MECA-H411: Mechanical vibrations

Satellite in microvibrations in orbit

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Thursday, 22 January 2026

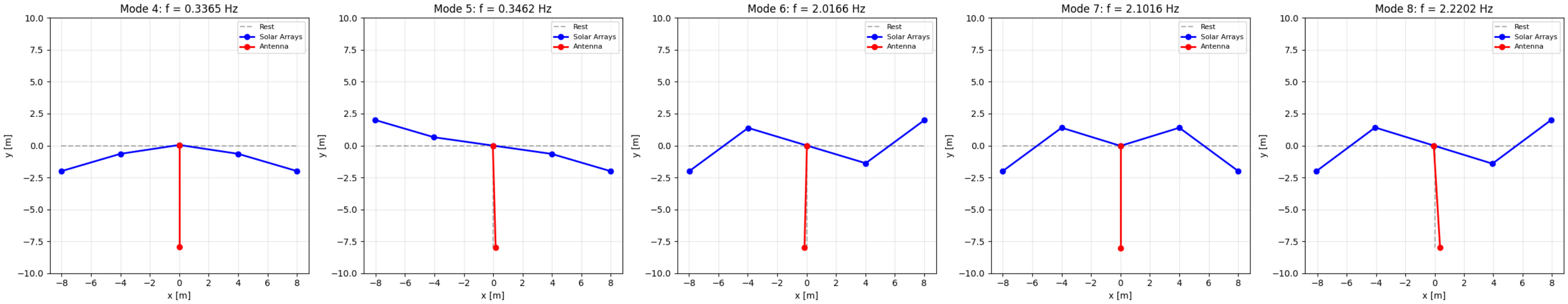
Q1.1 & Q1.2: Modal analysis

Rigid body modes (0Hz)

- 3 modes with $f_n = 0 \text{ Hz}$
- Free-free boundary conditions (satellite in orbit)
- Represent motion without elastic deformation:
    1. Translation along x
    2. Translation along y
    3. Rotation along z

First flexible modes

- Mode 4: $f = 0.33 \text{ Hz}$ (first symmetric bending)
- Mode 4: $f = 0.33 \text{ Hz}$ (first anti-symmetric bending)
- Mode 6: $f = 2.02 \text{ Hz}$ (antenna arm bending)



