Universidade Federal da Fronteira Sul - UFFS

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Professor: Milton Kist Disciplina: Calculo I

Lista 05 - Atividade

Questão 1

i) $\int x \text{ sen } 5x \, dx$

$u = x \Rightarrow du = dx$

$v = \int \text{sen}(5x)\, dx$

$dv = \text{sen}(5x)\, dx \leftarrow v = -\frac{1}{5} \cos(5x)$

$\int u\, dv = \int v\, du$

$\int x \text{ sen}(5x) = x \left(-\frac{1}{5} \cos(5x)\right) - \int \left(-\frac{1}{5} \cos(5x)\right) dx$

$\int x \text{ sen}(5x)\, dx = -\frac{1}{5} x \cos(5x) + \frac{1}{5} \int \cos(5x)\, dx$

$\int x \text{ sen}(5x)\, dx = -\frac{1}{5} x \cos(5x) + \frac{1}{5} \cdot \left(\frac{1}{5} \text{sen}(5x)\right) + C$

$\boxed{\int x \text{ sen}(5x)\, dx = -\frac{1}{5} x \cos(5x) + \frac{1}{25} \text{sen}(5x) + C}$

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ii) $\int x \ln 3x \, dx$ → vamos para formula

$u = \ln(3x$

$du = \frac{1}{x}$

$dv = x \cdot dx$

$v = \frac{x^2}{2}$

$= \frac{1}{2} \cdot x^2 \cdot \ln(3x) - \frac{1}{2} \int dx$

$\frac{1}{2} \cdot x^2 \cdot \ln(3x) - \frac{x^2}{4} + C$

//

iii) $\int x^2 e^x \, dx = x^2 \cdot e^x - \int e^x \cdot 2x \cdot dx = x^2 \cdot e^x - 2\int x \cdot e^x \, dx =$

$\int u\,dv = u \cdot v - \int v\,du$

$u = x^2 \to du = 2x\,dx$

$dv = e^x \cdot dx \to v = e^x$

$\int u\,dv = uv - \int v\,du$

$u = x \to du = 1\,dx$

$dv = e^x \to v = e^x$

$= x^2 \cdot e^x - 2\left(x \cdot e^x - \int e^x \cdot 1 \cdot dx\right) = x^2 \cdot e^x - 2x \cdot e^x + 2e^x =$

$= \boxed{e^x (x^2 - 2x + 2) + C}$

iv) $\int e^{3x} \cos 4x \, dx$

$u = \cos(4x)$

$du = -4\,\text{sen}(4x)\,dx$

$dv = e^{3x}\,dx$

$v = \dfrac{e^{3x}}{3}$

$$\int e^{3x} \cos(4x)\,dx = \frac{e^{3x}}{3}\cos(4x) + \frac{4}{3}\int e^{3x}\,\text{sen}(4x)\,dx$$

1ª equação

*Resolvendo a outra parte...

$\int e^{3x}\,\text{sen}(4x)\,dx$

$u = \text{sen}(4x)$

$du = 4\cos(4x)\,dx$

$dv = e^{3x}$

$v = \dfrac{e^{3x}}{3}$

$$\int e^{3x}\,\text{sen}(4x)\,dx = \frac{e^{3x}}{3}\,\text{sen}(4x) - \frac{4}{3}\int e^{3x}\cos(4x)\,dx$$

*substituindo o valor da integral na equação.

$$\int e^{3x} \cdot \cos(4x)\,dx = \frac{e^{3x}}{3}\cos(4x) + \frac{4}{3}\left(\frac{e^{3x}}{3}\,\text{sen}(4x) - \frac{4}{3}\int e^{3x}\cos(4x)\right)$$

$$\int e^{3x}\cos(4x)\,dx = \frac{e^{3x}}{3}\cos(4x) + \frac{4e^{3x}}{9}\,\text{sen}(4x) - \frac{16}{9}\int e^{3x}\cos(4x)\,dx$$

continuando o cálculo!

$$\int e^{3x}\cos(4x)\,dx + \frac{16}{9}\int e^{3x}\cos(4x)\,dx = \frac{e^{3x}}{3}\cos(4x) + \frac{4e^{3x}}{9}\,\text{sen}(4x)$$

$$\frac{25}{9}\int e^{3x}\cos(4x)\,dx = \frac{3e^{3x}\cos(4x) + 4e^{3x}\,\text{sen}(4x)}{9}$$

$$\int e^{3x}\cos(4x)\,dx = \frac{e^{3x}}{25} \cdot \left[3\cos(4x) + 4\,\text{sen}(4x)\right]$$

