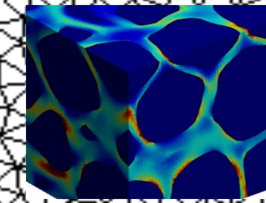

Structural Analysis and Finite Elements

Computer labworks – assembly and BC

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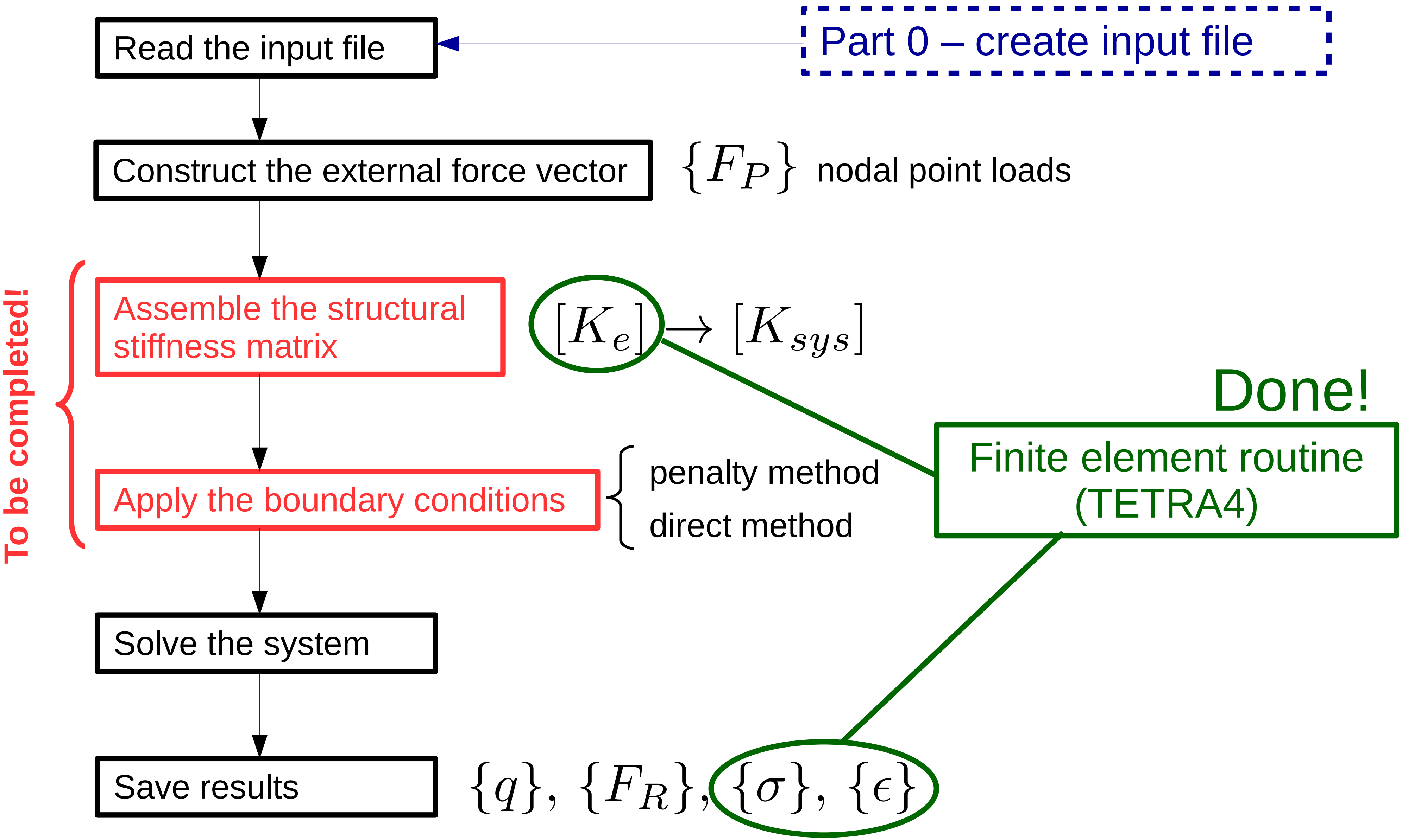
Part I – MatLab FE code



External force vector

Nodal point loads

$$\underset{[ndof \times 1]}{\mathbf{f}_P} = \begin{bmatrix} f_P^1 \\ f_P^2 \\ f_P^3 \\ \ldots \\ f_P^4 \end{bmatrix} \quad \underset{[nnode \times 3]}{dofpos} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \\ \ldots & \ldots & \ldots \\ (3 \times nnode - 2) & (3 \times nnode - 1) & 3 \times nnode \end{bmatrix}$$

allows the identification/positioning of the system dof: $f_y^2 \rightarrow f_P^5$

1. Initialize the nodal point load vector

$$\mathbf{f}_P = \mathbf{0}$$

2. Loop <u>on nodf</u> and fill nonzero terms

Assembly of the stiffness matrix

1. Initialize system stiffness matrix to sparse

$$\mathbf{K}_\sigma = sparse(ndof \times ndof)$$

2. Loop over all elements for <u>element stiffness matrix</u>

$$\mathbf{K}^e = \mathbf{B}^T \ \mathbf{H} \ \mathbf{B} \ V^e$$

Feed the right coordinates for each element!