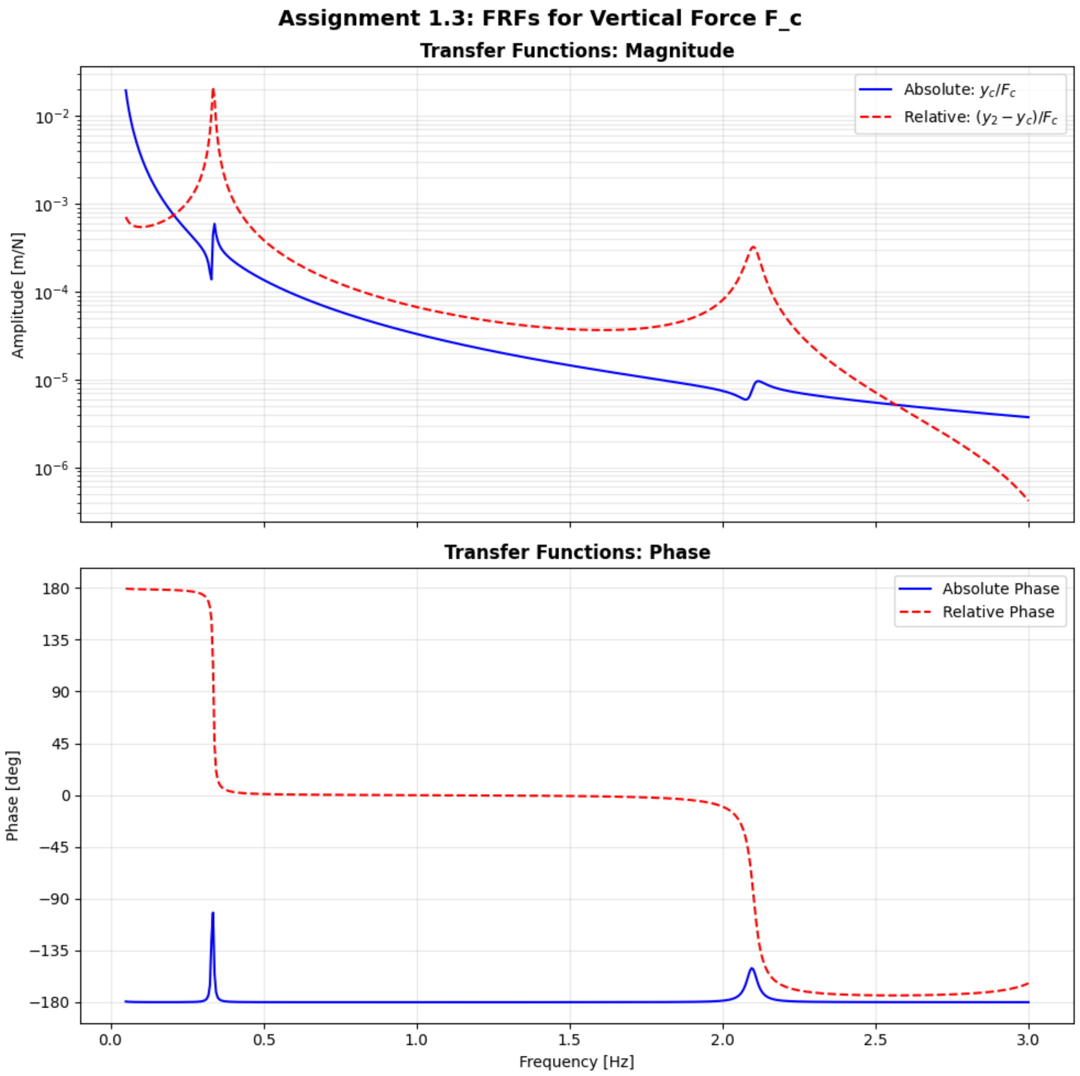
Q1.3: Frequency response, force excitation F_c

Transfer functions

- Absolute: $\frac{y_c}{F_c}$
- Relative: $\frac{y_2 - y_c}{F_c}$

Low-frequency behavior

- $y_c/F_c \to \infty$
  - Mass line behavior (inertia dominates)
  - Constant force
