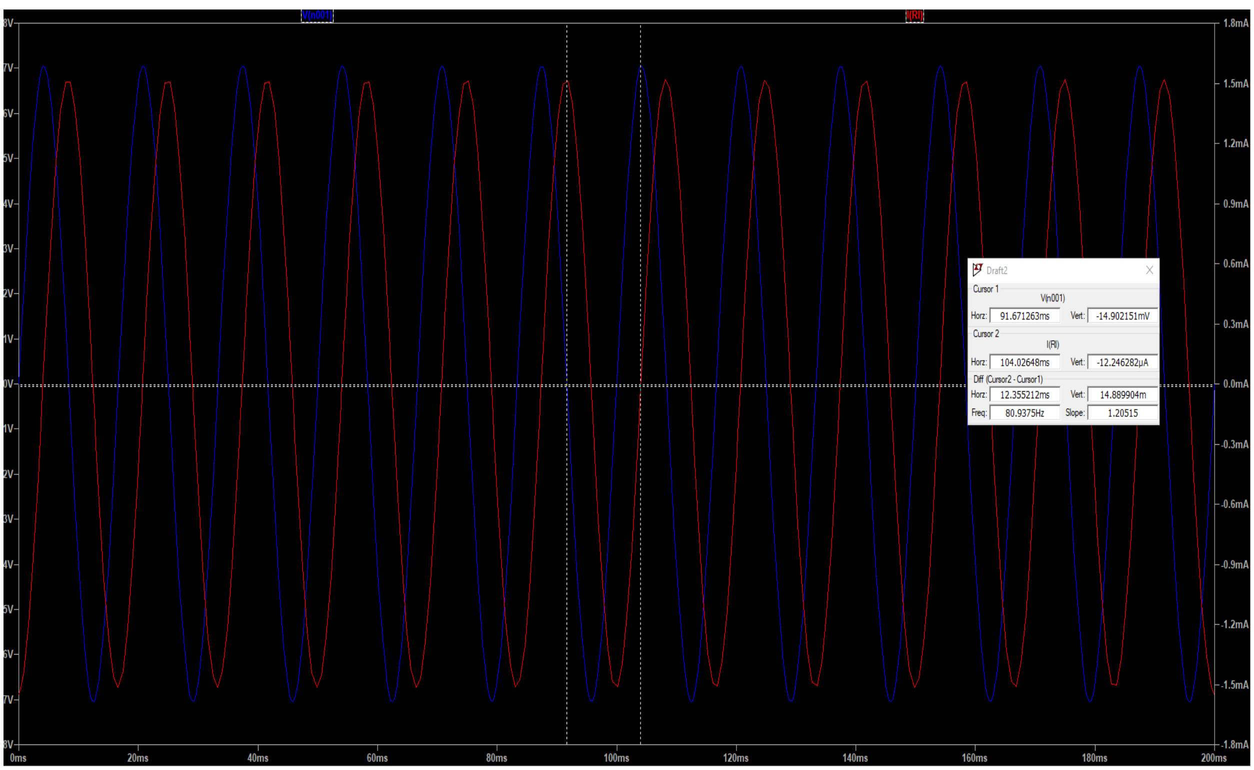

Figure 2b: Output of Source Voltage and Circuit Current (cursors used for Δt to get I_{angle})

