Fluid Dynamics

- Fluid at rest is kept in equilibrium by two kinds of forces gravity and pressure force.
- These are also present in the case of fluid motion.
- In addition there is the friction of fluid (viscous force).
- In this chapter we are going to neglect the viscous force compared to the other force and assume that the fluid is incompressible.

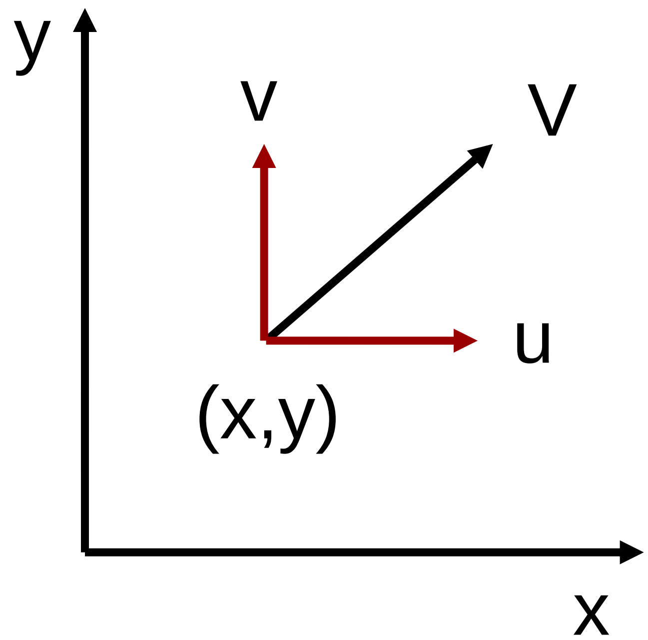
$$\sum F = m.a$$

For generalization:

1- For Cartesian Coordinates:

and for steady state condition:

$u = u(x,y)$ **and** $v = v(x,y)$



$$u = \frac{dx}{dt}$$ **and** $$a_x = \frac{du}{dt}$$

$$v = \frac{dy}{dt}$$ **and** $$a_y = \frac{dv}{dt}$$

$$\because du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy$$

and then

$$\because du = \frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy$$

Dividing the above equation by dt, we obtain:

$$\because a_x = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}$$

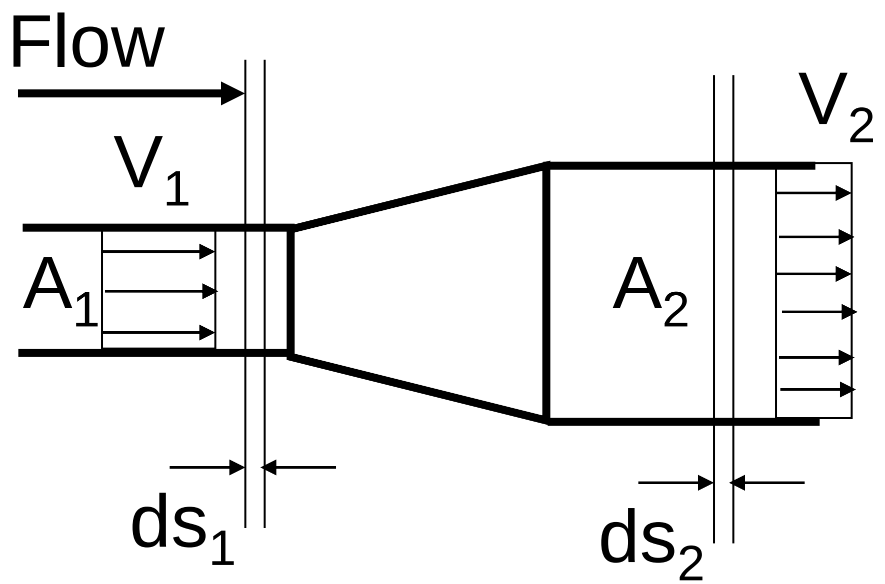
$$\because dv = \frac{\partial v}{\partial x} dx + \frac{\partial v}{\partial y} dy$$

Dividing the above equation by dt, we obtain:

$$\because a_y = u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y}$$

Equation of Continuity:

1- For One dimensional:



Conservation of mass

$$\rho_1 A_1 ds_1 = \rho_2 A_2 ds_2$$

Dividing by dt, we obtain:

$$\rho_1 A_1 \frac{ds_1}{dt} = \rho_2 A_2 \frac{ds_2}{dt}$$

$$\boxed{\therefore \rho_1 A_1 V_1 = \rho_2 A_2 V_2}$$

$$\therefore \dot{m} = \rho_1 A_1 V_1 = \rho_2 A_2 V_2 = \text{Const.}$$

Where: $\dot{m}$ is the mass flow rate.

