Data Consistency Conditions in Image Reconstruction from Projections

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Context

2D parallel projections:

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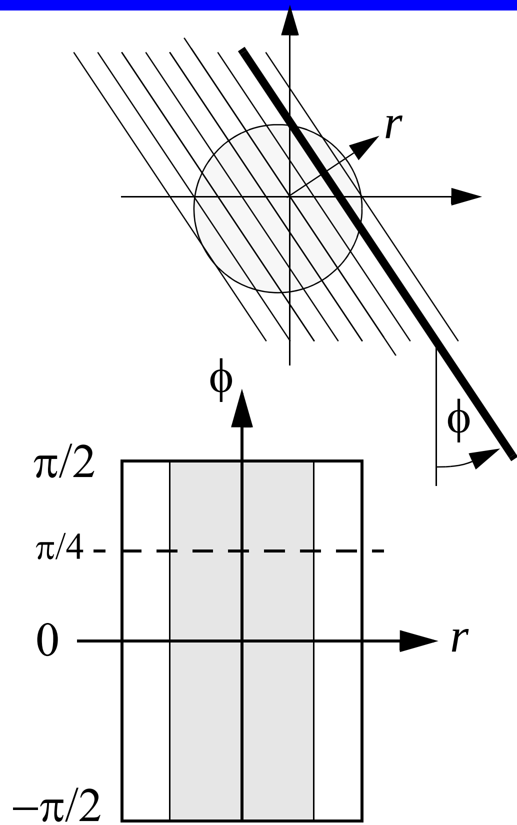


$p(\phi, \cdot)$ - parallel projection at angle ϕ

Context

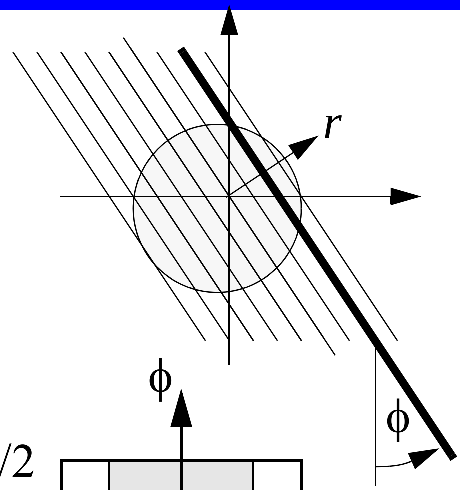
2D parallel projections:

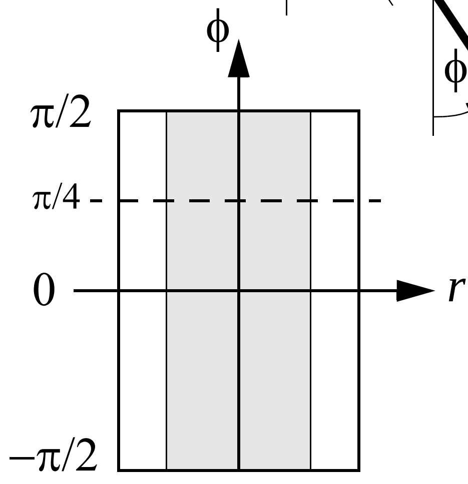
$$p(\phi, r) = \int f(r\alpha + s\beta)ds$$

$$\phi \in [-\pi/2, \pi/2]$$

$$\alpha = (\cos\phi, \sin\phi)$$

$$\beta = (-\sin\phi, \cos\phi)$$





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For short.... $p = \boldsymbol{R}f$

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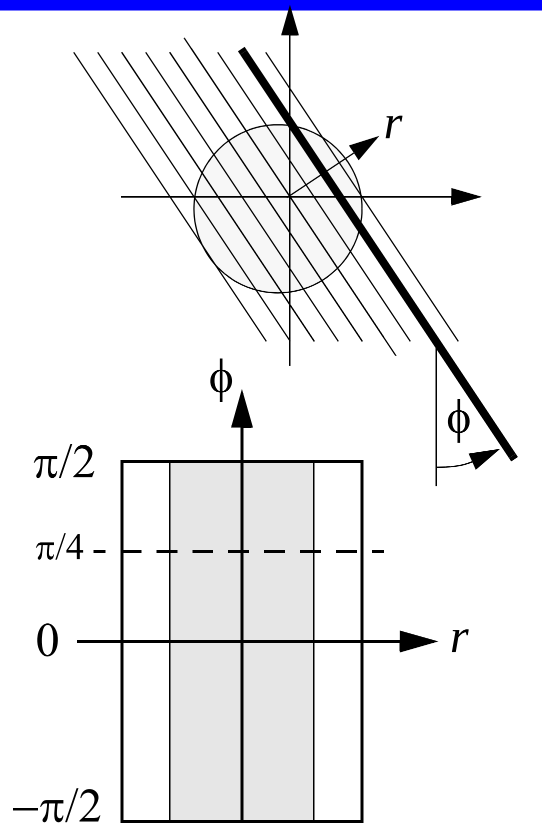

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We are interested in **truncated** parallel projections.

Data Consistency Conditions (DCC)

Helgason-Ludwig DCC

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If $p = \boldsymbol{R}f$ then $M_n(\phi) = a_0 U^n + a_1 U^{n-1} V + \ldots a_n V^n$

where $U = \cos\phi$ and $V = \sin\phi$

New DCC

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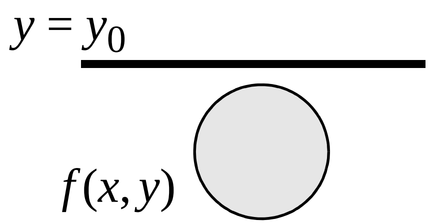
Take some line outside the target object, w.l.o.g. the line $y = y_0$ with $y_0 > 0$.



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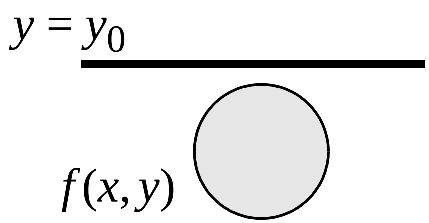

Let $B_0(x) = \int_0^{\pi} p(\phi, r^*) \dfrac{1}{\cos\phi}\, d\phi$, where $r^* = x\cos\phi + y_0 \sin\phi$.

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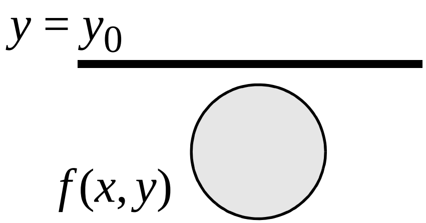

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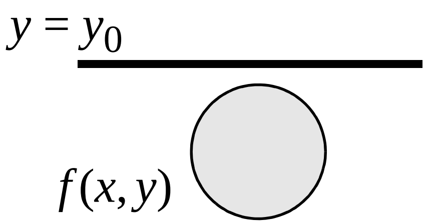

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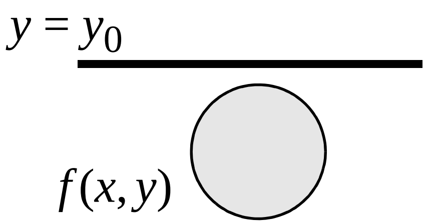

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If $p = \boldsymbol{R} f$ then $B_n(x) = b_0 + b_1 x + b_2 x^2 + \ldots + b_n x^n$

Comparing Helgason-Ludwig DCC and the new DCC

$$M_n(\phi) = \int p(\phi, r)\ r^n\ dr$$

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(limited-angle OK)

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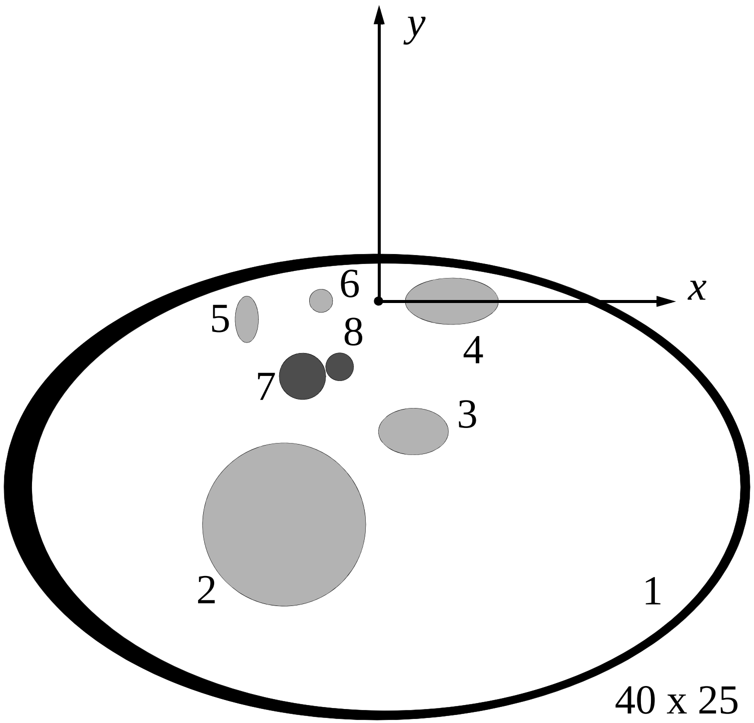
Simulations



