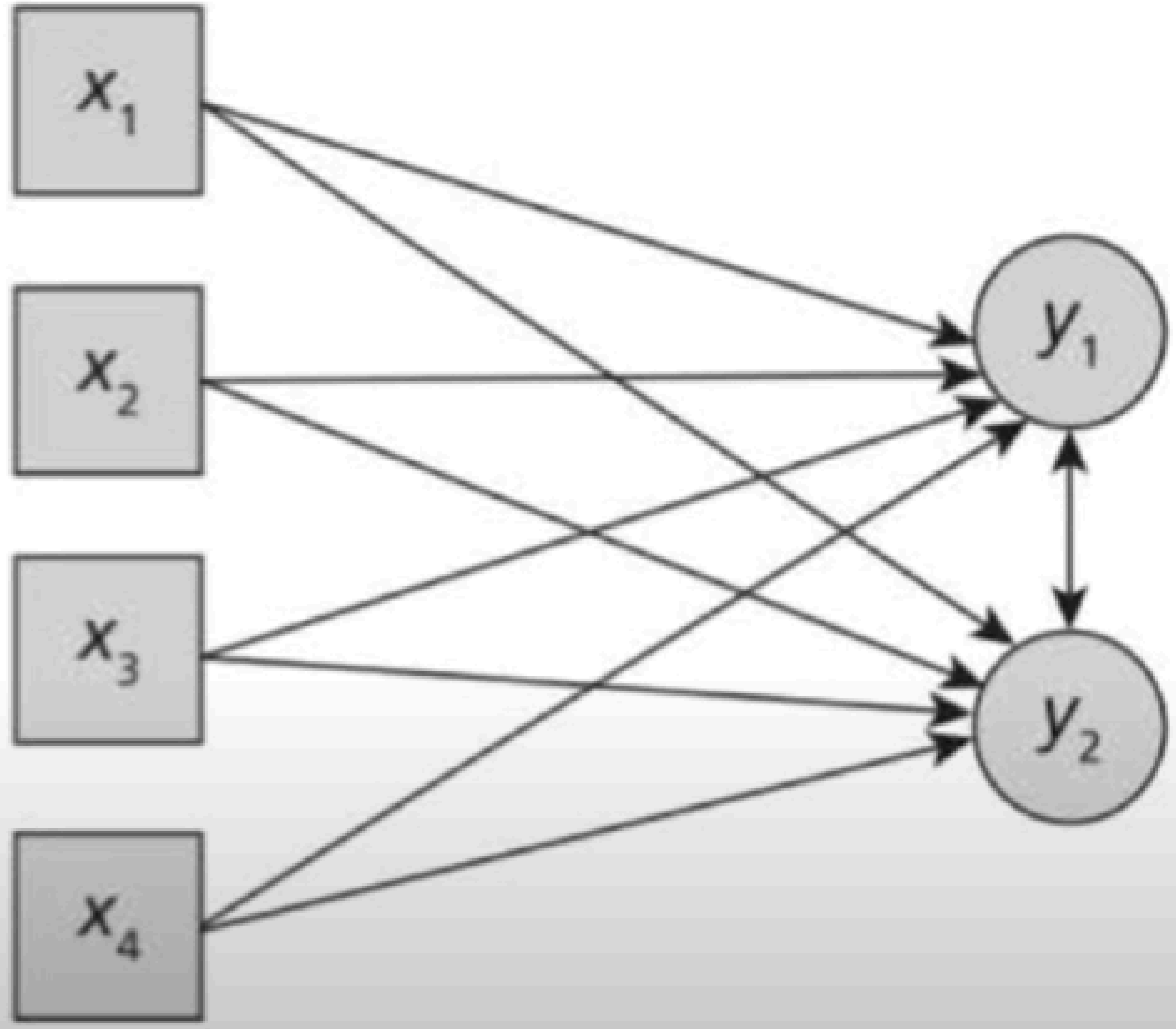
