

Atividade de Cálculo I

1- Questão

a) $f(x) = 4 - x^2$. $f'(-3)$, $f'(0)$, $f'(1)$

$$f(x) = 4 - x^2$$

$$f'(-3) = -2 \cdot (-3) = 6$$

$$f'(0) = -2 \cdot 0 = 0$$

$$f'(1) = -2 \cdot 1 = -2$$

b) $g(t) = \frac{1}{t^2}$; $g'(-1)$, $g'(2)$, $g'(\sqrt{3})$

$$g(t) = \frac{1}{t^2}$$

$$g'(t) = -\frac{2}{t^3}$$

$$g'(-1) = -\frac{2}{(-1)^3} = 2$$

$$g'(2) = -\frac{2}{2^3} = -\frac{1}{4}$$

$$g'(\sqrt{3}) = -\frac{2}{(\sqrt{3})^3} = -\frac{2}{3\sqrt{3}}$$

2- Questão

$$f(x) = x + \frac{9}{x}, \quad x = -3$$

* substituindo $x = -3$ na função.

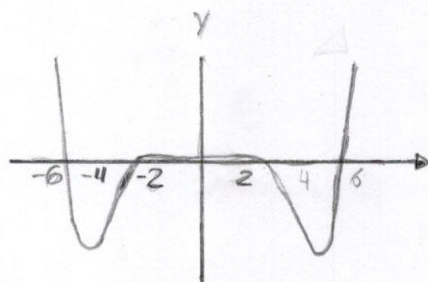
$$f(-3) = -3 + \frac{9}{-3} =$$

$$f(-3) = -3 - 3 =$$

$$f(-3) = -6$$

solução

* Gráfico da função



3- Questão

equação do enunciado!

$$v = 3t^2 - t^3$$

a) $f(x) = 3 \cdot 4^2 - 4^3$

$$f(x) = 36$$

b) $v(t) = 3t^2 - t^3$

$$v'(t) = 3 \cdot 2t - t^3$$

$$v'(t) = 6t - 3t^2$$

substituindo os tempos

$$v'(0) = 6 \cdot 0 - 3 \cdot 0^2 = 0 \text{ m.s}^{-1}$$

$$v'(1) = 6 \cdot 1 - 3 \cdot (1)^2 = 3 \text{ m.s}^{-1}$$

$$v'(2) = 6 \cdot 2 - 3 \cdot (2)^2 = 0 \text{ m.s}^{-1}$$

$$v'(3) = 6 \cdot 3 - 3 \cdot (3)^2 = 9 \text{ m.s}^{-1}$$

$$v'(4) = 6 \cdot 4 - 3 \cdot (4)^2 = 24 \text{ m.s}^{-1}$$

c) $f(t) = 3t^2 - t^3$

$$v'(t) = 6t - 3t^2$$

$$v''(t) = 6 - 6t$$

substituindo os tempos na função $v''(t)$

$$v''(0) = 6 - 6 \cdot (0) = 6 \text{ m.s}^{-2}$$

$$v''(1) = 6 - 6 \cdot (1) = 0 \text{ m.s}^{-2}$$

$$v''(2) = 6 - 6 \cdot (2) = -6 \text{ m.s}^{-2}$$

$$v''(3) = 6 - 6 \cdot (3) = -12 \text{ m.s}^{-2}$$

$$v''(4) = 6 - 6 \cdot (4) = -18 \text{ m.s}^{-2}$$

14 - Questões

a) $f(x) = 10(3x^2 + 7x - 3)^{10}$

$u = 3x^2 + 7x - 3$

$f(x) = 10 u^{10}$

$f' = 10 \cdot 10 \cdot u^9 \cdot u'$

$f' = 100 \cdot u^9 \cdot (6x + 7)$

$f' = 100(6x + 7) \cdot (3x^2 + 7x - 3)^9$

$$\begin{cases} u = 3x^2 + 7x - 3 \\ u' = 6x + 7 \end{cases}$$

b) $f(t) = (7t^2 + 6t)^7 \cdot (3t - 1)^4$

$f(t) = (7t^2 + 6t)^7 \cdot (3t - 1)^4 \Rightarrow f'(t) = 7(7t^2 + 6t)^6 \cdot (14t + 6) \cdot (3t - 1)^4 +$