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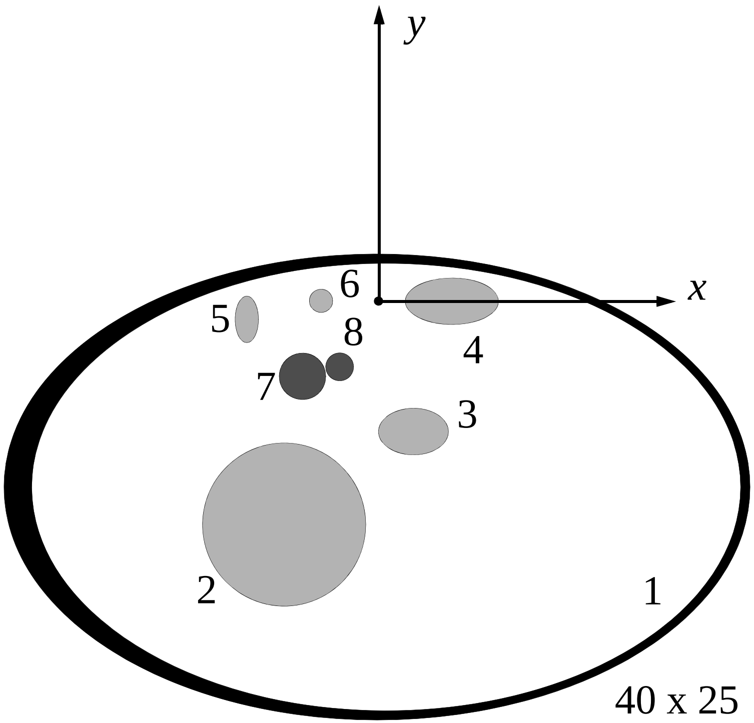

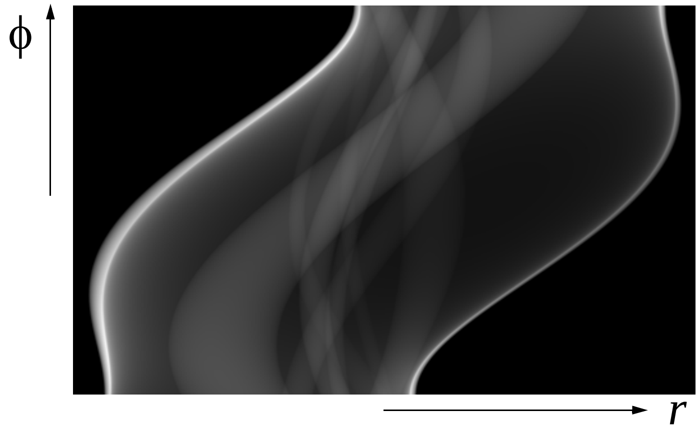

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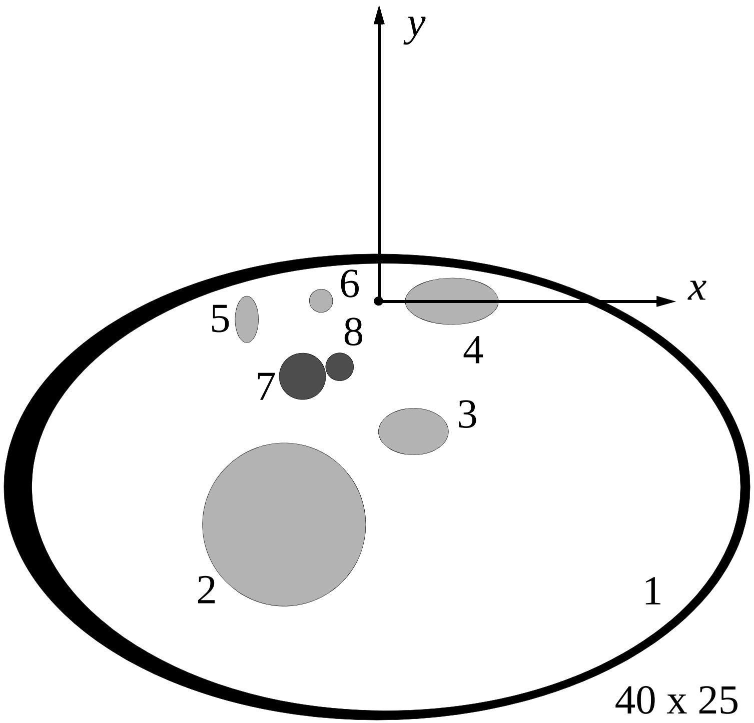

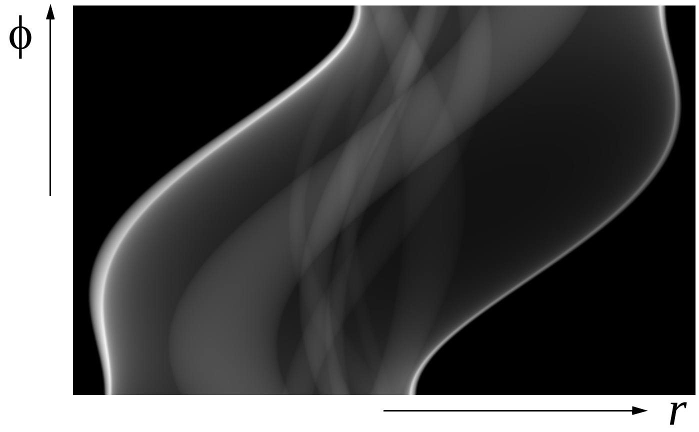

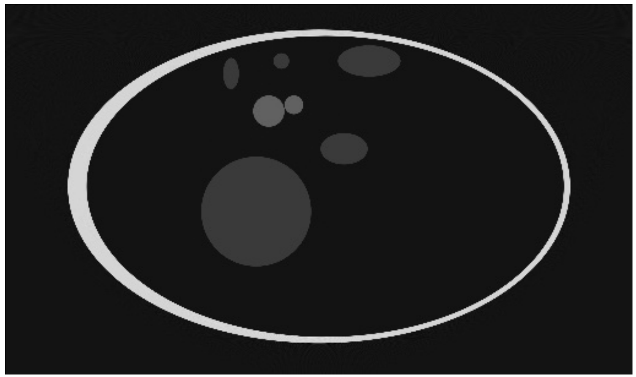

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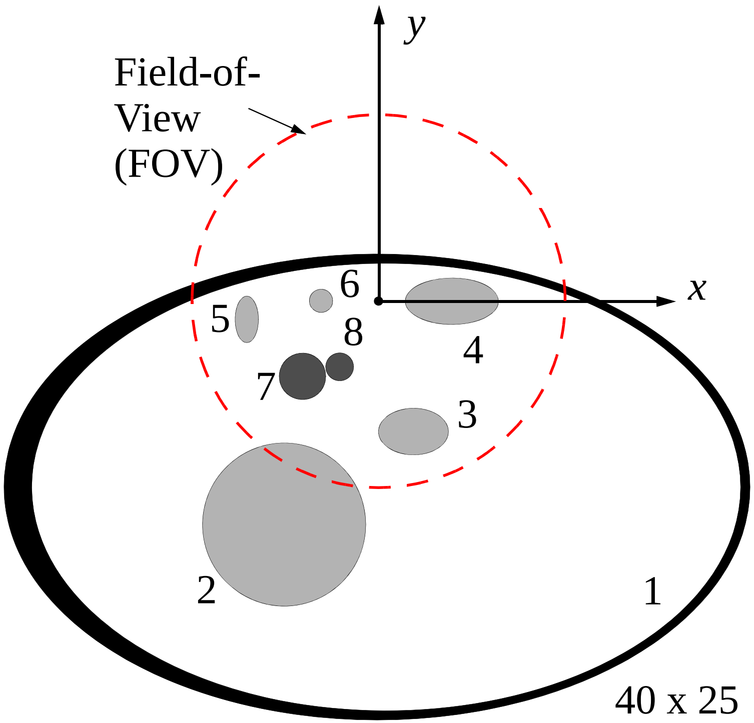

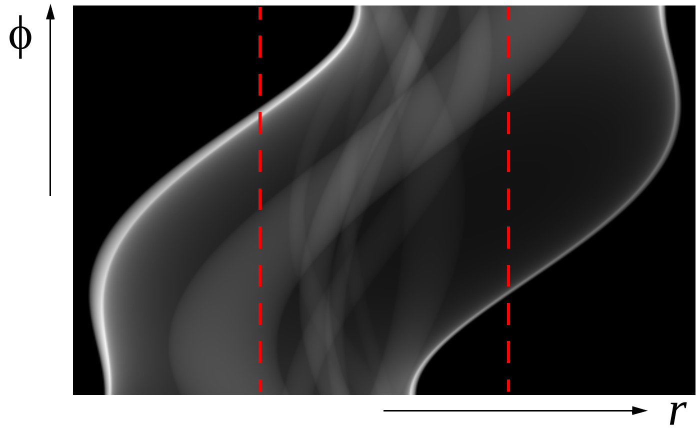

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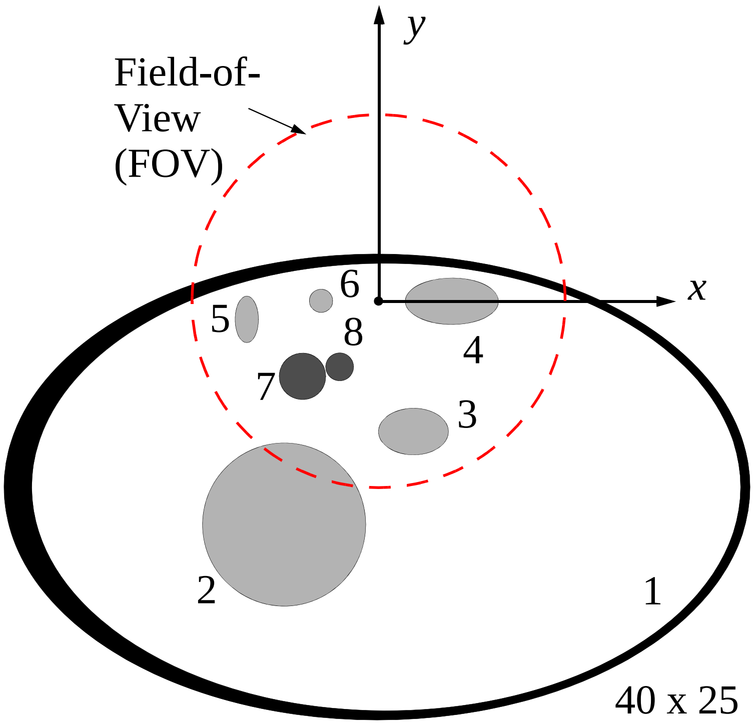

