1)

$$\bar{s} = (2t-3)\hat{i} + \hat{j} - t\hat{k}$$
$$\bar{r} = t^2\hat{i} - t\hat{j} + (2t+1)\hat{k}$$

نجد أولاً: $\bar{r}\cdot\bar{s}$

$$\bar{r}\cdot\bar{s} = (t^2)(2t-3) + (-t)(1) + (2t+1)(-t)$$

$$= 2t^3 - 3t^2 - t - 2t^2 - t$$

$$= 2t^3 - 5t^2 - 2t$$

ثم نجد المشتقة لـ $\bar{r}\cdot\bar{s}$

$$\frac{d}{dt}(\bar{r}\cdot\bar{s}) = 6t^2 - 10t - 2$$

عند $t = 1$

$$\therefore 6(1)^2 - 10(1) - 2 = -6$$

$$\therefore \frac{d}{dt}(\bar{r}\cdot\bar{s}) = -6$$

$$\bar{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ t^2 & -t & 2t+1 \end{vmatrix} \qquad \bar{s} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2t-3 & 1 & -t \end{vmatrix}$$

$$\bar{r} \times \bar{s} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ t^2 & -t & 2t+1 \\ 2t-3 & 1 & -t \end{vmatrix}$$

مركبة $\hat{i}$

$$(-t)(-t) - (2t+1)(1) = t^2 - 2t - 1$$

مركبة $\hat{j}$

$$t^2(-1) - (2t+1)(2t-3)$$

$$= -t^2 - 4t^2 + 4t + 3$$

مركبة $\hat{k}$

$$t^2(1) - (-t)(2t-3) = t^2 + 2t^2 - 3t$$

$$= 3t^2 - 3t$$

$$\therefore \bar{r} \times \bar{s} = (t^2 - 2t - 1)\hat{i} - (-t^3 - 4t^2 + 4t + 3)\hat{j} + (3t^2 - 3t)\hat{k}$$

$$= (t^2 - 2t - 1)\hat{i} + (t^3 + 4t^2 - 4t - 3)\hat{j} + (3t^2 - 3t)\hat{k}$$

$$\frac{d}{dt}(\bar{r} \times \bar{s}) = (2t - 2)\hat{i} + (3t^2 + 8t + 4)\hat{j} + (6t - 3)\hat{k}$$

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$1 = t$ is

$(2-2)\hat{i} + (3+8-4)\hat{j} + (6-3)\hat{k}$

$= 0\hat{j} + 7\hat{j} + 3k$

$\therefore \frac{d}{dt}(\bar{r} \times \bar{s}) = 7\hat{j} + 3\hat{k}$

$$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)$$

1) $f = 3x^2y - y^2z$

نشتق بالنسبة لـ x

$$\frac{\partial f}{\partial x} = 6xy$$

نشتق بالنسبة لـ y

$$\frac{\partial f}{\partial y} = 3x^2 - 2yz$$

نشتق بالنسبة لـ z

$$\frac{\partial f}{\partial z} = -y^2$$

عند $(1, -2, -1)$

$$\frac{\partial f}{\partial x} = 6(1)(-2) = -12$$

$$\frac{\partial f}{\partial y} = 3(1)^2 - 2(-2)(-1) = -1$$

$$\frac{\partial f}{\partial z} = -(-2)^2 = -4$$

$$\therefore \nabla f = (-12, -1, -4)$$

2) $f = 2z^2 - xy^2$

$\frac{\partial f}{\partial x} = -y^2$ x ثابت

$\frac{\partial f}{\partial y} = -2xy$ y ثابت

$\frac{\partial f}{\partial z} = 4z$ z ثابت

$(1, -2, -1)$ عند

$\frac{\partial f}{\partial x} = -(-2)^2 = -4$

$\frac{\partial f}{\partial y} = -2(1)(-2) = 4$

$\frac{\partial f}{\partial z} = 4(-1) = -4$

$\nabla f(1, -2, -1) = (-4, 4, -4)$

$$\bar{A} = 2x\hat{i} - 3xz\hat{j} + xz^2\hat{k}$$

$$\nabla \cdot \bar{A} = \frac{\partial}{\partial x}(2x) + \frac{\partial}{\partial y}(-3xz) + \frac{\partial}{\partial z}(xz^2)$$

* $\frac{\partial}{\partial x} = 2$
* $\frac{\partial}{\partial y} = 0$
* $\frac{\partial}{\partial x} = 2xz$

$$\therefore \nabla \cdot \bar{A} = 2 + 2xz$$

$(1, 1, 1)$ عند

$$\nabla \cdot \bar{A} = 2 + 2(1)(1) = 4$$

$$\nabla \times \bar{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2x & -3xz & xz^2 \end{vmatrix}$$

$\hat{i}$ عامل

$$\frac{\partial}{\partial y}(xz^2) - \frac{\partial}{\partial z}(-3xz) = 0 - (-3x)$$

$$= 3x$$

بالنسبة لـ $\hat{j}$

$$\frac{\partial}{\partial z}(2x) - \frac{\partial}{\partial x}(xz^2) = 0 - z^2 = -z^2$$

بالنسبة لـ $\hat{k}$

$$\frac{\partial}{\partial x}(-3xz) - \frac{\partial}{\partial y}(2x) = -3z - 0 = -3x$$

$$\nabla \times \bar{A} = 3x\hat{i} - (-z^2)\hat{j} - 3z\hat{k}$$

$$= 3x\hat{i} - (-z^2)\hat{j} - 3z\hat{k}$$

عند $(1,1,1)$

$$\nabla \times A = 3\hat{i} + \hat{j} - 3\hat{k}$$

$$\bar{B} = 2x^2 z\,\hat{i} + xy^3\,\hat{j} + 3yz^2\,\hat{k}$$

$$\nabla \cdot \bar{B} = \frac{d}{dx}(2x^2 z) + \frac{d}{dy}(-xy^3) + \frac{d}{dz}(3yz^2)$$

$$\nabla \cdot \bar{B} = 4xz - 3xy^2 + 6yz$$

$$\nabla \times \bar{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ 2x^2 z & -xy^3 & 3yz^2 \end{vmatrix}$$

$$\hat{i} = \frac{d}{dy}(3yz^2) - \frac{d}{dz}(-xy^3) = 3z^2 - 0 = 3z^2$$

$$\hat{j} = \frac{d}{dz}(2x^2 z) - \frac{d}{dx}(3yz^2) = 2x^2 - 0$$

$$\hat{k} = \frac{d}{dx}(-xy^3) - \frac{d}{dy}(2x^2 z) = -y^3 - 0$$

$$\nabla \times \bar{B} = 3z^2\,\hat{i} - 2x^2\,\hat{j} - y^3\,\hat{k}$$