    - ↳ Constant acceleration
      - ↳ Unbounded displacement
- $y_2 - y_c/F_c \to \text{cst}$
  - Stiffness-dominated region
  - Measures internal panel deformation
    - ↳ Remain finite (elastic strain energy)



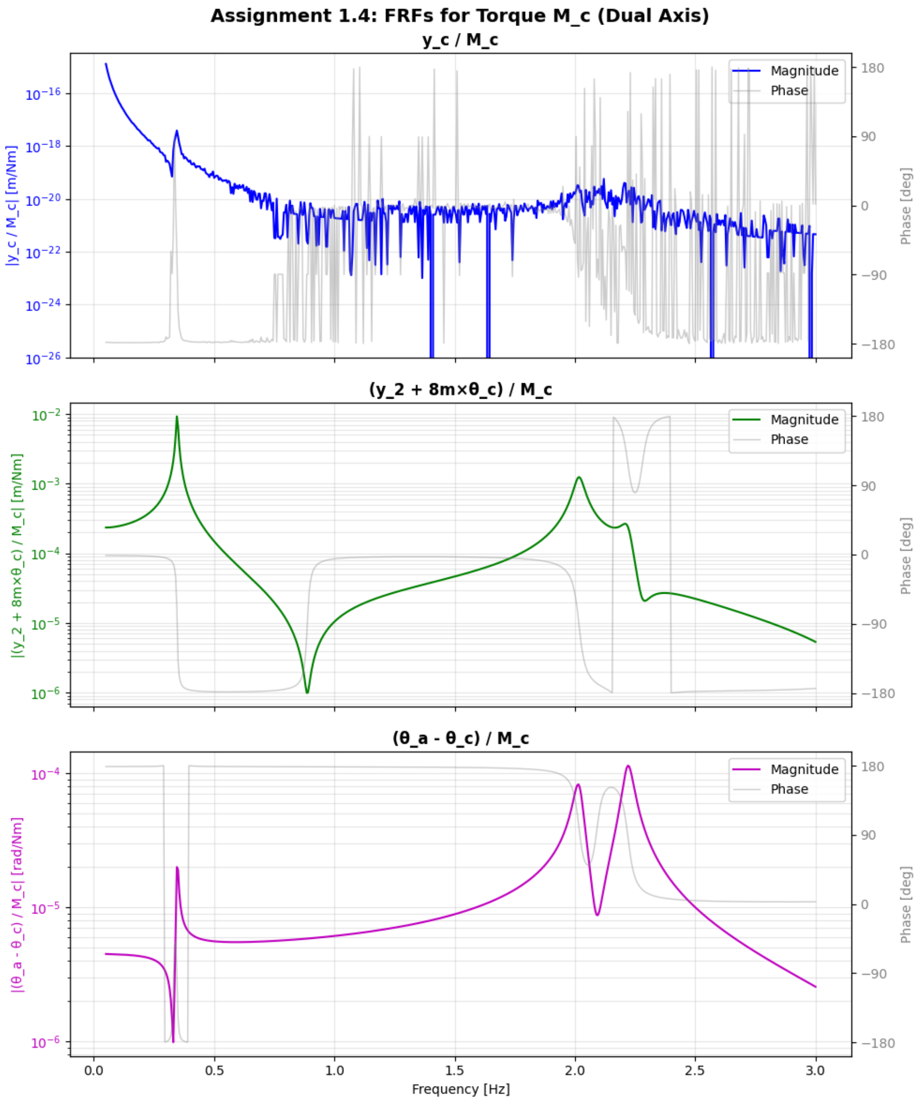
Q1.4: Frequency response, torque excitation M_c

