

Applied Learning Project: Designing PID Controller for 2nd Order System (RC Circuit) Using Root Locus Method

الموضوع: تصميم وحدة تحكم PID

التاريخ: / /

Lab project

$G_p(\text{plant}) \text{ T.F.} =$

$R_1 = R_2 = 1e6$

$C_1 = C_2 = 1e-7$

open loop second order T.F. = $\frac{\omega_n^2}{s(s + 2\zeta\omega_n)}$

$\zeta = \frac{3 \cdot R \cdot C}{2 \cdot \omega_n \cdot (R^2 \cdot C^2)} \rightarrow \frac{3(1000000)(1e-7)}{2(10)(1e-7)^2} = 1.5$

$\omega_n = \frac{1}{R \cdot C} = \frac{1}{1000000 \times (1e-7)} = 10$

open loop $\rightarrow \frac{100}{s(s + 2(1.5)(10))} = \frac{100}{s(s + 30)}$

system

G_{plant}

$\frac{100}{s(s + 30)}$

التاريخ:

15/9/20

2 → Draw root Locus

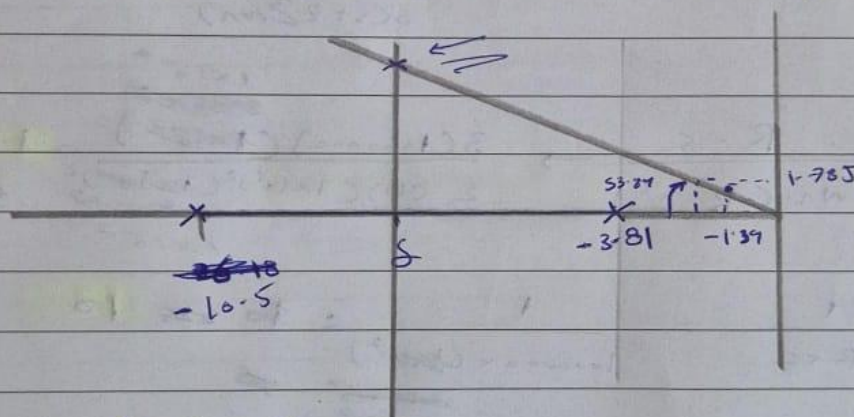
$$A_p = \frac{100}{s(s+30)} \rightarrow A_p \text{ closed-loop} = \frac{100}{s^2 + 2\alpha s + 100}$$

Number of zeros = 0

Number of poles = 2

$$s_1 = -3.81$$

$$s_2 = -26.18$$



$$p - z = 2$$

$$\sigma = \frac{-30}{2} = -15$$

$$\theta = \tan^{-1} \left(\frac{\sqrt{1 - \zeta^2}}{\zeta} \right)$$

$$q \rightarrow 0 \rightarrow 1$$

$$q = 0 \rightarrow 90^\circ \quad \frac{(2\zeta(0) + 1) \times 180}{2}$$

$$q = 1 \rightarrow 270^\circ \quad \frac{(2\zeta(1) + 1) \times 180}{2}$$

Requirements

$$MOS < \frac{15}{100}$$

5% ess

$$T_s < 3 \text{ seconds}$$

Assume 2% tolerance

$$T_s = \frac{3.91}{2 \cdot \omega_n}$$

$$3 = \frac{3.91}{0.59 \omega_n}$$

$$\omega_n = 2.21 \#$$

$$15 = e^{\frac{-\zeta^2}{1-\zeta^2}} \times 100$$

$$\zeta = 0.516$$

since Less than 15%

so I will choose 10%

$$10 = e^{\frac{-\zeta^2}{1-\zeta^2}} \times 100$$

$$\zeta = 0.59 \#$$

$$\theta = \tan^{-1} \left(\frac{\sqrt{1-\zeta^2}}{\zeta} \right)$$

$$\theta = \tan^{-1} \left(\frac{\sqrt{1-0.59^2}}{0.59} \right)$$

$$\theta = 53.87$$

$$s_{1,2} = -\zeta \omega_n \pm \omega_n \sqrt{1-\zeta^2}$$

$$s_{1,2} = -1.39 \pm 1.78j$$

$$s_{1,2} = \text{real} \pm \text{imaginary}$$

We will take $\zeta = 0.15$ in order to achieve the requirements

$$\zeta = 2 \omega_n$$

$$15 = 0.59 \omega_n$$

$$\omega_n = 25.423$$

$$\omega_d = \omega_n \sqrt{1-\zeta^2}$$

$$= 25.423 \sqrt{1-0.15^2}$$

$$\omega_d = 20.523$$

will achieve the requirements

$$s_{1,2} = -15 \pm 20.523j$$

2, Actual point you will design $s_{1,2} = -15 \pm 20.5z35$

$$1 + (CS) = 0$$



$$k_p + k_d + \frac{k_i}{s}$$

$$1 + \left[\frac{s k_p + k_d s^2 + k_i}{s} \times \frac{100}{s^2 + 30s + 100} \right] = 0$$

$$1 + \frac{100 k_p s + 100 k_d s^2 + 100 k_i}{s^2 + 30s + 100} = 0$$

$$\begin{aligned} s^3 + 30s^2 + 100k_p s + 100k_d s^2 + 100k_i &= 0 \\ s^3 + (30 + 100k_d)s^2 + (100k_p)s + 100k_i &= 0 \end{aligned}$$

$$1 + \left[\frac{s k_p + k_d s^2 + k_i}{s} \times \frac{100}{s^2 + 30s + 100} \right] = 0$$

$$1 + \frac{100 k_p s + 100 k_d s^2 + 100 k_i}{s^2 + 30s + 100} = 0$$

$$s^3 + 30s^2 + 100k_p s + 100k_d s^2 + 100k_i = 0$$

$$s^3 + (30 + 100k_d)s^2 + (100k_p)s + 100k_i = 0$$

التاريخ: / / الموضوع:

General form

$$(s+a)(s^2 + 2\zeta\omega_n s + \omega_n^2) = 0$$

assume $a = 10 \times 10^3 = 10(0.5)(2 \times 10^4) = 13059.35$

$$(s + 13059.35)(s^2 + 2.61s + 4.8841) = 0$$

$$s^3 + 2.61s^2 + 4.8841s + 13059.35s^2 + 13059.35 \times 2.61s + 13059.35 \times 4.8841 = 0$$

$$s^3 + [13059.35]s^2 + [2.61 + 13059.35 \times 2.61]s + 13059.35 \times 4.8841 = 0$$

$$s^3 + 37.61s^2 + 96.23s + 170.9435 = 0$$

Equating coefficients

$$s^3 + (30 + 100k_d)s^2 + (100k_p)s + 100k_i = 0$$

$100k_i = 13059.35$	$100k_i = 170.9435$
$k_i = 0.64$ or 0.836 #	$k_i = 1.71$ #
$100k_p = 38.9141$	$100k_p = 96.23$
$k_p = 0.3891$ #	$k_p = 0.9623$ #
$30 + 100k_d = 15.649$	$30 + 100k_d = 37.61$
	$100k_d = 7.61$
	$k_d = 0.0761$ #

Script_root_locus

```
R1 = 1e6;  
R2 = 1e6;  
C1 = 1e-7;  
C2 = 1e-7;  
num = [1/(R1*R2*C1*C2)];  
den = [1 (R1*C1+R2*C2+R1*C2)/(R1*R2*C1*C2) (1/(R1*R2*C1*C2))];  
G=tf(num,den)  
rlocus(G)
```