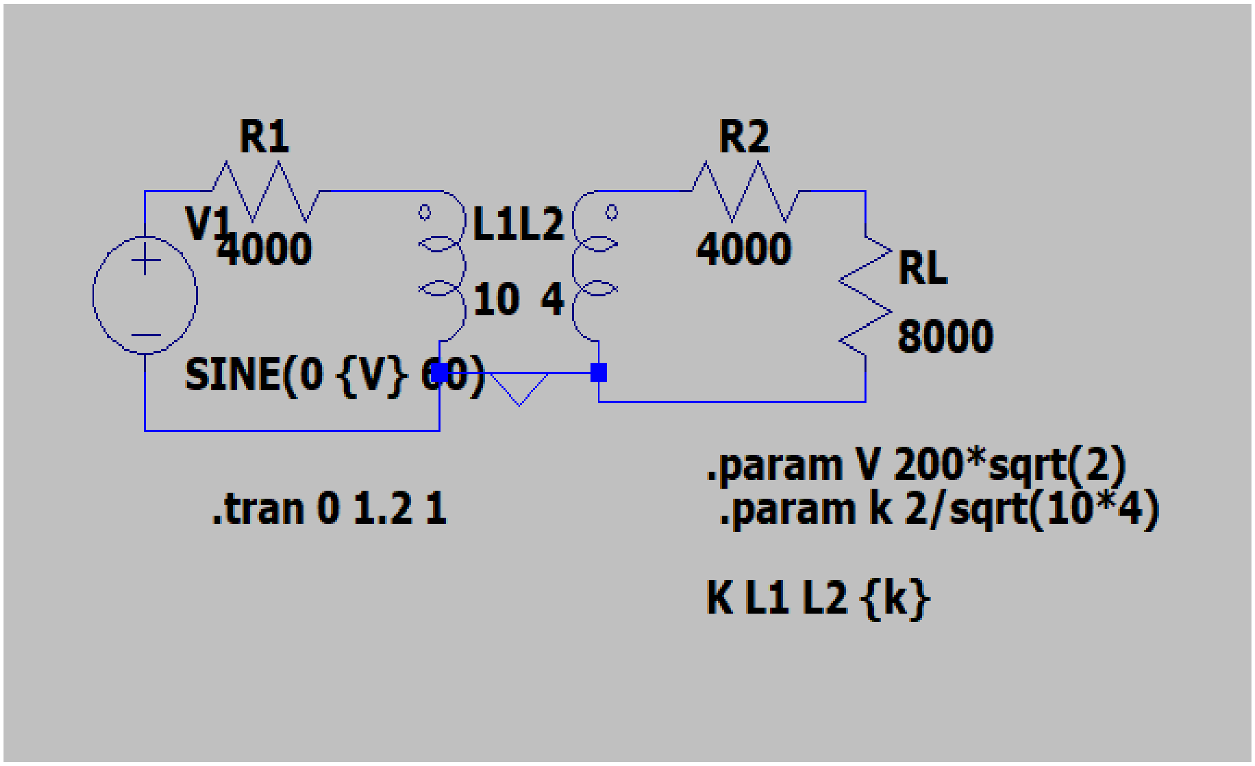

Figure 3a: Circuit 3 (Linear Transformer)