Transfer functions of interest

- Tip displacement: $\frac{y_2+8\theta_c}{M_c}$
  - Total vertical motion of solar panel tip
  - Critical for clearance & fatigue analysis
- Pointing error: $\frac{\theta_a-\theta_c}{M_c}$
  - Relative angle between antenna and body
  - Critical for communication accuracy

Physical relevance

- Impact on structural integrity
- Mission performance (pointing accuracy)



Q1.5: Modal truncation and validation

Truncation rule

$$f_{cutoff} = 1{,}5 \times f_{max} = 1{,}5 \times 3 = 4{,}5 \text{ Hz}$$

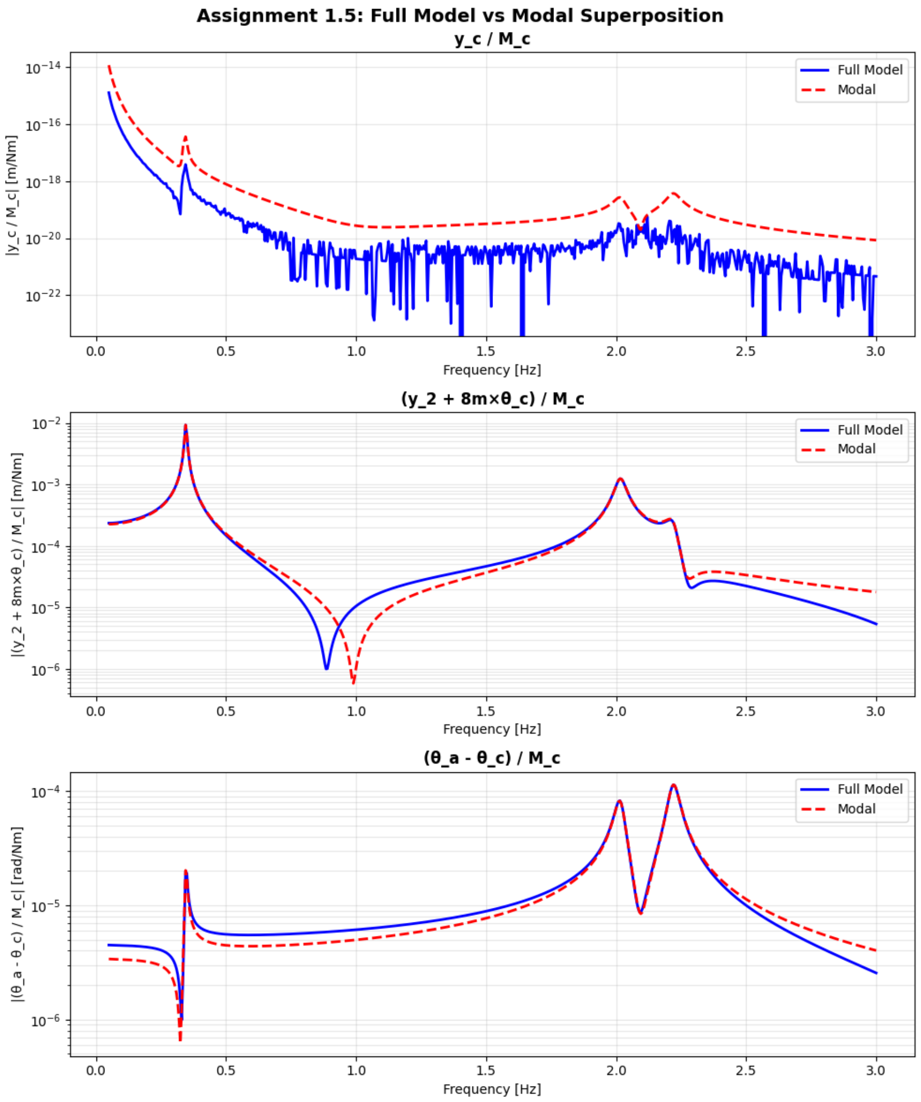
↳ 8 modes retained (3 rigid + 5 flexible)

