



Computer Vision; Image Classification; Large Networks



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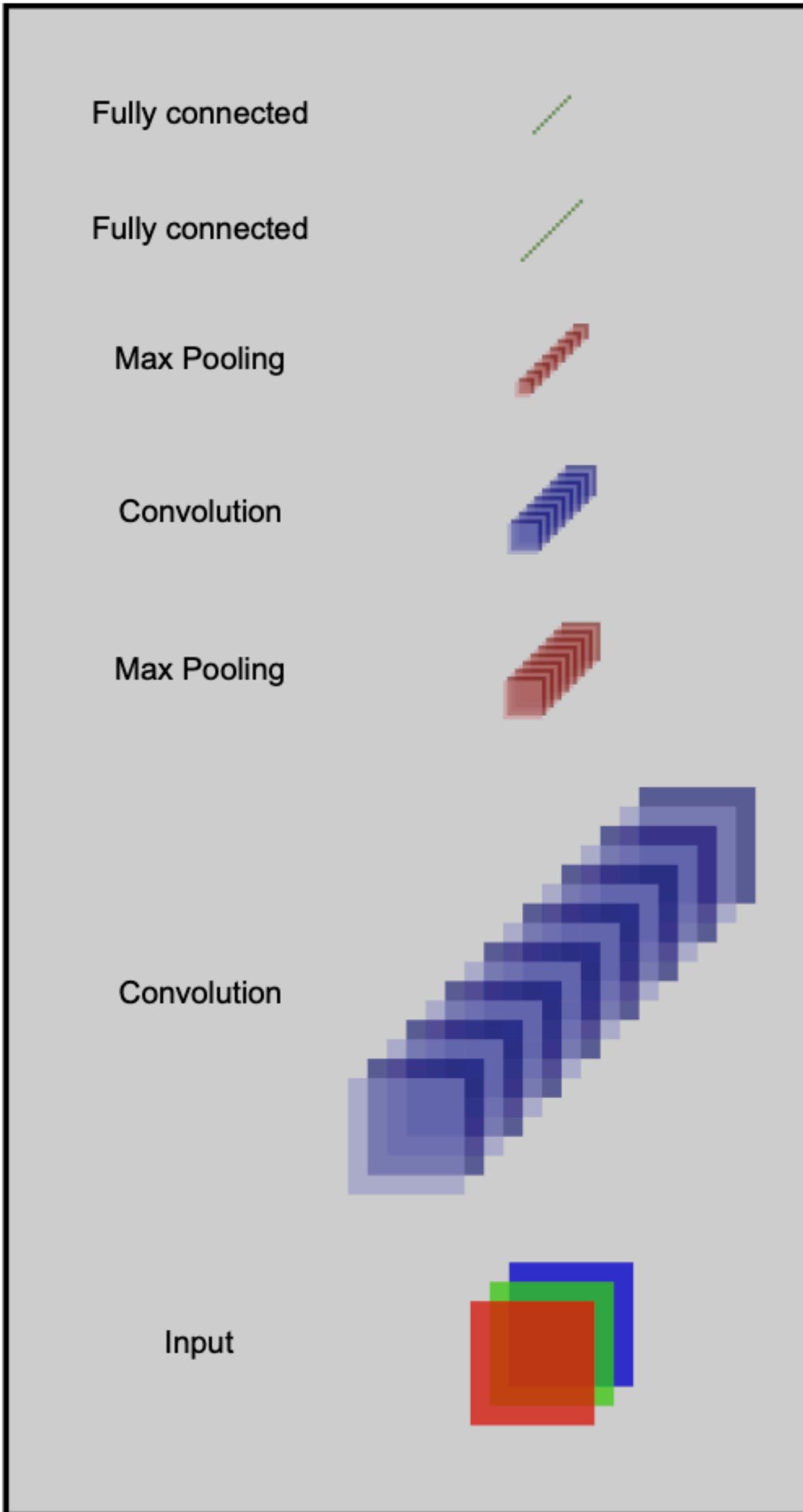
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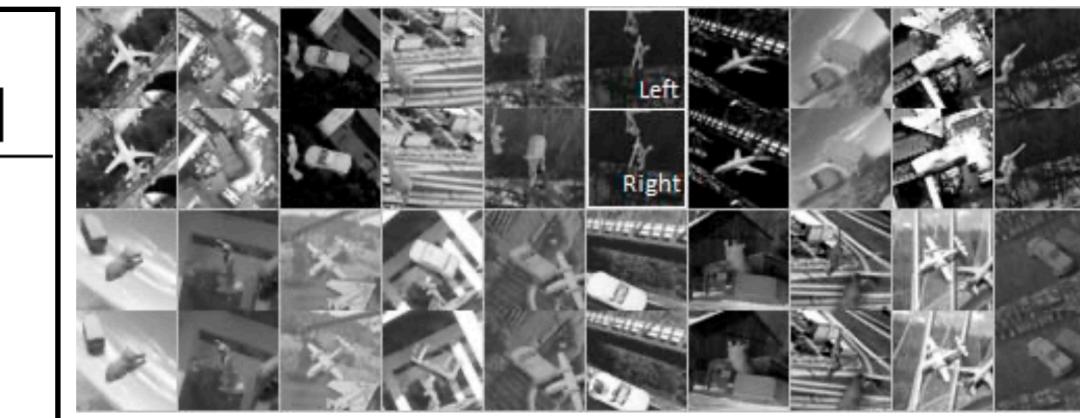


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Multi-column Deep Neural Networks for Image Classification



Method	Results on MNIST dataset.	
	Paper	Error rate[%]
CNN	[32]	0.40
CNN	[26]	0.39
MLP	[5]	0.35
CNN committee	[6]	0.27
MCDNN	this	0.23



Twenty NORB stereo images
(left image - up, right image - down)

Table 4. Average error rates of MCDNN for all experiments, plus results from the literature. * case insensitive

Data (task)	MCDNN error [%]	Published results Error[%] and paper
all (62)	11.63	
digits (10)	0.77	3.71 [12] 1.88 [23]
letters (52)	21.01	30.91[16]
letters* (26)	7.37	13.00 [4] 13.66[16]
merged (37)	7.99	
uppercase (26)	1.83	10.00 [4] 6.44 [9]
lowercase (26)	7.47	16.00 [4] 13.27 [16]

This is the first time human-competitive results are reported on widely used computer vision benchmarks. On the MNIST handwriting benchmark, this method is the first to achieve near-human performance. On a traffic sign recognition benchmark it outperforms humans by a factor of two.

