PIA: Week 1 Deliverable

Two Woman four men

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1 Problem Definition

The problem involves optimizing tile placement in the game of Rummikub. The goal is to determine the optimal way to place tiles from the player's hand onto the board, adhering to the game's rules, such as forming valid groups and runs. The objective is to maximize the number of tiles placed while ensuring that all constraints are satisfied, including valid combinations, joker usage, and the minimum score required for the first move.

This problem is relevant to **Operations Research (OR)** because it requires decision-making under constraints, aiming to maximize the outcome (tiles placed) given limited resources (tiles in hand). It aligns well with optimization techniques commonly used in OR, especially **Integer Programming**.

2 Model Classification

This is a **Mixed Integer Linear Programming (MILP)** problem. The model is linear because the objective function and constraints are expressed with linear relationships, such as summing tile usage and forming valid combinations. It is an integer problem because the decision variables are binary (indicating whether a tile is placed or a combination is formed) and integer (representing the number of tiles in valid groups or runs). The model is a single-objective optimization problem, as the goal is to maximize tile usage.

MILP is chosen because of the problem's linear nature, the binary decision variables, and the simplicity of the constraints, all of which fit well with this approach. Solvers like **PuLP**, which uses CBC by default, are well-suited for solving such problems efficiently.

3 Identification of Model Elements

Known Parameters

- $F = \{f_1, f_2, \ldots, f_n\}$: Set of tiles in the player's hand, where $f_j = (k_j, l_j)$ with $k_j \in C$ (number) and $l_j \in L$ (color).

- $C = \{1, 2, \ldots, 13\}$: Possible values of the tiles.
- $L = \{red, blue, yellow, black\}$: Available colors.
- $M = \{m_1, m_2, \ldots, m_p\}$: Set of tiles already on the board (for interactions in later turns).
- $P_{\min} = 30$: Minimum score required only for the first move.
- $K = 2$: Available jokers (if they are in F).
- g: Possible groups (valid sequences including M and F).
- s: Possible runs (valid sequences including M and F).

Decision Variables

- $x_j \in \{0, 1\}$: 1 if tile f_j is placed on the board; 0 if it remains in hand.
- $y_g \in \{0, 1\}$: 1 if group g is formed (set of at least 3 tiles of the same number in different colors).
- $z_s \in \{0, 1\}$: 1 if run s is formed (sequence of at least 3 consecutive numbers of the same color).
- $w \in \{0, 1\}$: 1 if a joker is used.
- $u \in \{0, 1\}$: 1 if it's the first move.

Objective Function

Maximize the total number of tiles placed:

$$\max \sum_{j=1}^{n} x_j$$

Constraints

1. **Group validity (y_g)**: A group g with number k is valid if it has at least 3 tiles of k in different colors, including M and F.

$$\sum_{m \in M \cap g} 1 + \sum_{f_j \in F \cap g} x_j \geq 3 \cdot y_g \quad \forall g$$

2. **Run validity (z_s)**: A run s must have at least 3 consecutive numbers, including M and F.

$$\sum_{m \in M \cap s} 1 + \sum_{f_j \in F \cap s} x_j \geq 3 \cdot z_s \quad \forall s$$

3. **Board consistency**:

$$x_j \leq \sum_{f_j \in g} y_g + \sum_{f_j \in s} z_s \quad \forall j \in F$$

This constraint ensures that if you place tile 7 red ($x_j = 1$) and 5-6 red already exist in M, you must activate $z_s = 1$ for the run 5-6-7.

4. **First Move**:

$$\sum_{j=1}^{n} k_j \cdot x_j \geq P_{\min}$$

This constraint is only active in the first turn.

5. **Use of jokers**:

$$w \leq 1 = u$$

If a joker is used, it must be part of a valid combination:

$$\sum_{g:J \in g} y_g + \sum_{s:J \in s} z_s \geq w$$

4 Initial Implementation Design

The model will be implemented in **Python** using the **PuLP** library for defining and solving the MILP model. **Pandas** will be used to structure the input data, including the tiles in hand and on the board. The code will be modular, with separate components for defining the parameters, decision variables, constraints, and objective function. The input will be provided via a structured file (e.g., CSV or JSON), and the output will include the optimal tile placement, the groups and runs formed, and any joker usage.

5 Project Planning

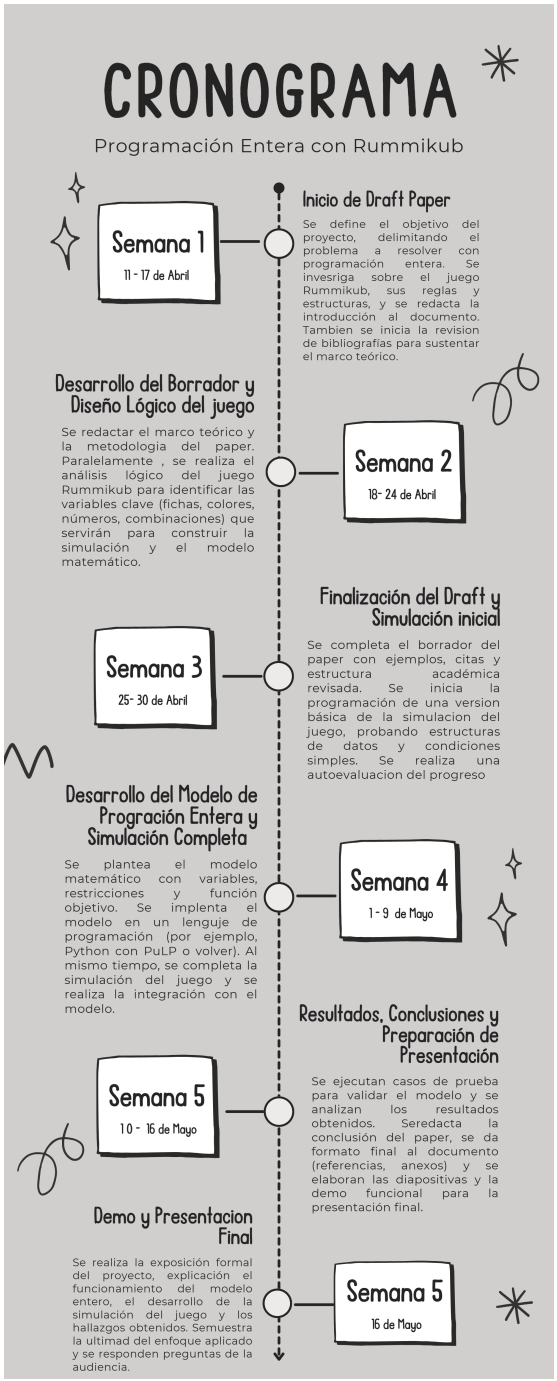

Figure 1: Schedule.