Universidade Federal da Fronteira Sul - UFFS

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Disciplina: Calculo I

Lista 04 - Atividade

1- Questão

i) $\int \left(9t^2 + \frac{1}{\sqrt{t^3}} \right) dt$

$\int (9t^2 \; dt + (1/\sqrt{1}) \int \frac{1}{t^3} \; dt =$

$9 \cdot \frac{t^2}{3} + (1\sqrt{1}) \cdot \int t^{-3} \; dt =$

$9 \cdot \frac{t^3}{5} + \frac{1}{\sqrt{1}} \cdot \frac{t^{-2}}{-2} + C =$

$3t^3 - \frac{1}{2\sqrt{1}} \cdot t^{-2} \; C =$

$3t^3 - \frac{1}{2t^2\sqrt{1}} + C$

* A Derivada seria,

$3t^3 - \frac{1}{2t^2\sqrt{1}} + \frac{C}{dt} = 9t^2 + \frac{1}{\sqrt{t^3}}$

//

ii) $\int (ax^4 + bx^3 + 3c)\, dx$

$\int (ax^4 + bx^3 + 3c)\, dx =$

$\int (ax^4)\, dx + \int (bx^3)\, dx + \int (3C)\, dx =$

$a\int (x^4)\, dx + b\int (x^3)\, dx + 3c\int dx =$

$\boxed{a\,(x^5)/5 + b\,(x^4)/4 + 3cx + d}$

iii) $\int \left(\sqrt{2y} - \frac{1}{\sqrt{2y}}\right) dy$

$I = \int \sqrt{2y} \int \sqrt{y}\, dy - \frac{1}{2} \int \frac{1}{\sqrt{y}}\, dy$

$I = \sqrt{2} \cdot \frac{2}{3} \sqrt{y}^3 - \frac{2}{\sqrt{2}} \cdot \sqrt{y} + C$

$\boxed{I = \frac{2\sqrt{2}}{3} \cdot \sqrt{y}^3 - \sqrt{2y} + C}$

iv) $\int \frac{x^5 + 2x^2 - 1}{x^4}\, dx$

$\int \left(\frac{x^5}{x^4} + \frac{2x^2}{x^4} + \frac{(-1)}{x^4}\right) dx$

$\int (x^5 \cdot x^{-4} + 2x^2 \cdot x^{-4} - 1 \cdot x^{-4})\, dx$

$\int (x + 2x^{-2} - x^{-4})\, dx$

* integrando

$\left(\frac{x^{1+1}}{1+1} + \frac{2x^{-2+1}}{-2+1} - \frac{x^{-4+1}}{-4+1}\right)$

$\left(\frac{x^2}{2} + \frac{2x^{-1}}{-1} - \frac{x^{-3}}{-3}\right)$

$\left(\frac{x^2}{2} - \frac{2}{x} + \frac{1}{3x^3} + C\right)$

Resposta:

$\boxed{\int \frac{x^5 + 2x^2 - 1}{x^4}\, dx = \frac{x^2}{2} - \frac{2}{x} + \frac{1}{3x^3} + C}$

V) $\int \left(\frac{e^t}{2} + \sqrt{t} + \frac{1}{t} \right) dt$

VI) $\int \frac{8x^4 - 9x^3 + 6x^2 - 2x + 1}{x^2} \, dx$

$\int (8x^4) \frac{dx}{x^2} - \int (9x^3) \frac{dx}{x^2} + \int (6x^2) \frac{dx}{x^2} - \int (2x) \frac{dx}{x^2} + \int (1) \frac{dx}{x^2} =$

$\int (8x^2) \, dx - \int (9x) \, dx + \int (6) \, dx - \int (2) . \frac{dx}{x} + \int (1) \frac{dx}{x^2} =$

$\int (8x^2) \, dx - \int (9x) \, dx + \int (6) \, dx - \int (2x^{-1}) \, dx + \int (x^{-2}) \, dx =$

$\boxed{\frac{8x^3}{3} - \frac{9x^2}{2} + 6x - 2 \ln (x) - \frac{1}{x} + C}$

VI) $\int \cos \theta \cdot tg\, \theta \; d\theta$

VII) $\int tg^2 x \; cosec^2 x \; dx$.

Questão 2

$$f(x) = x^{\frac{2}{3}} + x$$

* primitivando

$$F(x) = \int x^{2/3} + x \cdot dx = \frac{x^{\frac{2}{3}+1}}{\frac{2}{3}+1} + \frac{x^{1+1}}{1+1}$$

$$= \frac{x^{5/3}}{\frac{5}{3}} + \frac{x^2}{2}$$

$$= \frac{3x^{5/3}}{5} + \frac{x^2}{2}$$

$$\boxed{F(x) = \frac{3\sqrt[3]{x^5}}{5} + \frac{x^2}{2} + C}$$

$F(1) = 1$

logo,

$$1 = \frac{3\sqrt[3]{1^5}}{5} + \frac{1^2}{2} + C$$

$$1 = \frac{3}{5} + \frac{1}{2} + C$$

$$1 = \frac{11}{10} + C$$

$$1 - \frac{11}{10} = C$$

$$\frac{-1}{10} = C$$

Então a função procurada é

$$F(x) = \frac{3\sqrt[3]{x^5}}{5} + \frac{x^2}{2} - \frac{1}{10}$$

Questão 3.