Dataset	Best result of others [%]	MCDNN [%]	Relative improv. [%]
MNIST	0.39	0.23	41
NIST SD 19	see Table 4	see Table 4	30-80
HWDB1.0 on.	7.61	5.61	26
HWDB1.0 off.	10.01	6.5	35
CIFAR10	18.50	11.21	39
traffic signs	1.69	0.54	72
NORB	5.00	2.70	46

→ Latin characters

→ Chinese characters

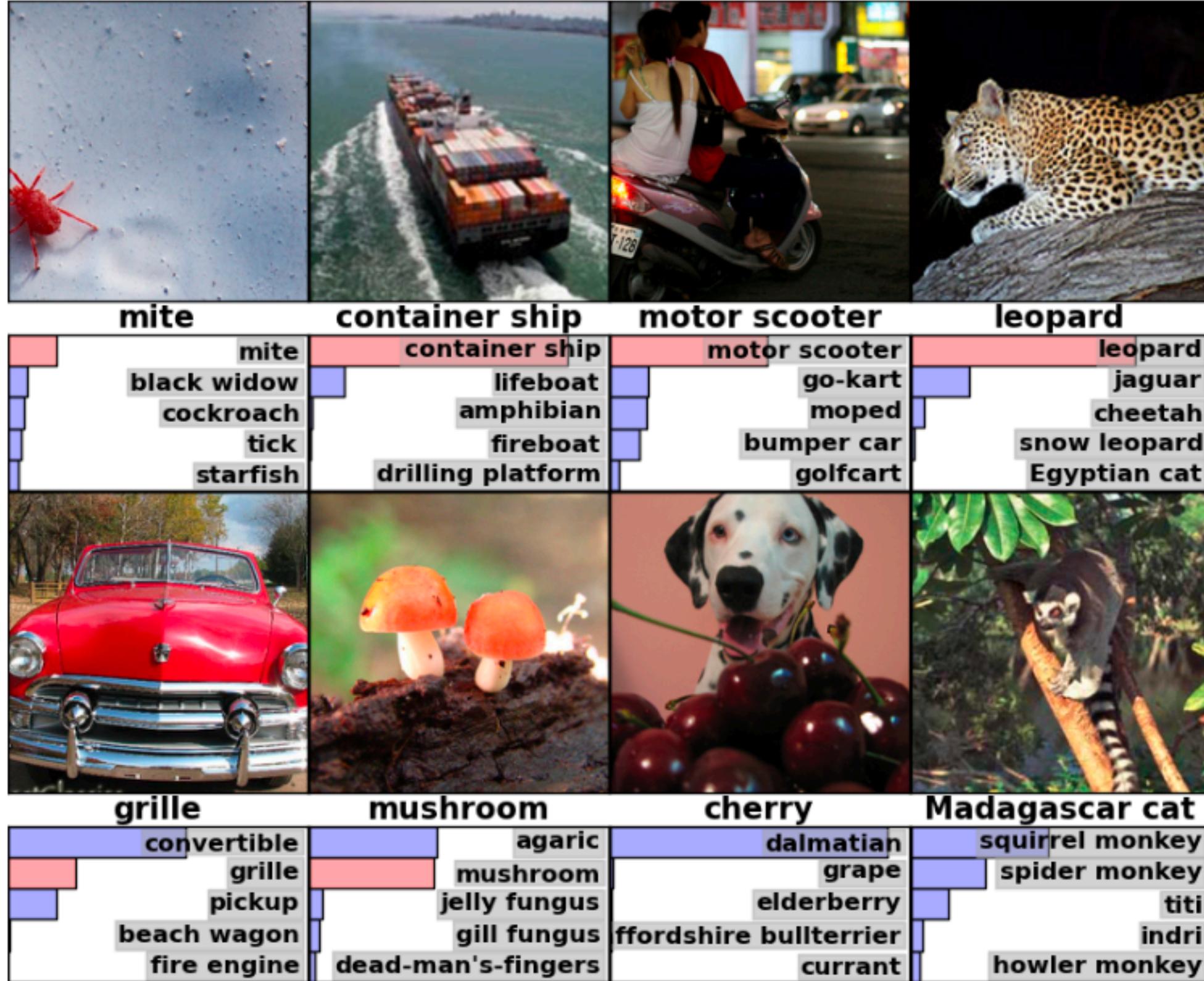


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ImageNet Classification with Deep Convolutional Neural Networks

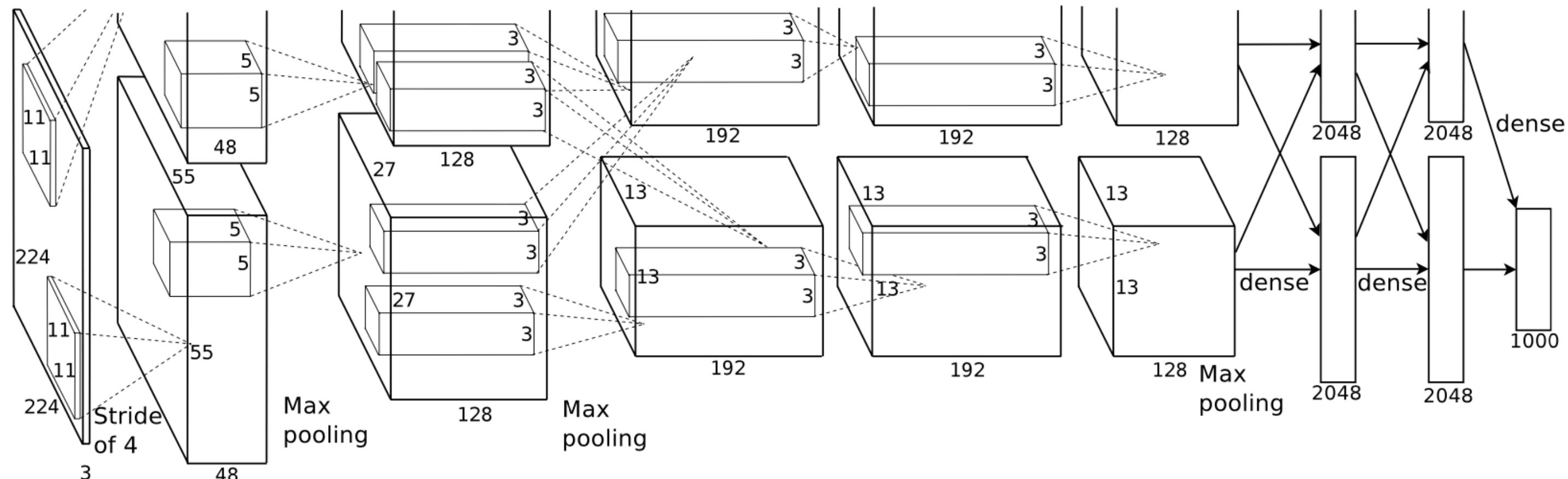


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"To learn about thousands of objects from millions of images, we need a model with a large learning capacity. However, the immense complexity of the object recognition task means that this problem cannot be specified even by a dataset as large as ImageNet, so our model should also have lots of prior knowledge to compensate for all the data we don't have."

