POLITECNICO DI MILANO

Applications of adaptive filtering

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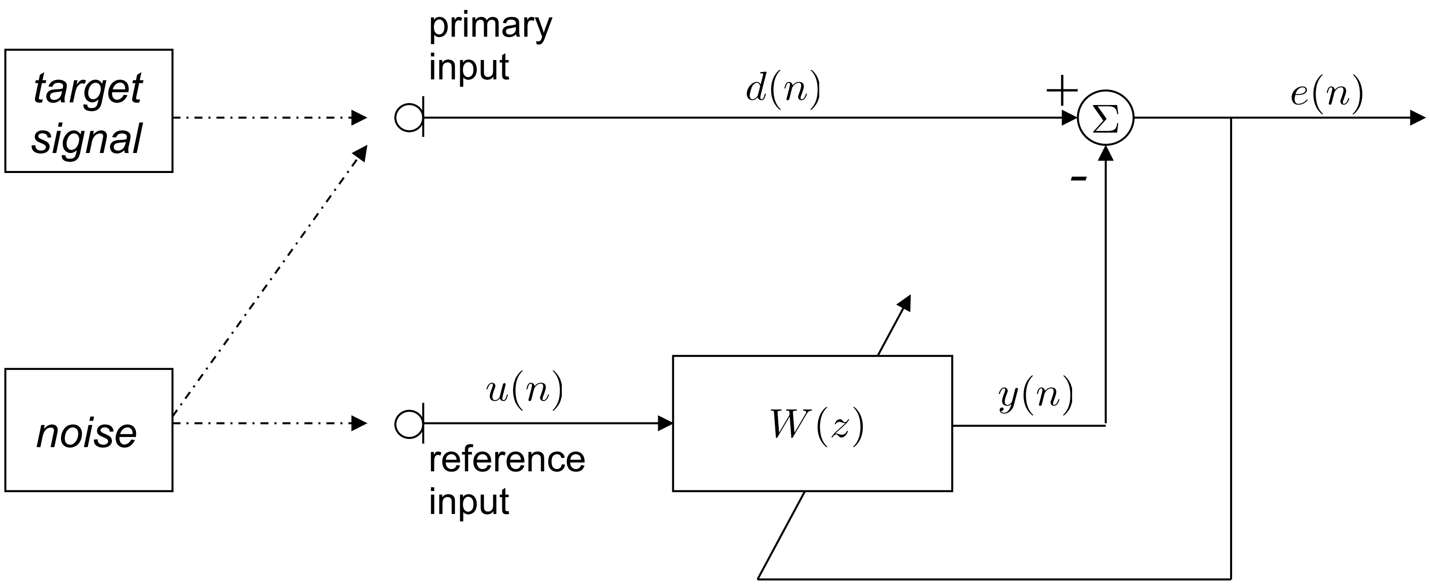
Summary

- Noise cancellation
- Acoustic echo cancellation
- Background noise removal
- Active noise control

Noise cancellation

- We consider to the following scheme:



- It refers to situations where it is required to cancel an interfering signal/noise from a given signal, which is a mixture of the target signal and the interference

Noise cancellation

- Methodology:
  - obtain an estimate of the interfering signal, measuring it with a microphone located in proximity of the noisy source
  - subtract it from the corrupted signal, by means of an adaptive filter
  - compute the residual error signal $e(n)$, containing an estimate of the target signal
- Working principle:
  - the adaptive filter produces an error signal that must be uncorrelated to the filter input (orthogonality principle)
  - thus, the error signal $e(n)$ will contain the part of the desired response which is not correlated with the filter input, i.e. the target signal
- Note that we cannot directly estimate the target signal subtracting the primary input and the reference noise signal. Indeed:
  - the noise signal propagates towards the primary input, i.e. it is filtered with an unknown impulse response
  - the goal of the adaptive filtering is to identify this impulse response, i.e., to predict the disturbing signal received by the reference input

Noise cancellation: an example

- A Formula-1 driver needs to communicate with the team boxes, without any interference from engine or mechanical parts of the car

- The noise coming from the engine and mechanical parts is very high: we have to suppress it

