MECA-H411: Mechanical vibrations

Satellite in microvibrations in orbit

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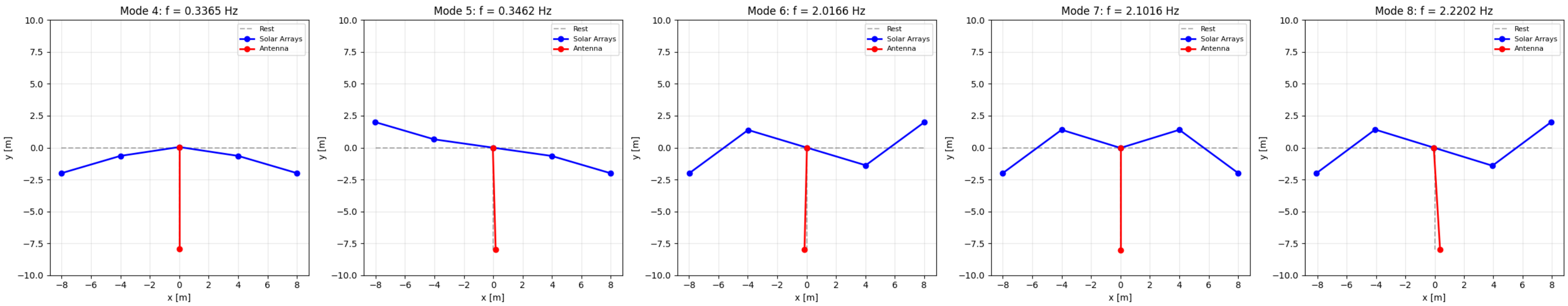
Q1.1 & Q1.2: Modal analysis

Rigid body modes (0Hz)

- 3 modes with $f_n \approx 0{,}00\ \mathrm{Hz}$
- Free-free boundary conditions (satellite in orbit)
- Represent motion without elastic deformation:
  1. Translation along x
  2. Translation along y
  3. Rotation around z

First flexible modes

- Mode 4: $f = 0{,}34\ \mathrm{Hz}$ (1st symmetric bending)
- Mode 5: $f = 0{,}35\ \mathrm{Hz}$ (1st anti-symmetric bending)
- Mode 6: $f = 2{,}02\ \mathrm{Hz}$ (2nd anti-symmetric bending)
- Mode 7: $f = 2{,}10\ \mathrm{Hz}$ (2nd symmetric bending)
- Mode 8: $f = 2{,}22\ \mathrm{Hz}$ (3rd anti-symmetric bending)