$+ (7t^2 + 6t)^7 \cdot 4(3t - 1)^3 \cdot 3 \rightarrow$

$\rightarrow f'(t) = (7t^2 + 6t)^6 \cdot (3t - 1)^3$

$\rightarrow [7(14t + 6)(3t - 1) + 12(7t^2 + 6t)]$

c) $f(x) = \sqrt[3]{(3x^2 + 6x - 2)^2}$

$$d) f(t) = \sqrt{\frac{2t+1}{t-1}}$$

$$a) f(m) = 2^{3m^2+6m} =$$

* Usando regra da cadeia.

$$f'(x) = \frac{d}{dx} (2^g) \cdot \frac{d}{dm} (3m^2+6m)$$

$$f'(x) = \ln(2) \cdot 2^g \cdot \frac{d}{dm} (3m^2+6m)$$

$$f'(m) = \ln(2) \cdot 2^g \cdot (3 \cdot 2m + 6)$$

* Desenvolvendo substituição

$$f'(x) = \ln(2) \cdot 2^{3m^2+6m} \cdot (3 \cdot 2m + 6)$$

$$f'(m) = \ln(2) \cdot 2^{3m^2+6m} \cdot (6m + 6) =$$

$$1) f(t) = e^{t/2} (t^2 + 5t)$$

* derivando

$$f'(t) = \frac{d}{dt} (e^{t/2} \cdot (t^2 + 5t))$$

$$f'(t) = e^{t/2} \cdot \frac{1}{2} \cdot (t^2 + 5t) + e^{t/2} \cdot \frac{d}{dt} (t^2 + 5t)$$

$$f'(t) = e^{t/2} \cdot \frac{1}{2} \cdot (t^2 + 5t) + e^{t/2} \cdot (2t + 5)$$

$$f'(t) = \frac{e^{t/2} \cdot t^2 + 9 e^{t/2} t + 5 e^{t/2}}{2} \quad * \text{ simplificamos!}$$

