Fourier Transforms

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Contents

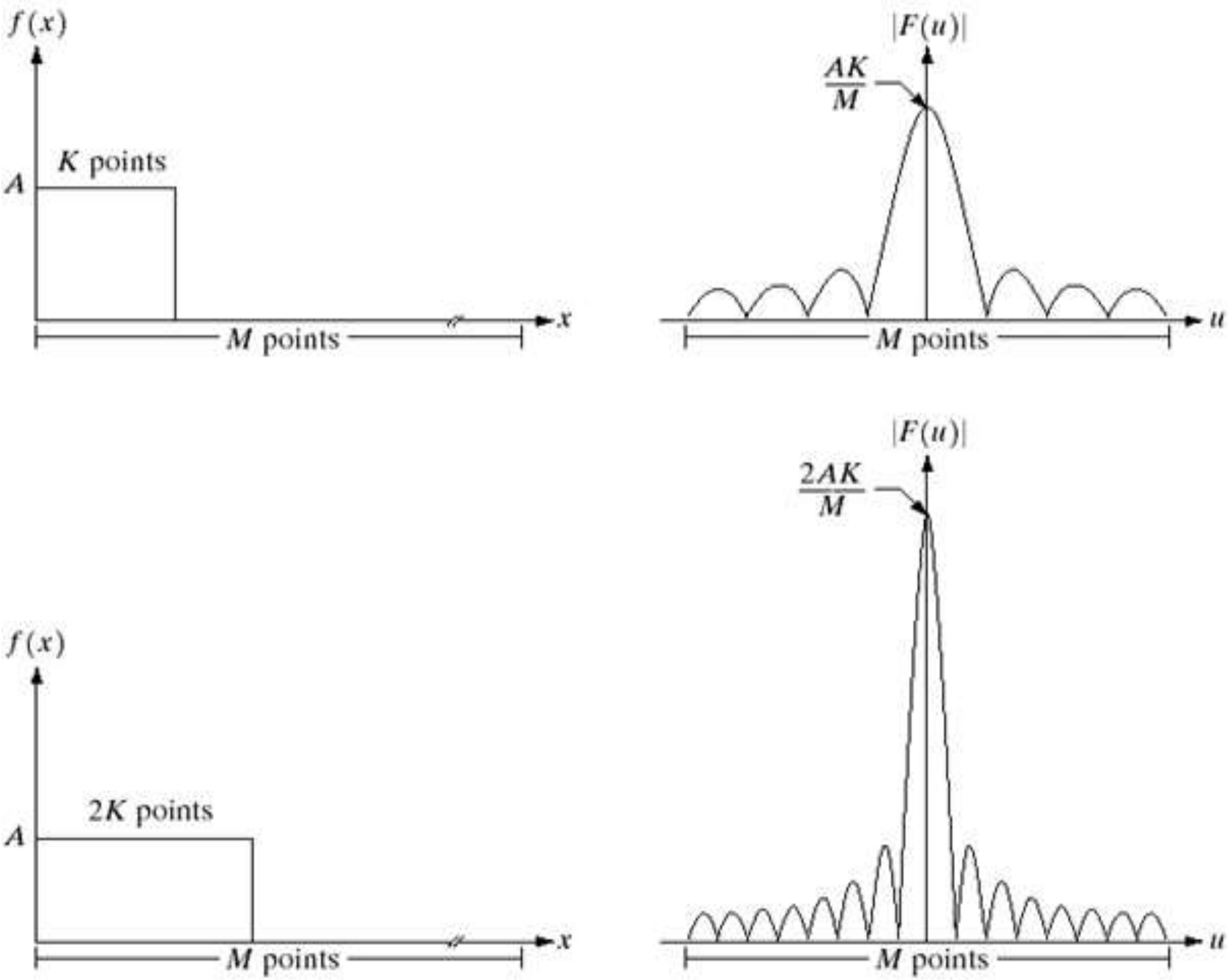
What is Fourier Transform

- ***Fourier Transform***, named after *Joseph Fourier*, is a mathematical transformation employed to transform signals between time(or spatial) domain and frequency domain.

- It is a tool that breaks a waveform (a function or signal) into an alternate representation, characterized by *sine and cosines*.

- It shows that any waveform can be re-written as *the weighted sum of sinusoidal* functions.

Spatial to Frequency Domain



a b
c d

(a)A discrete function of M points.