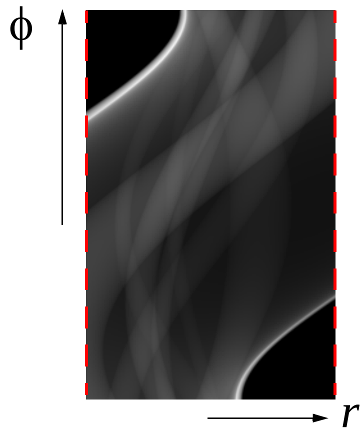

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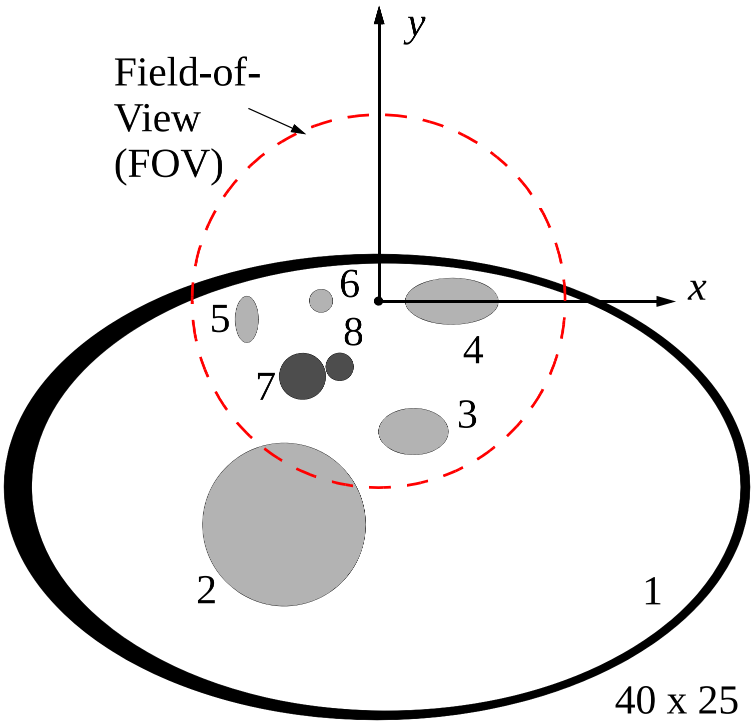

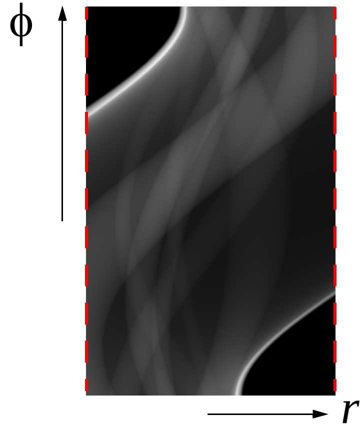

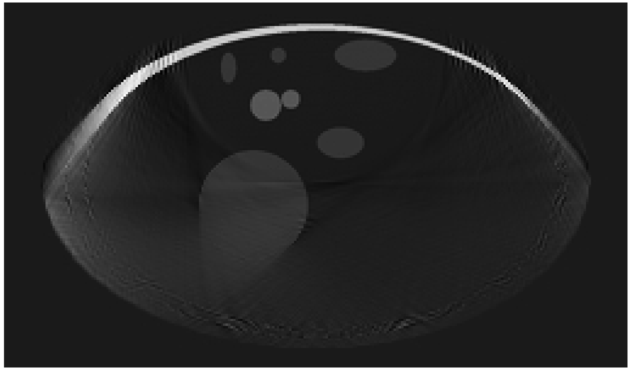

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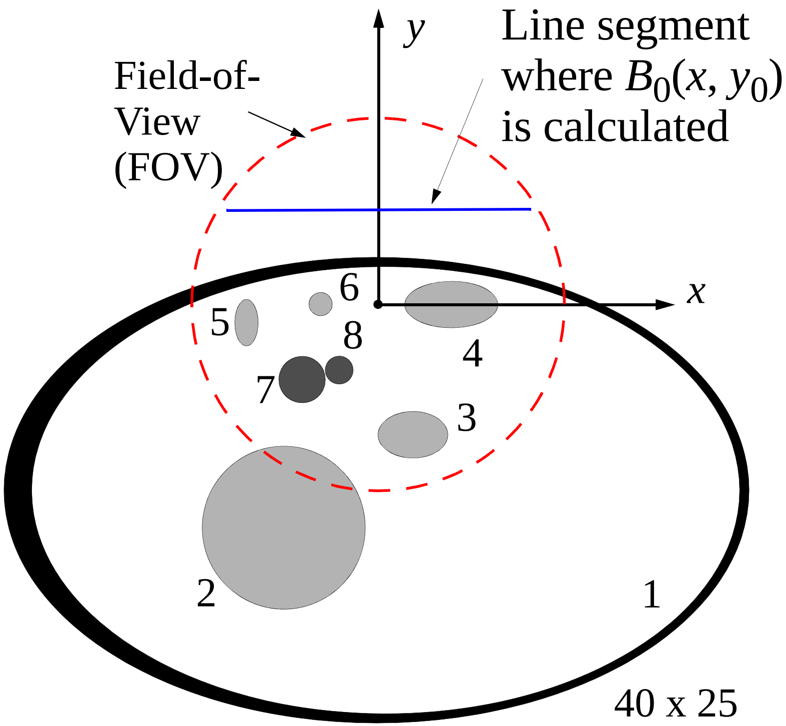

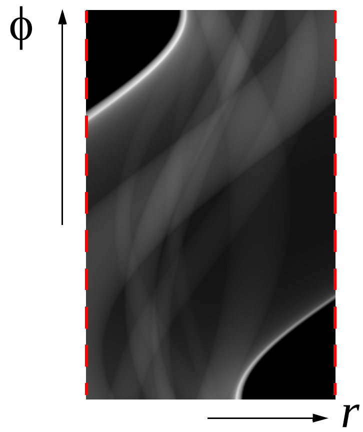

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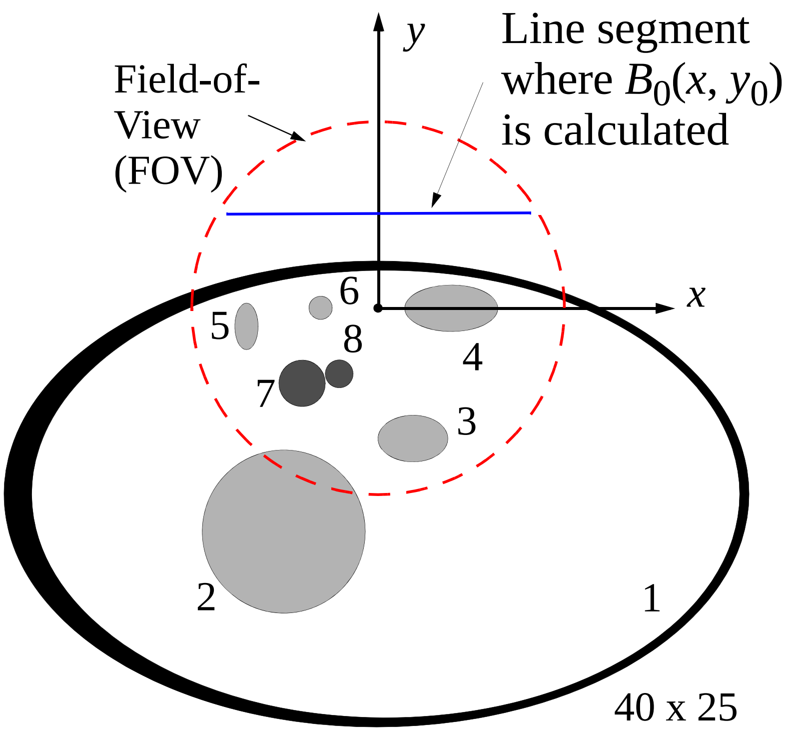

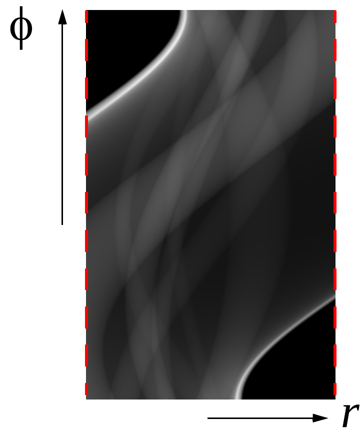

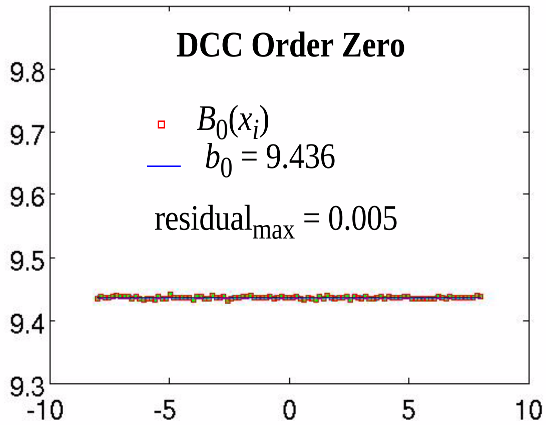

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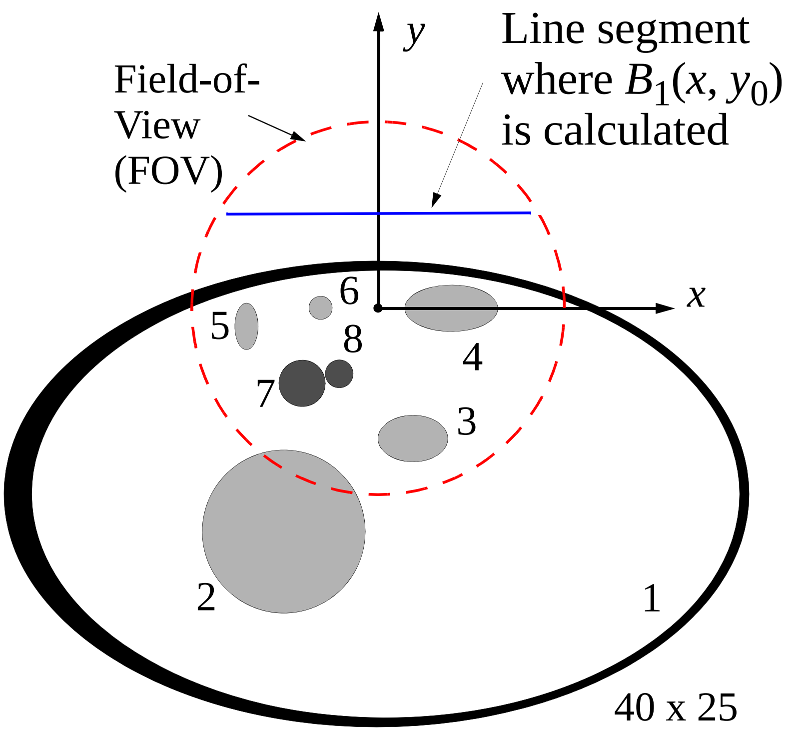