Damping approximation

- Global loss factor: $\eta = 0{,}02$
- Modal damping ratio: $\xi = \eta/2 = 0{,}01\ (1\%)$

Validation

- Modal superposition matches full model
- Excellent agreement near resonances
- Small differences due to damping model conversion (hysteric → viscous)



Q2.1 & Q2.2: TMD design strategy

Target mode identification

- For F_c excitation: Mode 4 $(0{,}33\ \mathrm{Hz})$ dominates
- First symmetric bending mode of solar arrays

Den Hartog optimal tuning

- Design parameters
  - Mass constraint: $m_{TMD} \leq 3\%$ of solar array mass
  - 2 symmetric TMDs (one per solar array)
  - Effective mass ratio: $\mu = \frac{2m_{TMD}}{M_{eff}}$
- Den Hartog formulas
  - Frequency tuning: $\nu_{opt} = \frac{1}{1+\mu}$
  - Damping tuning: $\xi_{opt} = \sqrt{\frac{3\mu}{8(1+\mu}}$

Parameter	Value
Target mode	Mode 4
Target frequency	0,33 Hz
m_{TMD}	0,686 kg
k_{TMD}	2,91 N/m
c_{TMD}	0,22 Ns/m

TMD frequency tuned slightly below target to account for mass loading effect

Q2.3: TMD efficiency, frequency domain

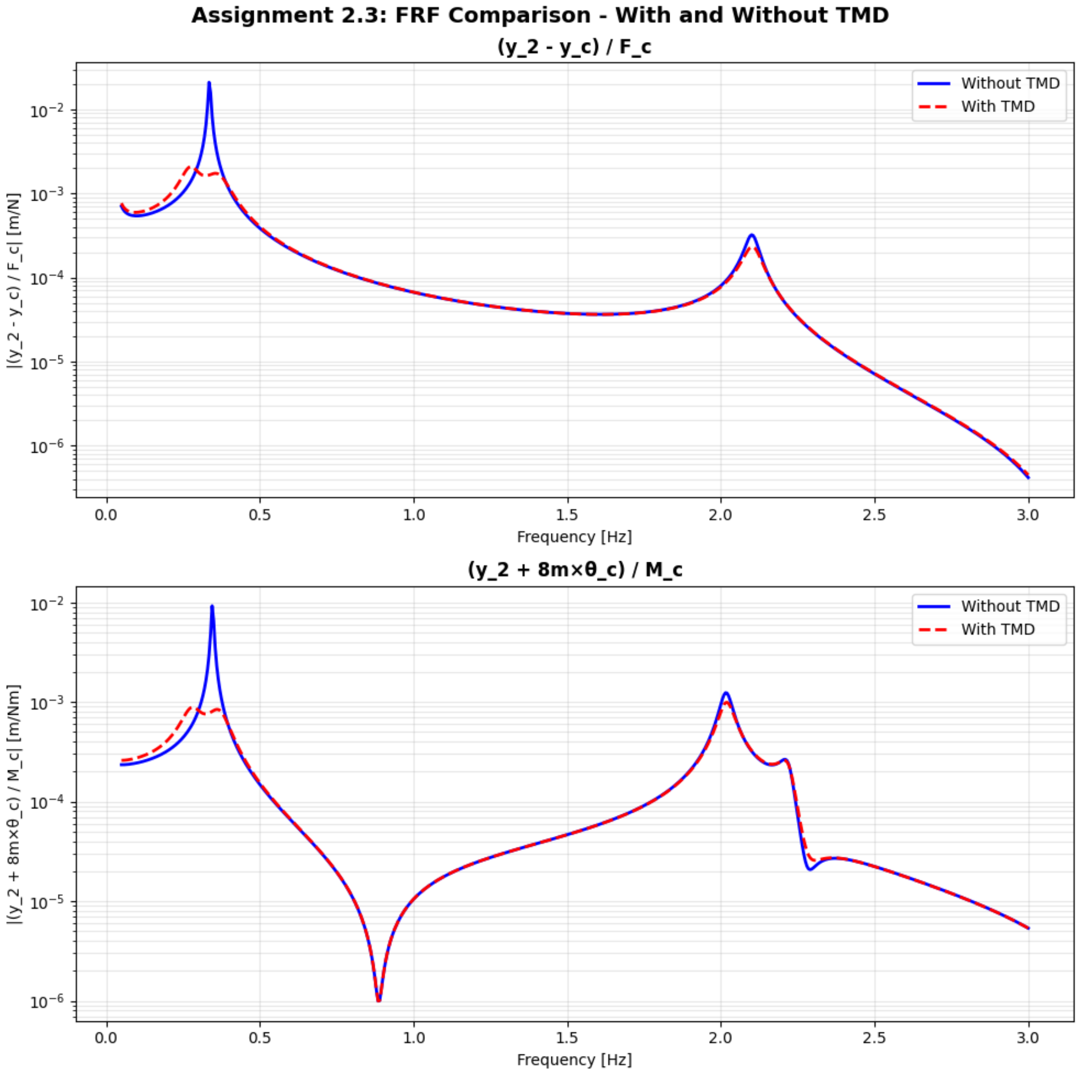
Key observations

- Peak splitting
  - original resonance peak splits into 2 equal smaller peaks
    ↳ confirms correct Den Hartog tuning

Effectiveness

- Force F_c: target mode 4 $(0{,}33\ \mathrm{Hz})$
  ↳ Massive amplitude reduction
- Torque M_c: also effective
  ↳ TMD covers both modes

The narrow frequency separation ($\Delta f = 0{,}01\ \mathrm{Hz}$) allows one TMD design to damp both loading cases



Q2.4: Equivalent damping estimation

Concept

What damping ratio ξ_{equiv} would produce the same peak reduction as TMD?

Method

$$\xi_{equiv} \approx \frac{H_{peak,original}}{H_{peak,TMD}} \times \xi_{original}$$

Results

Transfer function	$\xi_{original}$	ξ_{equiv}
$(y_2 - y_c)/F_c$	1%	$\sim 10\%$
$(y_2 - 8\theta_c)/M_c$	1%	$\sim 9{,}3\%$

Physical interpretation

- TMD acts as a massive increase in structural damping
- From 1% to $\sim 10\%$
  - 10× improvement in effective damping at tuned frequency
- The TMD is an efficient energy dissipation device that creates a “notch” in the FRF.