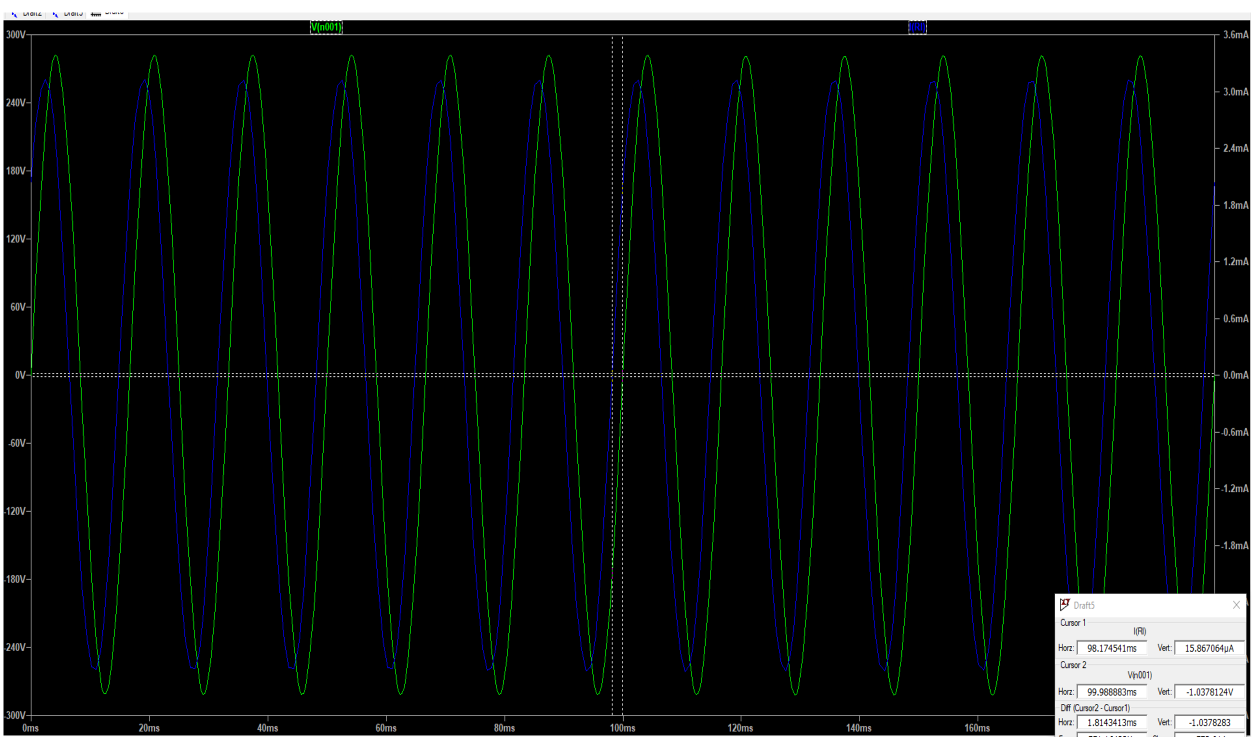

Figure 3b: Output of Source Voltage and Circuit Current (cursors used for Δt to get I_{angle})

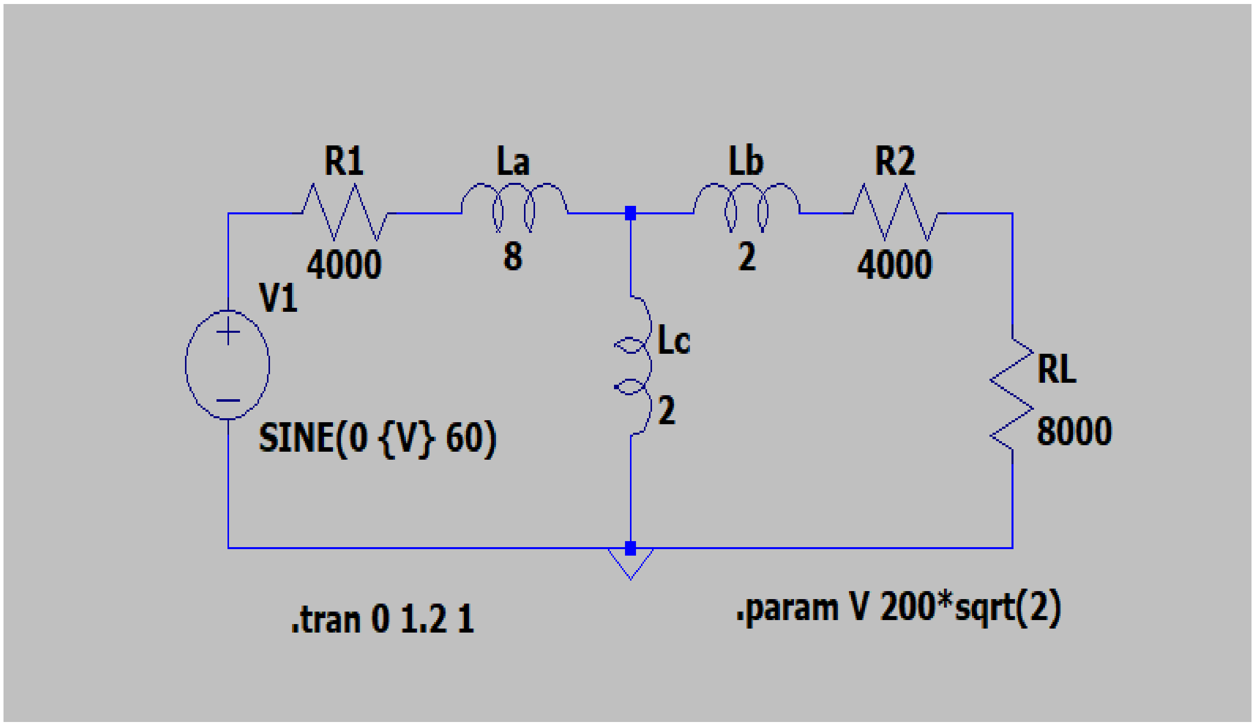

Figure 4a: Circuit 4 (Linear Transformer replaced with T equivalent)