"CNNs make strong and mostly correct assumptions about the nature of images (namely, **stationarity of statistics** and **locality of pixel dependencies**)."



Architecture: $I \in \mathbb{R}^{224 \times 224 \times 3} \mapsto p \in \{p \in \mathbb{R}^{1000} : \sum_i p_i = 1, p_i \geq 0 \forall i\}$

1x1 Conv: $X \in \mathbb{R}^{H \times W \times C} \mapsto Y \in \mathbb{R}^{H \times W \times F}$

$$Y(x, y) = WX(x, y), W \in \mathbb{R}^{F \times C}, X(x, y) \in \mathbb{R}^C, Y(x, y) \in \mathbb{R}^F$$

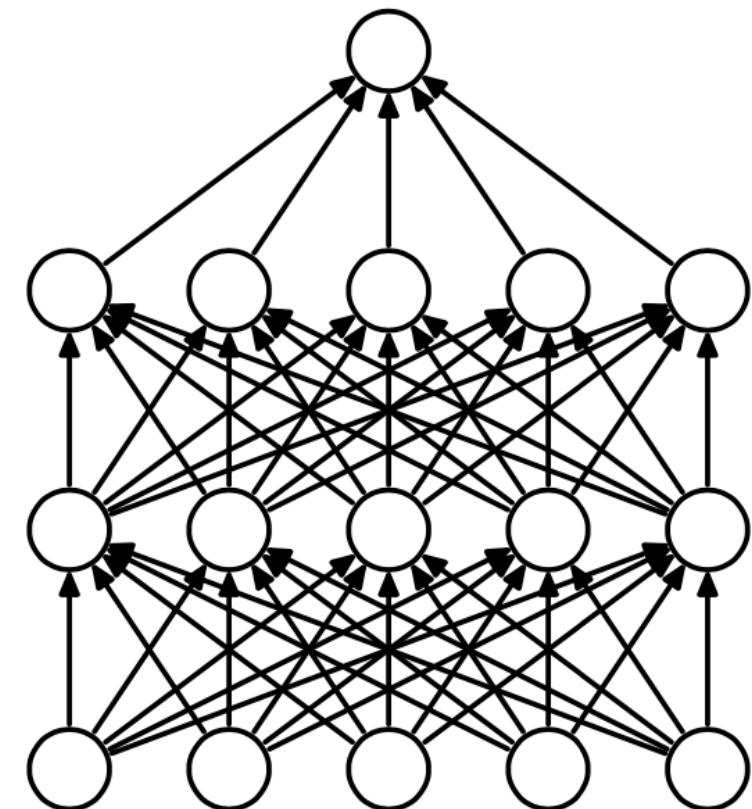
3x3 Conv: $Y(x, y) = \sum_{i=1,2,3} \sum_{j=1,2,3} W(i, j)X(s(x-2)+i, s(y-2)+j), W \in \mathbb{R}^{3 \times 3 \times F \times C}$

Krizhevsky, Alex, Ilya Sutskever, and Geoffrey E. Hinton. "Imagenet classification with deep convolutional neural networks." *Advances in neural information processing systems*. 2012.

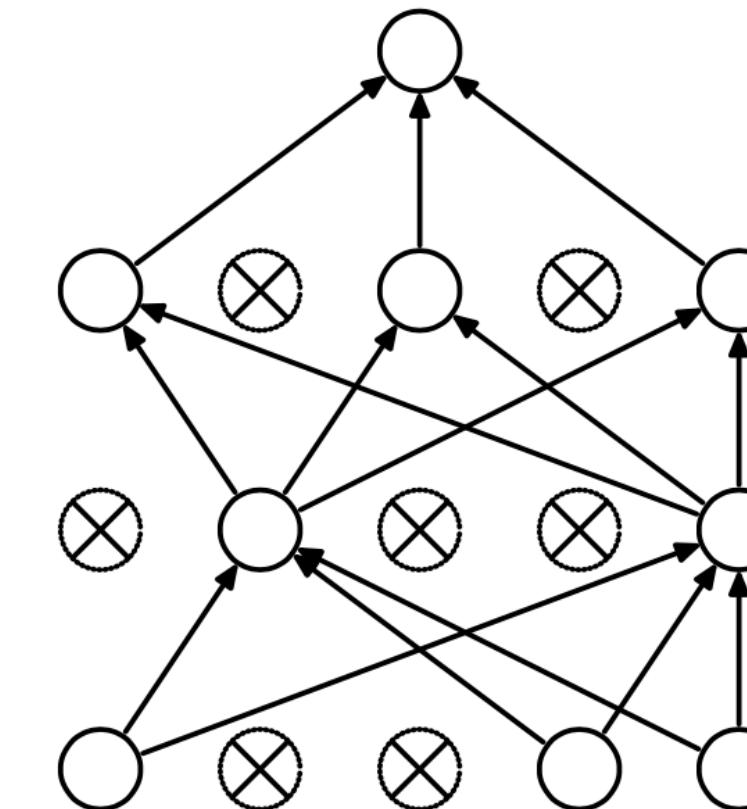
$$\text{Loss: } \mathcal{L}(\theta) = - \sum_{n=1}^N \log(p_{i_n}(I_n))$$

Model	Top-1	Top-5
Sparse coding [2]	47.1%	28.2%
SIFT + FVs [24]	45.7%	25.7%
CNN	37.5%	17.0%

Dropout: A Simple Way to Prevent Neural Networks from Overfitting

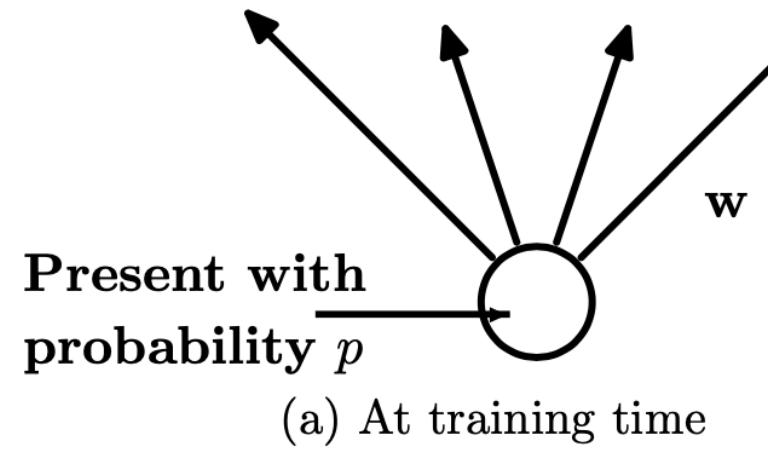

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(a) Standard Neural Net

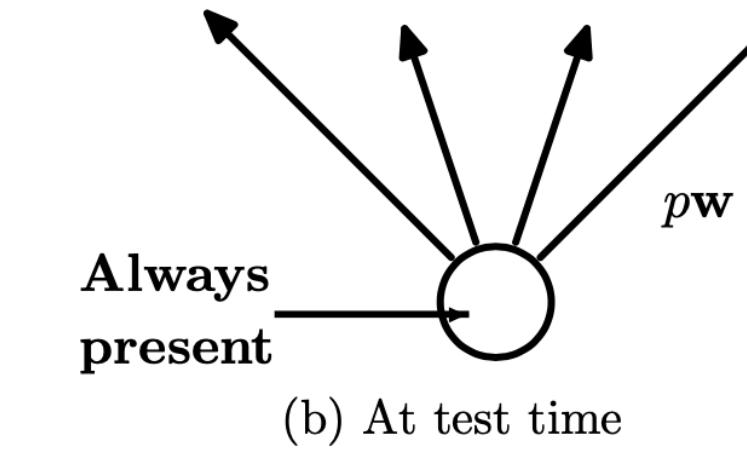


(b) After applying dropout.