V) $\int x^2 \cos x \, dx$

$u = x^2 \qquad du = 2x \, dx$

$dv = \cos x \, dx \qquad v = \operatorname{sen} x$

$\int u \, dv = uv - \int v \, du$

$\int x^2 \cos x \, dx = x^2 \operatorname{sen} x - \int \operatorname{sen} x . 2x \, dx$

$I = x^2 \operatorname{sen} x - 2 \underbrace{\int x \operatorname{sen} x \, dx}$ (I)

$u = x \qquad du = dx$

$dv = \operatorname{sen} x \, dx \qquad v = -\cos x$

* Aplicando metodo de integração.

$\int x \operatorname{sen} x \, dx = -x \cos x - \int (-\cos x) \, dx$

$I_1 = -x \cos x + \int \cos x \, dx$

$I_1 = -x \cos x + \operatorname{sen} x + C_1$

* substituindo em (I), obtemos a integral de

$I = x^2 \operatorname{sen} x - 2 I_1$

$I = x^2 \operatorname{sen} x - 2 . (-x \cos x + \operatorname{sen} x) + C$

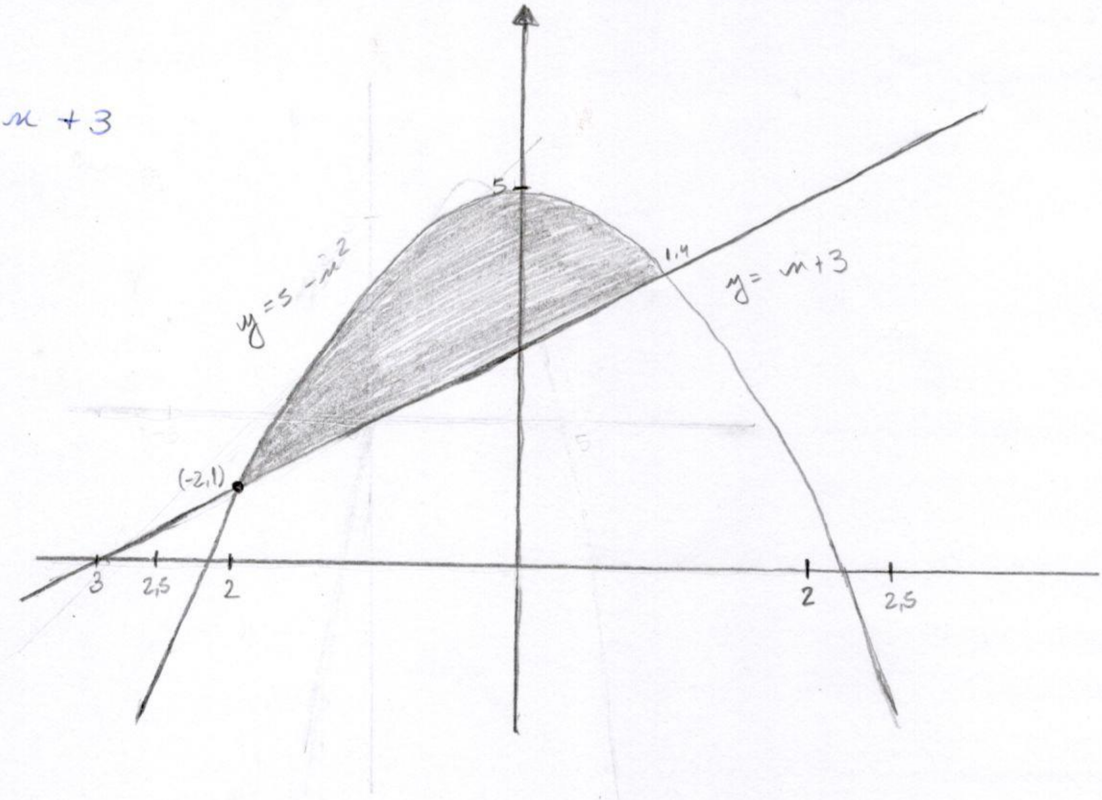
$I = x^2 \operatorname{sen} x + 2x \cos x - 2 \operatorname{sen} x + C$

$I = (x^2 - 2) \operatorname{sen} x + 2x \cos x + C$

$$\boxed{\int x^2 \cos x \, dx = (x^2 - 2) \operatorname{sen} x + 2x \cos x + C}$$