(b)Its Fourier spectrum

(c)A discrete function with twice the number of nonzero points

(d) Its Fourier spectrum

Fourier Transforms

Forward Fourier and Inverse Fourier transforms

- Given an image a and its Fourier transform A
  - Then *the forward transform* goes from the spatial domain (either continuous or discrete) to the frequency domain which is always continuous.

$$\text{Forward} - \qquad A = \mathfrak{F}\{a\}$$

  - The *inverse* goes from the frequency domain to the spatial domain.

$$\text{Inverse} - \qquad a = \mathfrak{F}^{-1}\{A\}$$

Fourier Transform

- The Fourier transform of F(u) of a *single variable, continuous function*, f(x)

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi ux} dx$$

- The Fourier transform of F(u,v) of a *double variable, continuous function*, f(x,y)

$$F(u,v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) e^{-j2\pi(ux+vy)} dxdy$$

Fourier Transform

- The Fourier Transform of a *discrete function of one variable*, f(x), x=0,1,2,. . . ,M-1,

$$F(u) = \frac{1}{M}\sum_{x=0}^{M-1} f(x)e^{-2j\pi ux/M}$$

u=0,1,2,. . . ,M-1.

- The concept of Frequency domain follows *Euler's formula*

$$e^{j\theta} = \cos\theta + j\sin\theta$$

Fourier Transform

- Fourier Transform (in one dimension)

$$F(u) = \frac{1}{M} \sum_{x=0}^{M-1} f(x) \left[\cos 2\pi ux / M - j \sin 2\pi ux / M \right]$$

- Each term of the Fourier transform is composed of the sum of all values of the function *f(x)*.
- The values of *f(x)* are multiplied by sine and cosines of various frequencies.
- Each of the M term of *F*(u) is called the *frequency component* of the transform.
- The domain (values of u) over which the values of F(u) range is appropriately called the *frequency domain*.

Properties of Fourier Transforms

- ***Linearity***
  - Scaling a function scales it's transform pair. Adding two functions corresponds to adding the two frequency spectrum.

If $h(x) \leftrightarrow H(f)$ then $ah(x) \leftrightarrow aH(f)$

If $h(x) \leftrightarrow H(f)$, $g(x) \leftrightarrow G(f)$ then $h(x)+g(x) \leftrightarrow H(f)+G(f)$

- ***Scaling Property***
  - If

$$f(t) \Leftrightarrow F(\omega)$$

  - Then

$$f(at) \Leftrightarrow \frac{1}{|a|} F(\omega / a)$$

Properties of Fourier Transforms

- ***Time Differentiation***
  - If
$$f(t) \Leftrightarrow F(\omega)$$
  - Then
$$\frac{df}{dt} \Leftrightarrow j\omega F(\omega)$$
- ***Convolution Property***
  - If
$$f_1(t) \Leftrightarrow F_1(\omega) \quad \text{and} \quad f_2(t) \Leftrightarrow F_2(\omega)$$
  - Then
$$f_1(t) * f_2(t) \Leftrightarrow F_1(\omega) F_2(\omega)$$
  - (where * is convolution) and
$$f_1(t) f_2(t) \Leftrightarrow \frac{1}{2\pi} F_1(\omega) * F_2(\omega)$$

Properties of Fourier Transforms

- ***Frequency-shift Property***
  - If

$$f(t) \Leftrightarrow F(\omega)$$

  - Then

$$f(t)e^{j\omega_0 t} \Leftrightarrow F(\omega-\omega_0)$$

- ***Time-Shift Property***
  - If

$$f(t) \Leftrightarrow F(\omega)$$

  - Then

$$f(t-t_0) \Leftrightarrow F(\omega)e^{-j\omega t_0}$$

  - In other words, a shift in time corresponds to a change in phase in the Fourier transform.

Fourier Transformation in Image processing

- Used to access the *geometric characteristics* of a spatial domain image.
  - Fourier domain decompose the image into its *sinusoidal components.*

- In most implementations
  - Fourier image is shifted in such a way that the *F(0,0)* represent the center of the image.
  - The further away from the center an image point is, the higher is its corresponding frequency.

Fourier Transformation in Image processing

- The Fourier Transform is used in a wide range in image processing
  - Image filtering,
  - Image applications
    - Image analysis,
    - Image filtering,
    - Image reconstruction, and
    - Image compression.

References

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Thank You