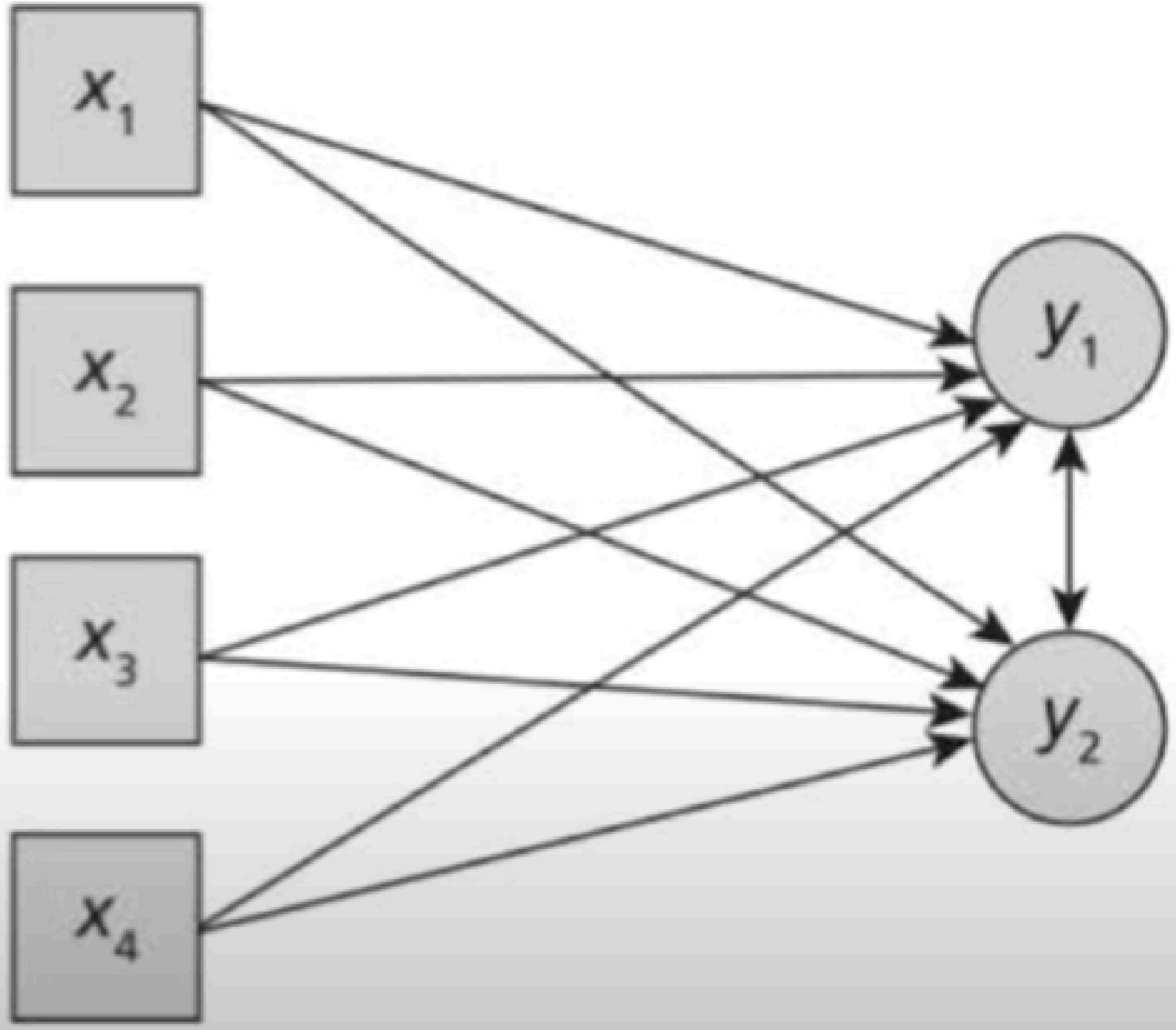
SOM
Self Organizing Feature Map

