Tomography reconstruction from 2D projections

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21^{th} December 2017

1 New results

As a reminder, for the last meeting, the sinogram has been derived for the unit disk only. Then, the idea was, using formulae relating the radon transform operator for a function f and its scaled and translated variations f_D and f_a respectively, to derive sinograms for any disk. Here are presented a few results for simple examples.

1.1 Scaled and centered disk

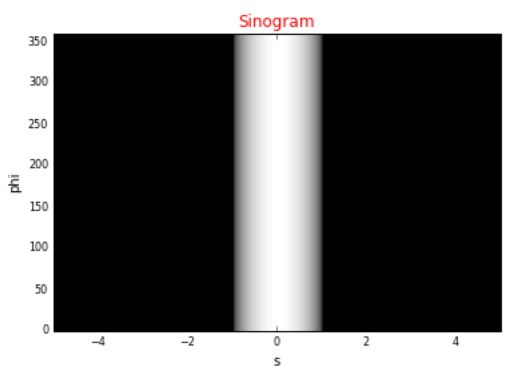

Figure 1: Unit disk

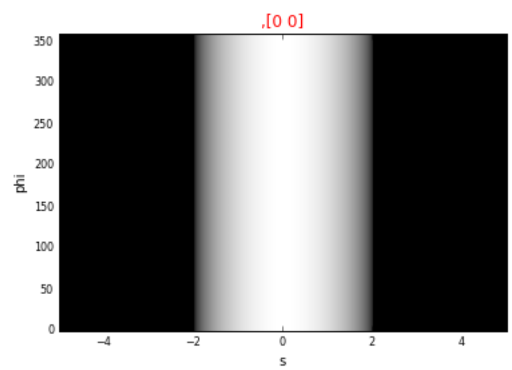

Figure 2: Centered disk, $r = 2$

As expected, the larger is the disk, the larger the sinogram we get, proportionally to the radius.

1.2 Translated unit disk

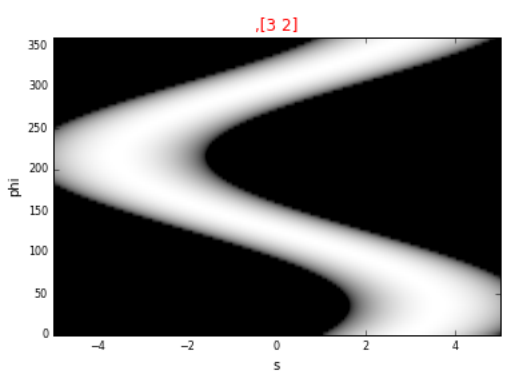

Figure 3: translated disk, $center = (3, 2)$

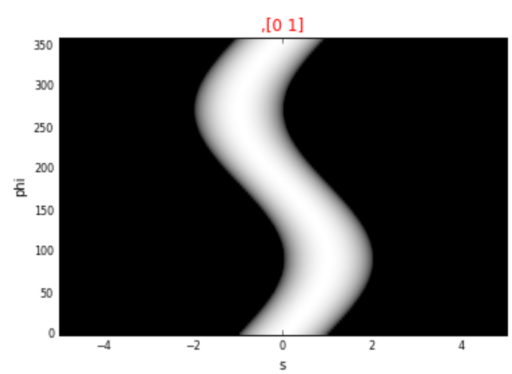

Figure 4: Centered disk, $r = 2$

For a translated disk, the sinogram has a sinusoidal shape. This can be explained by the translation formula of the previous report.
For any translation vector a, we have :

$$\begin{aligned}\Re f_a(\phi, s) &= \Re f(\phi, s - a.\alpha_\phi) \\ &= \Re f(\phi, s - a_0 * \cos\phi - a_1 * \sin\phi)\end{aligned}$$

Thus, the translated sinogram is the result of the composition of $\Re(B_1)$ (radon transform on the unit disk) with a sinusoidal function with parameters (a_0, a_1). It means that, the further from the center the translated disk is, the sharper is the sinogram.

1.3 Translated and scaled disk

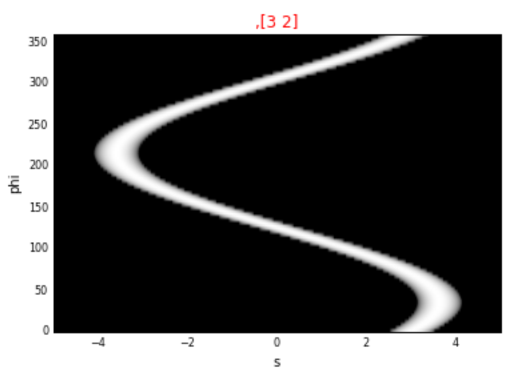

Figure 5: $r = 0.5$, $center = (3, 2)$

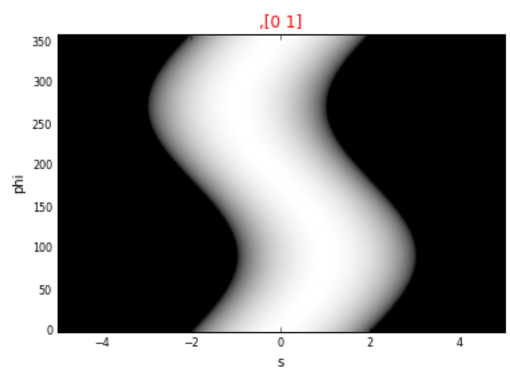

Figure 6: $r = 2$, $center = (0, 1)$

Here, we can clearly see the combination of both previous properties of the radon transform operator $\Re$.

Then, one can obviously add several disks to the image, so that the obtained final sinogram would be the sum of each disk's sinogram.

1.4 Fancy results

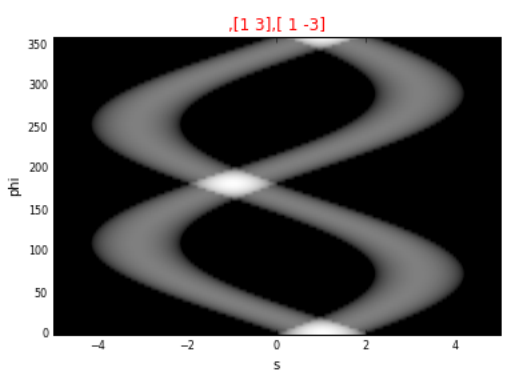

Figure 7

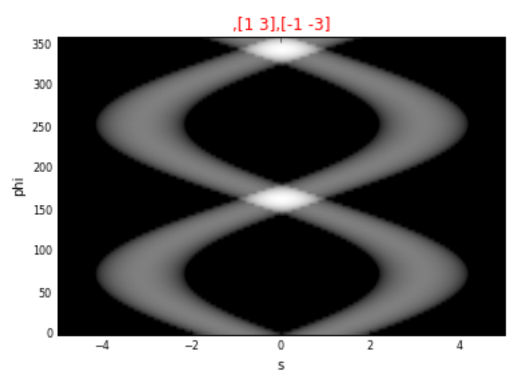

Figure 8

These are two examples of couples of symmetric disks with respect to the x-axis and the origin respectively.