$$\therefore \dot{m} = \rho A V$$

For constant ρ:

$$\mathbf{A_1V_1 = A_2V_2 = Q^o}$$

Where: Q^o **is the volume flow rate.**

Fluid Dynamics

One Dimensional Steady Flow

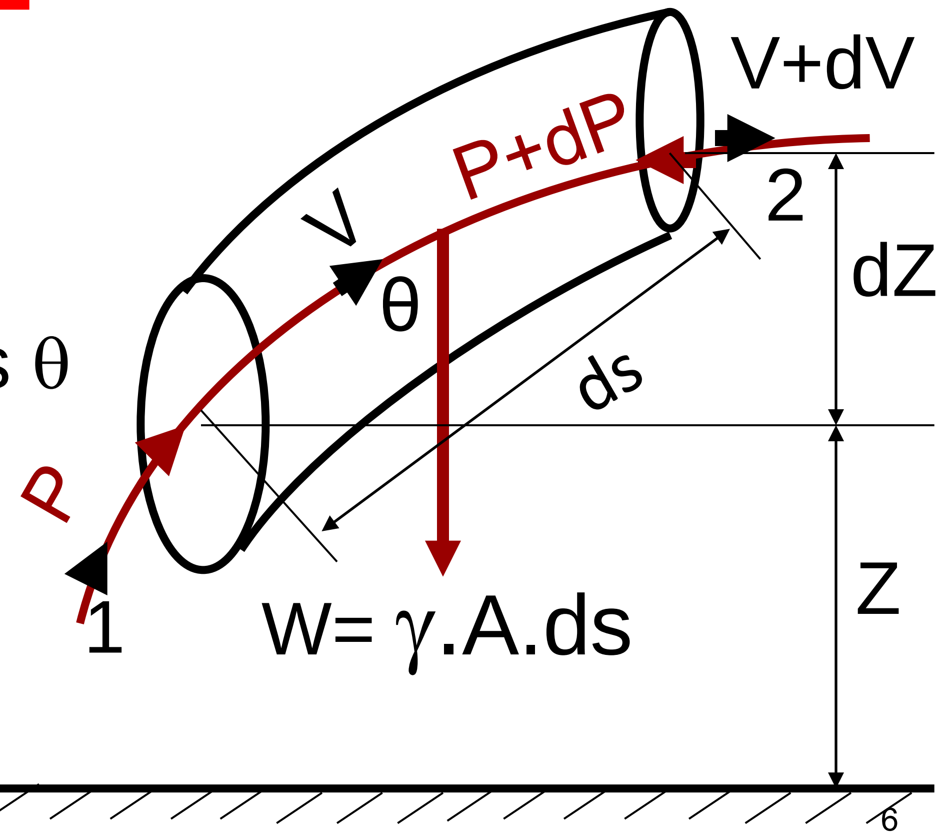
1- Euler's Equation (Energy equation)

Applying Newton's law:

$\sum F$ = mass x acceleration

$$P.A - (P+dP).A - \gamma.A.ds.\text{Cos}\,\theta$$

$$= \rho.A.ds.V.\frac{dV}{ds}$$



$$\boxed{\cos\theta = \frac{dz}{ds}}$$

$$P.A - (P+dP).A - \gamma.A.ds.\ \mathrm{Cos}\ \theta = \rho.A.ds.V.\ \frac{dV}{ds}$$

$$\because \cos\theta = \frac{dz}{ds}$$

$$P.A - (P+dP).A - \gamma.A.ds.\frac{dz}{ds} = \rho.A.ds.V.\frac{dV}{ds}$$

Dividing by γ. A.ds we obtain, where $\gamma = \rho.g$

$$-\frac{1}{\gamma}\frac{dP}{ds} - \frac{dz}{ds} = \frac{1}{g}\frac{d\left(V^2/2\right)}{ds}$$

$$\boxed{\therefore \frac{dP}{\gamma} + d\left(\frac{V^2}{2g}\right) + dz = 0}$$

Euler's Equation

2- Bernoulli's Equation

By integrating Eular's Equation for incompressible (ρ=constant), and one-dimensional fluid flow (take γ= constant)

$$\therefore \int \frac{dP}{\gamma} + \int d\left(\frac{V^2}{2g}\right) + \int dz = C$$

$$\therefore \frac{P}{\gamma} + \frac{V^2}{2g} + z = \text{constant} = H$$

Where: H is constant and termed as the total head

Bernoulli's equation can be taken as energy equation.

