Atividade de Calculo I

1- Questão

a) $f(x) = 4 - x^2 \ . \ f'(-3) \ . \ f'(0) \ . \ f'(1)$

$f'(x) = 0 - 2x$

$f'(-3) = -2 \cdot (-3) = 6$

$f'(0) = -2 \cdot 0 = 0$

$f'(1) = -2 \cdot 1 = -2$

b) $g(t) = \frac{1}{t^2}$; $g'(-1), g'(2), g'(\sqrt{3})$

$g(t) = 0$

$g'(0) = 0 - 1 = -1$

$g'(2) = 0 + 2 = 2$

$g'(\sqrt{3}) = 0 + \sqrt{3} = \sqrt{3}$

2- Questão

$f(x) = x + \frac{9}{x}$, $x = -3$

* substituindo $x=-3$ na função.

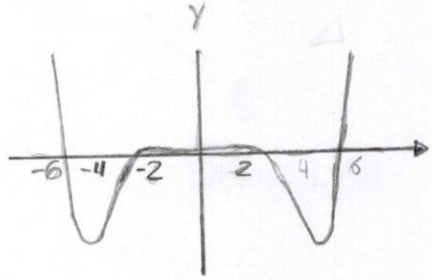
$f(-3) = -3 + \cancel{\frac{9}{-3}} =$

$f(-3) = -3 - 3 =$

$f(-3) = -6$

solução

* Gráfico da função



3- Questão equação do enunciado! $x = 3t^2 - t^3$

a) $f(x) = 3 \cdot 4^2 - 4^3$

$f(x) = 16$

b) $x(t) = 3t^2 - t^3$

$x'(t) = 3.2t - t^3$

$x'(t) = 6t - 3t^2$

substituindo os tempos

$x'(0) = 6.0 - 3.0^2 = 0\, m.s^{-1}$

$x'(1) = 6.1 - 3.(1)^2 = 3\, m.s^{-1}$

$x'(2) = 6.2 - 3.(2)^2 = 0\, m.s^{-1}$

$x'(3) = 6.3 - 3.(3)^2 = 9\, m.s^{-1}$

$x'(3) = 6.3 - 3.(3)^2 = 24\, m.s^{-1}$

$x'(4) = 6.4 - 3.(4)^2 = 24\, m.s^{-1}$

c) $f(x) = 3t^2 - t^3$

$x' = 6t - 3t^2$

$x' = 6 - 6t$

substituindo os tempos na função $x''(t)$

$x''(0) = 6 - 6.(0) = 6\, m.s^{-2}$

$x''(1) = 6 - 6.(1) = 0\, m.s^{-2}$

$x''(2) = 6 - 6.(2) = -6\, m.s^{-2}$

$x''(3) = 6 - 6.(3) = -12\, m.s^{-2}$

$x''(4) = 6 - 6.(4) = -18\, m.s^{-2}$

1- Questão

a) $f(x) = 10(3x^2 + 7x - 3)^{10}$

$u = 3x^2 + 7x - 3$

$f(x) = 10\, u^{10}$

$f' = 10 \cdot 10 \cdot u^9 \cdot u'$

$u = \underbrace{3x^2 + 7x - 3}$

$3 \cdot 2x + 7$

$6x + 7$

$f' = 100 \cdot u^9 \cdot (6x+7)$

$f' = 100\,(6x+7) \cdot (3x^2 + 7x - 3)^9$

b) $f(t) = (7t^2 + 6t)^7 \cdot (3t-1)^4$

$f(t) = (7t^2+6t)^7 \cdot (3t-1)^4 \Rightarrow f'(t) = 7\,(7t^2+6t)^6 \cdot (14t+6) \cdot (3t-1)^4 +$

$+ (7t^2+6t)^7 \cdot 4\,(3t-1)^3 \cdot 3 \rightarrow$

$\rightarrow f'(t) = (7t^2+6t)^6 \cdot (3t-1)^3$

$\rightarrow [7(14t+6)(3t-1) + 12(7t^2+6t)].$

c) $f(x) = \sqrt[3]{(3x^2+6x-2)^2}$

* Derivada $f(x) = \sqrt[3]{(3x^2+6x-2)^2}$

$\frac{d}{dx}\left(\sqrt[3]{(3x^2+6x-2)^2}\right) = \frac{4(3x^2+6x-2)(x+1)}{((3x^2+6x-2)^2)^{2/3}} =$