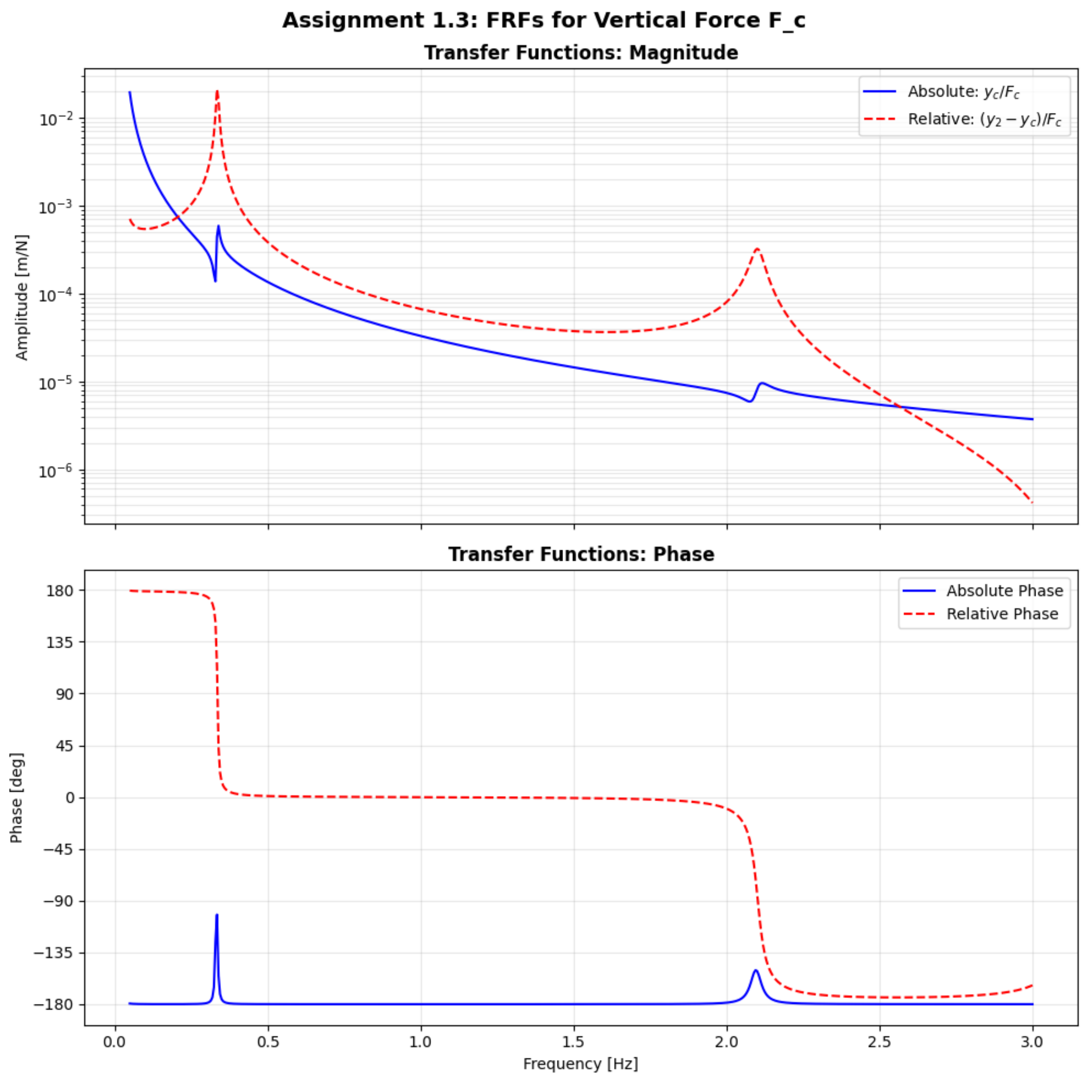
Q1.3: Frequency response, force excitation F_c

Transfer functions

- Absolute: $\frac{y_c}{F_c}$
- Relative: $\frac{y_2 - y_c}{F_c}$

Low-frequency behavior

- $y_c/F_c \to \infty$
    - Mass line behavior (inertia dominates)
    - Constant force
        ↳ Constant acceleration
            ↳ Unbounded displacement
- $(y_2 - y_c)/F_c \to \text{cst}$
    - Stiffness-dominated region
    - Measures internal panel deformation
        ↳ Remains finite (elastic strain energy)