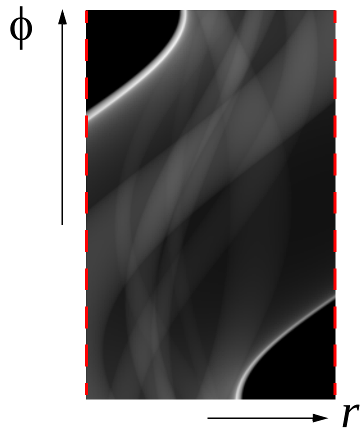

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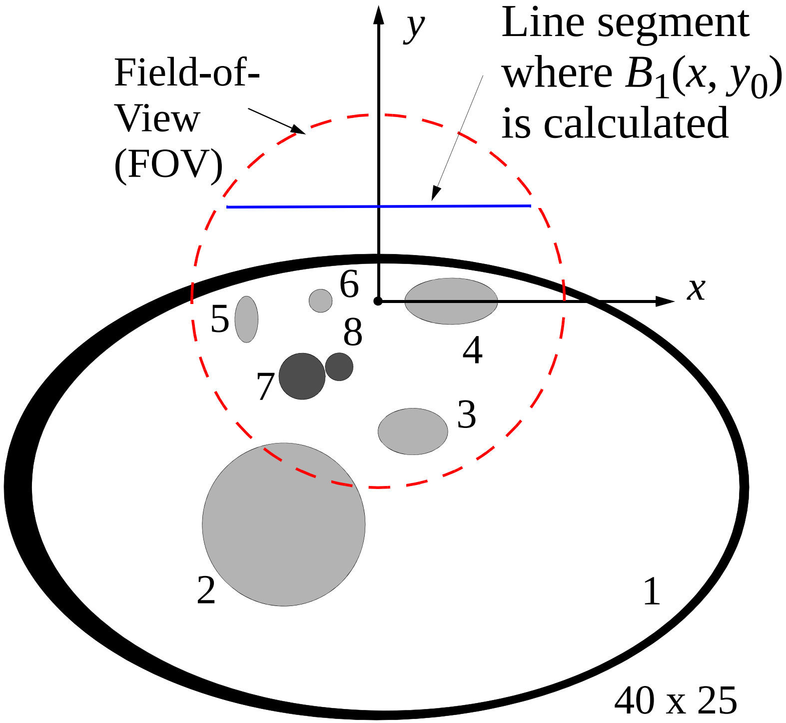

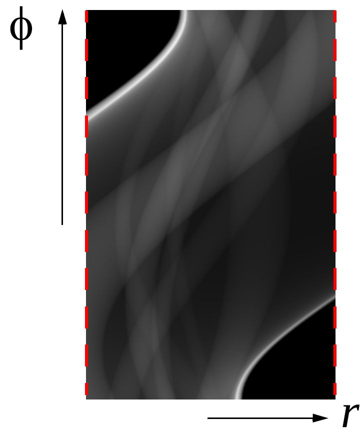

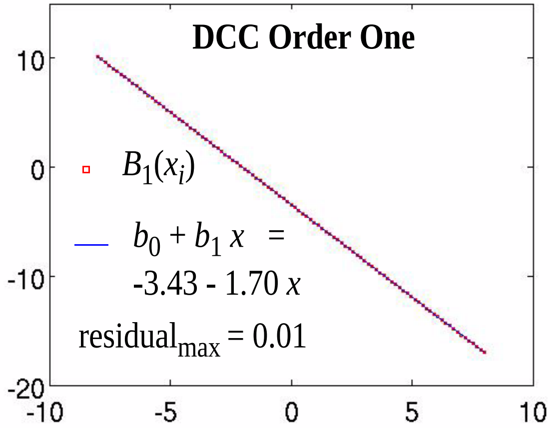

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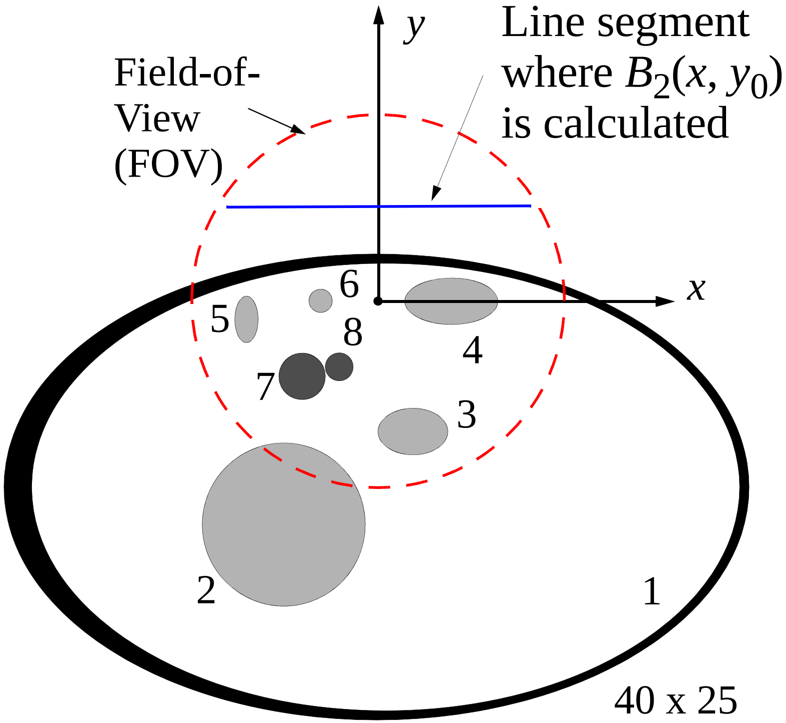

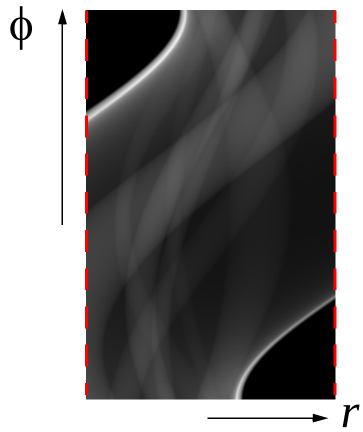

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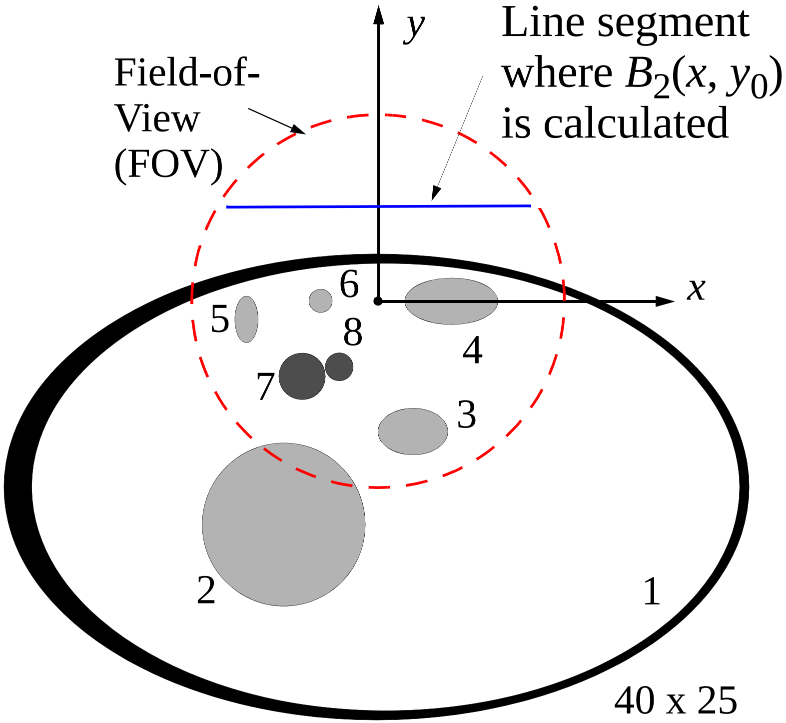