Questão 2

i) $y = 5 - x^2$ e $y = x + 3$



$x + 3 = 5 - x^2$

$x^2 + x + 3 - 5 = 0$

$x^2 + x - 2 = 0$

$S = -1$ e $P = -2$

$x' = -2$ e $x'' = 1$

$$\int_{-2}^{1} [5 - x^2 - x - 3]\, dx$$

$$\int_{-2}^{1} [2 - x^2 - x]\, dx$$

$$2x - \frac{x^3}{3} - \frac{x^2}{2} \Big| x = -2 \text{ até } 1$$

$$2.1 - \frac{1^3}{3} - \frac{1^2}{2} - [2.(-2) - (-2)^3/3 - (-2)^2/2] =$$

$2 - 1/3 - 1/2 - (-4 + 8/3 - 4/2) =$

$2 - 1/3 - 1/2 + 4 - 8/3 + 2 =$

$8 - 9/3 - 1/2 =$

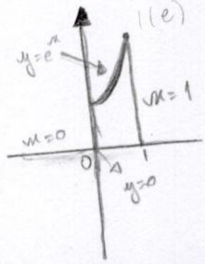
$8 - 3 - 1/2 =$

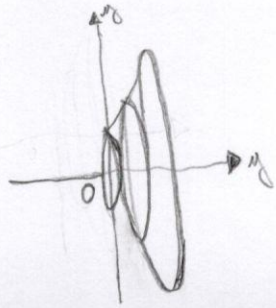
$5 - 1/2 =$

$10/2 - 1/2 = 9/2$

i) $x + y = 3$ e $y + x^2 = 3$

iii) $y = e^x$, $x = 0$, $x = 1$ e $y = 0$





*ARRUMANDO A FUNÇÃO.

$y = e^x$, $y = 0$, $x = 0$, $x = 1$

$$= \int_1^0 (e^x)^2 \pi = \frac{1}{2} \pi \left[e^{2x} \right]_0^1$$

$$= \int \frac{\pi}{2} \left[e^2 - e^0 \right] = \frac{\pi}{2} \left[e^2 - 1 \right]$$

V) $y = \text{sen } x$ e $y = -\text{sen } x$, $x \in [0, 2\pi]$

* Delimitada, organizando ficaria assim:

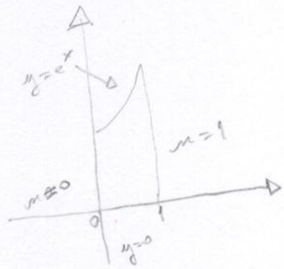
$\{(x,y) \mid 0 \leq x \leq 2\pi,\ \text{sin } x \leq y \leq \text{sin } x\}$

i) $\int_0^{2\pi} [y]_{-\text{sin } x}^{\text{sin } x} \, dx$

ii) $\int_0^{2\pi} 2 \text{ sin } x \, dx = [-2 \cos x]_0^{2x} = -2 \cos 2\pi + 2 \cos 0 = 4 u.c^2$

V) $x = y^2$ e $y = -\frac{1}{2} x$

VI) $y = e^x - 1$, $y = -x$ e $x = 1$.



$= \int_1^0 (e^x)^2 \pi = \frac{1}{2} \pi \left[e^{2x}\right]_0^1$

$= \int \frac{\pi}{2} \left[e^2 - e^0\right] = \frac{\pi}{2} \left[e^2 - 1\right]$

uestão 3

* regras

$$\boxed{sen^2 x + cos^2 x = 1}$$

$$\boxed{cos^2 x = 1 - sen^2 x}$$

$$\boxed{\int v^n \, dv = \frac{v^{n+1}}{n+1}}$$

$v = sen\, x$

$dv = cos\, x \, dx$

) $\int sen^6 x \; cos^3 x \; dx$

$= \int sen^6 x \, (1 - sen^2 x) \cos x \, dx$

$= \int sen^6 x \cos x \, dx - \int sen^8 x \cos x \, dx$

$= \boxed{\frac{sen^7 x}{7} - \frac{sen^9 x}{9} + C}$

iii) $\int \cos^5 (x) \, dx = \int \cos^4 (x) \cdot \cos(x) \, dx = \int (\cos^2 (x))^2 \cos(x) \, dx = \int (1 - \sin^2(x))^2 \cdot \cos(x) \, dx =$