1. Prevent overfitting (regularization technique)
2. Approximately combine exponentially many different neural network architectures efficiently



(a) At training time

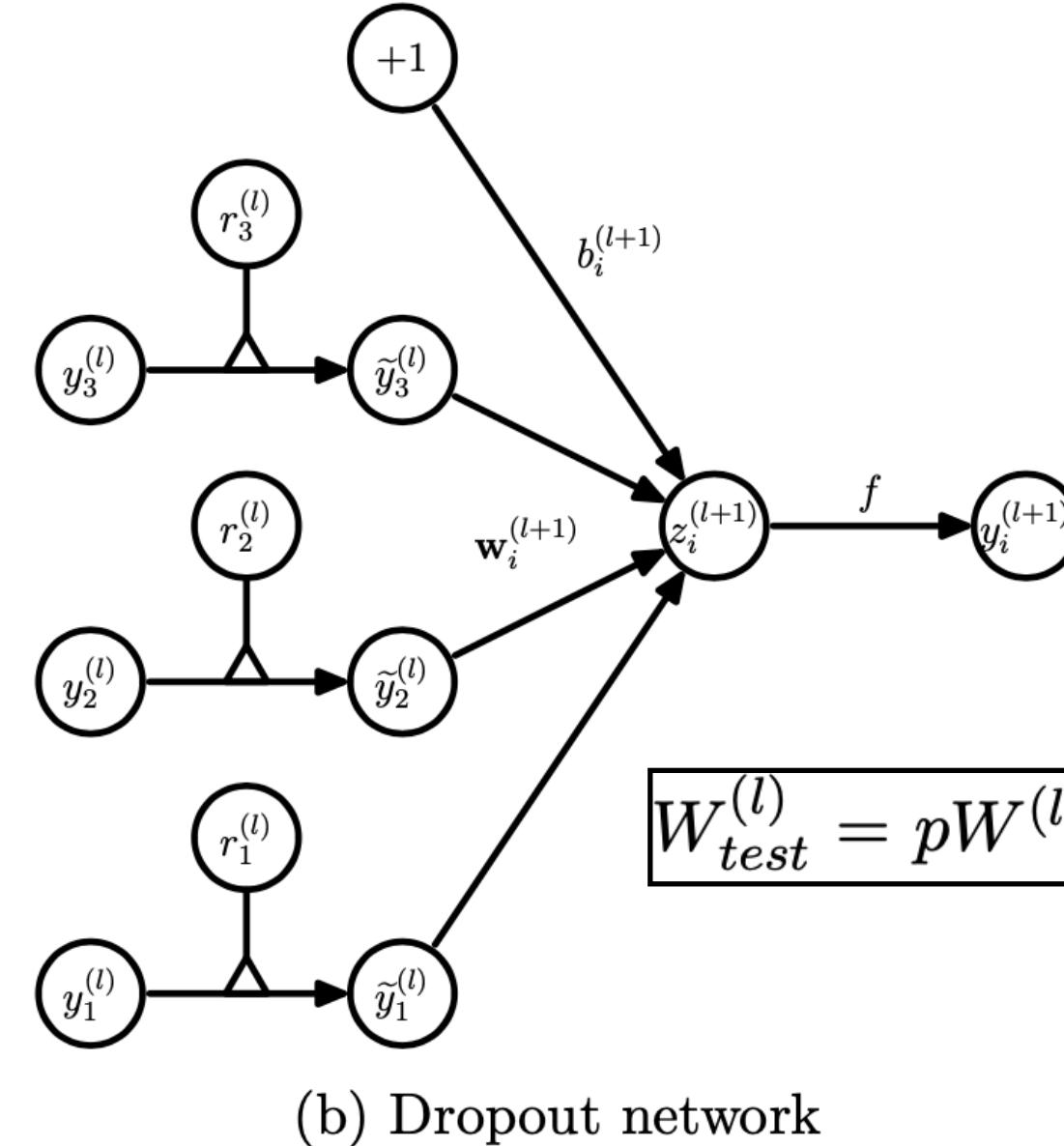
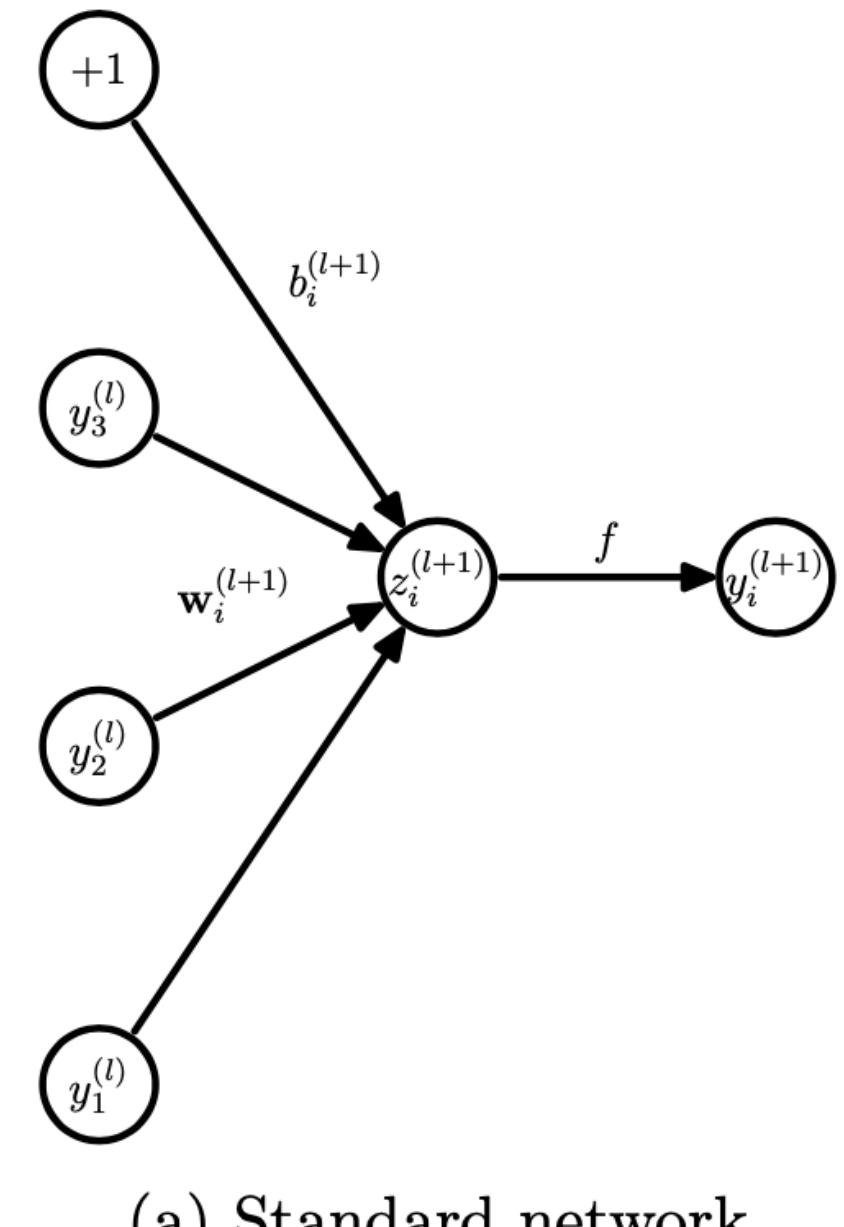