$= \frac{}{3((3x^2+6x-2)^2)^{2/3}} - \frac{d}{dx}((3x^2+6x-2)^2) =$

$\frac{d}{dx}((3x^2+6x-2)^2) = 2(3x^2+6x-2)(6x+6) =$

$= \frac{1}{3((3x^2+6x-2)^2)^{2/3}} \cdot 2(3x^2+6x-2)(6x+6) =$

simplificar $\frac{1}{3((3x^2+6x-2)^2)^{2/3}} \cdot 2(3x^2+6x-2)(6x+6) =$

$\frac{4(3x^2+6x-2)(x+1)}{((3x^2+6x-2)^2)^{2/3}} =$

$= \frac{4(3x^2+6x-2)(x+1)}{((3x^2+6x-2)^2)^{2/3}}$

d) $f(t) = \sqrt{\frac{2t+1}{t-1}} = \frac{d}{dx}\left(\sqrt{\frac{2t+1}{t+1}}\right) = \frac{d}{dx}(\sqrt{u}) \cdot \frac{d}{dx}\left(\frac{2t+1}{t+1}\right)$

$= \frac{1}{2\sqrt{\frac{2t+1}{t-1}}} \cdot \frac{d}{dt}\left(\frac{2t+1}{t+1}\right)$

$= \frac{d}{dt}(2t+1) = \underbrace{\frac{d}{dt}(2t)}_{\frac{2d(t)}{dt}} + \underbrace{\frac{d}{dt}(1)}_{0} = 2+0=2$

$2 \cdot 1 = 2$

$\frac{d}{dt}(t+1) = \frac{d}{dt}(t) + \frac{d}{dt}(1) = 1+0=0$

$= \frac{2(t+1)-1\cdot(2t+1)}{(t+1)^2}$

$= \frac{2(t-1)-(2t+1)}{(t-1)^2}$

$= \frac{1}{(t+1)^2}$

$= \frac{1}{2\sqrt{\frac{2t+1}{t+1}}} \cdot \frac{1}{(t+1)^2}$

$= \frac{1}{2\sqrt{\frac{2t+1}{t+1}}} \cdot \frac{1}{(t+1)^2}$

$= \frac{\sqrt{t+1}}{\sqrt{2t+1}\cdot 2} \cdot \frac{1}{(t+1)^2} =$

$= \frac{(t+1)^{\frac{1}{2}}}{2(t+1)^2 \cdot \sqrt{2t+1}}$

$= \frac{1}{2\sqrt{2t+1}\cdot(t+1)^{\frac{1}{2}+2}}$

$= \boxed{\frac{1}{2\sqrt{2t+1}\cdot(t+1)^{\frac{3}{2}}}}$

$\frac{t+1}{\sqrt{2t+1}\cdot 2(t+1)^2}$

e) $f(x) = 2^{3x^2+6x}$ =

* Usando regra da cadeia.

$f'(x) = \frac{d}{dg}(2^g) \cdot \frac{d}{dx}(3x^2+6x)$

$f'(x) = \ln(2) \cdot 2^g \cdot \frac{d}{dx} \cdot (3x^2+6x)$

$f'(x) = \ln(2) \cdot 2^g \cdot (3 \cdot 2x + 6)$

Devolvendo substituição

$f'(x) = \ln(2) \cdot 2^{3x^2+6x} \cdot (3 \cdot 2x + 6)$

$f'(x) = \ln(2) \cdot 2^{3x^2+6x} \cdot (6x+6)$

f) $f(t) = e^{t/2}\,(t^2 + 5t)$

* Derivando

$$f'(t) = \frac{d}{dt}\left(e^{\frac{t}{2}} \cdot (t^2+5t)\right)$$

$$f'(t) = e^{t/2} \cdot \frac{1}{2} \cdot (t^2+5t) + e^{\frac{t}{2}} \cdot \frac{d}{dt}(t^2+5t)$$

$$f'(t) = e^{\frac{t}{2}} \cdot \frac{1}{2} \cdot (t^2+5t) + e^{t/2} \cdot (2t+5)$$

$$f'(t) = \frac{e^{\frac{t}{2}} \cdot t^2 + 9\,e^{\frac{t}{2}}\,t}{2} + 5\,e^{t/2}$$ * simplicamos!

$$f'(t) = \frac{e^{\frac{t}{2}} \cdot t^2 + 9\,e^{t/2}\,t + 5\,e^{t/2}}{2}$$ //

g) $f(s) = \log_3 \sqrt{s+1}$

* Derivamos

$$f'(s) = \frac{d}{ds}\left(\log_3\left((s+1)^{\frac{1}{2}}\right)\right)$$

$$f'(s) = \frac{1}{2} \cdot \frac{d}{ds}\left(\log_3(s+1)\right)$$

$$f'(s) = \frac{1}{2} \cdot \frac{d}{dg}\left(\log_3(g)\right) \cdot \frac{d}{ds}(s+1)$$

