

Atividade de Cálculo I

1- Questão

a) $f(x) = 4 - x^2$. $f'(-3)$, $f'(0)$, $f'(1)$

$$f'(x) = 0 - 2x$$

$$f'(-3) = -2 \cdot (-3) = 6$$

$$f'(0) = -2 \cdot 0 = 0$$

$$f'(1) = -2 \cdot 1 = -2$$

b) $g(t) = \frac{1}{t^2}$; $g'(-1)$, $g'(2)$, $g'(\sqrt{3})$

$$g(t) = 0$$

$$g'(0) = 0 - 1 = -1$$

$$g'(2) = 0 + 2 = 2$$

$$g'(\sqrt{3}) = 0 + \sqrt{3} = \sqrt{3}$$

2- Questão

$$f(x) = x + \frac{9}{x}, \quad x = -3$$

* substituindo $x = -3$ na função.

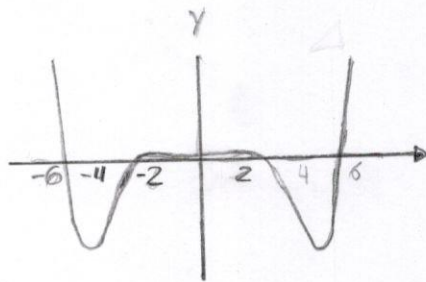
$$f(-3) = -3 + \frac{9}{-3} =$$

$$f(-3) = -3 - 3 =$$

$$f(-3) = -6$$

solução

* Gráfico da função



3- Questão

equação do enunciado!

$$v = 3t^2 - t^3$$

a) $f(x) = 3 \cdot 4^2 - 4^3$

$$f(x) = 56$$

b) $v(t) = 3t^2 - t^3$

$$v'(t) = 3 \cdot 2t - t^3$$

$$v'(t) = 6t - 3t^2$$

substituindo os tempos

$$v'(0) = 6 \cdot 0 - 3 \cdot 0^2 = 0 \text{ m.s}^{-1}$$

$$v'(1) = 6 \cdot 1 - 3 \cdot (1)^2 = 3 \text{ m.s}^{-1}$$

$$v'(2) = 6 \cdot 2 - 3 \cdot (2)^2 = 0 \text{ m.s}^{-1}$$

$$v'(3) = 6 \cdot 3 - 3 \cdot (3)^2 = 9 \text{ m.s}^{-1}$$

$$v'(4) = 6 \cdot 4 - 3 \cdot (4)^2 = 24 \text{ m.s}^{-1}$$

c) $f(x) = 3t^2 - t^3$

$$v'(t) = 6t - 3t^2$$

$$v''(t) = 6 - 6t$$

substituindo os tempos na função $v''(t)$

$$v''(0) = 6 - 6 \cdot (0) = 6 \text{ m.s}^{-2}$$

$$v''(1) = 6 - 6 \cdot (1) = 0 \text{ m.s}^{-2}$$

$$v''(2) = 6 - 6 \cdot (2) = -6 \text{ m.s}^{-2}$$

$$v''(3) = 6 - 6 \cdot (3) = -12 \text{ m.s}^{-2}$$

$$v''(4) = 6 - 6 \cdot (4) = -18 \text{ m.s}^{-2}$$

4 - Questões

$$a) f(x) = 10(3x^2 + 7x - 3)^{10}$$

$$u = 3x^2 + 7x - 3$$

$$f(x) = 10 u^{10}$$

$$f' = 10 \cdot 10 \cdot u^9 \cdot u'$$

$$f' = 100 \cdot u^9 \cdot (6x + 7)$$

$$f' = 100(6x + 7) \cdot (3x^2 + 7x - 3)^9$$

$$\begin{cases} u = 3x^2 + 7x - 3 \\ u' = 3 \cdot 2x + 7 \\ \quad 6x + 7 \end{cases}$$

$$b) f(t) = (7t^2 + 6t)^7 \cdot (3t - 1)^4$$

$$f(t) = (7t^2 + 6t)^7 \cdot (3t - 1)^4 \Rightarrow f'(t) = 7(7t^2 + 6t)^6 \cdot (14t + 6) \cdot (3t - 1)^4 +$$

$$+ (7t^2 + 6t)^7 \cdot 4(3t - 1)^3 \cdot 3 \rightarrow$$

$$\rightarrow f'(t) = (7t^2 + 6t)^6 \cdot (3t - 1)^3$$

$$\rightarrow [7(14t + 6)(3t - 1) + 12(7t^2 + 6t)] \cdot (7t^2 + 6t)^6 \cdot (3t - 1)^3$$

$$c) f(x) = \sqrt[3]{(3x^2 + 6x - 2)^2}$$

$$\text{Derivada } f(x) = \sqrt[3]{(3x^2 + 6x - 2)^2}$$

$$\frac{d}{dx} \left(\sqrt[3]{(3x^2 + 6x - 2)^2} \right) = \frac{4(3x^2 + 6x - 2)(x + 1)}{(3x^2 + 6x - 2)^{2/3}} =$$