- Consider the network shown in Figure which considers four training samples each vector of length 4 and two output units.
- Train the SOFM network by determining the class memberships of the input data
- Training Samples:
- X1: (1, 0, 1, 0)   X2: (1, 0, 0, 0)
- X3: (1, 1, 1, 1)   X4: (0, 1, 1, 0)



- Output Units: Unit 1, Unit 2
- Learning rate η(t) = 0.6
- Initial Weight matrix
- $\begin{bmatrix} Unit\ 1 \\ Unit\ 2 \end{bmatrix} = \begin{bmatrix} 0.3 & 0.5 & 0.7 & 0.2 \\ 0.6 & 0.5 & 0.4 & 0.2 \end{bmatrix}$



Iteration 1:

Training Sample x_1: (1, 0, 1, 0)

Weight matrix:

$$\begin{bmatrix} \text{Unit 1} \\ \text{Unit 2} \end{bmatrix} : \begin{bmatrix} 0.3 & 0.5 & 0.7 & 0.2 \\ 0.6 & 0.7 & 0.4 & 0.3 \end{bmatrix}$$

Compute Euclidean distance between x_1: (1, 0, 1, 0) and Unit 1 weights.

$$d^2 = (0.3 - 1)^2 + (0.5 - 0)^2 + (0.7 - 1)^2 + (0.2 - 0)^2 = 0.87$$

Compute Euclidean distance between x_1: (1, 0, 1, 0) and Unit 2 weights.

$$d^2 = (0.6 - 1)^2 + (0.7 - 0)^2 + (0.4 - 1)^2 + (0.3 - 0)^2 = 1.1$$

Unit 1 wins

$$w_j(t+1) = w_j(t) + \eta(t)(x_s - w_j(t))$$

Update the weights of the winning unit.

$$\begin{aligned} \text{New Unit 1 weights} &= [0.3\ \ 0.5\ \ 0.7\ \ 0.2] + 0.6\,([1\ \ 0\ \ 1\ \ 0] - [0.3\ \ 0.5\ \ 0.7\ \ 0.2]) \\ &= [0.3\ \ 0.5\ \ 0.7\ \ 0.2] + 0.6\,[0.7\ \ -0.5\ \ 0.3\ \ -0.2] \\ &= [0.3\ \ 0.5\ \ 0.7\ \ 0.2] + [0.42\ \ -0.30\ \ 0.18\ \ -0.12] \\ &= [0.72\ \ 0.2\ \ 0.88\ \ 0.08] \end{aligned}$$

$$\begin{bmatrix} \text{Unit 1} \\ \text{Unit 2} \end{bmatrix} : \begin{bmatrix} 0.72 & 0.2 & 0.88 & 0.08 \\ 0.6 & 0.7 & 0.4 & 0.3 \end{bmatrix}$$

Iteration 2:

Training Sample x_2: (1, 0, 0, 0)

Weight matrix:

$$\begin{bmatrix} \text{Unit 1} \\ \text{Unit 2} \end{bmatrix} : \begin{bmatrix} 0.72 & 0.2 & 0.88 & 0.08 \\ 0.6 & 0.7 & 0.4 & 0.3 \end{bmatrix}$$

Compute Euclidean distance between x_2: (1, 0, 0, 0) and Unit 1 weights.

$$d^2 = (0.72 - 1)^2 + (0.2 - 0)^2 + (0.88 - 0)^2 + (0.08 - 0)^2 = 0.74$$

Compute Euclidean distance between x_2: (1, 0, 0, 0) and Unit 2 weights.

$$d^2 = (0.6 - 1)^2 + (0.7 - 0)^2 + (0.4 - 0)^2 + (0.3 - 0)^2 = 0.9$$

Unit 1 wins

$$w_j(t+1) = w_j(t) + \eta(t)(x_s - w_j(t))$$

Update the weights of the winning unit:

$$\begin{aligned}
\text{New Unit 1 weights} &= [0.72\ \ 0.2\ \ 0.88\ \ 0.08] + 0.6\,([1\ \ 0\ \ 0\ \ 0] - [0.72\ \ 0.2\ \ 0.88\ \ 0.08]) \\
&= [0.72\ \ 0.2\ \ 0.88\ \ 0.08] + 0.6\,[0.28\ \ -0.2\ \ -0.88\ \ -0.08] \\
&= [0.72\ \ 0.2\ \ 0.88\ \ 0.08] + [0.17\ \ -0.12\ \ -0.53\ \ -0.05] \\
&= [0.89\ \ 0.08\ \ 0.35\ \ 0.03]
\end{aligned}$$

$$\begin{bmatrix} \text{Unit 1} \\ \text{Unit 2} \end{bmatrix} : \begin{bmatrix} 0.89 & 0.08 & 0.35 & 0.03 \\ 0.6 & 0.7 & 0.4 & 0.3 \end{bmatrix}$$

Iteration 3:

Training Sample x_3: (1, 1, 1, 1)

Weight matrix:

$$\begin{bmatrix} \text{Unit 1} \\ \text{Unit 2} \end{bmatrix} : \begin{bmatrix} 0.89 & 0.08 & 0.35 & 0.03 \\ 0.6 & 0.7 & 0.4 & 0.3 \end{bmatrix}$$

