First Mathematical Model of Rummikub

Two Woman four men

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Known Parameters

- $F = \{f_1, f_2, \ldots, f_n\}$: Set of tiles in the player's hand, where $f_j = (k_j, l_j)$ with $k_j \in C$ (number) and $l_j \in L$ (color).
- $C = \{1, 2, \ldots, 13\}$: Possible values of the tiles.
- $L = \{red, blue, yellow, black\}$: Available colors.
- $M = \{m_1, m_2, \ldots, m_p\}$: Set of tiles already on the board (for interactions in later turns).
- $P_{\min} = 30$: Minimum score required only for the first move.
- $K = 2$: Available jokers (if they are in F).
- g: Possible groups (valid sequences including M and F).
- s: Possible runs (valid sequences including M and F).

Decision Variables

- $x_j \in \{0, 1\}$: 1 if tile f_j is placed on the board; 0 if it remains in hand.
- $y_g \in \{0, 1\}$: 1 if group g is formed (set of at least 3 tiles of the same number in different colors).
- $z_s \in \{0, 1\}$: 1 if run s is formed (sequence of at least 3 consecutive numbers of the same color).
- $w \in \{0, 1\}$: 1 if a joker is used.
- $u \in \{0, 1\}$: 1 if it's the first move.

Objective Function

Maximize the total number of tiles placed:

$$\max \sum_{j=1}^{n} x_j$$

Constraints

1. **Group validity** (y_g): A group g with number k is valid if it has at least 3 tiles of k in different colors, including M and F.

$$\sum_{m \in M \cap g} 1 + \sum_{f_j \in F \cap g} x_j \geq 3 \cdot y_g \quad \forall g$$

2. **Run validity** (z_s): A run s must have at least 3 consecutive numbers, including M and F.

$$\sum_{m \in M \cap s} 1 + \sum_{f_j \in F \cap s} x_j \geq 3 \cdot z_s \quad \forall s$$

3. **Board consistency**:

$$x_j \leq \sum_{f_j \in g} y_g + \sum_{f_j \in s} z_s \quad \forall j \in F$$

This constraint ensures that if you place tile 7 red $(x_j = 1)$ and 5-6 red already exist in M, you must activate $z_s = 1$ for the run 5-6-7.

4. **First Move**:

$$\sum_{j=1}^{n} k_j \cdot x_j \geq P_{\min}$$

This constraint is only active in the first turn.

5. **Use of jokers**:

$$w \leq 1 = u$$

If a joker is used, it must be part of a valid combination:

$$\sum_{g: J \in g} y_g + \sum_{s: J \in s} z_s \geq w$$