$$f'(t) = \frac{e^{t/2} \cdot t^2 + 9 e^{t/2} t + 5 e^{t/2}}{2}$$

~~g)~~

$$g) f(s) = \log_3 \sqrt{s+1}$$

* derivamos

$$f'(s) = \frac{d}{ds} (\log_3 (s+1)^{1/2})$$

$$f'(s) = \frac{1}{2} \cdot \frac{d}{ds} (\log_3 (s+1))$$

$$f'(s) = \frac{1}{2} \cdot \frac{d}{dg} (\log_3 (g)) \cdot \frac{d}{ds} (s+1)$$

$$f'(s) = \frac{1}{2} \cdot \frac{1}{\ln(3)g} \cdot \frac{d}{ds} (s+1)$$

$$f'(s) = \frac{1}{2} \cdot \frac{1}{\ln(3)g} \cdot 1$$

$$f'(s) = \frac{1}{2} \cdot \frac{1}{\ln(3)g}$$

$$f'(s) = \frac{1}{2} \cdot \frac{1}{\ln(3) \cdot (s+1)}$$

* continuando

$$f'(s) = \frac{1}{2 \ln(3) \cdot (s+1)}$$

$$4) f(u) = \cos(\pi/2 - u)$$

* Derivando

$$f'(u) = \frac{d}{du} \left(\cos\left(\frac{\pi}{2} - u\right) \right)$$

$$f'(u) = \frac{d}{du} (\sin(u))$$

$$f'(u) = \cos(u)$$

$$ii) f(x) = \sin^3(3x^2 + 6x)$$

* Derivando

$$f'(x) = \frac{d}{dx} (\sin(3x^2 + 6x)^3)$$

$$f'(x) = \frac{d}{dg} (g^3) \cdot \frac{d}{dx} (\sin(3x^2 + 6x))$$

$$f'(x) = 3g^2 \cdot \cos(3x^2 + 6x) \cdot (3 \cdot 2x + 6)$$

$$f'(x) = 3 \sin(3x^2 + 6x)^2 \cdot \cos(3x^2 + 6x) \cdot (3 \cdot 2x + 6)$$

$$f'(x) = 3 \sin(3x^2 + 6x)^2 \cdot \cos(3x^2 + 6x) \cdot (6x + 6) =$$

$$J) f(x) = \frac{3 \sec^2 u}{u}$$

* Derivando

$$f'(x) = \frac{d}{dx} \left(\frac{3 \sec(x)^2}{x} \right)$$

$$f'(x) = \frac{\frac{d}{dx} (3 \sec(x)^2) \cdot x - 3 \sec(x)^2 \cdot \frac{d}{dx} (x)}{x^2}$$

$$f'(x) = \frac{3 \cdot 2 \sec(x) \cdot \tan(x) \cdot \sec(x) \cdot x - 3 \sec(x)^2 \cdot 1}{x^2}$$

$$f'(x) = \frac{6x \cdot \sin(x) - 3 \cos(x)}{\cos(x)^3 \cdot x^2}$$

$$K) f(\theta) = -\csc^2 \theta^3$$

* derivando

$$f'(\theta) = \frac{d}{d\theta} (-\csc(\theta^3)^2)$$

$$f'(\theta) = \frac{d}{d\theta} (-g^2) \cdot \frac{d}{d\theta} (\csc(\theta^3))$$

$$f'(\theta) = -2g \cdot (-\cot(\theta^3) \csc(\theta^3) \cdot 3\theta^2)$$

$$f'(\theta) = 2 \csc(\theta^3) \cdot (-\cot(\theta^3) \cdot \csc(\theta^3) \cdot 3\theta^2)$$

$$f'(\theta) = \frac{6\theta^2 \cdot \cos(\theta^3)}{\sin(\theta^3)^3}$$

//

Questão 5

→ calcular $f'(0)$, se $f(x) = e^{-x} \cos 3x$.

$$f'(x) = -x' e^{-x} \cdot \cos(3x) + \cos(3x)' e^{-x} =$$

$$= -e^{-x} \cos(3x) + (-3x') \sin(3x) e^{-x} =$$

$$= -\frac{1}{e^x} \cos(3x) - 3 \sin(3x) e^{-x} =$$

$$= -\frac{\cos(3x)}{e^x} - 3 \sin(3x) \cdot \frac{1}{e^x} =$$

$$= -\frac{\cos(3x)}{e^x} - \frac{3 \sin(3x)}{e^x} =$$

$$= \frac{-\cos(3x) - 3 \sin(3x)}{e^x}$$

$$f'(0) = -\cos(0) - 3 \sin(0) = \frac{-1-0}{1} = \boxed{-1}$$

6- Questão

* Mostrar que a função $y = x e^{-x}$ satisfaz a equação $xy' = \left(\frac{1-x}{x}\right) y$.

$$y = x \cdot e^{-x}$$

$$y' = -x \cdot e^{-x}$$

* Podemos substituir

$$xy' = (1-x)y$$

$$xy' = y - xy$$

$$-x^2 \cdot e^{-x} = x \cdot e^{-x} - x^2 \cdot e^{-x}$$

$$x \cdot e^{-x} (1-x) = 0$$

$$x=0 \quad //$$

Questão 7. $\rightarrow \frac{db}{dt} = b(m) = 10^4 - 2(10^3)t$

a) $b'(0) = 10^4 - 2(10^3)(0)$

$b'(0) = 10^4$ bactérias/hora

b) $b'(5) = 10^4 - 2(10^3)(5)$

$b'(5) = 0$ bactérias/hora

c) $b'(10) = 10^4 - 2(10^3)(10)$

$b'(10) = -10^4$ bactérias/hora