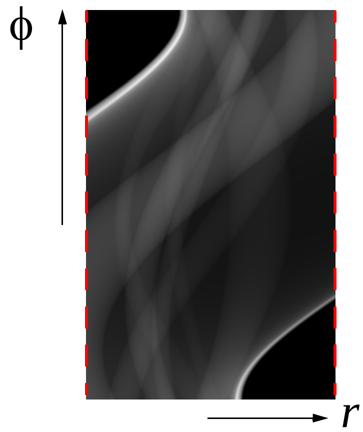

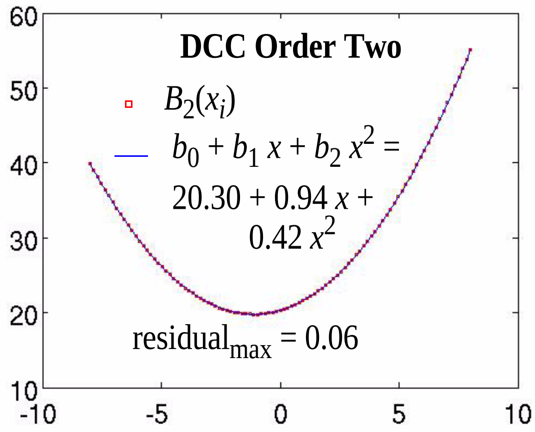

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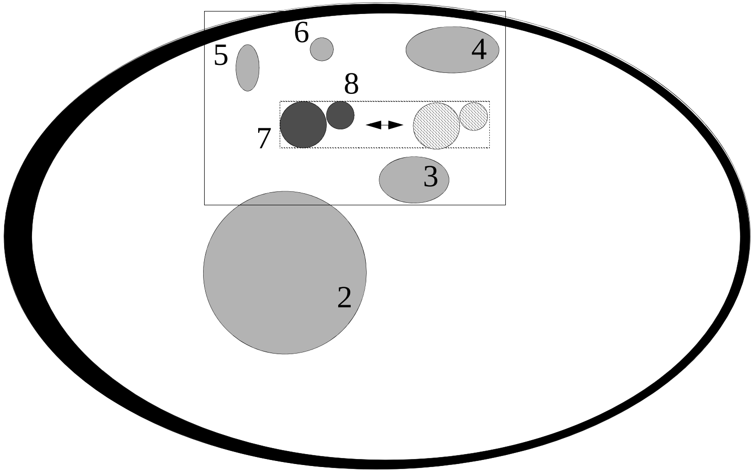

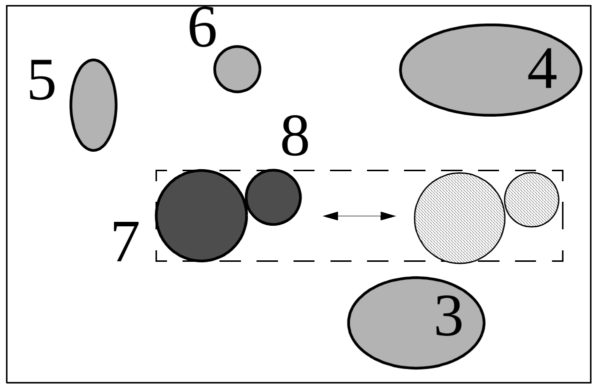

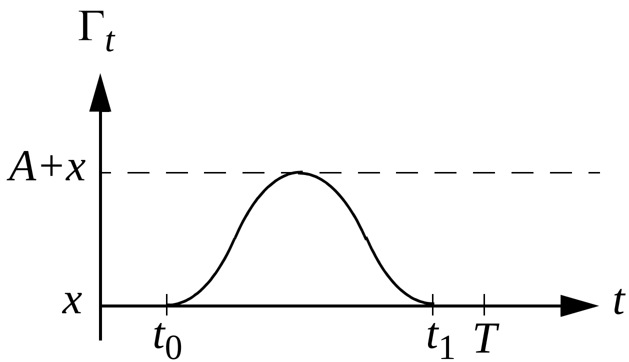

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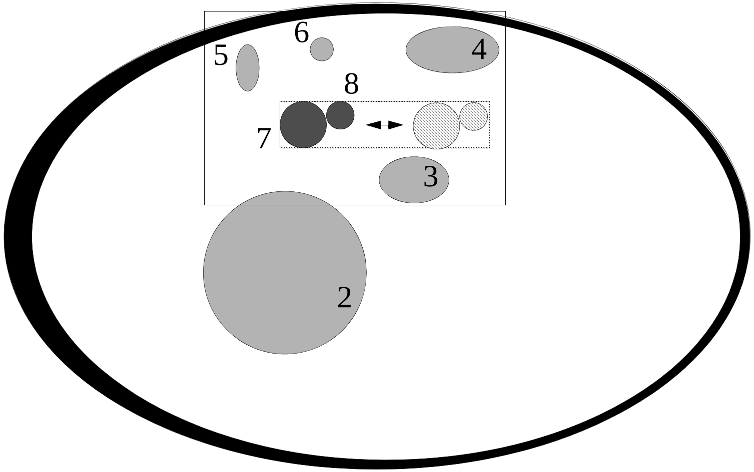
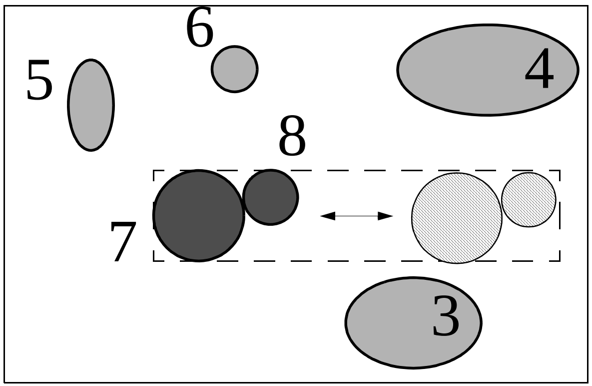
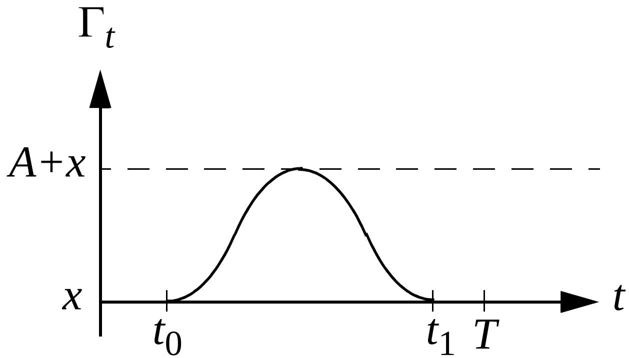
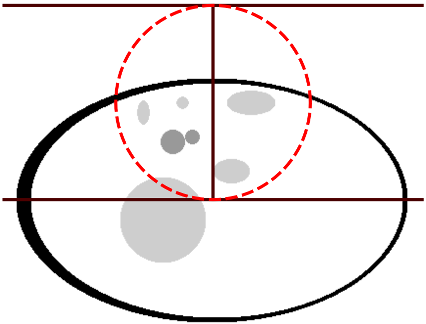


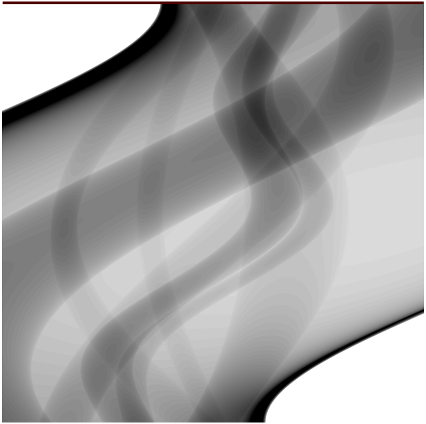




$$T = 18, \quad (t_0\,,\,t_1) = (2\,,\,17), \;\; A = 7$$

Dynamic phantom and parallel geometry





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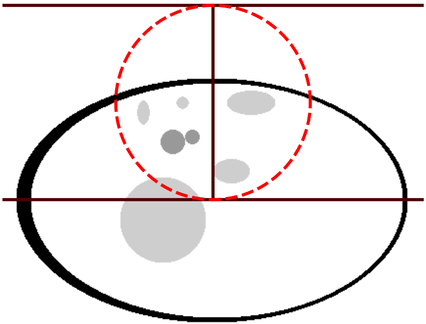

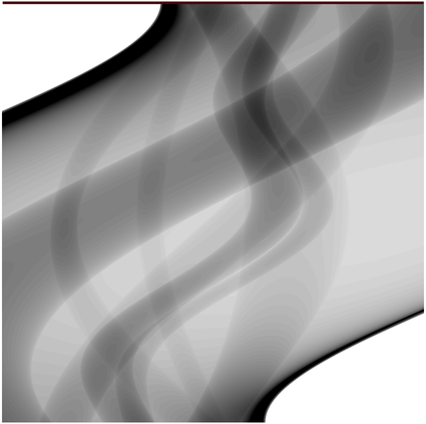

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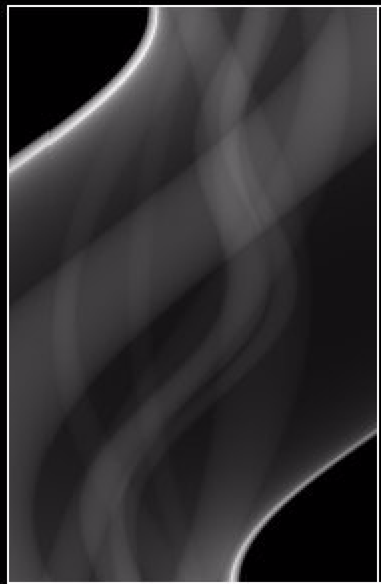

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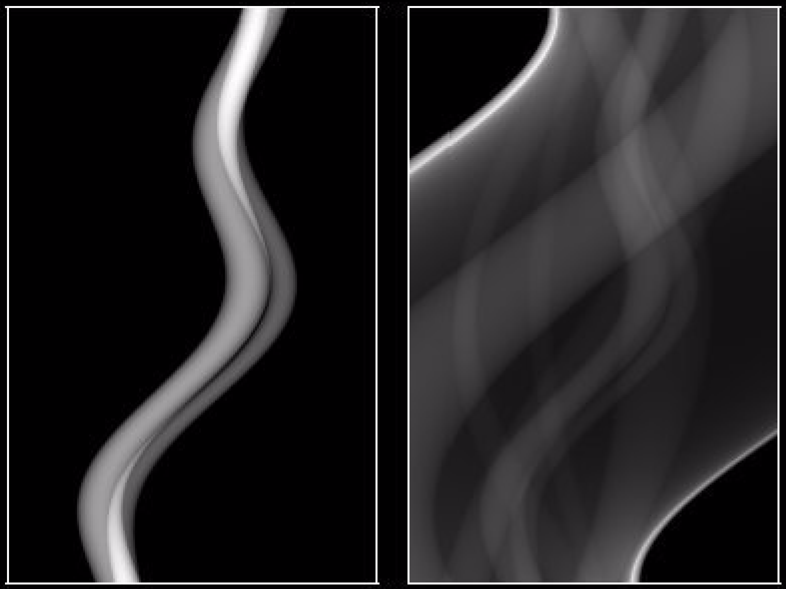