Feed the right material properties for each element!

$$\underset{[12 \times 12]}{[\mathbf{K}_e]} \begin{bmatrix} u_1 \\ v_1 \\ \dots \\ v_3 \\ w_3 \end{bmatrix}_e = \begin{bmatrix} f_1^x \\ f_1^y \\ \dots \\ f_3^y \\ f_3^z \end{bmatrix}_e$$

Assembly of the stiffness matrix

2A) Element stiffness matrix:

$$\begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} & \dots & k_{112} \\ k_{21} & k_{22} & k_{23} & k_{24} & \dots & k_{212} \\ k_{31} & k_{32} & k_{33} & k_{34} & \dots & k_{312} \\ k_{41} & k_{42} & k_{43} & k_{44} & \dots & k_{412} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ k_{121} & k_{122} & k_{123} & k_{124} & \dots & k_{1212} \end{bmatrix}_e \begin{bmatrix} u_1 \\ v_1 \\ w_1 \\ u_2 \\ \dots \\ w_3 \end{bmatrix}_e = \begin{bmatrix} f_1^x \\ f_1^y \\ f_1^z \\ f_2^x \\ \dots. \\ f_3^z \end{bmatrix}_e$$

expresses how $f^z_{1,\,e}$ varies as a function of $v_{1,\,e}$

- quantity with a physical meaning
- same meaning in the system matrix
- “challenge”: <u>find its correct position</u> in $\mathbf{K}_\sigma$

Assembly of the stiffness matrix

2B) Positioning of element stiffness matrix terms in $\mathbf{K}_\sigma$

$k_{32,\,e}$ expresses how $f^z_{1,\,e}$ varies as a function of $v_{1,\,e}$

$$dofpos = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \\ \dots & \dots & \dots \\ (3\times nnode-2) & (3\times nnode-1) & 3\times nnode \end{bmatrix}$$

$$\mathbf{f}_\sigma = \begin{Bmatrix} f_1 \\ f_2 \\ \dots \\ f_j \\ \dots \\ f_n \end{Bmatrix} \qquad \mathbf{u}_\sigma = \begin{Bmatrix} u_1 \\ u_2 \\ \dots \\ u_i \\ \dots \\ u_n \end{Bmatrix}$$

jth line of $\mathbf{K}_\sigma$

ith column of $\mathbf{K}_\sigma$

Assembly of the stiffness matrix

$k_{32,\,e}$ expresses how $f^z_{1,\,e}$ varies as a function of $v_{1,\,e}$

$$\mathbf{u}_\sigma = \begin{Bmatrix} u_1 \\ u_2 \\ \cdots \\ u_i \\ \cdots \\ u_n \end{Bmatrix}$$

i^th column of $\mathbf{K}_\sigma$

$$\mathbf{f}_\sigma = \begin{Bmatrix} f_1 \\ f_2 \\ \cdots \\ f_j \\ \cdots \\ f_n \end{Bmatrix}$$

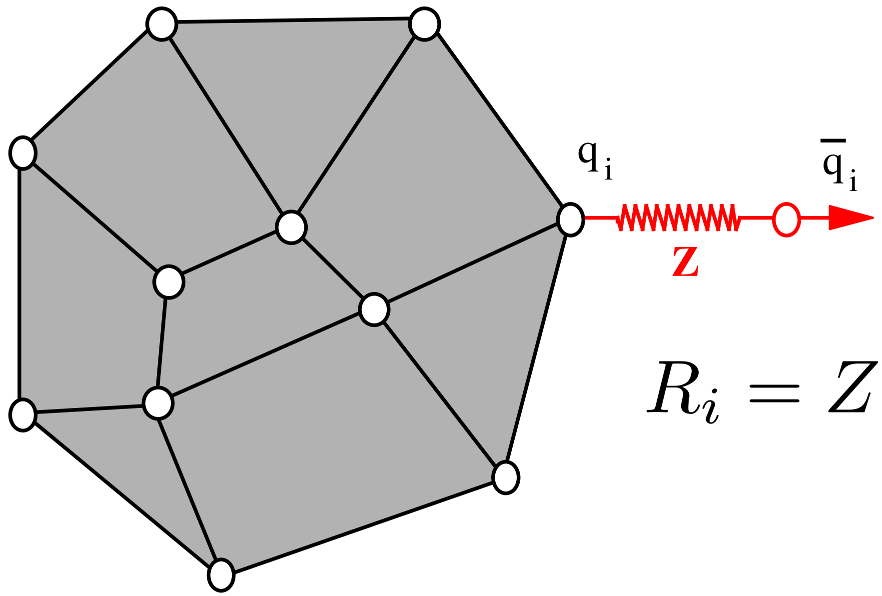
j^th line of $\mathbf{K}_\sigma$

Stiffness contribution to the system stiffness matrix (update formula):