(b) At test time

Each hidden unit in a neural network trained with dropout must learn to work with a randomly chosen sample of other units
 $L \rightarrow$ number of hidden layers
 $l \in \{1, \dots, L\}$

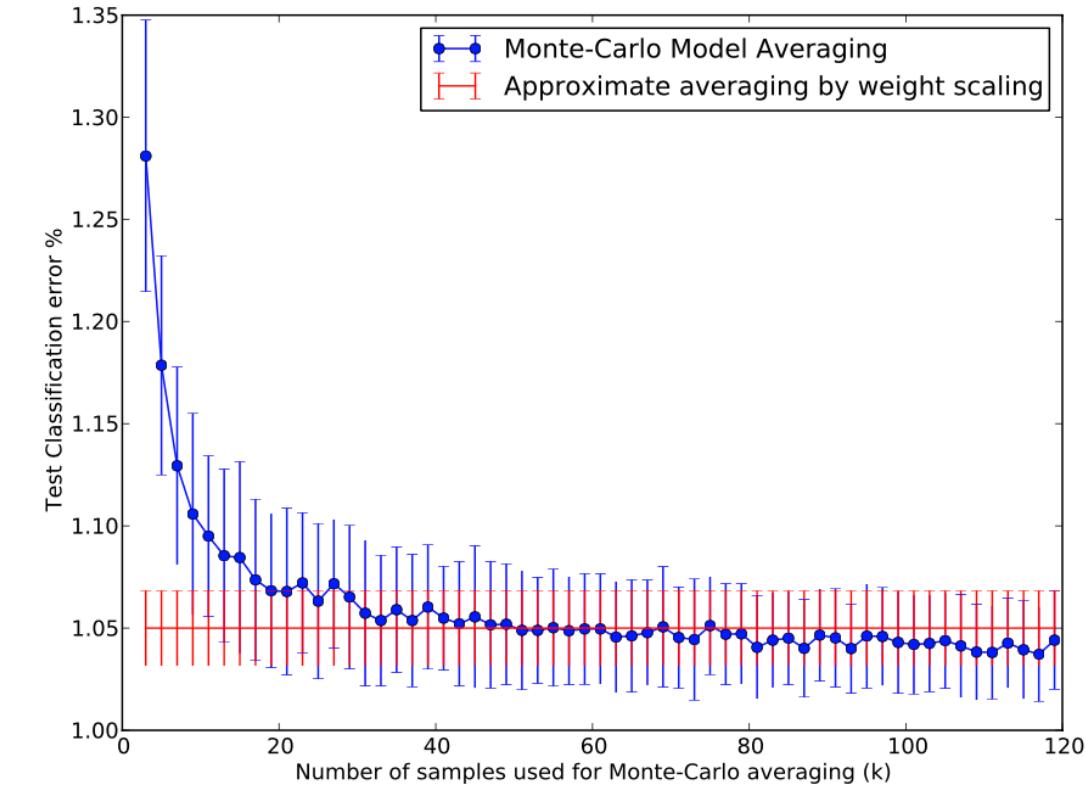
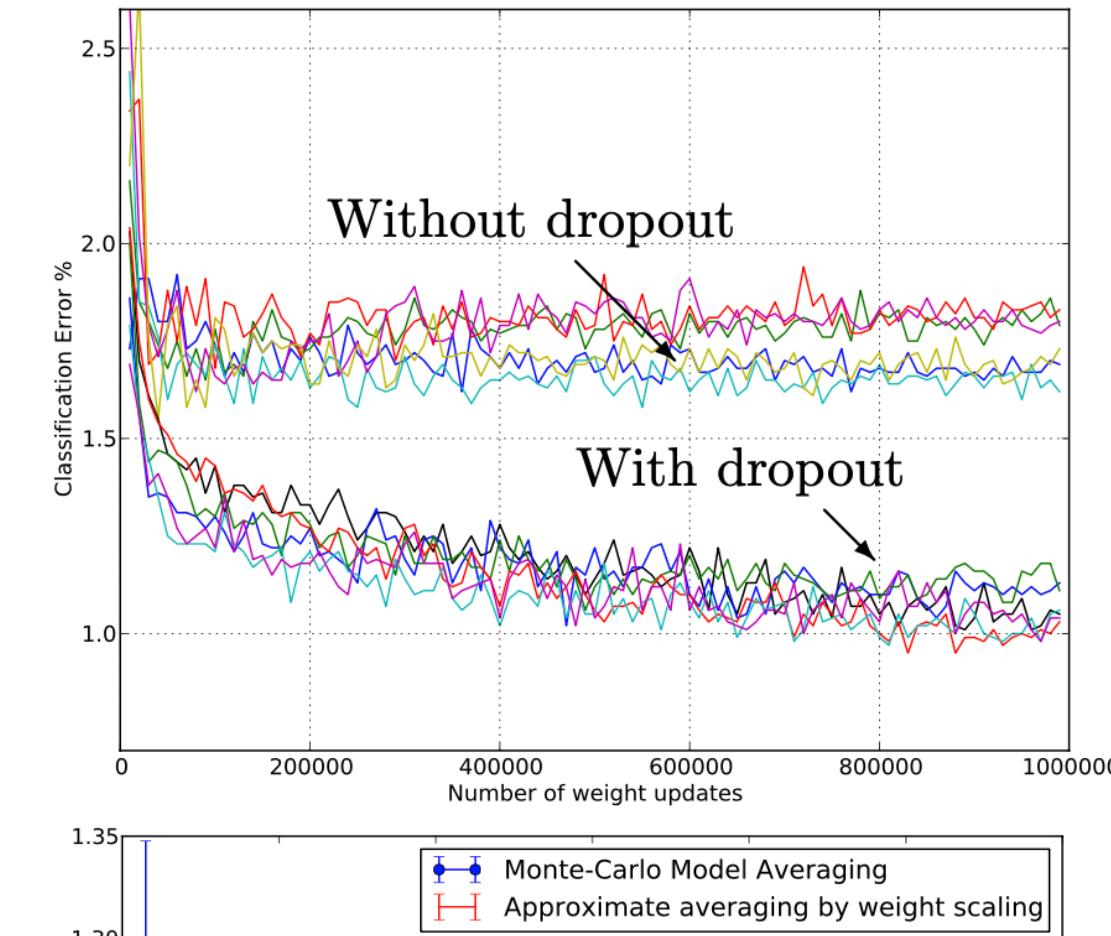
$$l \in \{1, \dots, L\}$$