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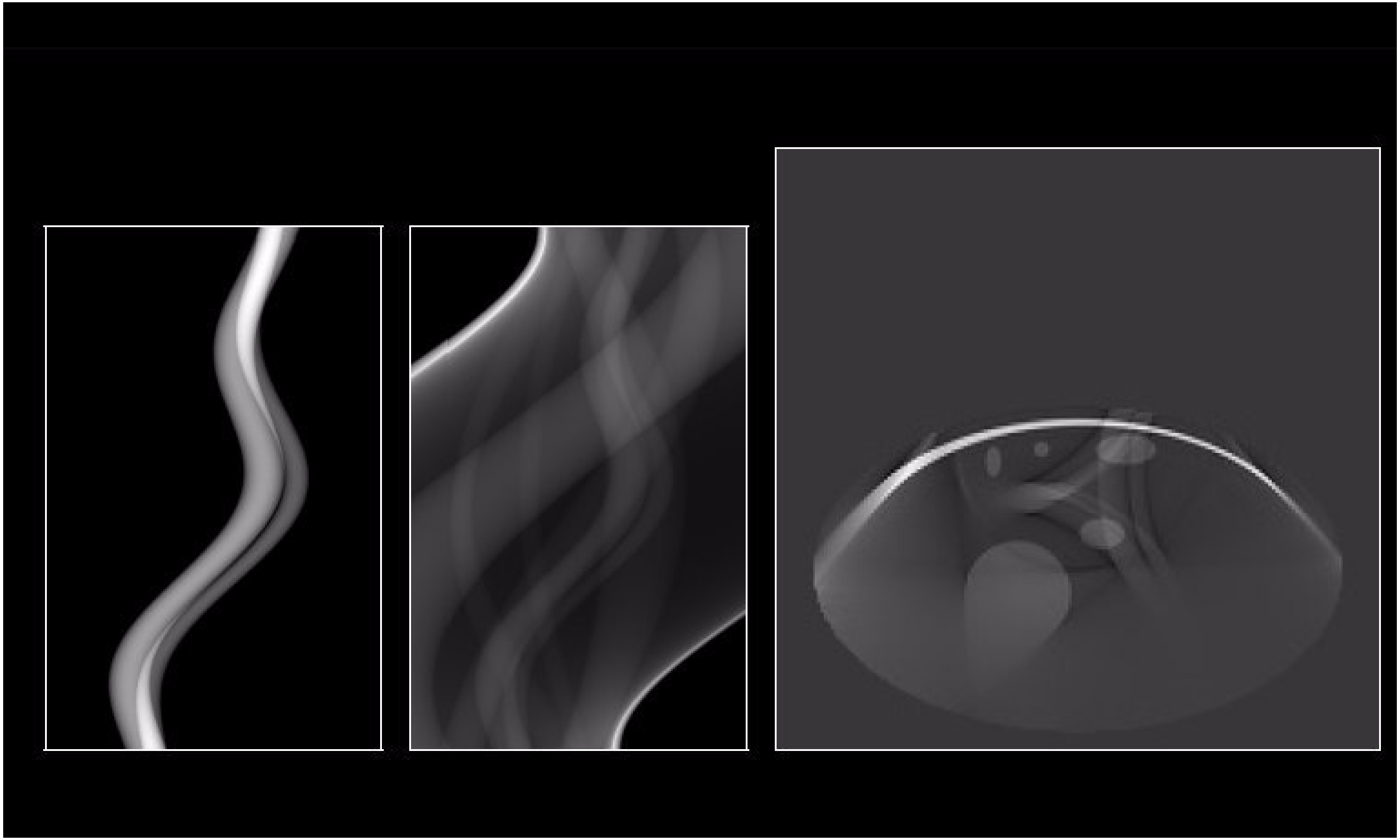

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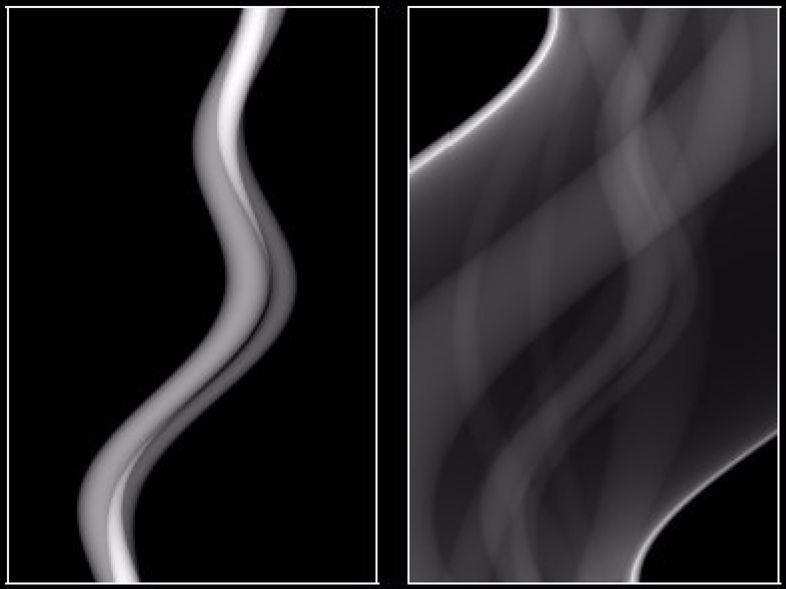

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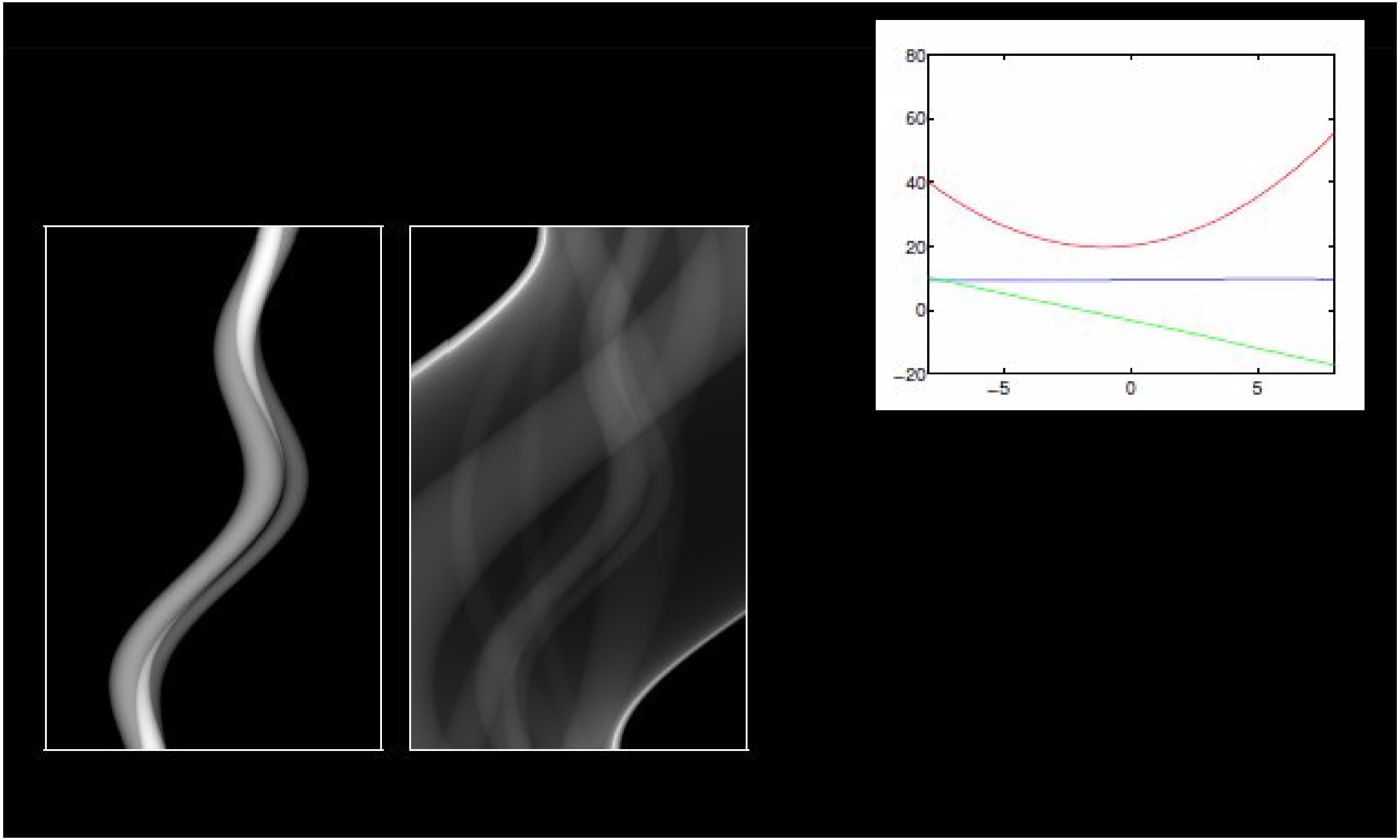