$\sin^2(x) + \cos^2(x) = 1$

$\cos^2(x) = 1 - \sin^2(x)$

$u = \sin(x)$

$du = \cos(x) \, dx$

$= \int (1-u^2)^2 \, du = \int (1 - 2u^2 + u^4) \, du =$

$= u - 2 \cdot \frac{u^3}{3} + \frac{u^5}{5} = \boxed{\sin(x) - \frac{2}{3} \sin^3(x) + \frac{1}{5} \sin^5(x) + C}$

u) $\int sen^2 (\pi x) \cos^5 (\pi x)\, dx$

usando função trigonométrica!
*vamos substituir.

$$\int sen^2 (\pi x) \cos^5 (\pi x)\, dx = \frac{1}{\pi} \int sen^2\, t \cos^5\, t\, dt$$

*vamos separar o $\cos^5 t = \cos^4 t \cos t = (1 - sen^2 t)^2 \cos t$:

$$\int sen^2 (\pi x) \cos^5 (\pi x)\, dx = \frac{1}{\pi} \int sen^2 t\, (1 - sen^2 t)^2 \cos t\, dt$$

*substituindo pela integral: $u = sen\, t$

$$\int sen^2 (\pi x) \cos^5 (\pi x)\, dx = \frac{1}{\pi} \int u^2 (1 - u^2)^2\, du = \frac{1}{\pi} \int u^2 (1 - 2u^2 + u^4)\, du$$

Assim,

$$\int sen^2 (\pi x) \cos^5 (\pi x)\, dx = \frac{1}{\pi} \int (u^2 - 2u^4 + u^6)\, du$$

*calculando integral

$$\int sen^2 (\pi x) \cos^5 (\pi x)\, dx = \frac{1}{\pi} \left(\frac{u^3}{3} - \frac{2u^5}{5} + \frac{u^7}{7} \right) + C$$

*voltando com a variavel original com $u = sen\, (\pi x)$.

$$\int sen^2 (\pi x) \cos^5 (\pi x)\, dx = \frac{1}{\pi} \left(\frac{1}{3} \cdot sen^3 . (\pi x) - \frac{2}{5} sen^5 (\pi x) + \frac{1}{7} sen^7 (\pi x) \right) + C$$

*RESPOSTA

$$\frac{1}{\pi} \left(\frac{1}{3} sen^3 (\pi x) - \frac{2}{5} sen^5 (\pi x) + \frac{1}{7} sen^7 (\pi x) \right) + C$$

IV) $\int \text{sen } \underline{8x} \cos \underline{5x}\, dx$

resultado de $8x + 5x = 13x$

$8x - 5x$

* Usei essa linha de pensamento

$\sin A \cos B$

$= \frac{1}{2} \left(\sin(A-B) + \sin(A+B)\right)$

$= \frac{1}{2} \int \text{sen}(3x) + \text{sen}(13x)\, dx$

$= \frac{1}{2}\left(-\frac{1}{3}\cos(3x) - \frac{1}{13}\cos(13x)\right)$

$= \boxed{-\frac{1}{6}\cos(3x) - \frac{1}{26}\cos(13x) + C}$

V) $\int \cos 7\theta \cos 5\theta\, d\theta$

$\int \cos 7\theta \cos 5\theta\, d\theta = \int \frac{1}{2}\left[\cos(7\theta - 5\theta) + \cos(7\theta + 5\theta)\right] d\theta$

$\int \cos 7\theta \cos 5\theta\, d\theta = \frac{1}{2}\int \left[\cos(2\theta) + \cos(12\theta)\right] d\theta$

$\int \cos 7\theta \cos 5\theta\, d\theta = \frac{1}{2}\int \cos(2\theta)\, d\theta + \frac{1}{2}\int \cos(12\theta)\, d\theta$

$\int \cos 7\theta \cos 5\theta\, d\theta = \frac{1}{4}\int 2\cos(2\theta)\, d\theta + \frac{1}{24}\int 12\cos(12\theta)\, d\theta$

$\boxed{\int \cos 7\theta \cos 5\theta\, d\theta = \frac{1}{4}\text{sen } 2\theta + \frac{1}{24}\text{sen } 12\theta + C}$

VI) $\int_0^{\pi/2} \text{sen}^2\, \cos^2 x\, dx$

$\int_0^{\pi/2} \text{sen}^2(x) \cos^2(x)\, dx = \frac{1}{4} \cdot \int \left[2\,\text{sen}(x)\cos(x)\right]^2 dx = \frac{1}{4} \cdot \int \text{sen}^2(2x)\, dx =$

$\left(\frac{1}{8}\right) \cdot \int (1 - \cos(4x))\, dx = \frac{x}{8} - \frac{\text{sen } 4x}{32} + K$

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