$$\begin{aligned} z_i^{(l+1)} &= \mathbf{w}_i^{(l+1)} \mathbf{y}^l + b_i^{(l+1)}, \\ y_i^{(l+1)} &= f(z_i^{(l+1)}), \\ r_j^{(l)} &\sim \text{Bernoulli}(p), \\ \tilde{\mathbf{y}}^{(l)} &= \mathbf{r}^{(l)} * \mathbf{y}^{(l)}, \\ z_i^{(l+1)} &= \mathbf{w}_i^{(l+1)} \tilde{\mathbf{y}}^{(l)} + b_i^{(l+1)}, \\ y_i^{(l+1)} &= f(z_i^{(l+1)}). \end{aligned}$$



$\|\mathbf{w}\|_2 \leq c$
max-norm regularization

Dropout with linear regression is equivalent to ridge regression.



Data Set	Domain	Dimensionality	Training Set	Test Set
MNIST	Vision	784 (28×28 grayscale)	60K	10K
SVHN	Vision	3072 (32×32 color)	600K	26K
CIFAR-10/100	Vision	3072 (32×32 color)	60K	10K
ImageNet (ILSVRC-2012)	Vision	65536 (256×256 color)	1.2M	150K
TIMIT	Speech	2520 (120-dim, 21 frames)	1.1M frames	58K frames
Reuters-RCV1	Text	2000	200K	200K
Alternative Splicing	Genetics	1014	2932	733

Hinton, Geoffrey E., et al. "Improving neural networks by preventing co-adaptation of feature detectors." *arXiv preprint arXiv:1207.0580* (2012).

Srivastava, Nitish, et al. "Dropout: a simple way to prevent neural networks from overfitting." *The journal of machine learning research* 15.1 (2014): 1929-1958.



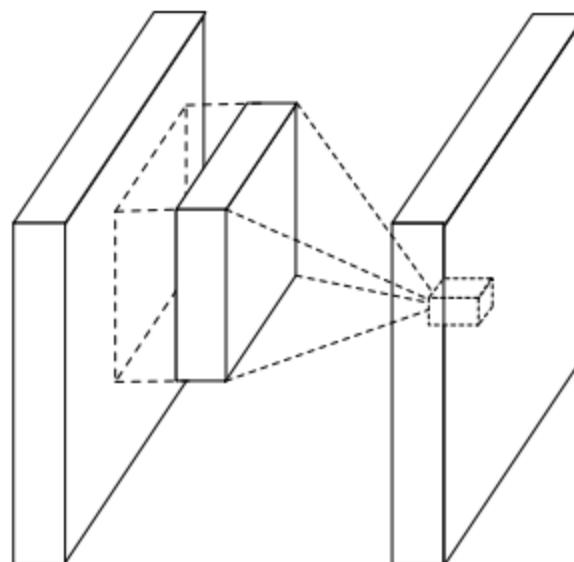
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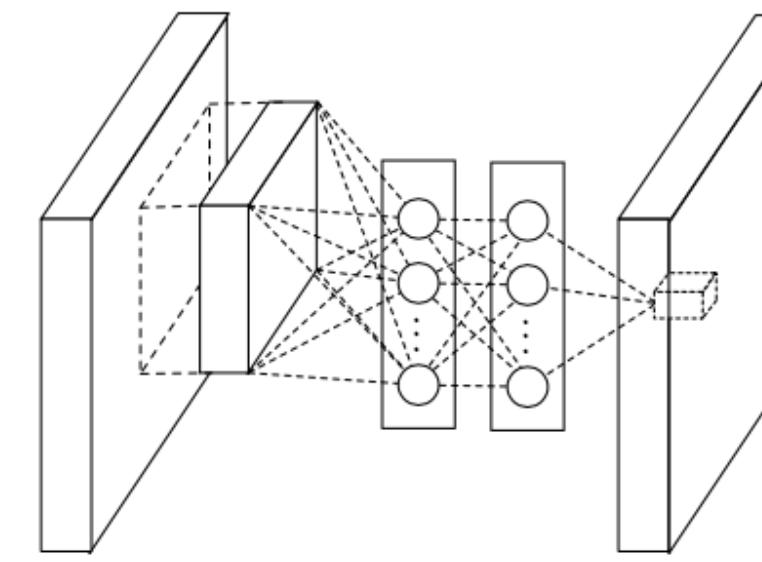
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Network In Network

1) MLP Convolution Layers (1x1 Convolutions)



(a) Linear convolution layer



(b) Mlpconv layer

$$\begin{aligned}
 f_{i,j,k_1}^1 &= \max(w_{k_1}^1{}^T x_{i,j} + b_{k_1}, 0). && \text{Input patch centered at location } (i, j) \\
 &\vdots && \text{ReLU activation function} \\
 f_{i,j,k_n}^n &= \max(w_{k_n}^n{}^T f_{i,j}^{n-1} + b_{k_n}, 0).
 \end{aligned}$$

