1- Questão

i) $\int \left(9t^2 + \frac{1}{\sqrt{t^3}}\right) dt$

$\int (9t^2 \, dt + (1/\sqrt{1}) \int \frac{1}{t^3} \, dt =$

$9 \cdot \frac{t^2}{3} + (1\sqrt{1}) \cdot \int t^{-3} \, dt =$

$9 \cdot \frac{t^3}{5} + \frac{1}{\sqrt{1}} \cdot \frac{t^{-2}}{-2} + C =$

$3t^3 - \frac{1}{2\sqrt{1}} \cdot t^{-2} \quad C =$

$\boxed{3t^3 - \frac{1}{2t^2\sqrt{1}} + C}$

$*$ A Derivada seria,

$3t^3 - \frac{1}{2t^2\sqrt{1}} + \frac{C}{dt} = 9t^2 + \frac{1}{\sqrt{t^3}}$ //

ii) $\int (ax^4 + bx^3 + 3c) \, dx$

$\int (ax^4 + bx^3 + 3c) \, dx =$

$\int (ax^4) \, dx + \int (bx^3) \, dx + \int (3c) \, dx =$

$a \int (x^4) \, dx + b \int (x^3) \, dx + 3c \int dx =$

$\boxed{a \, (x^5)/5 + b \, (x^4)/4 + 3cx + d}$

iii) $\int \left(\sqrt{2y} - \frac{1}{\sqrt{2y}} \right) dy$

$I = \int \sqrt{2y} \int \sqrt{y} \, dy - \frac{1}{2} \int \frac{1}{\sqrt{y}} \, dy$

$I = \sqrt{2} \cdot \frac{2}{3} \cdot \sqrt{y}^3 - \frac{2}{\sqrt{2}} \cdot \sqrt{y} + c$

$\boxed{I = \frac{2\sqrt{2}}{3} \cdot \sqrt{y}^3 - \sqrt{2y} + c}$

iv) $\int \frac{x^5 + 2x^2 - 1}{x^4} \, dx$

$\int \left(\frac{x^5}{x^4} + \frac{2x^2}{x^4} + \frac{(-1)}{x^4} \right) dx$

$\int (x^5 \cdot x^{-4} + 2x^2 \cdot x^{-4} - 1 \cdot x^{-4}) \, dx$

$\int (x + 2x^{-2} - x^{-4}) \, dx$

* integrando

$\left(\frac{x^{1+1}}{1+1} + \frac{2x^{-2+1}}{-2+1} - \frac{x^{-4+1}}{-4+1} \right)$

$\left(\frac{x^2}{2} + \frac{2x^{-1}}{-1} - \frac{x^{-3}}{-3} \right)$

$\left(\frac{x^2}{2} - \frac{2}{x} + \frac{1}{3x^3} + c \right)$

▶ Resposta:

$\boxed{\int \frac{x^5 + 2x^2 - 1}{x^4} \, dx = \frac{x^2}{2} - \frac{2}{x} + \frac{1}{3x^3} + c}$

V) $\int \left(\frac{e^t}{2} + \sqrt{t} + \frac{1}{t} \right) dt$

VI) $\int \frac{8x^4 - 9x^3 + 6x^2 - 2x + 1}{x^2} \, dx$

$\int (8x^4) \frac{dx}{x^2} - \int (9x^3) \frac{dx}{x^2} + \int (6x^2) \frac{dx}{x^2} - \int (2x) \frac{dx}{x^2} + \int (1) \frac{dx}{x^2} =$

$\int (8x^2) \, dx - \int (9x) \, dx + \int (6) \, dx - \int (2) \cdot \frac{dx}{x} + \int (1) \frac{dx}{x^2} =$

$\int (8x^2) \, dx - \int (9x) \, dx + \int (6) \, dx - \int (2x^{-1}) \, dx + \int (x^{-2}) \, dx =$

$$\boxed{\frac{8x^3}{3} - \frac{9x^2}{2} + 6x - 2 \ln (x) - \frac{1}{x} + c}$$

VII) $\int \cos \theta \cdot tg\,\theta \; d\theta$

VIII) $\int tg^2 x \; \cosec^2 x \; dx.$

Questão 2

$$f(x) = x^{\frac{2}{3}} + x$$

* primitivando

$$F(x) = \int x^{\frac{2}{3}} + x \cdot dx = \frac{x^{\frac{2}{3}+1}}{\frac{2}{3}+1} + \frac{x^{1+1}}{1+1}$$

$$= \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + \frac{x^2}{2}$$

$$= \frac{3x^{\frac{5}{3}}}{5} + \frac{x^2}{2}$$

$$\boxed{F(x) = \frac{3\sqrt[3]{x^5}}{5} + \frac{x^2}{2} + C}$$

$$F(1) = 1$$

logo,

$$1 = \frac{3\sqrt[3]{1^5}}{5} + \frac{1^2}{2} + C$$

$$1 = \frac{3}{5} + \frac{1}{2} + C$$

$$1 = \frac{11}{10} + C$$

$$1 - \frac{11}{10} = C$$

$$\frac{-1}{10} = C$$

ENTÃO   A FUNÇÃO PROCURADA É

$$F(x) = \frac{3\sqrt[3]{x^5}}{5} + \frac{x^2}{2} - \frac{1}{10}$$

Questão 3.

