

Lista 05 - Atividade

Questão 1

i) $\int x \sin 5x \, dx$

$u = x \Rightarrow du = dx$

$$v = \int \sin(5x) \, dx$$

$$dv = \cos(5x) \, dx \quad \leftarrow v = -\frac{1}{5} \cos(5x)$$

$$\int u \, dv = \int v \, du$$

$$\int x \sin(5x) \, dx = x \left(-\frac{1}{5} \cos(5x) \right) - \int \left(-\frac{1}{5} \cos(5x) \right) dx$$

$$\int x \sin(5x) \, dx = -\frac{1}{5} x \cos(5x) + \frac{1}{5} \int \cos(5x) \, dx$$

$$\int x \sin(5x) \, dx = -\frac{1}{5} x \cos(5x) + \frac{1}{5} \cdot \left(\frac{1}{5} \sin(5x) \right) + C$$

$$\boxed{\int x \sin(5x) \, dx = -\frac{1}{5} x \cos(5x) + \frac{1}{25} \sin(5x) + C}$$

ii) $\int x \ln 3x \, dx$  vamos para fórmula

$u = \ln(3x)$

$du = \frac{1}{x}$

$dv = x \cdot dx$

$v = \frac{x^2}{2}$

$$= \frac{1}{2} \cdot x^2 \cdot \ln(3x) - \frac{1}{2} \int dx$$

$$\frac{1}{2} \cdot x^2 \cdot \ln(3x) - \frac{x^2}{4} + C$$

$$\text{iii) } \int x^2 e^x dx = x^2 \cdot e^x - \int e^x \cdot 2x \cdot dx = x^2 \cdot e^x - 2 \int x \cdot e^x dx =$$

$$\int u dv = uv - \int v du$$

$$u = x^2 \rightarrow du = 2x dx$$

$$dv = e^x \cdot dx \rightarrow v = e^x$$

$$\int u dv = uv - \int v du$$

$$u = x \rightarrow du = 1 dx$$

$$dv = e^x \rightarrow v = e^x$$

$$= x^2 \cdot e^x - 2 \left(x \cdot e^x - \int e^x \cdot 1 \cdot dx \right) = x^2 \cdot e^x - 2x \cdot e^x + 2e^x =$$

$$= \boxed{e^x (x^2 - 2x + 2) + C}$$

$$\text{iv) } \int e^{3x} \cos 4x dx$$

$$u = \cos(4x)$$

$$du = -4 \sin(4x) dx$$

$$dv = e^{3x} dx$$

$$v = \frac{e^{3x}}{3}$$

$$\int e^{3x} \cos(4x) dx = \frac{e^{3x}}{3} \cos(4x) + \frac{4}{3} \int e^{3x} \sin(4x) dx$$

↑
a equação

* Resolvendo a outra parte...

$$\int e^{3x} \sin(4x) dx$$

$$u = \sin(4x)$$

$$du = 4 \cos(4x) dx$$

$$dv = e^{3x}$$

$$v = \frac{e^{3x}}{3}$$

$$\int e^{3x} \sin(4x) dx = \frac{e^{3x}}{3} \sin(4x) - \frac{4}{3} \int e^{3x} \cos(4x) dx$$

* Substituindo o valor da integral na equação.

$$\int e^{3x} \cos(4x) dx = \frac{e^{3x}}{3} \cos(4x) + \frac{4}{3} \left(\frac{e^{3x}}{3} \sin(4x) - \frac{4}{3} \int e^{3x} \cos(4x) dx \right)$$

$$\int e^{3x} \cos(4x) dx = \frac{e^{3x}}{3} \cos(4x) + \frac{4e^{3x}}{9} \sin(4x) - \frac{16}{9} \int e^{3x} \cos(4x) dx$$

continuando
o cálculo!

$$V) \int x^2 \cos x \, dx$$

$$u = x^2 \quad du = 2x \, dx$$

$$dv = \cos x \, dx \quad v = \sin x$$

$$\int u \, dv = uv - \int v \, du$$

$$\int x^2 \cos x \, dx = x^2 \sin x - \int \sin x \cdot 2x \, dx$$

$$I = x^2 \sin x - 2 \int x \sin x \, dx \quad (I)$$

$$u = x \quad du = dx$$

$$dv = \sin x \, dx \quad v = -\cos x$$

* Aplicando método de integração.