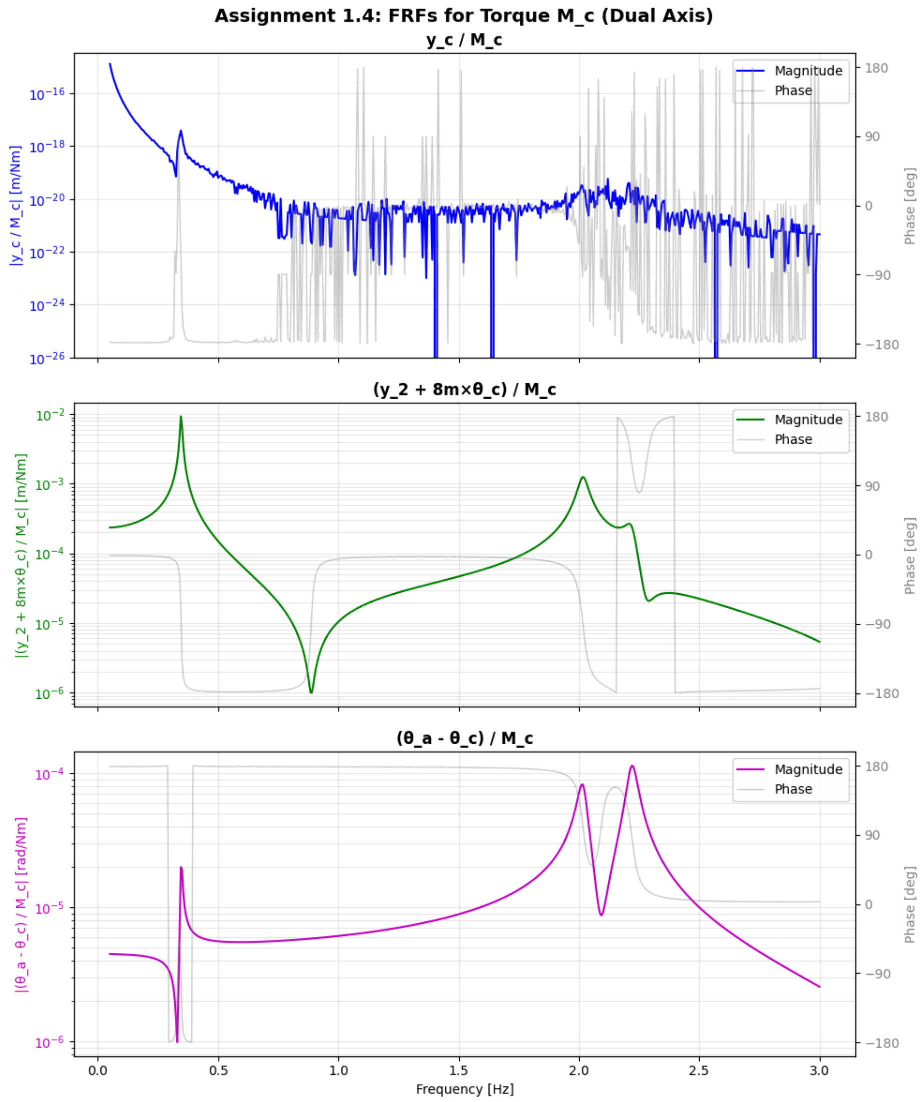
Q1.4: Frequency response, torque excitation M_c

Transfer functions of interest

- Tip displacement:
$$\frac{y_2 + 8m \cdot \theta_c}{M_c}$$
  - Total vertical motion of solar panel tip
  - Critical for clearance & fatigue analysis
- Pointing error:
$$\frac{\theta_a - \theta_c}{M_c}$$
  - Relative angle between antenna and body