$$\mathbf{K}_\sigma(j,\,i) = \mathbf{K}_\sigma(j,\,i) + k_{32,\,e}$$

naturally considers all element contributions to the stiffness matrix

Application of BC – penalty method



$$R_i = Z\left(\overline{q}_i - q_i\right)$$

from pdof

$$\begin{bmatrix} K_{11} & \dots & K_{1i} & \dots & K_{1n} \\ \dots & \dots & \dots & \dots & \dots \\ K_{i1} & \dots & K_{ii} & \dots & K_{in} \\ \dots & \dots & \dots & \dots & \dots \\ K_{n1} & \dots & K_{ni} & \dots & K_{nn} \end{bmatrix} \begin{Bmatrix} q_1 \\ \dots \\ q_i \\ \dots \\ q_n \end{Bmatrix} = \begin{Bmatrix} f_1 \\ \dots \\ f_i + R_i \\ \dots \\ f_n \end{Bmatrix}$$

Application of BC – penalty method

$$\begin{bmatrix} K_{11} & \dots & K_{1i} & \dots & K_{1n} \\ \dots & \dots & \dots & \dots & \dots \\ K_{i1} & \dots & K_{ii} & \dots & K_{in} \\ \dots & \dots & \dots & \dots & \dots \\ K_{n1} & \dots & K_{ni} & \dots & K_{nn} \end{bmatrix} \begin{Bmatrix} q_1 \\ \dots \\ q_i \\ \dots \\ q_n \end{Bmatrix} = \begin{Bmatrix} f_1 \\ \dots \\ f_i + R_i \\ \dots \\ f_n \end{Bmatrix}$$

$$R_i = Z\left(\overline{q}_i - q_i\right)$$

from pdof

$$\begin{bmatrix} K_{11} & \dots & K_{1i} & \dots & K_{1n} \\ \dots & \dots & \dots & \dots & \dots \\ K_{i1} & \dots & K_{ii} + Z & \dots & K_{in} \\ \dots & \dots & \dots & \dots & \dots \\ K_{n1} & \dots & K_{ni} & \dots & K_{nn} \end{bmatrix} \begin{Bmatrix} q_1 \\ \dots \\ q_i \\ \dots \\ q_n \end{Bmatrix} = \begin{Bmatrix} f_1 \\ \dots \\ f_i + Z\,\overline{q}_i \\ \dots \\ f_n \end{Bmatrix}$$

Application of the BC – direct method

1. Group known terms on RHS

$$\begin{bmatrix} K_{11} & \dots & \boxed{0} & \dots & K_{1n} \\ \dots & \dots & 0 & \dots & \dots \\ K_{i1} & \dots & 0 & \dots & K_{in} \\ \dots & \dots & 0 & \dots & \dots \\ K_{n1} & \dots & 0 & \dots & K_{nn} \end{bmatrix} \begin{Bmatrix} q_1 \\ \dots \\ q_i \\ \dots \\ q_n \end{Bmatrix} = \begin{Bmatrix} f_1 - K_{1i}\,\overline{q}_i \\ \dots \\ f_i - K_{ii}\,\overline{q}_i \\ \dots \\ f_n - K_{ni}\,\overline{q}_i \end{Bmatrix}$$

2. "Remove" equation corresponding to the known quantity

$$\begin{bmatrix} K_{11} & \dots & 0 & \dots & K_{1n} \\ \dots & \dots & 0 & \dots & \dots \\ 0 & 0 & \boxed{1} & 0 & 0 \\ \dots & \dots & 0 & \dots & \dots \\ K_{n1} & \dots & 0 & \dots & K_{nn} \end{bmatrix} \begin{Bmatrix} q_1 \\ \dots \\ q_i \\ \dots \\ q_n \end{Bmatrix} = \begin{Bmatrix} f_1 - K_{1i}\,\overline{q}_i \\ \dots \\ \boxed{\overline{q}_i} \\ \dots \\ f_n - K_{ni}\,\overline{q}_i \end{Bmatrix}$$

$$\boxed{q_i = \overline{q}_i}$$