k_ℓ is used to index the channels of the feature map

2) Global Average Pooling

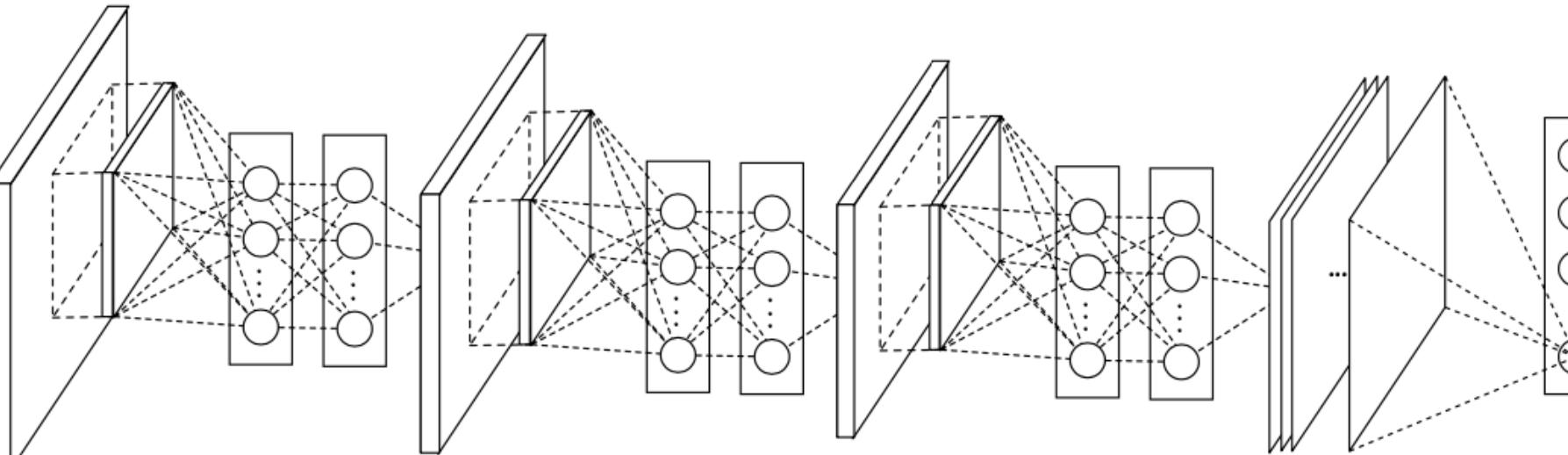


Table 3: Test set error rates for SVHN of various methods.

Method	Test Error
Stochastic Pooling [11]	2.80%
Rectifier + Dropout [18]	2.78%
Rectifier + Dropout + Synthetic Translation [18]	2.68%
Conv. maxout + Dropout [8]	2.47%
NIN + Dropout	2.35%
Multi-digit Number Recognition [19]	2.16%
DropConnect [15]	1.94%

Table 1: Test set error rates for CIFAR-10 of various methods.

Method	Test Error
Stochastic Pooling [11]	15.13%
CNN + Spearmint [14]	14.98%
Conv. maxout + Dropout [8]	11.68%
NIN + Dropout	10.41%
CNN + Spearmint + Data Augmentation [14]	9.50%
Conv. maxout + Dropout + Data Augmentation [8]	9.38%
DropConnect + 12 networks + Data Augmentation [15]	9.32%
NIN + Dropout + Data Augmentation	8.81%

Table 2: Test set error rates for CIFAR-100 of various methods.

Method	Test Error
Learned Pooling [16]	43.71%
Stochastic Pooling [11]	42.51%
Conv. maxout + Dropout [8]	38.57%
Tree based priors [17]	36.85%
NIN + Dropout	35.68%

Table 4: Test set error rates for MNIST of various methods.

Method	Test Error
2-Layer CNN + 2-Layer NN [11]	0.53%
Stochastic Pooling [11]	0.47%
NIN + Dropout	0.47%
Conv. maxout + Dropout [8]	0.45%

Table 5: Global average pooling compared to fully connected layer.

Method	Testing Error
mlpconv + Fully Connected	11.59%
mlpconv + Fully Connected + Dropout	10.88%
mlpconv + Global Average Pooling	10.41%



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Very Deep Convolutional Networks for Large-Scale Image Recognition



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ConvNet Configuration					
A	A-LRN	B	C	D	E
11 weight layers	11 weight layers	13 weight layers	16 weight layers	16 weight layers	19 weight layers
input (224×224 RGB image)					
conv3-64	conv3-64 LRN	conv3-64 conv3-64	conv3-64 conv3-64	conv3-64 conv3-64	conv3-64 conv3-64
maxpool					
conv3-128	conv3-128	conv3-128 conv3-128	conv3-128 conv3-128	conv3-128 conv3-128	conv3-128 conv3-128
maxpool					
conv3-256 conv3-256	conv3-256 conv3-256	conv3-256 conv3-256	conv3-256 conv3-256 conv1-256	conv3-256 conv3-256 conv3-256	conv3-256 conv3-256 conv3-256
maxpool					
conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512 conv1-512	conv3-512 conv3-512 conv3-512	conv3-512 conv3-512 conv3-512
maxpool					
conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512 conv1-512	conv3-512 conv3-512 conv3-512	conv3-512 conv3-512 conv3-512
maxpool					
FC-4096					
FC-4096					
FC-1000					
soft-max					

Local Response Normalization (LRN):

Create competition for big activities amongst neuron outputs computed using different kernels.

$$b_{x,y}^i = a_{x,y}^i / \left(k + \alpha \sum_{j=\max(0,i-n/2)}^{\min(N-1,i+n/2)} (a_{x,y}^j)^2 \right)^\beta$$

activity of a neuron computed by applying kernel i
at position (x, y) and then applying the ReLU nonlinearity

$$k = 2, n = 5, \alpha = 10^{-4}, \beta = 0.75$$

Data Augmentation:

- 1) Image translations (random crops) and horizontal reflection
- 2) Altering the intensities of the RGB channels in training images (object identity is invariant to changes in the intensity and color of the illumination)

$$I_{xy} = [I_{xy}^R, I_{xy}^G, I_{xy}^B]^T + [p_1, p_2, p_3] [\underbrace{\alpha_1 \lambda_1, \alpha_2 \lambda_2, \alpha_3 \lambda_3}_\text{eigenvector and eigenvalue of the } 3 \times 3 \text{ covariance matrix of RGB pixel values}]^T$$

3) Scale jittering

Each training image is individually rescaled by randomly sampling S from a certain range $[S_{\min}, S_{\max}]$
 S is the smallest side of an isotropically-rescaled training image.