  - Critical for communication accuracy

Physical relevance

- Impact on structural integrity
- Mission performance (pointing accuracy)



Q1.5: Modal truncation and validation

Truncation rule

$$f_{cutoff} = 1{,}5 \times f_{max} = 1{,}5 \times 3 = 4{,}5 \text{ Hz}$$

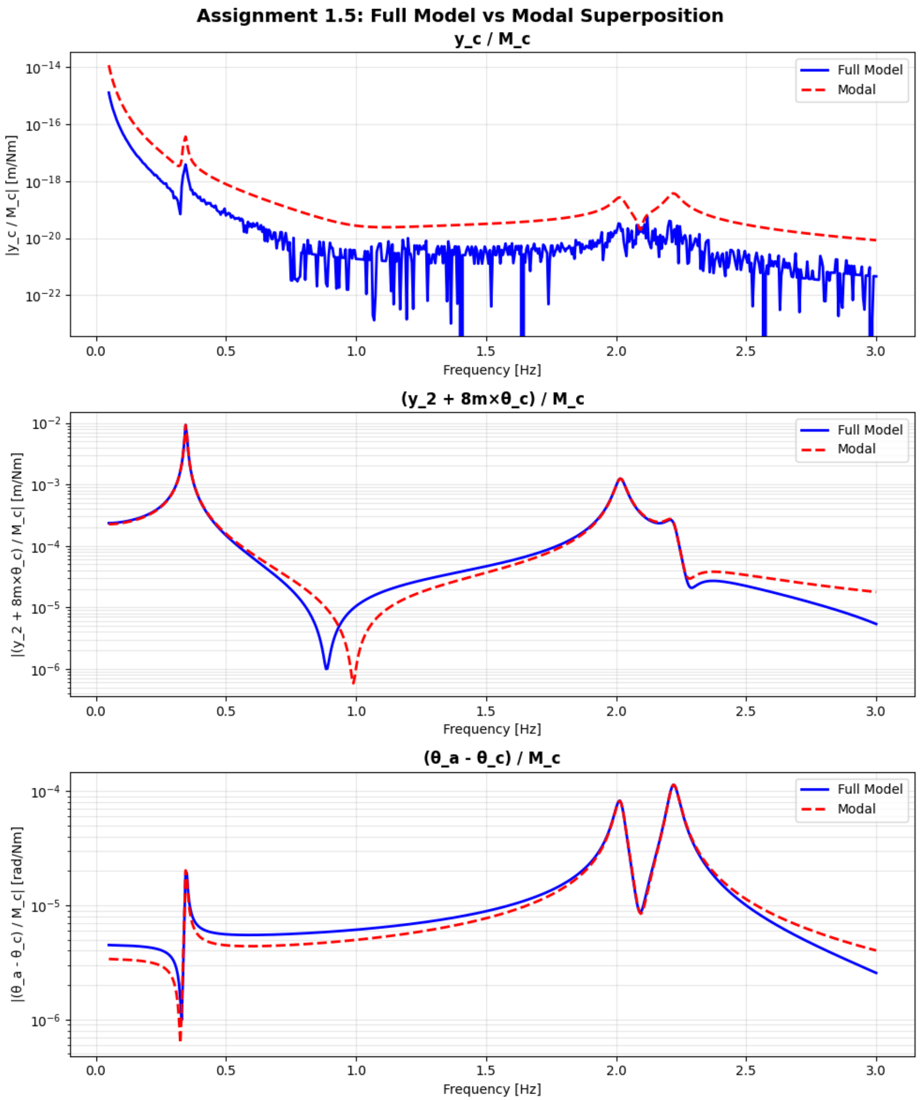
↳ 8 modes retained (3 rigid + 5 flexible)