$$\int x \sin x \, dx = -x \cos x - \int (-\cos x) \, dx$$

$$I_1 = -x \cos x + \int \cos x \, dx$$

$$I_1 = -x \cos x + \sin x + C_1$$

* substituindo em (I), obtemos a integral de

$$I = x^2 \sin x - 2I_1$$

$$I = x^2 \sin x - 2 \cdot (-x \cos x + \sin x) + C$$

$$I = x^2 \sin x + 2x \cos x - 2 \sin x + C$$

$$I = (x^2 - 2) \sin x + 2x \cos x + C$$

$$\boxed{\int x^2 \cos x \, dx = (x^2 - 2) \sin x + 2x \cos x + C}$$

Questão 2

i) $y = 5 - x^2$ e $y = x + 3$

$$x + 3 = 5 - x^2$$

$$x^2 + x + 3 - 5 = 0$$

$$x^2 + x - 2 = 0$$

$$S = -1 \text{ e } P = -2$$

$$x' = -2 \text{ e } x'' = 1$$

$$\int_{-2}^1 [5 - x^2 - x - 3] dx$$

$$\int_{-2}^1 [2 - x^2 - x] dx$$

$$2x - \frac{x^3}{3} = \frac{x^2}{2} \mid x = -2 \text{ até } 1$$

$$2 \cdot 1 - \frac{1^3}{3} - \frac{1^2}{2} - [2 \cdot (-2) - \frac{(-2)^3}{3} - \frac{(-2)^2}{2}] =$$

