CS305
System Modelling and Simulation

Section #2

Nour Talaat

Random Number Generators (RNGs)

- We will discuss them in detail in chapter 7
- For now, know that an RNG initially takes a seed ***S*** or a starting point R_0
- Outputs a random number R_i for every run, given R_{i-1}
- $R_i = f(R_{i-1})$
- Example: $R_i = (R_{i-1} * 25) \% 74$
  - $R_0 = 12$ (given or assumed)
  - $R_1 = (12 * 25) \% 74 = 4$
  - $R_2 = (4 * 25) \% 74 = 26$
  - $R_3 = (26 * 25) \% 74 = 58$
  - etc.

Important Equations

- Average = sum(items) / number(items)
- Average Queuing Time = sum(waiting_time)/number(entities)
- Average Service Time = sum(service_time)/number(entities)
- Average Interarrival Time= sum(interarrival_time)/(number(entities)-1)
- Average Queuing Time (for the entities that do wait) = sum(waiting_time)/number(entities_waited)
- Average Time in System = sum(entity_system_time)/number(entities)
- Probability of Waiting = number(entities_waited)/number(entities)
- Probability of Server Idling = sum(idling_time)/total_time

Exercise #1

1. For a job shop the interarrival times of jobs are distributed as follows :

Time Between Arrivals (Hours)	*Probability*
0	.23
1	.37
2	.28
3	.12

Following random values are uniformly distributed between 0 and 999. Use these values to generate the first 10 values of the interarrival times.

1	2	3	4	5	6	7	8	9	10
786	903	240	874	308	237	490	128	787	364

Exercise #1

Solution steps:

1. **Calculate cumulative probabilities**
2. Scale randomly generated numbers
3. Map interarrival times

Interarrival Time (Hours)	Probability	Cumulative Probability
0	0.23	0.23
1	0.37	0.60
2	0.28	0.88
3	0.12	1.00

Exercise #1

Solution steps:

1. Calculate cumulative probabilities
2. **Scale randomly generated numbers**
3. Map interarrival times

#	Random Number	Scaled Random Number
1	786	0.786
2	903	0.903
3	240	0.240
4	874	0.874
5	308	0.308
6	237	0.237
7	490	0.490
8	128	0.128
9	787	0.787
10	364	0.364

Solution steps:

1. Calculate cumulative probabilities
2. Scale randomly generated numbers
3. **Map interarrival times**

Exercise #1

Interarrival Time (Hours)	Cumulative Probability
0	0.23
1	0.60
2	0.88
3	1.00

#	Scaled Random Number	Mapped Interarrival Time
1	0.786	**2**
2	0.903	**3**
3	0.240	**1**
4	0.874	**2**
5	0.308	**1**
6	0.237	**1**
7	0.490	**1**
8	0.128	**0**
9	0.787	**2**
10	0.364	**1**

Exercise #2

2. Pizza delivery time is distributed as follows:

Delivery Time (Minutes)	20	25	30	35	40
Probability	0.15	0.25	0.35	0.20	0.05

Use the following random number generator to generate the first 10 values of the delivery times.

$$R_i = (37 \times R_{i-1}) \mod 107, \qquad \text{where } R_0 = 35$$

Solution steps:

1. **Calculate cumulative probabilities**
2. Generate random numbers
3. Scale random numbers
4. Map delivery time

Exercise #2

Delivery Time (Minutes)	Probability	Cumulative Probability
20	0.15	0.15
25	0.25	0.40
30	0.35	0.75
35	0.20	0.95
40	0.05	1.00

Solution steps:

1. Calculate cumulative probabilities
2. **Generate random numbers**
3. Scale random numbers
4. Map delivery time

Exercise #2

In [2]:

```
# Initialize array 'r' with the first value = 35
r = [35]
# Loop for 10 times
for i in range(10):
    # For every iteration calculate value R = (37 * R[i]) % 107
    rnew = (37 * r[i]) % 107
    # Insert the new value into the array 'r'
    r.append(rnew)
# Loop over array 'r' and print all values in it
for i in range(len(r)):
    print(f'R{i}: {r[i]}')
```

```
R0: 35
R1: 11
R2: 86
R3: 79
R4: 34
R5: 81
R6: 1
R7: 37
R8: 85
R9: 42
R10: 56
```

Solution steps:

1. Calculate cumulative probabilities
2. Generate random numbers
3. **Scale random numbers**
4. Map delivery time

Exercise #2

R	Random Number	Scaled Random Number
1	11	0.10
2	86	0.80
3	79	0.73
4	34	0.31
5	81	0.75
6	1	0.01
7	37	0.34
8	85	0.79
9	42	0.39
10	56	0.52

Solution steps:

1. Calculate cumulative probabilities
2. Generate random numbers
3. Scale random numbers
4. **Map delivery time**

Exercise #2

Delivery Time (Minutes)	Cumulative Probability
20	0.15
25	0.40
30	0.75
35	0.95
40	1.00

R	Scaled Random Number	Mapped Delivery Time
1	0.10	20
2	0.80	35
3	0.73	30
4	0.31	25
5	0.75	35
6	0.01	20
7	0.34	25
8	0.79	35
9	0.39	25
10	0.52	30

Exercise #3

3. For a job shop with single queue and single server the interarrival times of the jobs and their service time is provided as follows:

Job	1	2	3	4	5	6	7	8	9	10
Interarrival Time	0	4	2	3	2	3	3	4	2	1
Service Time	3	4	2	3	4	5	2	2	3	4

Construct the simulation table, then calculate the following statistics :

(a) Average time in queue.

(b) Average time in system.

(c) Average queue length.

(d) Server utilization.

Solution steps:

1. Construct simulation table
2. Calculate statistics:
    a. Average time in queue
    b. Average time in system
    c. Average queue length
    d. Server utilization

Exercise #3

Simulation Table

Simulation Table									
Entity	Time	Clock	Time	Clock	Time	Number	Clock	Time	Time
Customer	Interarrival Time	Arrival Time	Service Time	Time Service Begins	Waiting Time (Queue)	Customers in Queue	Time Service Ends	Time Customer Spends in System	Server Idle Time
1	0	0	3	0	0	0	3	3	0
2	4	4	4	4	0	0	8	4	1
3	2	6	2	8	2	1	10	4	0
4	3	9	3	10	1	1	13	4	0
5	2	11	4	13	2	1	17	6	0
6	3	14	5	17	3	1	22	8	0
7	3	17	2	22	5	1	24	7	0
8	4	21	2	24	3	2	26	5	0
9	2	23	3	26	3	2	29	6	0
10	1	24	4	29	5	2	33	9	0
Average	2.4	12.9	3.2	15.3	2.4	1.1	18.5	5.6	0.1
Sum	24	129	32	153	24	11	185	56	1

Exercise #3

Solution steps:

1. Construct simulation table
2. Calculate statistics:
    a. **Average time in queue**
    b. Average time in system
    c. Average queue length
    d. Server utilization

- sum(waiting_time) = 24 minutes
- number(customers) = 10 customers
- Average time in queue = 24/10 = 2.4 minutes

Exercise #3

Solution steps:

1. Construct simulation table
2. Calculate statistics:
    a. Average time in queue
    b. **Average time in system**
    c. Average queue length
    d. Server utilization