$$= \frac{4(3x^2 + 6x - 2)(x + 1)}{3((3x^2 + 6x - 2)^{2/3})} - \frac{d}{dx} ((3x^2 + 6x - 2)^2) =$$

$$\frac{d}{dx} ((3x^2 + 6x - 2)^2) = 2(3x^2 + 6x - 2)(6x + 6) =$$

$$= \frac{2(3x^2 + 6x - 2)(6x + 6)}{3((3x^2 + 6x - 2)^{2/3})} =$$

$$\text{Simplificando } \frac{1}{3((3x^2 + 6x - 2)^{2/3})} \cdot 2(3x^2 + 6x - 2)(6x + 6) =$$

$$\frac{4(3x^2 + 6x - 2)(x + 1)}{(3x^2 + 6x - 2)^{2/3}} =$$

$$= \frac{4(3x^2 + 6x - 2)(x + 1)}{(3x^2 + 6x - 2)^{2/3}}$$

$$d) f(t) = \sqrt{\frac{2t+1}{t-1}} = \frac{d}{du} \left(\sqrt{\frac{2t+1}{t+1}} \right) = \frac{d}{du} (\sqrt{u}) \cdot \frac{d}{dt} \left(\frac{2t+1}{t+1} \right)$$

$$= \frac{1}{2\sqrt{\frac{2t+1}{t-1}}} \cdot \frac{d}{dt} \left(\frac{2t+1}{t+1} \right) \rightarrow = \frac{d}{dt} (2t+1) = \frac{d}{dt} (2t) + \frac{d}{dt} (1) = 2 + 0 = 2$$

$$\Delta \frac{d}{dt} (t+1) = \frac{d}{dt} (t) + \frac{d}{dt} (1) = 1 + 0 = 1$$

$$= \frac{1}{2\sqrt{\frac{2t+1}{t+1}}} \cdot \frac{1}{(t+1)^2}$$

$$= \frac{1}{2\sqrt{\frac{2t+1}{t+1}}} \cdot \frac{1}{(t+1)^2}$$

$$= \frac{\sqrt{t+1}}{\sqrt{2t+1} \cdot 2} \cdot \frac{1}{(t+1)^2}$$

$$\frac{t+1}{\sqrt{2t+1} \cdot 2 (t+1)^2}$$

$$f(u) = 2^{3m^2+6m} =$$

* Usando regra da cadeia.

$$f'(x) = \frac{d}{dx} (2^u) \cdot \frac{d}{dm} (3m^2+6m)$$

$$f'(x) = \ln(2) \cdot 2^u \cdot \frac{d}{dm} (3m^2+6m)$$

$$f'(m) = \ln(2) \cdot 2^u \cdot (3 \cdot 2m + 6)$$

Devolvendo substituição

$$f'(x) = \ln(2) \cdot 2^{3m^2+6m} \cdot (3 \cdot 2m + 6)$$

$$f'(m) = \ln(2) \cdot 2^{3m^2+6m} \cdot (6m + 6)$$

$$f(t) = e^{t/2} (t^2 + 5t)$$

* derivando

$$f'(t) = \frac{d}{dt} (e^{t/2} \cdot (t^2 + 5t))$$

$$f'(t) = e^{t/2} \cdot \frac{1}{2} \cdot (t^2 + 5t) + e^{t/2} \cdot \frac{d}{dt} (t^2 + 5t)$$

$$f'(t) = e^{t/2} \cdot \frac{1}{2} \cdot (t^2 + 5t) + e^{t/2} \cdot (2t + 5)$$

$$f'(t) = \frac{e^{t/2} \cdot t^2 + 9 e^{t/2} t + 5 e^{t/2}}{2} \quad * \text{ simplifiquemos! }$$