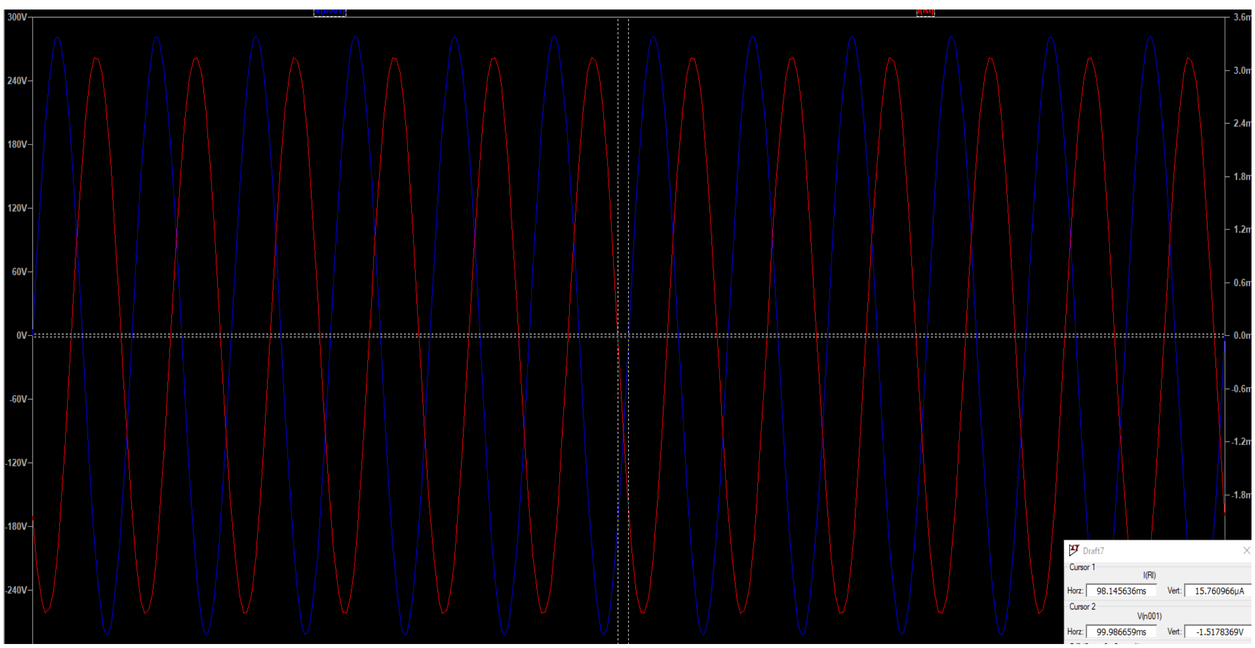

Figure 3b: Output of Source Voltage and Circuit Current (cursors used for Δt to get I_{angle})

Inferences

In part1 it was interesting to see the effect of where the dots are in mutually coupled inductors. Based on the results table 1, it is evident that in case 1 (series aiding connection) there is much less current in the circuit. Conversely, for case 2 (series opposing connection) there is much more current in the circuit. The best explanation for this is to consider the relationship between total inductance(self inductances + mutual inductances) and current. When the connection is aided, the M term is added in the equation. With more inductance it makes sense to see less current. On the other hand with opposing connections, the M term is subtracted. Hence, less total inductance allows for more current flow.

For part 2 the experiment was simple and successful due to the current values being exactly identical (some error due to rounding). By shorting the two previously magnetically coupled circuits an inductor is added to account for the behavior of the mutual inductance. After converting to the T-equivalent circuit, Kirchoff Voltage Laws remained true.