Q3.1 & Q3.2: Time domain analysis, without TMD

Main body rotation $\theta_c(t)$

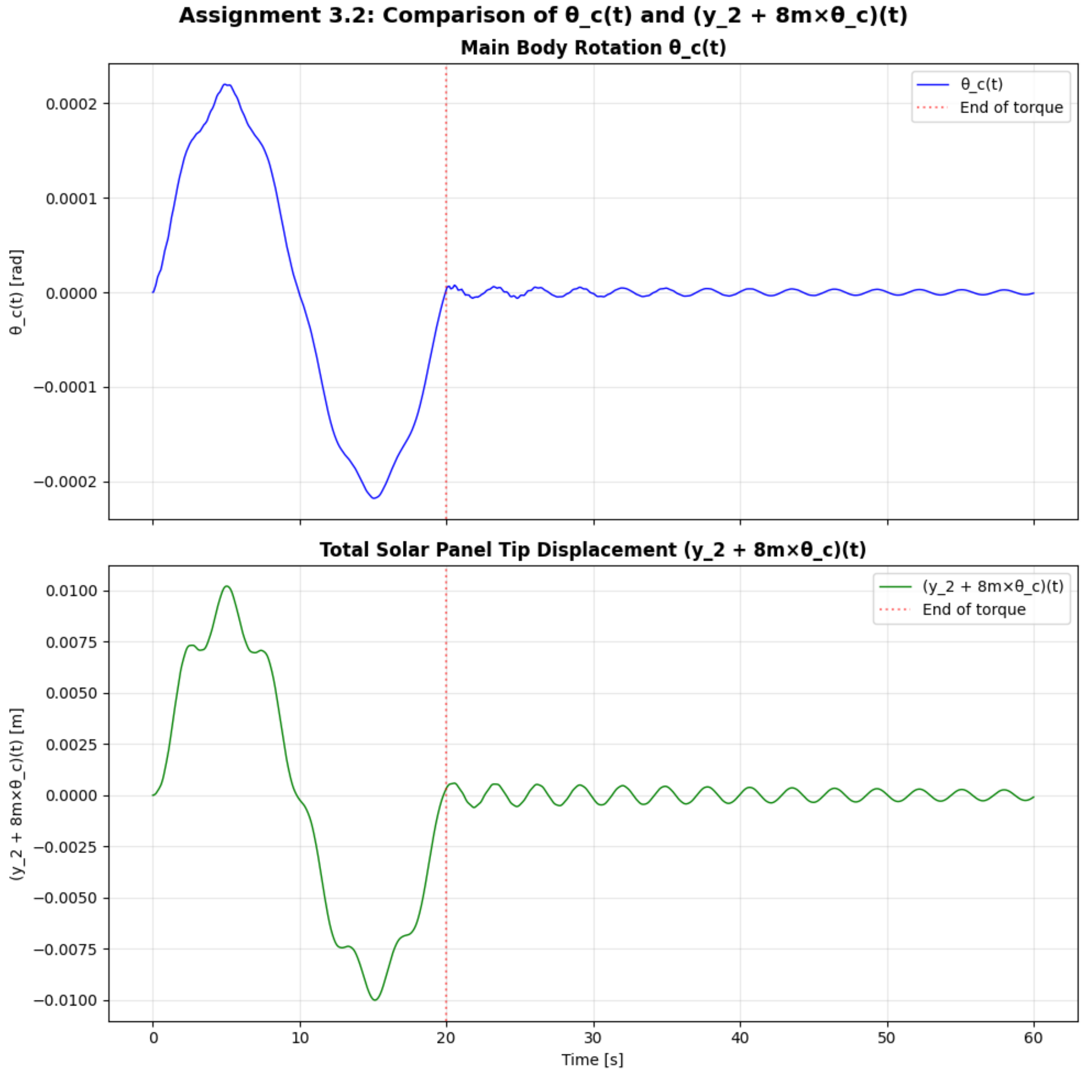
- Follows global torque profile (Bang-Bang)
- Smooth trajectory with small high-frequency ripples
- Dominated by rigid body inertia

Solar panel tip $(y_2 + 8\theta_c)$

- Much more “chaotic” behavior
- Strong high-frequency oscillations
- Cantilever effect: panels lag during rotation

Physical meaning

- Flexible modes are excited by the maneuver
- Long-lasting oscillations → fatigue risk



Q3.3: TMD effect in time domain

Method

- Use equivalent damping ξ_{equiv} *(from Q2.4)*
- Apply to targeted flexible modes

Results

- Faster decay of oscillations
- Slight peak reduction, strong cycle reduction

Main observation

- TMD engages after several cycles
  ↳ less dramatic than frequency domain,
  but faster energy dissipation

Impact

- Reduced fatigue cycles → extended solar array lifetime.