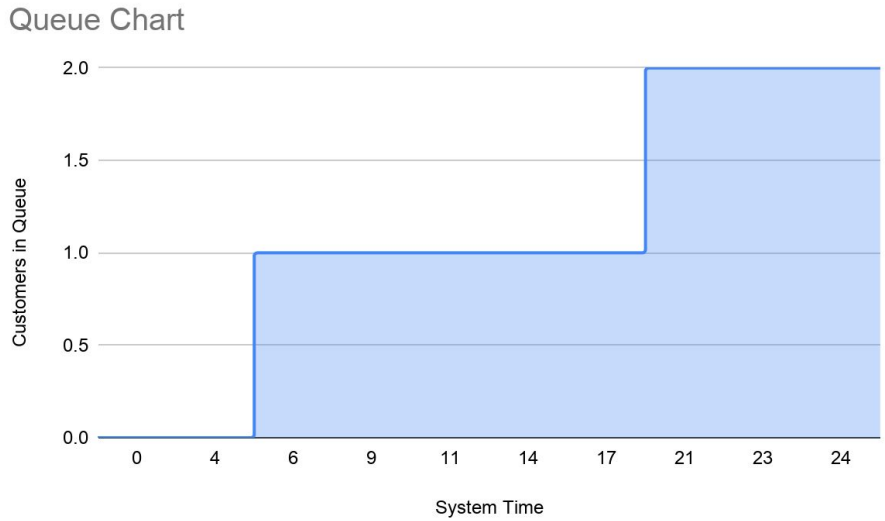
- sum(customer_system_time) = 56 minutes
- number(customers) = 10 customers
- Average time in system = 56/10 = 5.6 minutes

Solution steps:

1. Construct simulation table
2. Calculate statistics:
   a. Average time in queue
   b. Average time in system
   c. **Average queue length**
   d. Server utilization

Exercise #3

- Construct a queue chart



- Sum of queue length at all points= 11 customers
- Total number of customers = 10
- Average queue length = 11/10 = 1.1 customers

Exercise #3

Solution steps:

1. Construct simulation table
2. Calculate statistics:
    a. Average time in queue
    b. Average time in system
    c. Average queue length
    d. **Server utilization**

- Server idle time = 1 minute
- Run time = 33 minutes
- Server utilization = 1 - (idle time/run time)
  = 1 - (1/33)
  = 0.96
  = **96%**

Exercise #4

Example 2: Random Service Times

An automated telephone information service spends either 3, 6, or 10 minutes with each caller. The proportion of calls for each service length is 30%, 45%, and 25%

- Simulate the call time for 10 callers

Exercise #4

Solution steps:

1. **Generate 10 random numbers**
2. Calculate cumulative probabilities
3. Map call times

In [5]:

```
# Initialize empty array 'random_nums'
random_nums = []
# Loop for 10 times
for i in range(10):
    # Generate random number using the function 'random()'
    random_num = random()
    # Insert the new value into the array 'random_nums'
    random_nums.append(random_num)
# Loop over array 'random_nums' and print all values in it
for i in range(len(random_nums)):
    print(f'R{i}: {random_nums[i]}')
```

```
R0: 0.27139014876253365
R1: 0.15634215657987782
R2: 0.5136391387037177
R3: 0.7610090652049514
R4: 0.12504849131962048
R5: 0.20368810459234998
R6: 0.9960015872930557
R7: 0.5883199446380185
R8: 0.5855688016070569
R9: 0.15221783276115108
```

Exercise #4

Solution steps:

1. Generate 10 random numbers
2. **Calculate cumulative probabilities**
3. Map call times

Call Time (Minutes)	Probability	Cumulative Probability
3	0.30	0.30
6	0.45	0.75
10	0.25	1.00

Exercise #4

Solution steps:

1. Generate 10 random numbers
2. Calculate cumulative probabilities
3. **Map call times**

Call Time (Minutes)	Cumulative Probability
3	0.30
6	0.75
10	1.00

R	Random Number	Mapped Call Time
1	0.27	3
2	0.15	3
3	0.51	6
4	0.76	10
5	0.12	3
6	0.20	3
7	0.99	10
8	0.59	6
9	0.58	6
10	0.15	3

Questions?

Assignment

Sheets:

- Questions 5 and 6 in the second chapter

Due next week (March 16th, 2021)

Submit on Google Classroom