Tomography reconstruction from 2D projections

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1 New results

As a reminder, for the last meeting, the sinogram has been derived for the unit disk only. Then, the idea was, using formulae relating the radon transform operator for a function f and its scaled and translated variations f_r and $f_{\mathbf{a}}$ respectively, to derive sinograms for any disk. Here are presented a few results for simple examples.

1.1 Scaled and centered disk

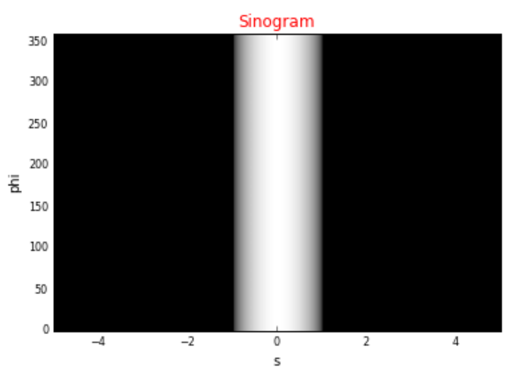

Figure 1: Unit disk

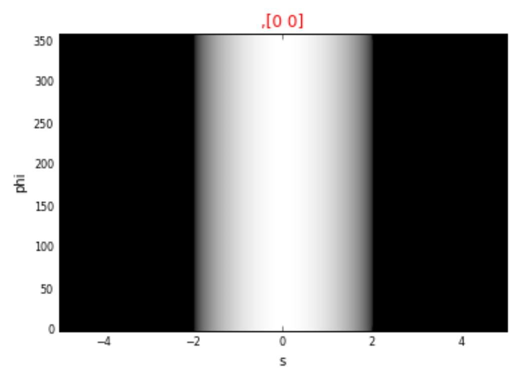

Figure 2: Centered disk, $r = 2$

As expected, the larger is the disk, the larger the sinogram we get, proportionally to the radius.

1.2 Translated unit disk

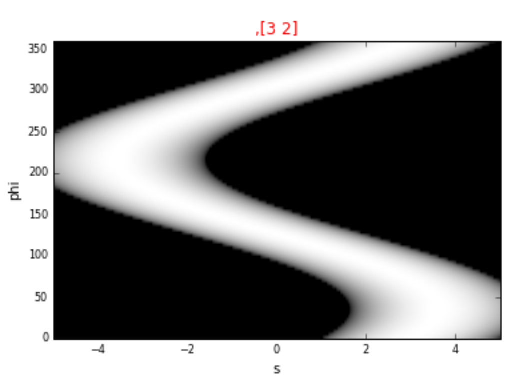

Figure 3: translated disk, $center = (3, 2)$

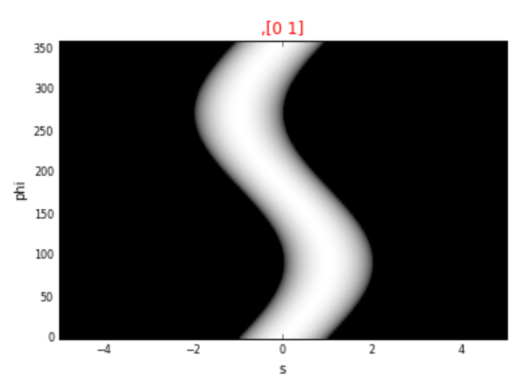

Figure 4: translated disk, $center = (0, 1)$

For a translated disk, the sinogram has a sinusoidal shape. This can be explained by the translation formula of the previous report.
For any translation vector $\mathbf{a} = (a_0, a_1)$, we have :

$$\begin{aligned} \Re f_{\mathbf{a}}(\phi, s) &= \Re f(\phi, s - \mathbf{a}.\alpha_\phi) \\ &= \Re f(\phi, s - a_0 * \cos\phi - a_1 * \sin\phi) \end{aligned}$$

Thus, the translated sinogram is the result of the composition of $\Re(B_1)$ (radon transform on the unit disk) with a sinusoidal function with parameters (a_0, a_1). It means that, the further from the center the translated disk is, the sharper is the sinogram variation.

1.3 Translated and scaled disk

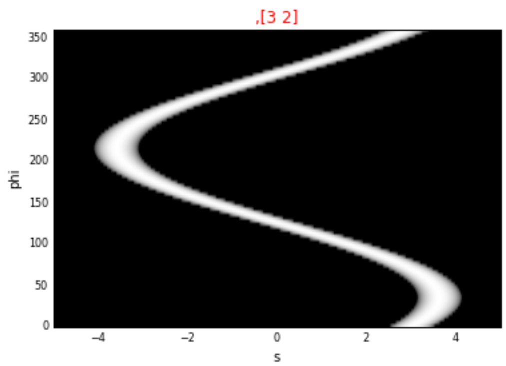

Figure 5: $r = 0.5$, $center = (3, 2)$

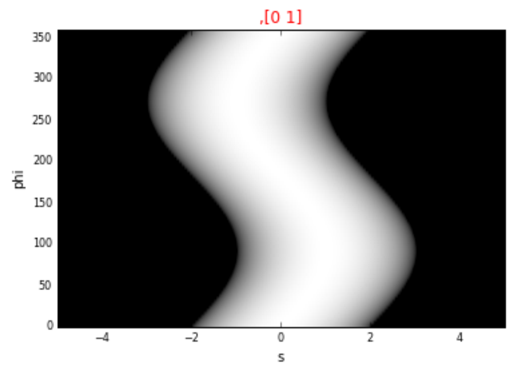

Figure 6: $r = 2$, $center = (0, 1)$

Here, we can clearly see the combination of both previous properties of the radon transform operator $\Re$.

$$\Re f_{\mathbf{a},r}(\phi, s) = r \times \Re f(\phi, \frac{s - \mathbf{a}.\alpha_\phi}{r})$$

Then, one can obviously add several disks to the image, so that the obtained final sinogram would be the sum of each disk's sinogram.

1.4 Fancy results

These are two examples of couples of symmetric disks with respect to the x-axis and the origin respectively.

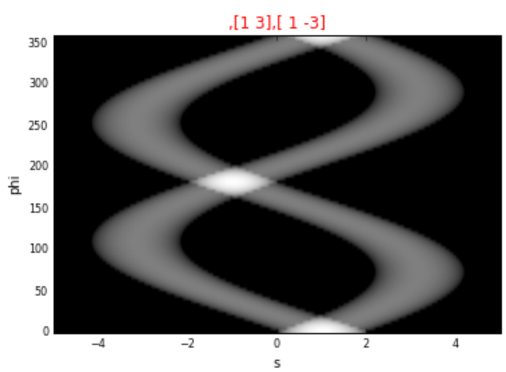

Figure 7

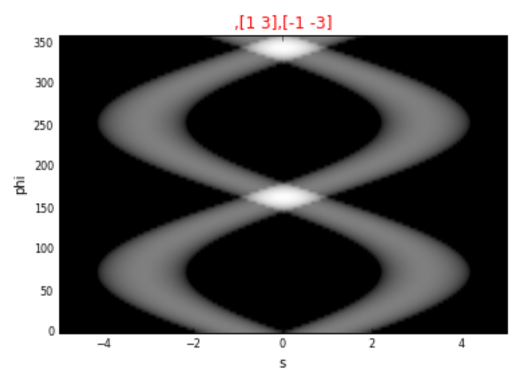

Figure 8