$$f'(s) = \frac{1}{2} \cdot \frac{1}{\ln(3)g} \cdot \frac{d}{ds}(s+1)$$

$$f'(s) = \frac{1}{2} \cdot \frac{1}{\ln(3)g} \cdot 1$$

$$f'(s) = \frac{1}{2} \cdot \frac{1}{\ln(3)g}$$

$$f'(s) = \frac{1}{2} \cdot \frac{1}{\ln(3) \cdot (s+1)}$$

* continuando

$$f'(s) = \frac{1}{2\ln(3) \cdot (s+1)}$$ //

h) $f(u) = \cos\ (\pi/2\ - u)$

* Derivando

$$f'(u) = \frac{d}{du}\left(\cos\left(\frac{\pi}{2} - u\right)\right)$$

$$f'(u) = \frac{d}{du}\ (\sin(u))$$

$$f'(u) = \cos(u)$$

i) $f(x) = \text{sen}^3\ (3x^2 + 6x)$

* Derivando

$$f'(x) = \frac{d}{dx}\ (\sin(3x^2 + 6x)^3)$$

$$f'(x) = \frac{d}{dg}(g^3) \cdot \frac{d}{dx}\left(\sin\left(3x^2 + 6x\right)\right)$$

$$f'(x) = 3g^3 \cdot \cos(3x^2 + 6x) \cdot (3 \cdot 2x + 6)$$

$$f'(x) = 3\sin(3x^2 + 6x)^2 \cdot \cos \cdot (3x^2 + 6x) \cdot (3 \cdot 2x + 6$$

$$f'(x) = 3\sin(3x^2 + 6x)^2 \cdot \cos(3x^2 + 6x) \cdot (6x + 6)$$

j) $f(x) = \frac{3 \sec^2 x}{x}$

* Derivando

$$f'(x) = \frac{d}{dx}\left(\frac{3 \sec(x)^2}{x}\right)$$

$$f'(x) = \frac{\frac{d}{dx}(3\sec(x)^2) \cdot x - 3\sec(x)^2 \cdot \frac{d}{dx}(x)}{}$$

$$f'(x) = \frac{3 \cdot 2\sec(x) \cdot \tan(x) \cdot \sec(x) \cdot x - 3\sec(x)^2 \cdot 1}{x^2}$$

$$f'(x) = \frac{6x \cdot \sin(x) - 3\cos(x)}{\cos(x)^3 \cdot x^2}$$

=

K) $f(\theta) = -\operatorname{cosec}^2 \theta^3$

* Derivando

$$f'(\theta) = \frac{d}{d\theta}\left(-\csc(\theta^3)^2\right)$$

$$f'(\theta) = \frac{d}{dg}(-g^2) \cdot \frac{d}{d\theta}(\csc(\theta^3))$$

$$f'(\theta) = -2g \cdot (-\cot(\theta^3)\csc(\theta^3) \cdot 3\theta^2)$$

$$f'(\theta) = 2\csc(\theta^3) \cdot (-\cot(\theta^3) \cdot \csc(\theta^3) \cdot 3\theta^2)$$

$$f'(\theta) = \frac{6\theta^2 \cdot \cos(\theta^3)}{\sin(\theta^3)^3}$$

//

Questão 5

→ Calcular $f'(0)$, se $f(x) = e^{-x} \cos 3x$.

$f'(x) = -x' \, e^{-x} . \cos(3x) + \cos(3x)' \, e^{-x} =$

$= -e^{-x} \cos(3x) + (-3x') \operatorname{sen}(3x) \, e^{-x} =$

$= -\frac{1}{e^{x}} \cos(3x) - 3 \operatorname{sen}(3x) \, e^{-x} =$

$= \frac{-\cos(3x)}{e^{x}} - 3 \operatorname{sen}(3x) \cdot \frac{1}{e^{x}} =$

$= \frac{-\cos(3x)}{e^{x}} - \frac{3 \operatorname{sen}(3x)}{e^{x}} =$

$= \frac{-\cos(3x) - 3\operatorname{sen}(3x)}{e^{x}}$

$f'(0) = -\cos(0) - 3\operatorname{sen}(0) = \frac{-1-0}{1} = -1$

6- Questão

* Mostrar que a função $y = x \, e^{-x}$ satisfaz a equação $xy' = (1-x)y$.

$y = x . e^{-x}$

$y' = -x . e^{-x}$

* Podemos substituir

$xy = (1-x)y$

$xy' = y - xy$

$-x^2 . e^{-x} = x . e^{-x} - x^2 . e^{-x}$

$x . e^{-x} (-x) = 0$

$X = 0$ //

Questão 7. $\rightarrow$ $\frac{db}{dt} = b(m) = 10^4 - 2(10^3)t$

a) $b'(0) = 10^4 - 2(10^3)(0)$

$b'(0) = 10^4$ bacterias/hora

b) $b'(5) = 10^4 - 2(10^3).(5)$

$b'(5) = 0$ bacterias/hora

c) $b'(10) = 10^4 - 2(10^3).(10)$

$b'(10) = -10^4$ bacterias/hora