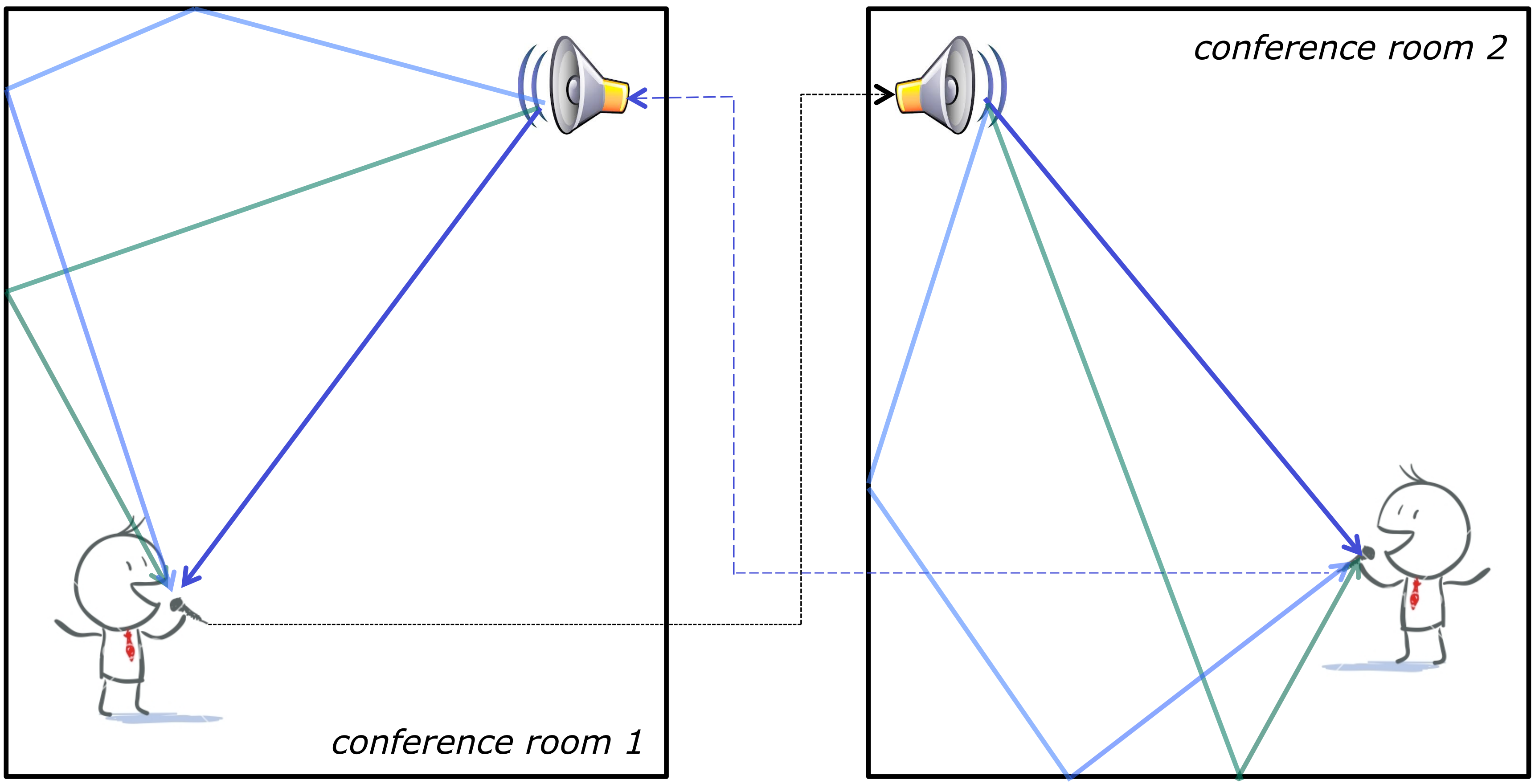
- Solution:
  - place a microphone where the noise originates, acquiring the reference noise signal $u(n)$
  - perform adaptive filtering of the signal $u(n)$, comparing the filter output with primary input $d(n)$, constituting the desired response

- LMS implementation:

  1. filter the reference input: $y(n) = \mathbf{w}^T(n)\mathbf{u}(n)$
  2. estimation of the target signal as $e(n) = d(n) - y(n)$
  3. update the filter: $\mathbf{w}(n+1) = \mathbf{w}(n) + \mu\mathbf{u}(n)e(n)$