$$2 - 1/3 - 1/2 - (-4 + 8/3 - 4/2) =$$

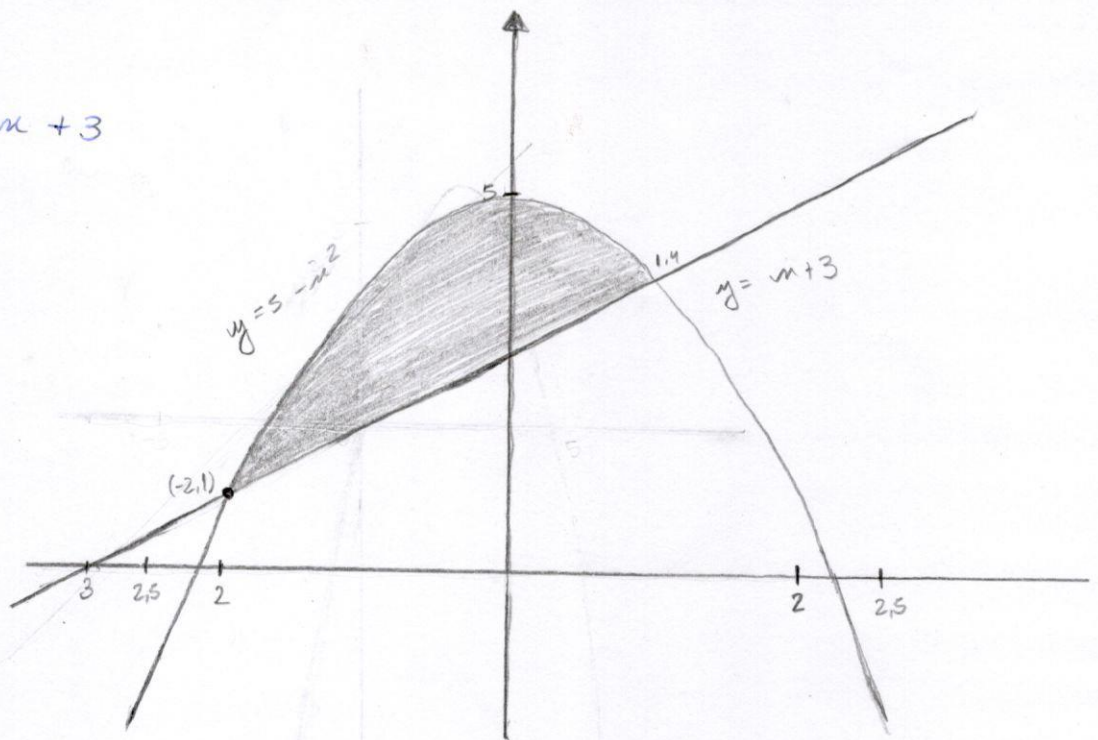
$$2 - 1/3 - 1/2 + 4 - 8/3 + 2 =$$

$$8 - 9/3 - 1/2 =$$

$$8 - 3 - 1/2 =$$

$$5 - 1/2 =$$

$$10/2 - 1/2 = 9/2$$



$$i) m + y = 3 \quad e \quad y + m^2 = 3$$

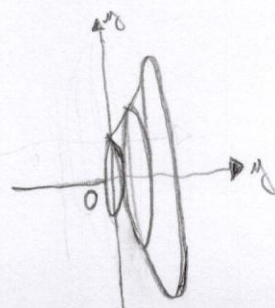
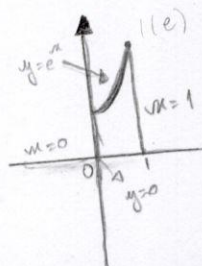
$$iii) y = e^m, m=0, m=1 \quad e \quad y=0$$

* ARRUMANDO A FUNÇÃO.

$$y = e^m, y=0, m=0, m=1$$

$$= \int_0^1 (e^m)^2 \pi = \frac{1}{2} \pi [e^{2m}]_0^1$$

$$= \int \frac{\pi}{2} [e^2 - e^0] = \frac{\pi}{2} [e^2 - 1]$$



$$v) y = \sin x \text{ e } y = -\sin x, x \in [0, 2\pi]$$

* Delimitada, organizando ficaria assim:

$$\{(x, y) \mid 0 \leq x \leq 2\pi, \sin x \leq y \leq -\sin x\}$$

$$i) \int_0^{2\pi} [\sin x]_{-\sin x}^{\sin x} dx$$

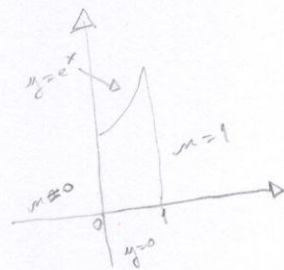
$$ii) \int_0^{2\pi} 2 \sin x dx = [-2 \cos x]_0^{2\pi} = -2 \cos 2\pi + 2 \cos 0 = 4u.c^2$$

$$v) x = y^2 \text{ e } y = -\frac{1}{2} x$$

$$vii) y = e^x - 1, y = -x \text{ e } x=1.$$

$$= \int_1^0 (e^x)^2 \pi = \frac{1}{2} \pi [e^{2x}]_0^1$$

$$= \int \frac{\pi}{2} [e^2 - e^0] = \frac{\pi}{2} [e^2 - 1]$$



questão 3

$$\int \sin^6 x \cos^3 x \, dx$$

$$\int \sin^6 x (1 - \sin^2 x) \cos x \, dx$$

$$\int \sin^6 x \cos x \, dx - \int \sin^8 x \cos x \, dx$$

$$\left\{ \frac{\sin^7 x}{7} - \frac{\sin^9 x}{9} + C \right\}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\cos^2 x = 1 - \sin^2 x$$

$$\int v^n \, dv = \frac{v^{n+1}}{n+1}$$

$$v = \sin x$$

$$dv = \cos x \, dx$$

$$\text{ii) } \int \cos^5(x) \, dx = \int \cos^4(x) \cdot \cos(x) \, dx = \int (\cos^2(x))^2 \cos(x) \, dx = \int (1 - \sin^2(x))^2 \cos(x) \, dx =$$

$\sin^2(x) + \cos^2(x) = 1$
 $\cos'(x) = 1 - \sin^2(x)$

$u = \sin(x)$
 $du = \cos(x) \, dx$

$$= \int (1 - u^2)^2 \, du = \int (1 - 2u^2 + u^4) \, du =$$

$$= u - 2 \cdot \frac{u^3}{3} + \frac{u^5}{5} = \sin(x) - \frac{2}{3} \sin^3(x) + \frac{1}{5} \sin^5(x) + C$$

$$a) \int \sin^2(\pi x) \cos^5(\pi x) dx$$

usando Função trigonométrica!
*vamos substituir.