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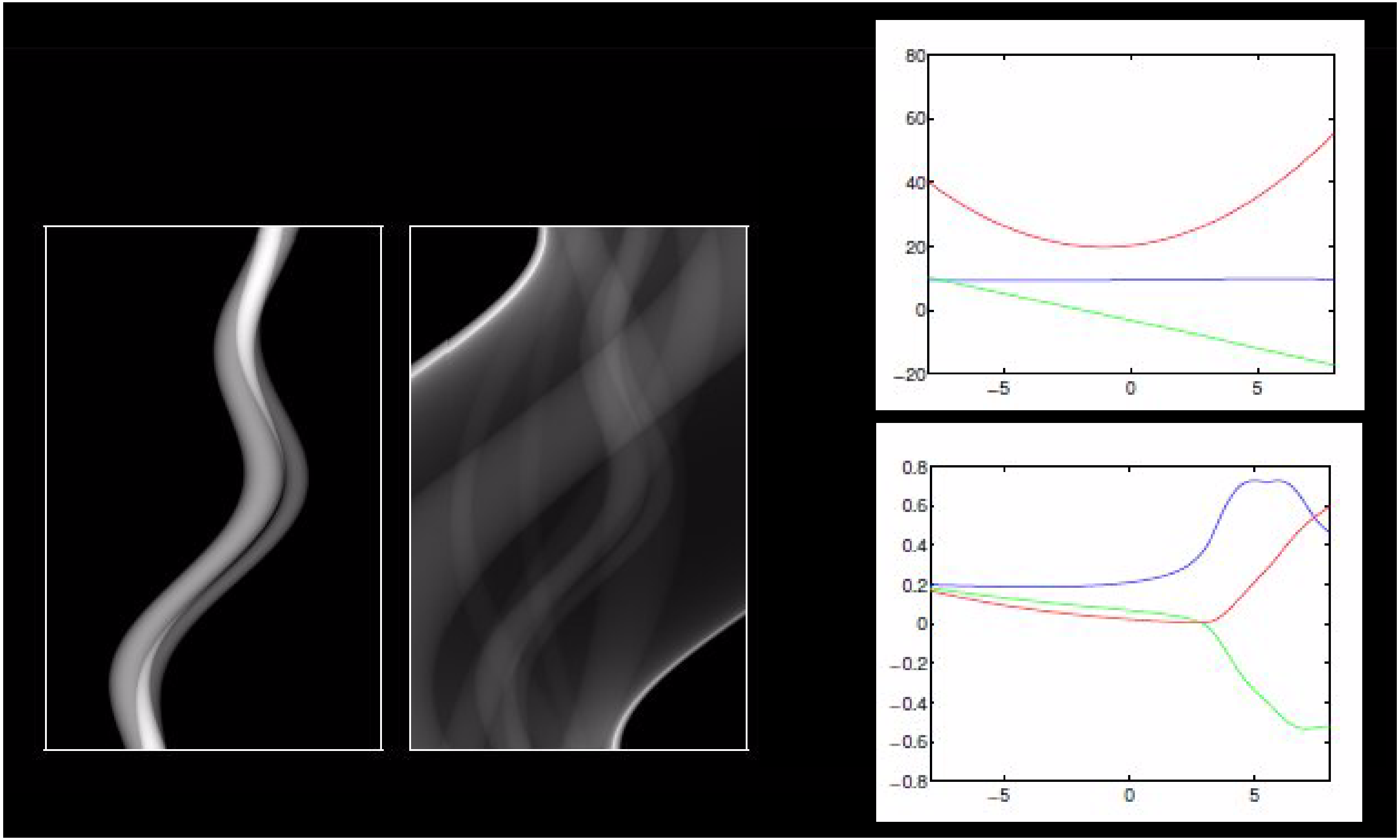

Simulations

Search for the (non-linear) parameters (t_0, t_1) and A such that the DCC are "best satisfied" (are polynomials of the correct degree), when the moving component is subtracted from the data.

(The alternative, not using DCC, is to solve for (t_0, t_1), A, and $f(x, y)$ - which combines a small non-linear problem with a huge linear problem.)

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The non-linear estimation found $(\tilde{t}_0, \tilde{t}_1) = (1.98, 16.99)$ and $\tilde{A} = 7.03$

(The true values were $(t_0^*, t_1^*) = (2, 17)$ and $A^* = 7$)

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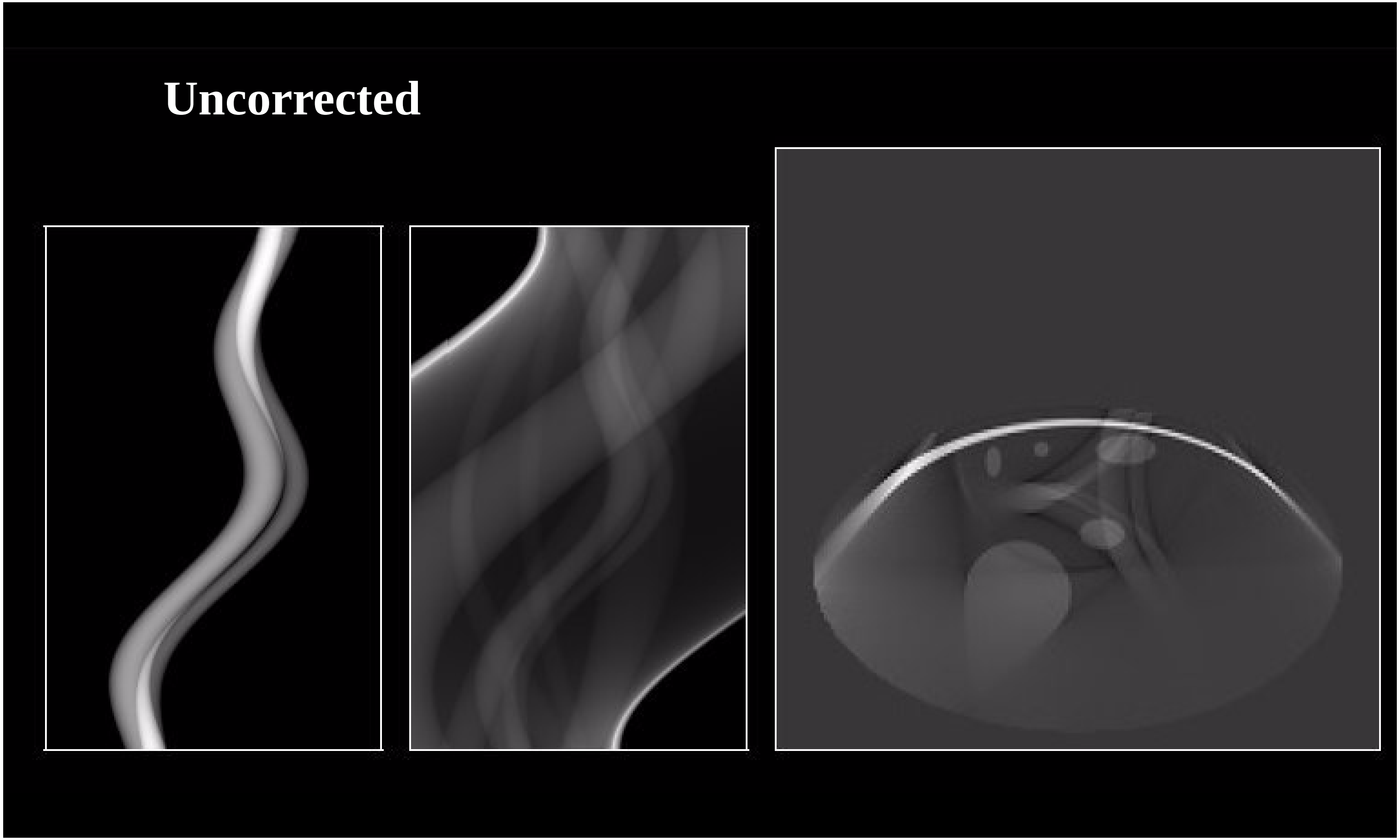
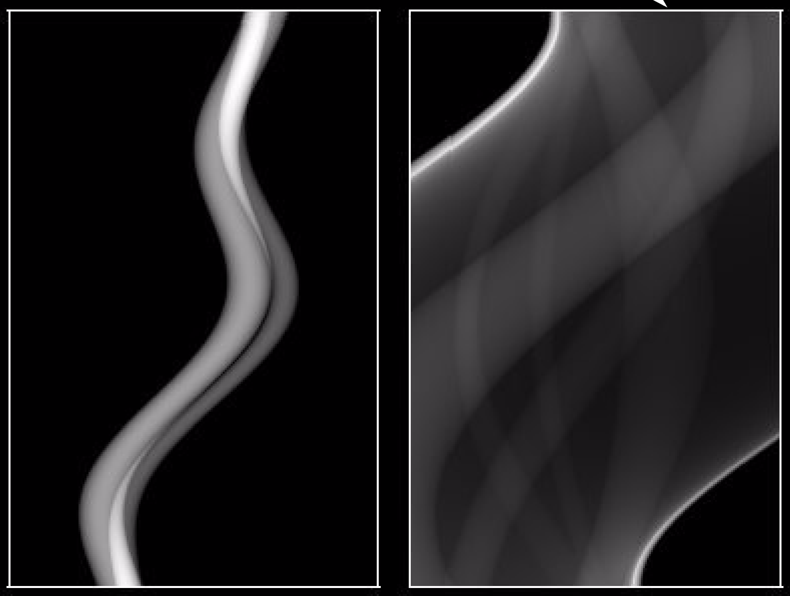