$$\int f(x)\,dx = x^2 + \frac{1}{2}\cos 2x + C$$

* Derivando

$y = x^2 + \frac{1}{2}\cos 2x + C$

$y' = 2x + \frac{1}{2}\cdot(-\sin 2x \cdot 2)$

$y' = 2x - \sin 2x$

* Verificando

$$\int 2x - \sin 2x \, dx$$

$2 \cdot \frac{x^2}{2} - \frac{1}{2}\cdot(-\cos 2x) + C$

$x^2 + \frac{1}{2}\cos 2x + C$

Questão 4

i) $\int (2x^2 + 2x - 3)^{10} (2x+1)\,dx$

* substituição

$2x^2 + 2x - 3 = u$

$\Rightarrow (4x + 2)\,dx = du$

$\Rightarrow 2\cdot(2x+1)\,dx = du$

$\Rightarrow (2x+1)\,dx = \frac{1}{2}\,du$

* substituindo na integral

$I = \int u^{10} \cdot \frac{1}{2}\,du$

$I = \frac{1}{2}\int u^{10}\,du$

$I = \frac{1}{2}\cdot\frac{u^{10+1}}{10+1} + C$

$I = \frac{1}{2}\cdot\frac{u^{11}}{11} + C$

$I = \frac{u^{11}}{22} + C$

continuação

$I = \frac{1}{22}\cdot(2x^2 + 2x - 3)^{11} + C$

$$\Rightarrow \boxed{\int (2x^2 + 2x - 3)^{10}(2x+1)\,dx = \frac{1}{22}\cdot(2x^2+2x-3)^{11} + C}$$ //

ii) $\int 5x\sqrt{4-3x^2}\,dx$

* Mudanças de variável.

$u = 4-3x^2 \Rightarrow du = -6x\,dx \Rightarrow x\,dx = -\frac{du}{6}$

* Montando a conta.

$$\int 5x\sqrt{4-3x^2} = 5\int \underbrace{\sqrt{4-3x^2}}_{=\sqrt{u}}\,\underbrace{x\,dx}_{=\frac{du}{6}} = -\frac{5}{6}\int \sqrt{u}\,du.$$

* Como $\sqrt{u} = u^{\frac{1}{2}}$, a integral simples.

$$\int u^{\frac{1}{2}}\,du = \frac{u^{3/2}}{\frac{3}{2}} + C = \frac{2}{3}\cdot u^{3/2} + C, \quad \text{com } C \in \mathbb{R}$$

Portanto,

$$-\frac{5}{6}\int\sqrt{u}\,du = -\frac{5}{6}\cdot\left(\frac{2}{3}u^{3/2}+C\right) = \frac{-10}{18}u^{3/2} - \frac{5}{6}C = -\frac{5}{6}u^{\frac{3}{2}} + K,$$

com $K = -\frac{5}{6}C$

$$\boxed{\int 5x\sqrt{4-3x^2}\,dx = -\frac{5}{9}(4-3x^2)^{\frac{3}{2}} + K, \quad \text{com } K \in \mathbb{R}}.$$

iii) $\int (e^{2t}+2)^{1/3}\,e^{2t}\,dt$

$e^{2t} + 2 = u$

* Ficamos:

$u = e^{2t} + 2$

$\frac{du}{dt} = 2\cdot e^{2t} + 0$

$du = 2e^{2t}\,dt$

$\frac{du}{2} = e^{2t}\,dt$

* Portanto,

$\int u^{1/3}\,\frac{du}{2}$

$\frac{1}{2}\int u^{1/3}\,du$

* continuando

integral de uma potencia

$\int u^{(\frac{1}{3})}\,du = u^{(1/3+1)/(1/3+1)}$

$= u^{(1/3+3/3)/(1/3+3/3)}$

$= u^{\frac{4}{3}/4/3}$

* Então,

$\frac{1}{2}\,u^{(4/3)/(4/3)}$

$\frac{1}{2}\,u^{(4/3)\cdot(3/4)}$

$\frac{1}{2}\cdot\frac{3}{4}\cdot u^{4/3}$

$\frac{3}{8}\cdot u^{4/3}$

* continuando

Agora só substituir

$\frac{3}{8}\cdot(e^{2t}+2)^{4/3} + K$

IV) $\int sen^4 x \cos x \, dx$

$u = sen\, x$

$\frac{du}{dx} = \cos x$

$dx = \frac{du}{\cos x}$

$\int u^4 . \cancel{\cos} x . \frac{du}{\cancel{\cos} x}$

$\int u^4 . du = \frac{u^5}{5} + C = \frac{sen^5 x}{5} + C$

V) $\int \frac{sen\, x}{\cos^5 x} . dx$

VI) $\int e^x \cos 2e^x \, dx$

* variavel $y = 2e^x$, vamos de ...

$\rightarrow dy = \frac{d}{dx} \cdot (2e^x)$

$\rightarrow dy = 2e^x \, dx$

Portanto, a integral é:

$\rightarrow \int e^x \cos(2e^x) \, dx = \int \frac{1}{2} \cdot (2e^x) \cos(2e^x) \, dx$

$\rightarrow \int e^x \cos(2e^x) \, dx = \frac{1}{2} \int \cos y \, dy$

$\rightarrow \int e^x \cos(2e^x) \, dx = \frac{1}{2} (\sin y + c)$

$\rightarrow \boxed{\int e^x \cos(2e^x) \, dx = \frac{1}{2} \sin(2e^x) + C}$