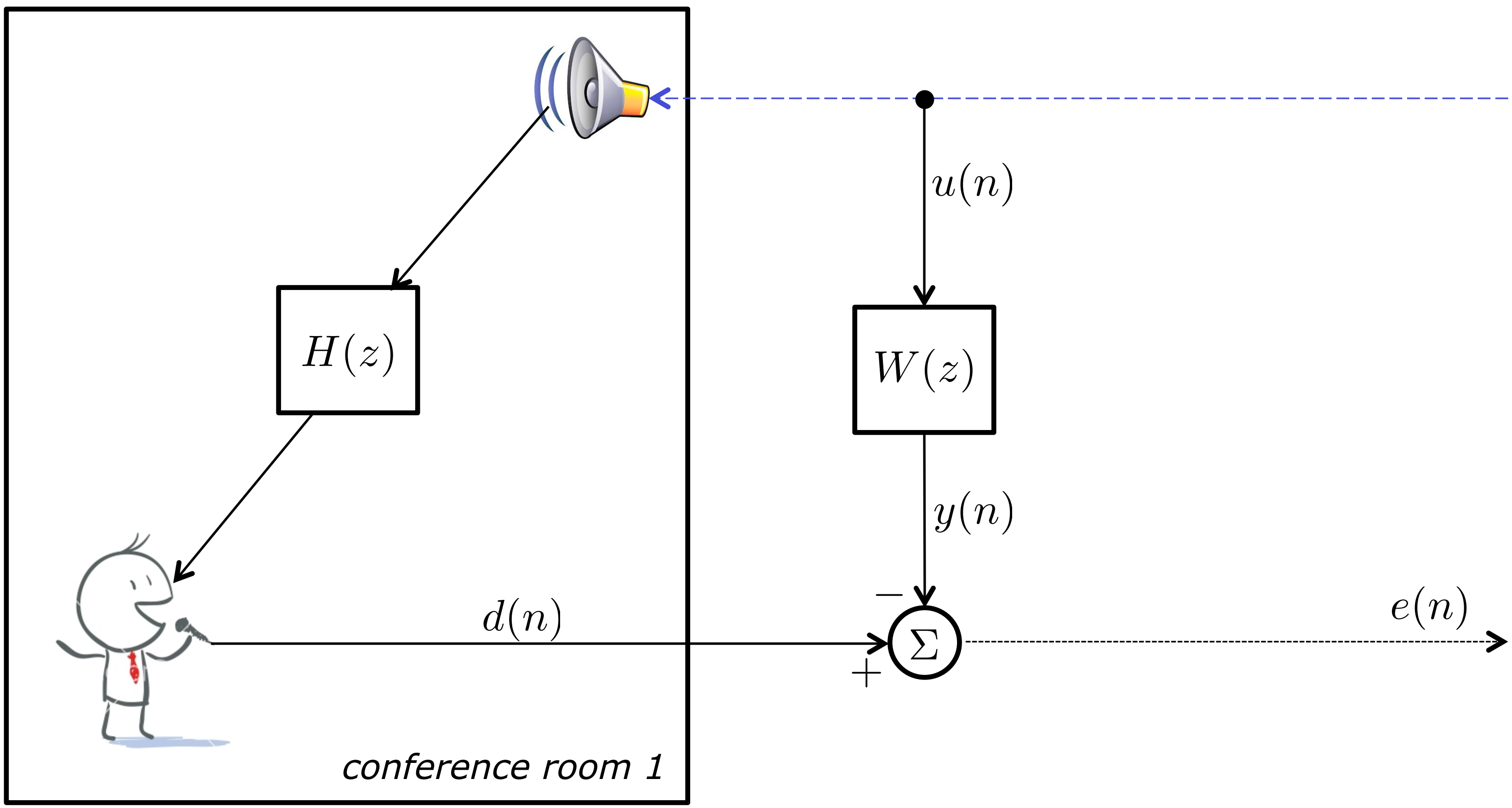
Acoustic echo cancellation

- Scenario:



Acoustic echo cancellation

- Echo cancellation through adaptive filtering:

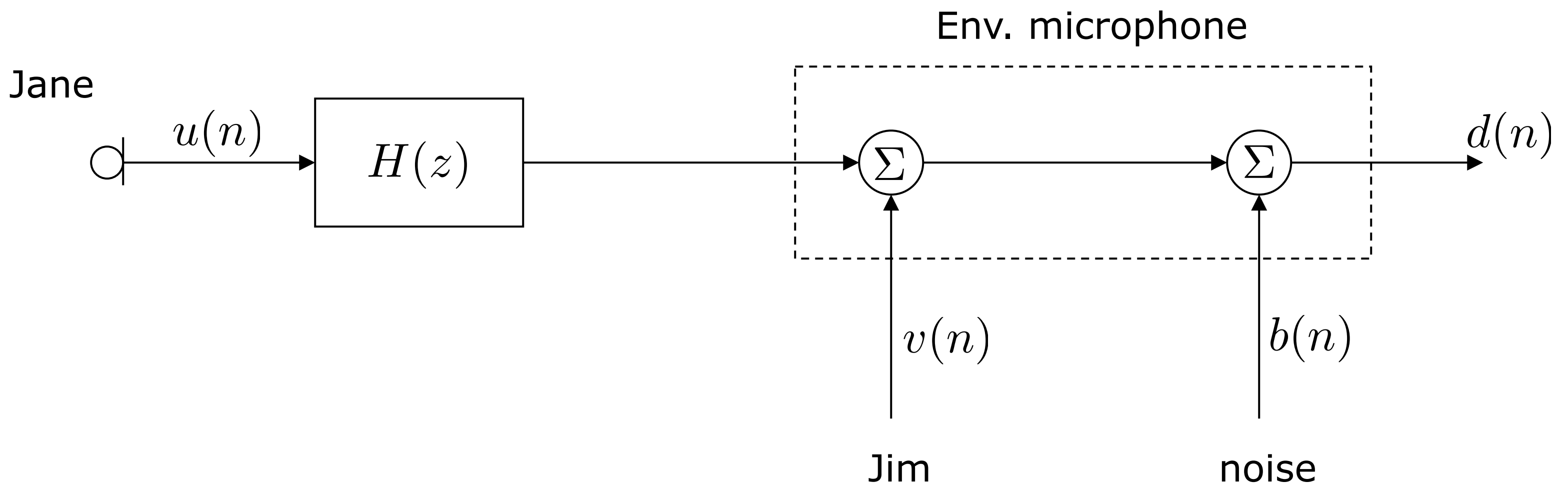


Acoustic echo cancellation

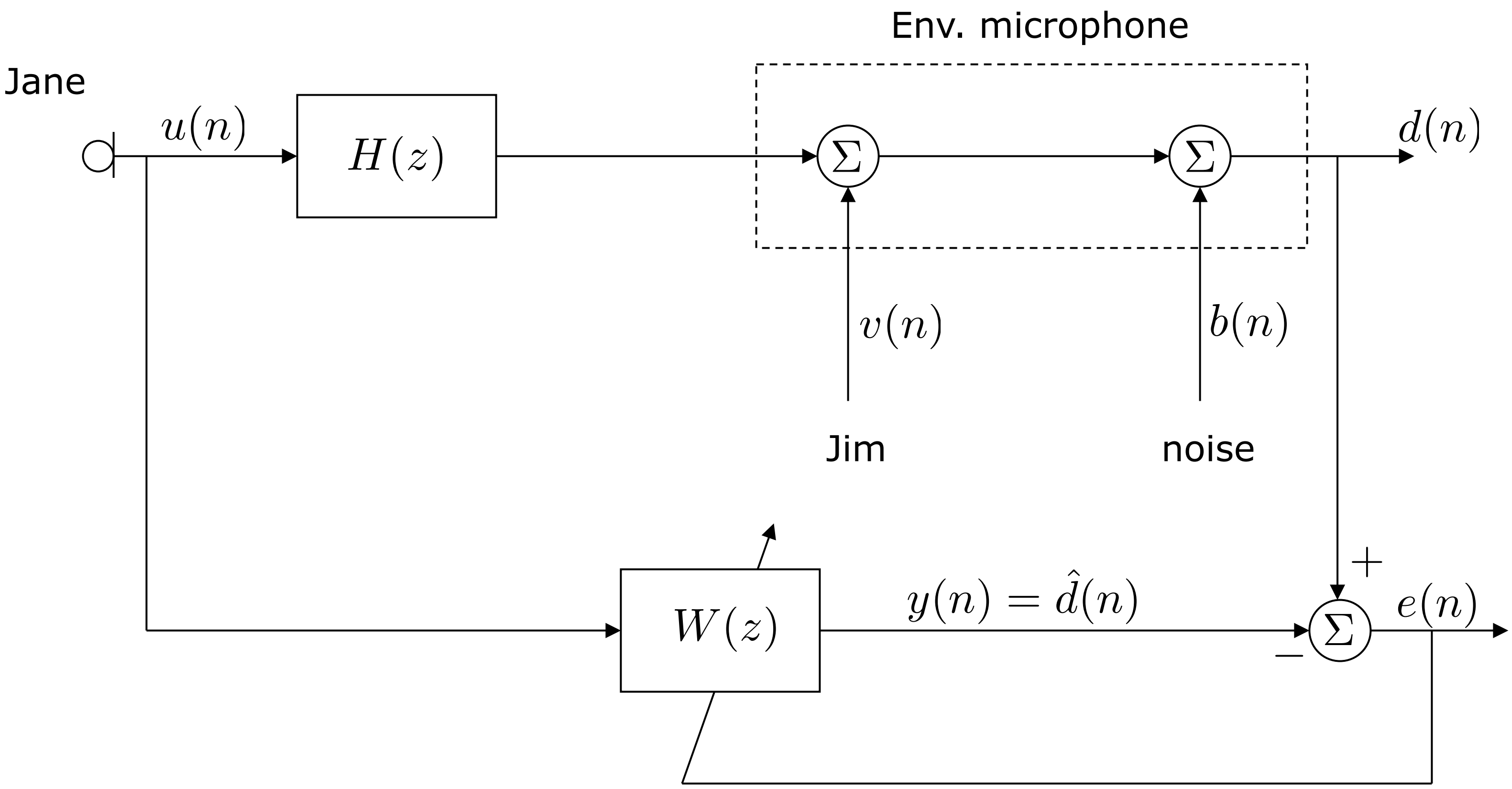
- **Another scenario**: in a reverberant room two people (Jane and Jim) are speaking at the same time. Person 1 (Jane) is provided a wearable microphone. Another microphone acquires the signal from the environment