Simulations

Inconsistent component subtracted



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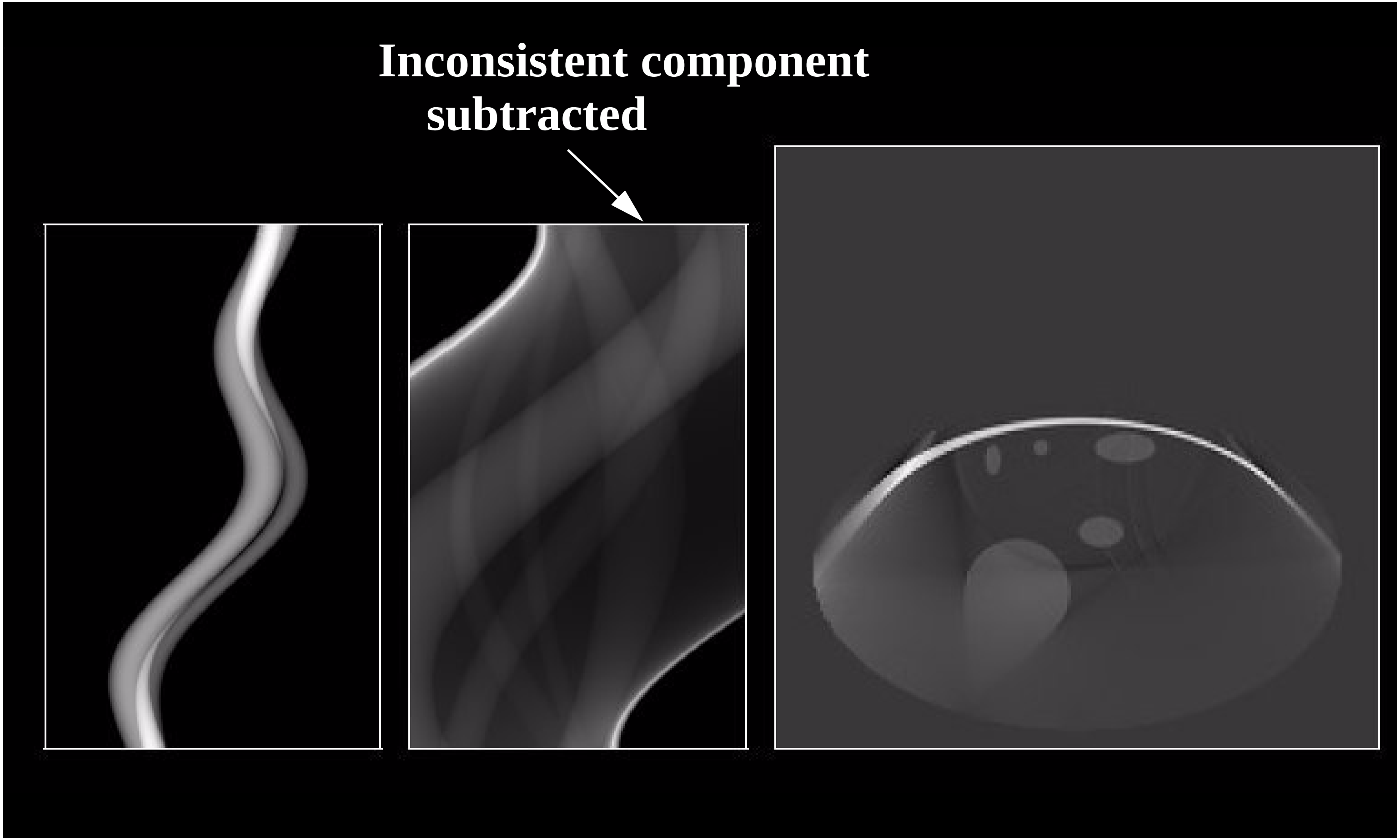
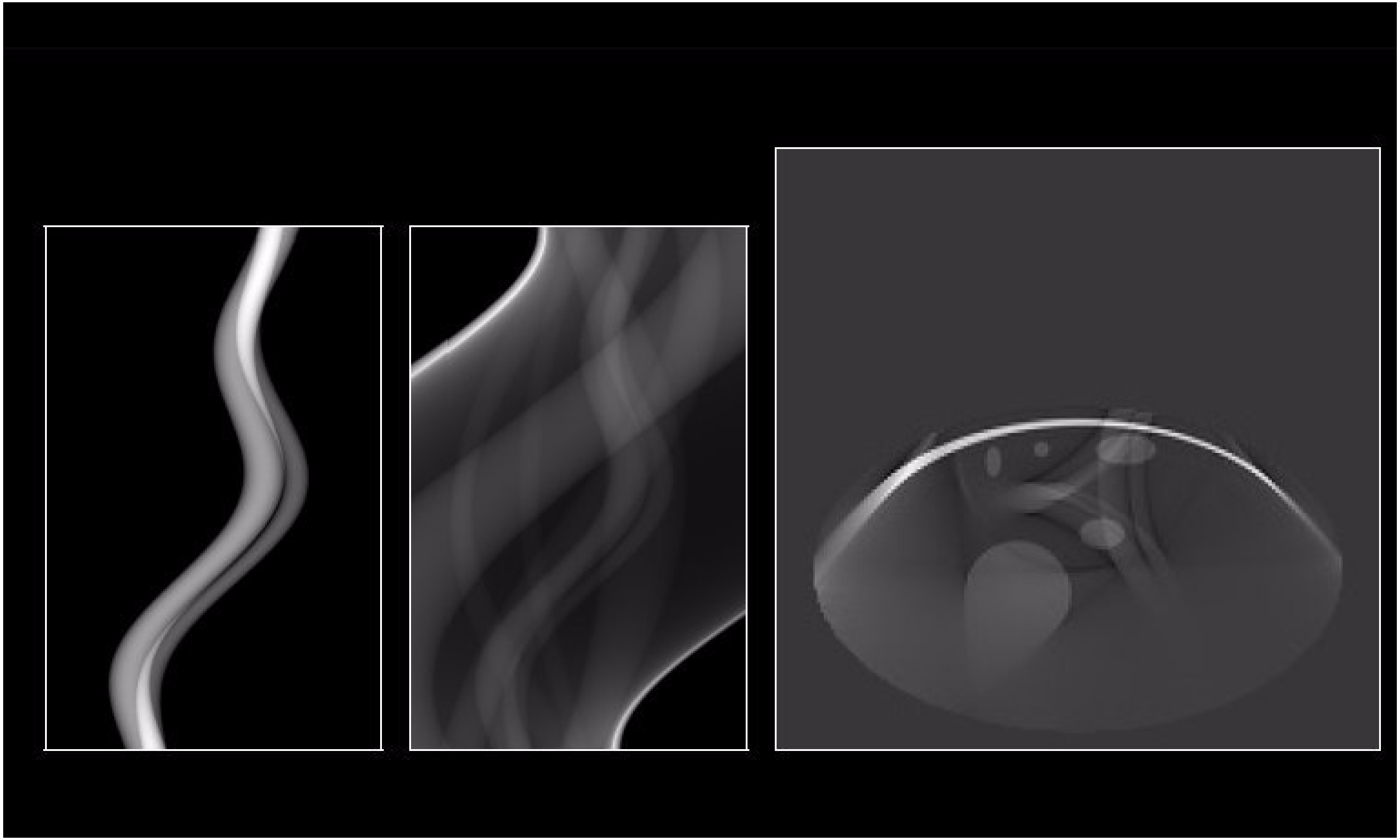
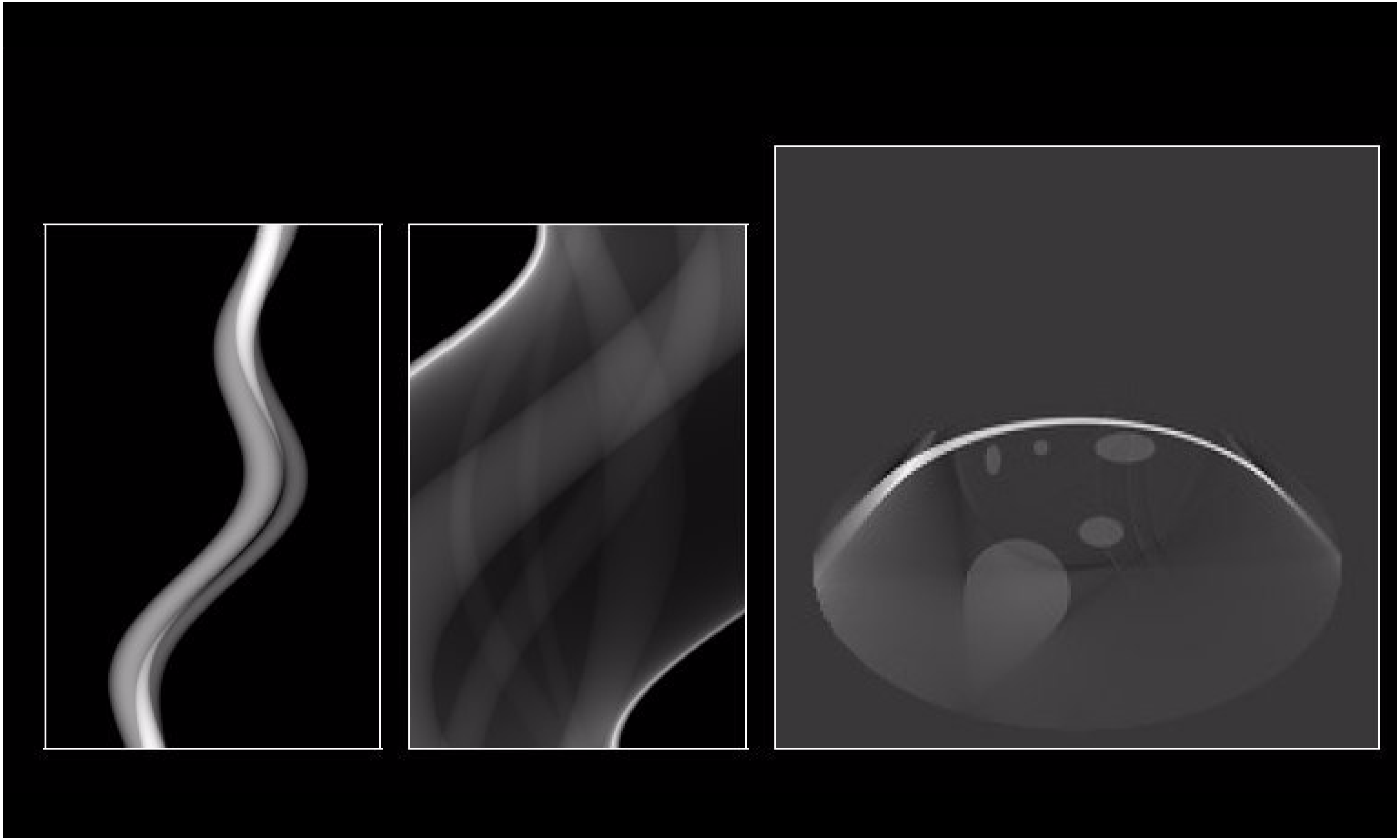


Simulations



Simulations



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- New DCC for truncated projections have been described (2014)
- DCC were successfully used to estimate non-linear motion parameters

Thanks for your attention!