Calculus :

→ It's a branch of math that focuses on the study of Continuous change.

1. Derivatives
2. Integerals
3. Limits

Whey it's crucial in ML ?

1. Optimization : gradient descent to optimize model parameters.
2. Backpropagation : Neural Networks.
3. loss function : to minimizing error in model.
4. Model design :
5. Understanding algorithms : linear Regression, NN, SVM, K-Means, logistic Regression, PCA...

1 Limits :

→ Used to describe the behavior of a fuction as the values of X approach, or become closer and closer to some particular number.

$$\lim_{x \to a} f(x) = L$$

ex. $\lim_{x \to 2} \boxed{\dfrac{x^3 - 2x^2}{x - 2}}^{g(x)} = \dfrac{x^2\cancel{(x-2)}}{\cancel{(x-2)}} = \boxed{4}$

$g(x)$ ⇒ undefined at $x=2$ since $x=2$ makes the denominator 0.

Rules of limits :

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1. $\lim_{x \to a} \overset{\ln \text{ أو }}{c} f(x) = \overset{\ln}{c} \lim_{x \to a} f(x)$

2. $\lim_{x \to a} [f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$

3. $\lim_{x \to a} (f(x) \cdot g(x)) = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$

4. $\lim_{x \to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$ ⇒ where $\lim_{x \to a} g(x) \neq 0$

5. $\lim_{x \to a} [f(x)]^{g(x)} = [\lim_{x \to a} f(x)]^{\lim_{x \to a} g(x)}$

Famous limits :

① $\lim_{x\to\infty} a^x = \begin{cases} a > 1 \Rightarrow \infty \text{ (not exist)} \\ a < 1 \Rightarrow 0 \text{ (exist} \Rightarrow \text{zero)} \\ a = 1 \Rightarrow \text{Unspecified} \end{cases}$

② $\lim_{x\to\infty} \frac{P(x)^{\text{degree}(m)}}{Q(x)^{\text{degree}(n)}} \Rightarrow \begin{cases} m > n \Rightarrow \infty \\ m < n \Rightarrow 0 \\ m = n \Rightarrow \frac{\text{Coff } x^m}{\text{Coff } x^n} \end{cases}$

③ $\lim_{x\to 0} \frac{\sin x}{x} = \boxed{1} \Rightarrow$ proof l'Hopital

④ $\lim_{x\to 0} \frac{\tan x}{x} = \boxed{1}$

Ex. $\lim_{x\to 0} \frac{(2)\sin 2x}{(2x)\, x} = 2 \lim_{x\to 0} \frac{\sin 2x}{2x}$

$= 2 \cdot 1 = \boxed{2}$

L'Hopital Rule :

$$\lim_{x\to a} \frac{f(x)}{g(x)} \Longrightarrow \frac{0}{0} \text{ أو } \frac{\infty}{\infty}$$

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$$= \lim_{x\to a} \frac{f'(x)}{g'(x)} \Longrightarrow \text{لو } \frac{0}{0} \text{ أو } \frac{\infty}{\infty} \text{ بنكرر تاني}$$

Ex: $\lim_{x\to 0} \frac{e^{x} - e^{-x} - 2x}{x - \sin x} = \frac{e^{0} - e^{-0} - 2\times 0}{0 - \underbrace{\sin 0}_{0}} = \frac{0}{0}$

using L'Hopital Rule $\lim_{x\to 0} \frac{e^{x} + e^{-x} - 2}{1 - \cos x} = \frac{e^{0} + e^{-0} - 2}{1 - \cos 0} = \frac{0}{0}$

$$\lim_{x\to 0} \frac{e^{x} - e^{-x}}{\sin x} = \frac{0}{0} \Rightarrow \lim_{x\to 0} \frac{e^{x} + e^{-x}}{\cos x}$$

$$= \boxed{2}$$

[2] Differentiation:

$F(x) = x^2$ by difinition find $f'(x)$

Sol.

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 - x^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{x^2} + \cancel{\Delta x^2} + 2x\cancel{\Delta x} - \cancel{x^2}}{\cancel{\Delta x}} = \lim_{\Delta x \to 0} \frac{\cancel{\Delta x} + 2x}{1} = \boxed{2x}$$

$f(x)$	$f'(x)$	$f(x)$	$f'(x)$
c	0	$\ln x$	$\frac{1}{x}$
x^n	nx^{n-1}	$\log_a x$	$\frac{1}{x} \cdot \frac{1}{\ln a}$
x^{-1} أو $\frac{1}{x}$	$-x^{-2}$ أو $\frac{-1}{x^2}$	$\sin(x)$	$\cos(x)$
$x^{\frac{1}{2}}$ أو $\sqrt{x}$	$\frac{1}{2}x^{-\frac{1}{2}}$ أو $\frac{1}{2\sqrt{x}}$	$\cos(x)$	$-\sin(x)$
e^x	e^x	$\tan(x)$	$\sec^2(x)$
a^x	$a^x \cdot \ln a$	$\cot(x)$	$-\csc^2(x)$
		$\sec(x)$	$\sec(x)\tan(x)$
		$\csc(x)$	$-\csc(x)\cot(x)$

Rules:

[1] $\frac{d}{dx}\left[f(x) \pm g(x)\right] = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x)$

[2] $\frac{d}{dx}\left[c f(x)\right] = c\frac{d}{dx}f(x)$

[3] $\frac{d}{dx}\left[f(x) \cdot g(x)\right] = f(x) \cdot g'(x) + g(x)\, f'(x)$

[4] $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$

[5] $\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$

[6] $\frac{d}{dx}\left[f(x)\right]^n = n \cdot \left[f(x)\right]^{n-1} \cdot f(x)$

$\boxed{7}$ $\frac{d}{dx}\left[f(x)\right]^{g(x)}$ → using ln

→ Using Rules

$$y = f(x)^{g(x)} \Rightarrow y' = \underline{g(x).f(x)^{g(x)-1}.f'(x)} + \underline{f(x)^{g(x)}.\ln f(x).g'(x)}$$

Power function

exponintial function

Ex. $y = x^{\sin(x)}$

Sol.

$$y' = \sin(x).x^{\sin(x)-1} + x^{\sin(x)}.\ln x.\cos(x)$$

Ex. $y = \left(\tan^{-1}x\right)^{x^2}$

Sol

$$y' = x^2.\left[\tan^{-1}(x)\right]^{x^2-1}.\frac{1}{1+x^2} + \left(\tan^{-1}(x)\right)^{x^2}.\ln\tan^{-1}(x).2x$$

$\boxed{8}$ Parametric Differentiation:

$$y = Y(t) \quad , \quad x = X(t)$$

$$y \rightarrow t \leftarrow x$$

$$\frac{dy}{dx} = ?? \Rightarrow \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

From chain Rule

$$\frac{dy}{\cancel{dt}} \cdot \frac{\cancel{dt}}{dx}$$

$$x = X(t)$$

In general: $$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

Ex. $y = t - \cos 2t \quad , \quad x = \sin^2 t$ at $t = \frac{\pi}{4}$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{1 + 2\sin 2t}{2\sin t \cdot \cos t} = \frac{1 + 2\sin 2t}{\sin 2t}$$

At $t = \pi$ $$= \frac{1+2}{1} = \boxed{3}$$