$$f'(t) = \frac{e^{t/2} \cdot t^2 + 9 e^{t/2} t + 5 e^{t/2}}{2} //$$

~~g)~~

$$g) f(s) = \log_3 \sqrt{s+1}$$

* derivamos

$$f'(s) = \frac{d}{ds} (\log_3 ((s+1)^{1/2}))$$

$$f'(s) = \frac{1}{2} \cdot \frac{d}{ds} (\log_3 (s+1))$$

$$f'(s) = \frac{1}{2} \cdot \frac{d}{dg} (\log_3 (g)) \cdot \frac{d}{ds} (s+1)$$

$$f'(s) = \frac{1}{2} \cdot \frac{1}{\ln(3)g} \cdot \frac{d}{ds} (s+1)$$

$$f'(s) = \frac{1}{2} \cdot \frac{1}{\ln(3)g} \cdot 1$$

$$f'(s) = \frac{1}{2} \cdot \frac{1}{\ln(3)g}$$

$$f'(s) = \frac{1}{2} \cdot \frac{1}{\ln(3) \cdot (s+1)}$$

* continuando

$$f'(s) = \frac{1}{2 \ln(3) \cdot (s+1)} //$$

$$4) f(u) = \cos\left(\frac{\pi}{2} - u\right)$$

* Derivando

$$f'(u) = \frac{d}{du} \left(\cos\left(\frac{\pi}{2} - u\right) \right)$$

$$f'(u) = \frac{d}{du} (\sin(u))$$

$$f'(u) = \cos(u)$$

$$i) f(x) = \sin^3(3x^2 + 6x)$$

* Derivando

$$f'(x) = \frac{d}{dx} (\sin(3x^2 + 6x)^3)$$

$$f'(x) = \frac{d}{dg} (g^3) \cdot \frac{d}{dx} (\sin(3x^2 + 6x))$$

$$f'(x) = 3g^2 \cdot \cos(3x^2 + 6x) \cdot (3 \cdot 2x + 6)$$

$$f'(x) = 3 \sin^2(3x^2 + 6x) \cdot \cos(3x^2 + 6x) \cdot (3 \cdot 2x + 6)$$

$$f'(x) = 3 \sin^2(3x^2 + 6x) \cdot \cos(3x^2 + 6x) \cdot (6x + 6) =$$

$$j) f(x) = \frac{3 \sec^2 u}{u}$$

* Derivando

$$f'(x) = \frac{d}{dx} \left(\frac{3 \sec (x)^2}{x} \right)$$

$$f'(x) = \frac{\frac{d}{dx} (3 \sec (x)^2) \cdot x - 3 \sec (x)^2 \cdot \frac{d}{dx} (x)}{x^2}$$

$$f'(x) = \frac{3 \cdot 2 \sec (x) \cdot \tan (x) \cdot \sec (x) \cdot x - 3 \sec (x)^2 \cdot 1}{x^2}$$

$$f'(x) = \frac{6x \cdot \sin (x) - 3 \cos (x)}{\cos (x)^3 \cdot x^2}$$

=

$$k) f(\theta) = - \csc^2 \theta^3$$

* derivando

$$f'(\theta) = \frac{d}{d\theta} (-\csc (\theta^3)^2)$$

$$f'(\theta) = \frac{d}{d\theta} (-g^2) \cdot \frac{d}{d\theta} (\csc (\theta^3))$$

$$f'(\theta) = -2g \cdot (-\cot (\theta^3) \csc (\theta^3) \cdot 3\theta^2)$$

$$f'(\theta) = 2 \csc (\theta^3) \cdot (-\cot (\theta^3) \cdot \csc (\theta^3) \cdot 3\theta^2)$$

$$f'(\theta) = \frac{6\theta^2 \cdot \cos (\theta^3)}{\sin (\theta^3)^3}$$

//

Questão 5

→ Calcular $f'(0)$, se $f(x) = e^{-x} \cos 3x$.

$$\begin{aligned} f'(x) &= -x' e^{-x} \cdot \cos(3x) + \cos(3x)' e^{-x} = \\ &= -e^{-x} \cos(3x) + (-3x') \sin(3x) e^{-x} = \\ &= -\frac{1}{e^x} \cos(3x) - 3 \sin(3x) e^{-x} = \\ &= -\frac{\cos(3x)}{e^x} - 3 \sin(3x) \cdot \frac{1}{e^x} = \\ &= -\frac{\cos(3x)}{e^x} - \frac{3 \sin(3x)}{e^x} = \\ &= \frac{-\cos(3x) - 3 \sin(3x)}{e^x} \end{aligned}$$

$$f'(0) = -\cos(0) - 3 \sin(0) = \frac{-1 - 0}{1} = \boxed{-1}$$

6- Questão

* Mostrar que a função $y = x e^{-x}$ satisfaz a equação $xy' = \left(\frac{1-x}{x}\right) y$.

$$\begin{aligned} y &= x \cdot e^{-x} \\ y' &= -x \cdot e^{-x} \end{aligned}$$

* Podemos substituir

$$xy' = (1-x)y$$

$$xy' = y - xy$$

$$-x^2 \cdot e^{-x} = x \cdot e^{-x} - x^2 \cdot e^{-x}$$

$$x \cdot e^{-x} (-x) = 0$$

$$x = 0 //$$

questão 7. $\rightarrow \frac{db}{dt} = b(m) = 10^4 - 2(10^3)t$

a) $b'(0) = 10^4 - 2(10^3)(0)$

$b'(0) = 10^4$ bacterias / hora

b) $b'(5) = 10^4 - 2(10^3)(5)$

$b'(5) = 0$ bacterias / hora

c) $b'(10) = 10^4 - 2(10^3)(10)$

$b'(10) = -10^4$ bacterias / hora