Compute Euclidean distance between x_3: (1, 1, 1, 1) and Unit 1 weights.

$$d^2 = (0.89 - 1)^2 + (0.08 - 1)^2 + (0.35 - 1)^2 + (0.03 - 1)^2$$

$$= 2.2$$

Compute Euclidean distance between x_3: (1, 1, 1, 1) and Unit 2 weights.

$$d^2 = (0.6 - 1)^2 + (0.7 - 1)^2 + (0.4 - 1)^2 + (0.3 - 1)^2$$

$$= 1.1$$

Unit 2 wins

$$w_j(t+1) = w_j(t) + \eta(t)(x_s - w_j(t))$$

Update the weights of the winning unit:

$$\begin{aligned}\text{New Unit 2 weights} &= [0.6\ \ 0.7\ \ 0.4\ \ 0.3] + 0.6\,([1\ \ 1\ \ 1\ \ 1] - [0.6\ \ 0.7\ \ 0.4\ \ 0.3]) \\ &= [0.6\ \ 0.7\ \ 0.4\ \ 0.3] + 0.6\,[0.4\ \ 0.3\ \ 0.6\ \ 0.7] \\ &= [0.6\ \ 0.7\ \ 0.4\ \ 0.3] + [0.24\ \ 0.18\ \ 0.36\ \ 0.42] = [0.84\ \ 0.88\ \ 0.76\ \ 0.72]\end{aligned}$$

$$\begin{bmatrix}\text{Unit 1}\\ \text{Unit 2}\end{bmatrix} : \begin{bmatrix}0.89 & 0.08 & 0.35 & 0.03\\ 0.84 & 0.88 & 0.76 & 0.72\end{bmatrix}$$

Iteration 4:

Training Sample x_4: (0, 1, 1, 0)

Weight matrix:

$$\begin{bmatrix} \text{Unit 1} \\ \text{Unit 2} \end{bmatrix} : \begin{bmatrix} 0.89 & 0.08 & 0.35 & 0.03 \\ 0.84 & 0.88 & 0.76 & 0.72 \end{bmatrix}$$

Compute Euclidean distance between x_4: (0, 1, 1, 0) and Unit 1 weights.

$$d^2 = (0.89 - 0)^2 + (0.08 - 1)^2 + (0.35 - 1)^2 + (0.03 - 0)^2$$

$$= 2.06$$

Compute Euclidean distance between x_1: (0, 1, 1, 0) and Unit 2 weights.

$$d^2 = (0.84 - 0)^2 + (0.88 - 1)^2 + (0.76 - 1)^2 + (0.72 - 0)^2$$

$$= 1.3$$

$$w_j(t+1) = w_j(t) + \eta(t)(x_s - w_j(t))$$

Unit 2 wins

Update the weights of the winning unit:

$$\begin{aligned}
\text{New Unit 2 weights} &= [0.84 \;\; 0.88 \;\; 0.76 \;\; 0.72] + 0.6\,([0 \;\; 1 \;\; 1 \;\; 0] - [0.84 \;\; 0.88 \;\; 0.76 \;\; 0.72]) \\
&= [0.84 \;\; 0.88 \;\; 0.76 \;\; 0.72] + 0.6\,[-0.84 \;\; 0.12 \;\; 0.24 \;\; -0.72] \\
&= [0.84 \;\; 0.88 \;\; 0.76 \;\; 0.72] + [-0.5 \;\; 0.07 \;\; 0.14 \;\; -0.43] = [0.34 \;\; 0.95 \;\; 0.9 \;\; 0.29]
\end{aligned}$$

$$\begin{bmatrix} \text{Unit 1} \\ \text{Unit 2} \end{bmatrix} : \begin{bmatrix} 0.89 & 0.08 & 0.35 & 0.03 \\ 0.34 & 0.95 & 0.9 & 0.29 \end{bmatrix}$$

Best mapping units for each of the sample taken are:

x_1: (1, 0, 1, 0) $\rightarrow$ Unit 1

x_2: (1, 0, 0, 0) $\rightarrow$ Unit 1

x_3: (1, 1, 1, 1) $\rightarrow$ Unit 2

x_4: (0, 1, 1, 0) $\rightarrow$ Unit 2

This process is continued for many epochs until the feature map does not change.