Co-Simulation-Test Case: Math 003

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1 Test Description

The test case was designed by C. Clauß from Fraunhofer IIS EAS in Dresden.

1.1 Mathematical Equations

$$x_1 = \begin{cases} 0 & t < 1 \quad \text{or} \quad 2 \leq t < 5 \\ 1 & else \end{cases} \tag{1}$$

$$x_2 = \begin{cases} 0 & t < 3 \quad \text{or} \quad 4 \leq t < 6 \\ 1 & else \end{cases} \tag{2}$$

$$x_3 = \begin{cases} 3 & x_1 = 1 \quad \text{and} \quad x_2 < 0.01 \quad \text{and} \quad x_4 < 2.5 \\ -3 & x_1 < 0.001 \quad \text{and} \quad x_2 > 0 \quad \text{and} \quad x_4 > -2.5 \\ 0 & else \end{cases} \tag{3}$$

$$\dot{x}_4 = 2x_3 \tag{4}$$

Note, in the conditions for variable x_3 the tests for the digital signals are not done with tests for 0 or 1, but instead using tests that allow for small variations in variable x_1 and x_2. This is important for Newton-based algorithms that construct Jacobian matrixes based on difference-quotient approximations, hereby adding small offsets to the variables. With the more relaxed tests, the model is more robust for such solution methods.

1.2 Requested solution

Solution for variables x_1, x_2, x_3, x_4 is to be obtained for the time interval $t \in [0, T]$, $T = 10$.

1.3 Expected results

The problem has an exact solution, shown in Figure 1.

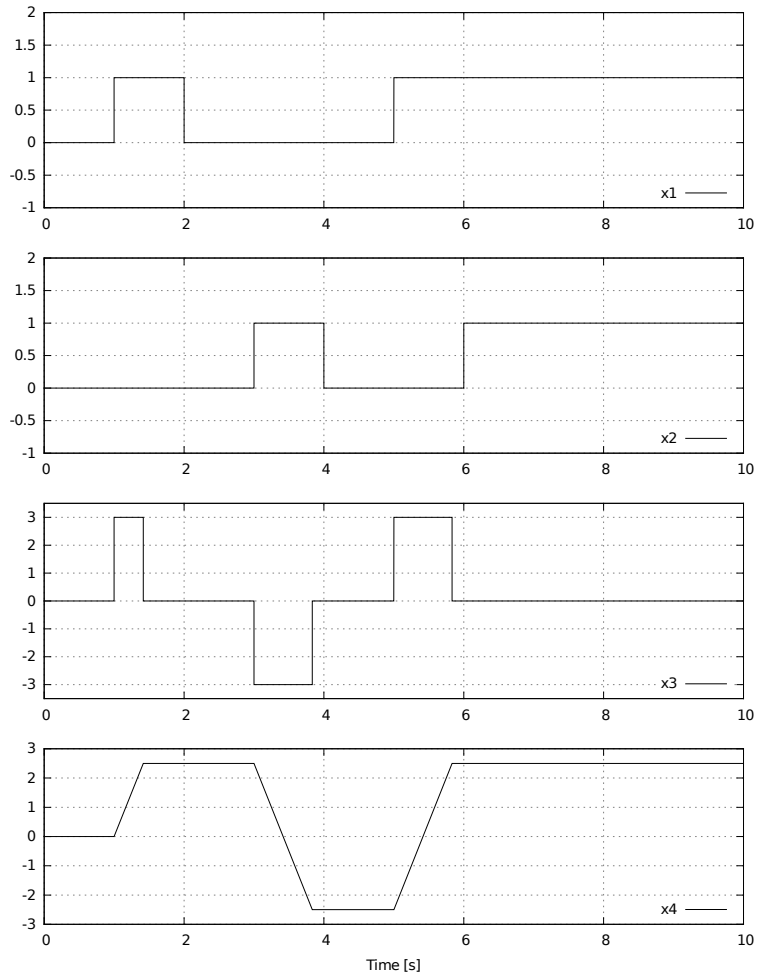

Abbildung 1: Exact solution

The state events occur at $t = 1 + 2.5/6 = 1.4166667$, $t = 3 + 5/6 = 3.833333$ and $t = 5 + 5/6 = 5.833333$.

t	x1	x2	x3	x4
0	0	0	0	0
0.99999999	0	0	0	0
1	1	0	3	0
1.416666657	1	0	3	2.5
1.41667	1	0	0	2.5
1.99999999	1	0	0	2.5
2	0	0	0	2.5
2.99999999	0	0	0	2.5
3	0	1	-3	2.5
3.83333333	0	1	-3	-2.5
3.83334	0	1	0	-2.5
3.99999999	0	1	0	-2.5
4	0	0	0	-2.5
4.99999999	0	0	0	-2.5
5	1	0	3	-2.5
5.83333333	1	0	3	2.5
5.83334	1	0	0	2.5
5.99999999	1	0	0	2.5
6	1	1	0	2.5
10	1	1	0	2.5

Abbildung 2: Data table with exact solution

1.4 Reference Modelica Model

The equations can be solved directly in a coupled manner with Modelica. The source code for a fully coupled solution in Modelica[1] is given below in Listing 1.

```
within ;
model Math_003 "Math_003.mo"
    Real x1;
    Real x2;
    Real x3;
    Real x4;
    initial equation
        x4 = 0;
    equation
        x1 = if ((time < 1) or (time >= 2 and time < 5)) then 0
             else 1;
        x2 = if ((time < 3) or (time >= 4 and time < 6)) then 0
             else 1;
        x3 = noEvent(
                if (x1 > 0 and x2 <= 0.01 and x4 < 2.5) then 3
                elseif (x1 <= 0.001 and x2 > 0 and x4 > -2.5) then -3
                else 0
            );
        der(x4) = 2*x3;
    annotation(
        experiment(
            StopTime=10,
            StartTime=0,
            Interval= 0.01, Tolerance = 1e-06)
        );
end Math_003;
```

Listing 1: Modelica Code Listing

2 Co-Simulation

2.1 Decomposition

For the purpose of testing Co-Simulation masters the test case is split into three parts.

Part	Cycle	Input	Equations	Output
1	1	–	Equations (1) and (2)	x_1, x_2
2	2	x_1, x_2, x_4	Equation (3)	x_3
3	2	x_3	Equation (4)	x_4

Only part 2 and 3 are coupled (in a cycle).

2.2 Evaluation Order

Cycle 1 shall be evaluated first. For non-iterative co-simulation master algorithms, part 2 shall be evaluated before part 3.

[1] Note, the `noEvent()` clause around the condition for variable x_3 is needed for OpenModelica, which otherwise gets stuck in resolving the state events. For other simulators (e.g. SimulationX 3.6 or newer) this is not needed.