$$\int \sin^2(\pi x) \cos^5(\pi x) dx = \frac{1}{\pi} \int \sin^2 t \cos^5 t dt$$

*vamos separar o $\cos^5 t = \cos^4 t \cos t = (1 - \sin^2 t)^2 \cos t$:

$$\int \sin^2(\pi x) \cos^5(\pi x) dx = \frac{1}{\pi} \int \sin^2 t (1 - \sin^2 t)^2 \cos t dt$$

*substituindo pela integral, $u = \sin t$

$$\int \sin^2(\pi x) \cos^5(\pi x) dx = \frac{1}{\pi} \int u^2 (1 - u^2)^2 du = \frac{1}{\pi} \int u^2 (1 - 2u^2 + u^4) du$$

Assim,

$$\int \sin^2(\pi x) \cos^5(\pi x) dx = \frac{1}{\pi} \int (u^2 - 2u^4 + u^6) du$$

*calculando integral

$$\int \sin^2(\pi x) \cos^5(\pi x) dx = \frac{1}{\pi} \left(\frac{u^3}{3} - \frac{2u^5}{5} + \frac{u^7}{7} \right) + C$$

*voltando com a variável original com $u = \sin(\pi x)$.

$$\int \sin^2(\pi x) \cos^5(\pi x) dx = \frac{1}{\pi} \left(\frac{1}{3} \sin^3(\pi x) - \frac{2}{5} \sin^5(\pi x) + \frac{1}{7} \sin^7(\pi x) \right) + C$$

*RESPOSTA

$$\frac{1}{\pi} \left(\frac{1}{3} \sin^3(\pi x) - \frac{2}{5} \sin^5(\pi x) + \frac{1}{7} \sin^7(\pi x) \right) + C$$

$$\begin{aligned}
 V) \int \sin 8x \cos 5x \, dx & \quad \text{resultado de } 8x + 5x = 13x \\
 & \quad \swarrow \quad \searrow \\
 & \quad 8x - 5x \quad 5x \\
 & = \frac{1}{2} \int \sin(3x) + \sin(13x) \, dx \\
 & = \frac{1}{2} \left(-\frac{1}{3} \cos(3x) - \frac{1}{13} \cos(13x) \right) \\
 & = \boxed{-\frac{1}{6} \cos(3x) - \frac{1}{26} \cos(13x) + C}
 \end{aligned}$$

* Usei essa linha de pensamento

$$\sin A \cos B = \frac{1}{2} [\sin(A-B) + \sin(A+B)]$$

$$V) \int \cos 7\theta \cos 5\theta \, d\theta$$

$$\int \cos 7\theta \cos 5\theta \, d\theta = \int \frac{1}{2} [\cos(7\theta - 5\theta) + \cos(7\theta + 5\theta)] \, d\theta$$

$$\int \cos 7\theta \cos 5\theta \, d\theta = \frac{1}{2} \int [\cos(2\theta) + \cos(12\theta)] \, d\theta$$

$$\int \cos 7\theta \cos 5\theta \, d\theta = \frac{1}{2} \int \cos(2\theta) \, d\theta + \frac{1}{2} \int \cos(12\theta) \, d\theta$$

$$\int \cos 7\theta \cos 5\theta \, d\theta = \frac{1}{4} \int 2 \cos(2\theta) \, d\theta + \frac{1}{24} \int 12 \cos(12\theta) \, d\theta$$

$$\boxed{\int \cos 7\theta \cos 5\theta \, d\theta = \frac{1}{4} \sin 2\theta + \frac{1}{24} \sin 12\theta + C}$$

$$VI) \int_0^{\pi/12} \sin^2 x \cos^2 x \, dx$$

$$\int_0^{\pi/12} \sin^2(x) \cos^2(x) \, dx = \frac{1}{4} \cdot \int [2 \sin(x) \cos(x)]^2 \, dx = \frac{1}{4} \cdot \int \sin^2(2x) \, dx =$$

$$\left(\frac{1}{8}\right) \cdot \int (1 - \cos(4x)) \, dx = \frac{x}{8} - \sin \frac{4x}{32} + K //$$