- **Goal:** obtain an estimate of Jim's speech

- **Data model:**



- Solution:


Env. microphone
Jane
$u(n)$
$H(z)$
Σ
Σ
$d(n)$
$v(n)$
$b(n)$
Jim
noise
$W(z)$
$y(n) = \hat{d}(n)$
$+$
$-$
Σ
$e(n)$

- **Architecture:** given the signal $u(n)$ (acquired at Jane's microphone) and the desired signal $d(n)$ (environment), we obtain $y(n)$, an estimation of $d(n)$

$$y(n) = \mathbf{w}^T \mathbf{u}(n)$$

$$e(n) = y(n) - d(n) = \mathbf{w}^T \mathbf{u}(n) - \mathbf{h}^T \mathbf{u}(n) + v(n) + b(n)$$

- After the transient, the output signal converges to the desired signal, therefore the error signal contains "all the information in $d(n)$ that cannot be estimated from the knowledge of $u(n)$ and $d(n)$": the error signal statistically converges to $v(n) + b(n)$

- Comment: the error signal is used as segregated signal everytime we are provided only the signal to be cancelled.

Acoustic echo cancellation
Echo Return Loss Enhancement (ERLE)

- In order to measure the effectiveness of an echo canceller, the Echo Return Loss Enhancement (measured in dB) is usually given

- ERLE measures the ratio between the energy of the residual echo after and before cancellation

$$\mathrm{ERLE}|_{\mathrm{dB}} = -10\log_{10}\left(\frac{\sum_n [e(n)-v(n)]^2}{\sum_n [d(n)]^2}\right)$$

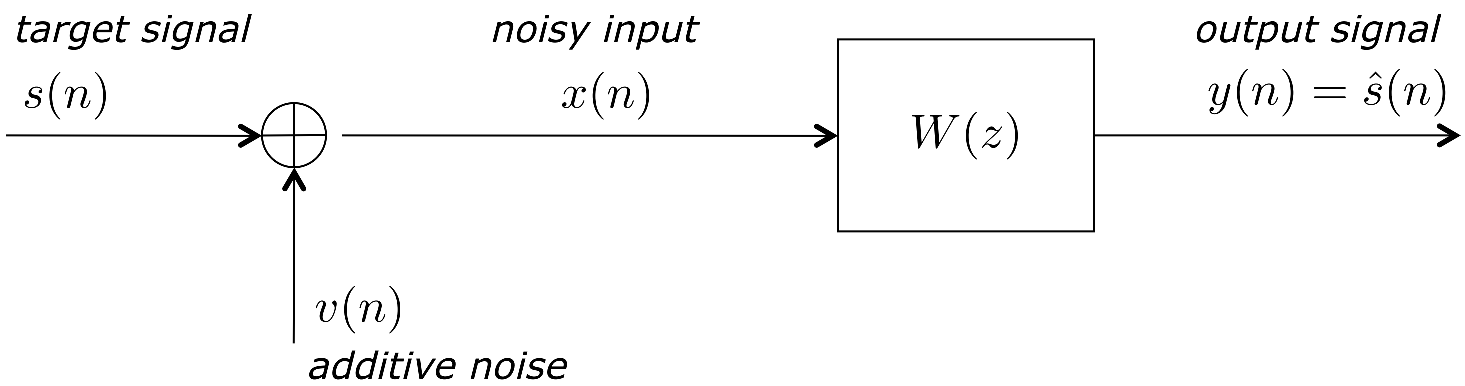
- As described above ERLE is an integral measure, i.e. it does not account for non stationary sound. For this reason, ERLE is usually measured at time intervals