$$\int f(x) \, dx = x^2 + \frac{1}{2} \cos 2x + C$$

* Derivando

$$y = x^2 + \frac{1}{2} \cos 2x + C$$

$$y' = 2x + \frac{1}{2} \cdot (- \sin 2x \cdot 2)$$

$$y' = 2x - \sin 2x$$

* Verificando

$$\int 2x - \sin 2x \, dx$$

$$2 \cdot \frac{x^2}{2} - \frac{1}{2} \cdot (- \cos 2x) + C$$

$$x^2 + \frac{1}{2} \cos 2x + C$$

Questão 4

i) $\int (2x^2 + 2x - 3)^{10} \, (2x + 1) \, dx$

* substituição

$$2x^2 + 2x - 3 = u$$

$\Rightarrow (4x + 2) \, dx = du$

$\Rightarrow 2 \cdot (2x + 1) \, dx = du$

$\Rightarrow (2x + 1) \, dx = \frac{1}{2} \, du$

* substituindo na integral

$$I = \int u^{10} \cdot \frac{1}{2} \, du$$

$$I = \frac{1}{2} \int u^{10} \, du$$

$$I = \frac{1}{2} \cdot \frac{u^{10+1}}{10+1} + C$$

$$I = \frac{1}{2} \cdot \frac{u^{11}}{11} + C$$

$$I = \frac{u^{11}}{22} + C$$

continuação

$$I = \frac{1}{22} \cdot (2x^2 + 2x - 3)^{11} + C$$

$$\Rightarrow \boxed{\int (2x^2 + 2x - 3)^{10} \, (2x + 1) \, dx = \frac{1}{22} \cdot (2x^2 + 2x - 3)^{11} + C}$$

ii) $\int 5x \sqrt{4-3x^2}\, dx$

* Mudanças de variavel.

$u = 4 - 3x^2 \implies du = -6x\, dx \implies x\, dx = -\dfrac{du}{6}$

* Mantendo a consta.

$$\int 5x\sqrt{4-3x^2} = 5\int \underbrace{\sqrt{4-3x^2}}_{=\sqrt{u}}\ \underbrace{x\, dx}_{=\frac{du}{6}} = -\frac{5}{6}\int \sqrt{u}\, du.$$

* Como $\sqrt{u} = u^{\frac{1}{2}}$, a integral simples.

$$\int u^{\frac{1}{2}}\, du = \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2}{3}\cdot u^{\frac{3}{2}} + C, \quad \text{com } C \in \mathbb{R}$$

Portanto,

$$-\frac{5}{6}\int \sqrt{u}\, du = -\frac{5}{6}\cdot\left(\frac{2}{3}u^{\frac{3}{2}} + C\right) = \frac{-10}{18}u^{\frac{3}{2}} - \frac{5}{6}C = -\frac{5}{6}u^{\frac{3}{2}} + K,$$

com $K = -\frac{5}{6}C$

$$\boxed{\int 5x\sqrt{4-3x^2}\, dx = -\frac{5}{9}(4-3x^2)^{\frac{3}{2}} + K, \qquad \text{com } K \in \mathbb{R}}$$

iii) $\int (e^{2t}+2)^{\frac{1}{3}}\, e^{2t}\, dt$

$e^{2t} + 2 = u$

* Ficamos:

$u = e^{2t} + 2$

$\dfrac{du}{dt} = 2\cdot e^{2t} + 0$

$du = 2e^{2t}\, dt$

$\dfrac{du}{2} = e^{2t}\, dt$

* Portanto,

$\int u^{\frac{1}{3}}\, \dfrac{du}{2}$

$\dfrac{1}{2}\int u^{1/3}\, du$

*continuando

► integral de uma potencia

$\int u^{(\frac{1}{3})}\, du = u^{(1/3 + 1)\ /\ (1/3 + 1)}$

$= u^{(1/3 + 3/3)\ /\ (1/3 + 3/3)}$

$= u^{\frac{4}{3}/\frac{4}{3}}$

* Então,

$\dfrac{1}{2} u^{(4/3)/(4/3)}$

$\dfrac{1}{2} u^{(4/3)\cdot(3/4)}$

$\dfrac{1}{2}\cdot\dfrac{3}{4}\cdot u^{4/3}$

$\dfrac{3}{8}\cdot u^{4/3}$

*continuando

► Agora so substituir

$\dfrac{3}{8}\cdot\left(e^{2t}+2\right)^{4/3} + K$

IV) $\int \operatorname{sen}^4 x \cos x \, dx$

$\int u^4 \cdot \cos x \cdot \dfrac{du}{\cos x}$

$\int u^4 \cdot du = \dfrac{u^5}{5} + C = \dfrac{\operatorname{sen}^5 x}{5} + C$

V) $\int \dfrac{\operatorname{SEN} x}{\cos^5 x} \cdot dx$

VI) $\int e^x \cos 2e^x \, dx$

* variavel $y = 2e^x$, vamos de...

$\to dy = \dfrac{d}{dx} \cdot (2e^x)$

$\to dy = 2e^x \, dx$

Portanto, a integral é:

$\to \int e^x \cos(2e^x) \, dx = \int \dfrac{1}{2} \cdot (2e^x) \cos(2e^x) \, dx$

$\to \int e^x \cos(2e^x) \, dx = \dfrac{1}{2} \int \cos y \, dy$

$\to \int e^x \cos(2e^x) \, dx = \dfrac{1}{2}(\sin y + c)$

$\to \boxed{\int e^x \cos(2e^x) \, dx = \dfrac{1}{2}\sin(2e^x) + C}$