Damping approximation

- Global loss factor: $\eta = 0{,}02$
- Modal damping ratio: $\xi = \eta/2 = 0{,}01\ (1\%)$

Validation

- Modal superposition matches full model
- Excellent agreement near resonances
- Small differences due to damping model conversion (hysteretic → viscous)



Q2.1 & Q2.2: TMD design strategy

Target mode identification

- For F_c excitation: Mode 4 ($0{,}34\ \mathrm{Hz}$) dominates
- First symmetric bending mode of solar arrays

Den Hartog optimal tuning

- Design parameters
  - Mass constraint: $m_{TMD} \leq 3\%$ of solar array mass
  - 2 symmetric TMDs (one per solar array)
  - Effective mass ratio: $\mu = \frac{2m_{TMD}}{M_{eff}}$
- Den Hartog formulas
  - Frequency tuning: $\nu_{opt} = \frac{1}{1+\mu}$
  - Damping tuning: $\xi_{opt} = \sqrt{\frac{3\mu}{8\cdot(1+\mu)}}$

Parameter	Value
Target mode	Mode 4
Target frequency	0,34 Hz
m_{TMD}	0,686 kg
k_{TMD}	2,447 N/m
c_{TMD}	0,520 Ns/m

TMD frequency tuned slightly below target to account for mass loading effect

Q2.3: TMD efficiency, frequency domain

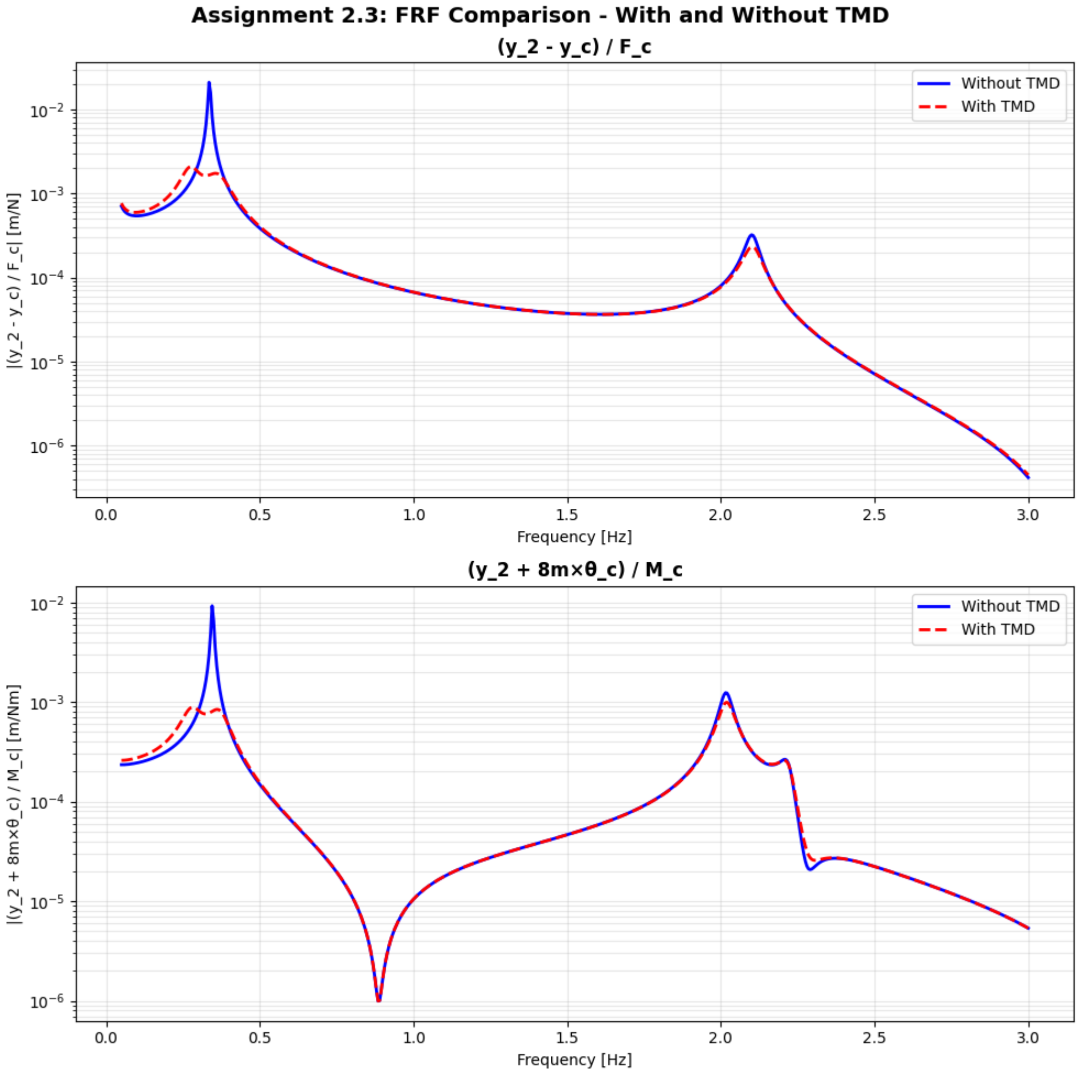
Key observations

- Peak splitting
  - original resonance peak splits into 2 equal smaller peaks
    ↳ confirms correct Den Hartog tuning

Effectiveness

- Force F_c: target mode 4 $(0{,}34\ \mathrm{Hz})$
  ↳ Massive amplitude reduction
- Torque M_c: also effective
  ↳ TMD covers both modes

The narrow frequency separation $(\Delta f = 0{,}01\ \mathrm{Hz})$ *allows one TMD design to damp both loading cases*



Q2.4: Equivalent damping estimation

Concept

What damping ratio ξ_{equiv} would produce the same peak reduction as TMD?

Method

$$\xi_{equiv} \approx \frac{H_{peak,original}}{H_{peak,TMD}} \times \xi_{original}$$

Results

Transfer function	$\xi_{original}$	ξ_{equiv}
$(y_2 - y_c)/F_c$	1%	$\sim 10\%$
$(y_2 + 8\mathrm{m} \cdot \theta_c)/M_c$	1%	$\sim 9{,}3\%$

Physical interpretation

- TMD acts as a massive increase in structural damping
- From 1% to $\sim 10\%$
  - $10\times$ improvement in effective damping at tuned frequency
- The TMD is an efficient energy dissipation device that creates a “notch” in the FRF.