- Typically, instantaneous ERLE is measured by averaging $[e(n)-v(n)]^2$ and $[d(n)]^2$ over 1/20s long windows

Background noise removal

- **Problem statement**: we want to reduce hissing noise in an analog recording

- **Data model**:



- **Assumptions**:
  - $s(n)$ and $v(n)$ are independent
  - An estimate of the power spectrum $P_v(\omega)$ of $v(n)$ is available

Background noise removal

- Goal: find the filter $W(z)$ that minimizes the MSE:

$$J = E\left\{|s(n) - y(n)|^2\right\}$$

- The signal $s(n)$ constitutes the desired response of the Wiener filter

- For the orthogonality principle, the MSE is minimum when

$$E\left\{[s(n) - y(n)]x(m)\right\} = 0 \quad \forall n\,, \forall m$$

- Wiener-Hopf equations:

$$\begin{aligned} r_{sx}(n-m) &= E\left\{\sum_k w(k)x(n-k)x(m)\right\} \\ &= \sum_k w(k) r_x(n-k-m) \end{aligned}$$

- Working in the frequency domain, we have that

$$r_{sx}(n) \longleftrightarrow S^*(\omega)X(\omega) = P_{sx}(\omega) \qquad \textit{cross-spectrum}$$

$$r_{x}(n) \longleftrightarrow X^*(\omega)X(\omega) = P_{x}(\omega) \qquad \textit{power spectrum}$$

- The Wiener-Hopf equations thus become:

$$r_{sx}(n-m) = \sum_k w(k) r_x(n-k-m) \quad \longleftrightarrow \quad P_{sx}(\omega) = W(\omega)P_x(\omega)$$

- And the optimum filter is given by

$$\boxed{W(\omega) = \frac{P_{sx}(\omega)}{P_x(\omega)}}$$

Background noise removal

- As the input signal $s(n)$ and the noise $v(n)$ are uncorrelated, it follows that:

$$\begin{aligned} r_{sx}(n) &= E\left\{s(k)x(k+n)\right\} \\ &= E\left\{s(k)[s(k+n)+v(k+n)]\right\} \\ &= E\left\{s(k)s(k+n)\right\} + E\left\{s(k)v(k+n)\right\} \\ &= E\left\{s(k)s(k+n)\right\} \\ &= r_s(n) \end{aligned}$$

- Therefore, in the frequency domain: $P_{sx}(\omega) = P_s(\omega)$

- Moreover, using again the uncorrelatedness of $s(n)$ and $v(n)$, we have:

$$P_x(\omega) = P_s(\omega) + P_v(\omega)$$

Background noise removal

- It follows that the optimum filter can be rewritten as

$$W(\omega) = \frac{P_{sx}(\omega)}{P_x(\omega)} = \frac{P_s(\omega)}{P_s(\omega) + P_v(\omega)}$$

- To implement the filter, we need to estimate the power spectra of signal and noise:

  - noise power spectrum can be estimated when signal is absent
  - signal power spectrum is estimated at each time frame as

$$\hat{P}_s(\omega) = P_x(\omega) - \hat{P}_v(\omega) \quad \Longrightarrow \quad W(\omega) = \frac{\hat{P}_s(\omega)}{\hat{P}_s(\omega) + \hat{P}_v(\omega)}$$

- The above equation does not account for phase. Zero-phase filters are commonly used

Background noise removal spectral attenuation factor

- Let us consider two extreme situations:

  - If $P_s(\omega) << P_v(\omega)$ then $|W(\omega)| \to 0$

  the noise overcomes the signal, all the filter can do is to attenuate the input signal at those frequencies

  - If $P_s(\omega) >> P_v(\omega)$ then $|W(\omega)| \to 1$

  the signal overcomes the noise, the filter leaves the signal undistorted

- From the above considerations, we can conclude that the Wiener filter behaves as a spectral attenuator, as its absolute value is between 0 and 1.

Active noise control

- ANC refers to situations where acoustic anti-noise waves are generated from electronic circuits

- Example: cancellation of noise in narrow ducts, such as ventilation systems