$$\boxed{\therefore \frac{P}{\gamma} + \frac{V^2}{2g} + z = H}$$

Where H is the total energy per unit weight

Kinetic Energy $\frac{1}{2}mV^2 * \frac{g}{g}$

Term $\frac{V^2}{2g}$ Energy Per **unit weight**

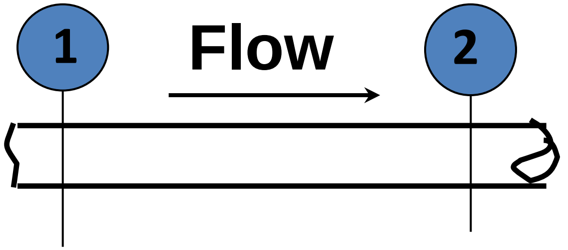
Potential Energy mgz

Term z **Per unit weight**

Pressure Energy $mg * \frac{P}{\gamma}$

Term $\frac{P}{\gamma}$ **Per unit weight**

$$\therefore \frac{P}{\gamma} + \frac{V^2}{2g} + z = H = \text{Energy/weight}$$



The Bernoulli's equation is for Ideal fluid flow flow.

$$\boxed{E_1 = E_2}$$

But Bernoulli's equation for real fluid (viscous) flow.

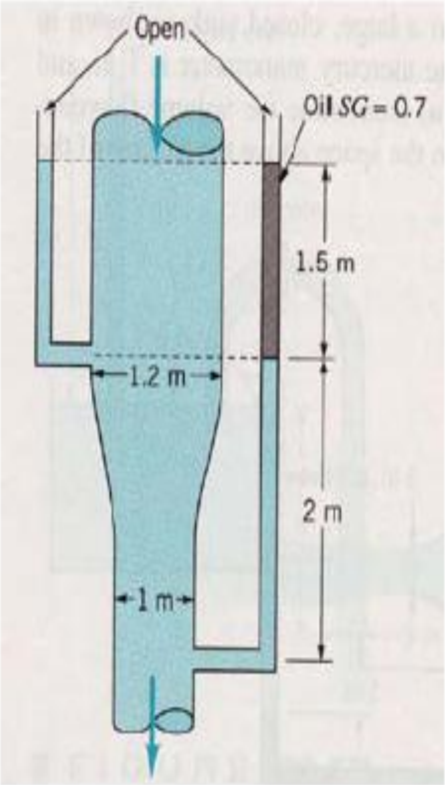
$$\boxed{E_1 = E_2 + \text{Losses}_{1\text{-}2}}$$

The losses are determined or calculated experimentally

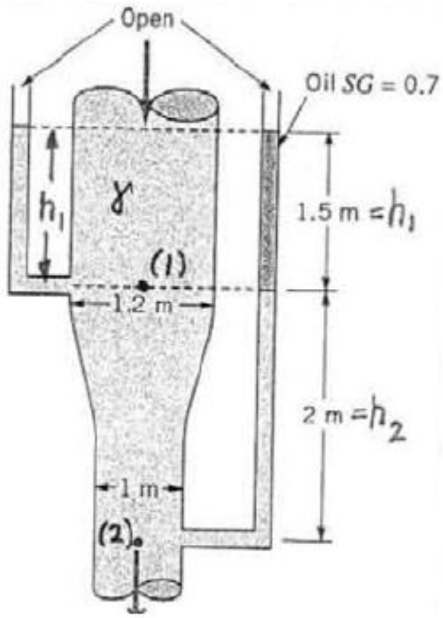
If $E_1 = E_2$ Ideal fluid flow or no flow

If $E_1 = E_2 + \text{Losses}_{1\text{-}2}$ Real fluid flow or there is a flow

1. Water flows steadily downward in the pipe shown in figure with negligible losses. Determine the flow rate. (ρ_{water}=1000kg/m^3)



1) Total=10 points



(3) $$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2 \quad (1)$$

(1) $$z_1-z_2=2m \quad (2)$$

(2) $$A_2V_2=A_1V_1 \Rightarrow D_2^2V_2=D_1^2V_1 \quad (3)$$

(0.5) $$\Rightarrow V_1=0.694V_2$$

$$p_1=\gamma h_1;\, p_2=\gamma h_2+\gamma_{oil}h_1 \Rightarrow$$

(2) $$\frac{p_2-p_1}{\gamma}=\left(h_2-h_1\right)+\frac{\gamma_{oil}}{\gamma}h_1 \quad (4)$$

(0.5) $$=2-1.5+0.7\times1.5=1.55\,m$$

(1), (2), (3), and (4)>>>>>>>>>>>>

(0.5) $$V_2=4.13 \quad m/s$$

(0.5) $$\Rightarrow Q=\frac{\pi D_2^2}{4}V_2=3.24\ m^3/s$$