Q3.1 & Q3.2: Time domain analysis, without TMD

Main body rotation $\theta_c(t)$

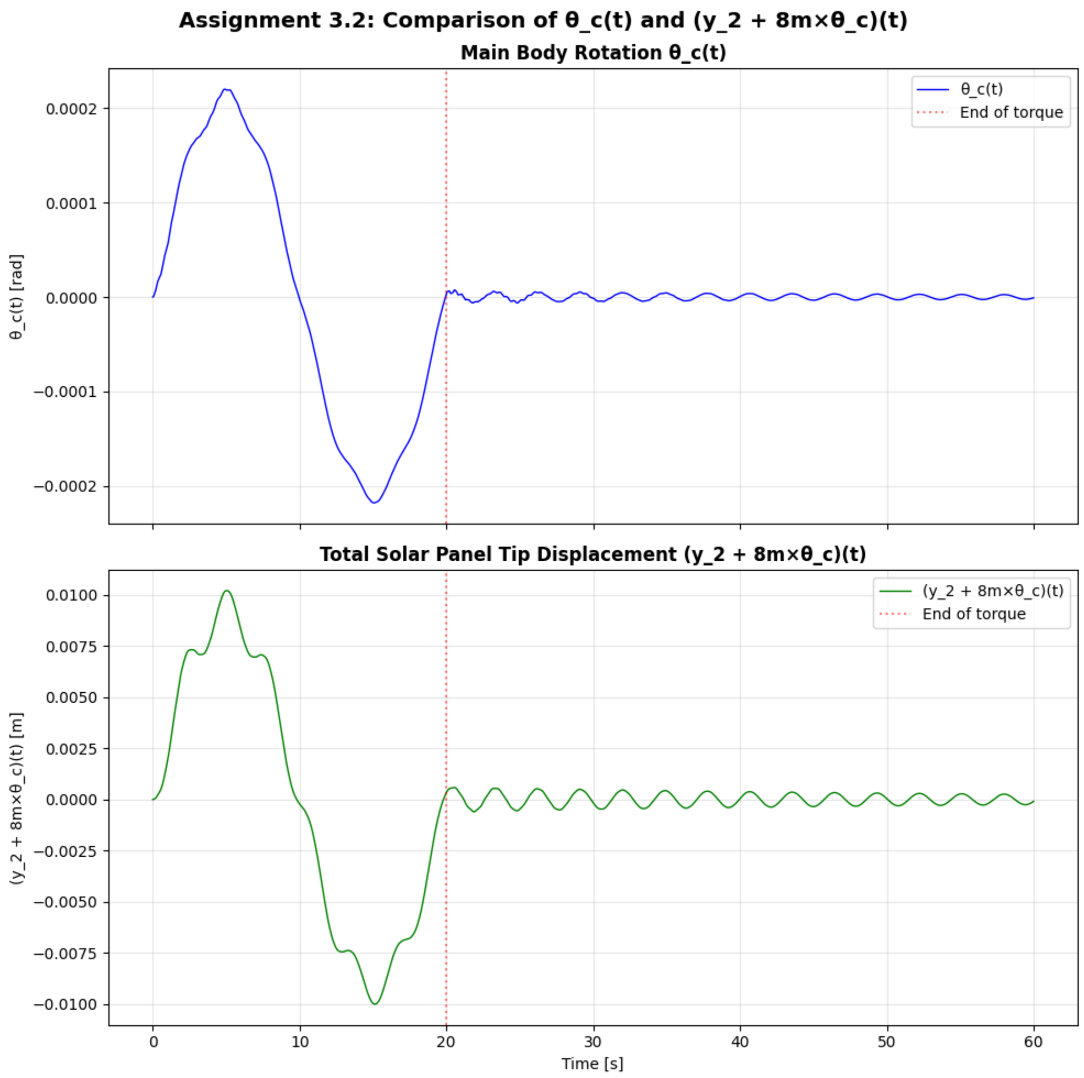
- Follows global torque profile (Bang-Bang)
- Smooth trajectory with small high-frequency ripples
- Dominated by rigid body inertia

Solar panel tip $(y_2 + 8m \cdot \theta_c)$

- Much more “chaotic” behavior
- Strong high-frequency oscillations
- Cantilever effect: panels lag during rotation

Physical meaning

- Flexible modes are excited by the maneuver
- Long-lasting oscillations → fatigue risk



Q3.3: TMD effect in time domain

Method

- Use equivalent damping ξ_{equiv} *(from Q2.4)*
- Apply to targeted flexible modes

Results

- Faster decay of oscillations
- Slight peak reduction, strong cycle reduction

Main observation

- TMD engages after several cycles
  ↳ less dramatic than frequency domain, but faster energy dissipation

Impact

- Reduced fatigue cycles → extended solar array lifetime.