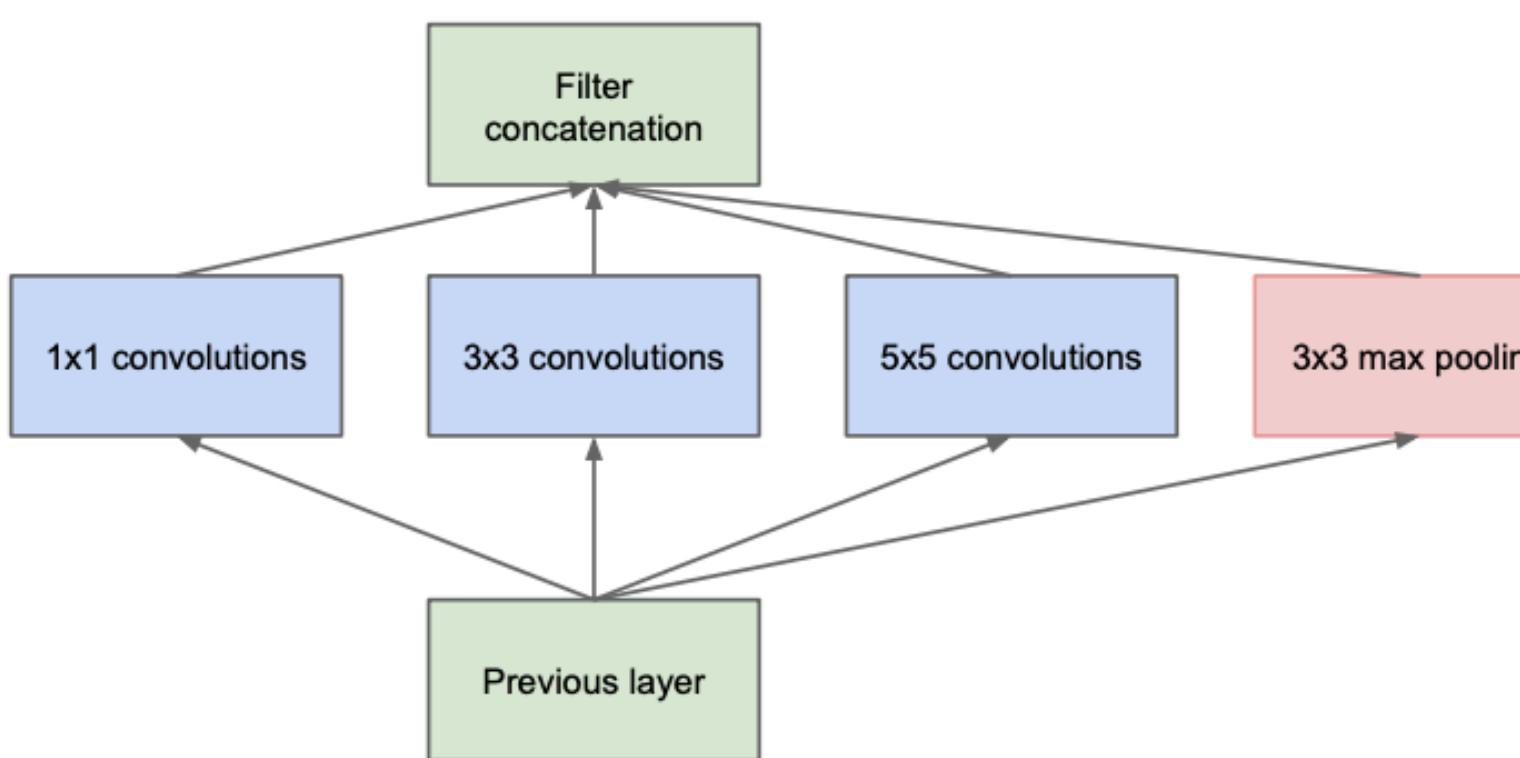


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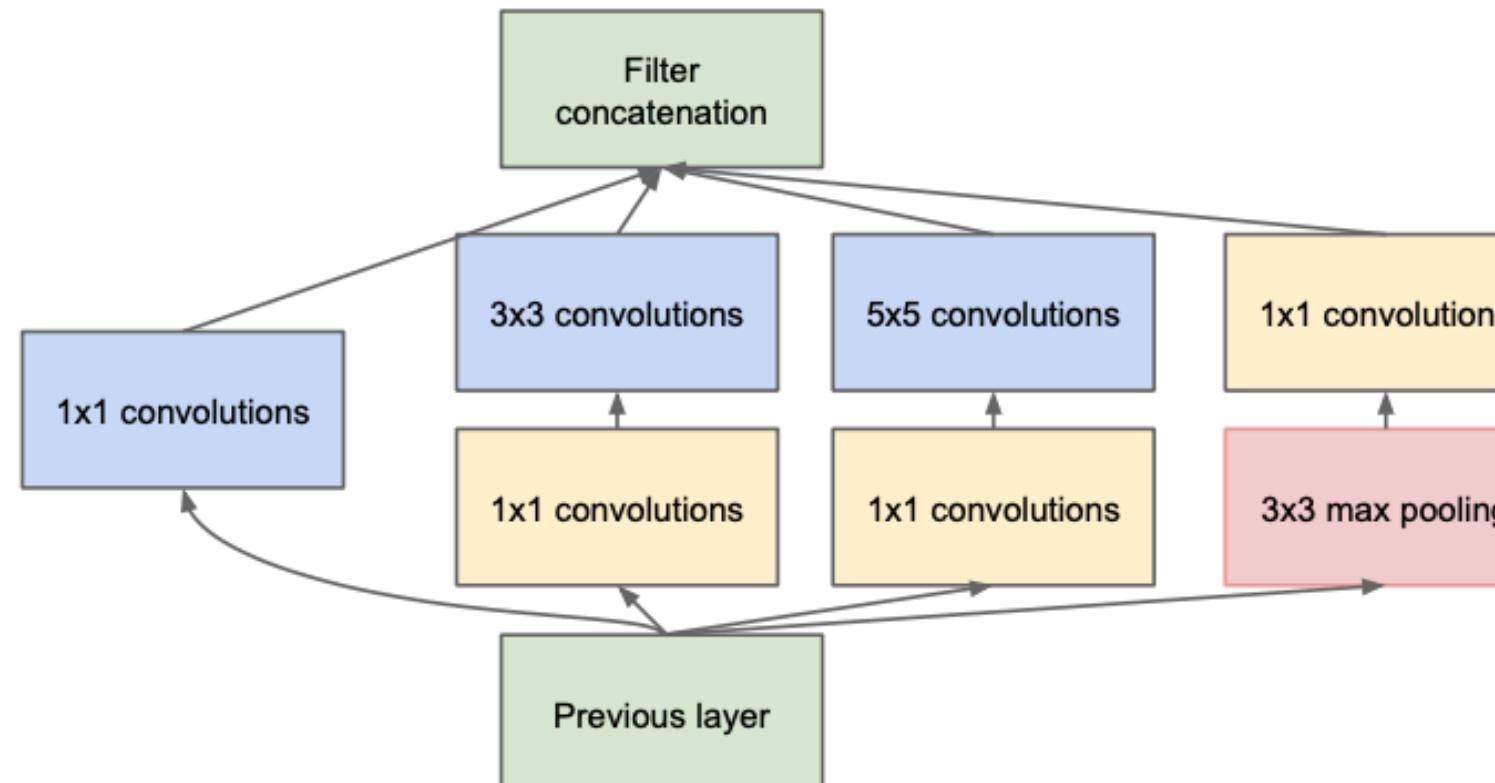


Going Deeper with Convolutions

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(a) Inception module, naïve version



(b) Inception module with dimensionality reduction

type	patch size/stride	output size	depth	#1x1	#3x3 reduce	#3x3	#5x5 reduce	#5x5	pool proj	params	ops
convolution	7×7/2	112×112×64	1							2.7K	34M
max pool	3×3/2	56×56×64	0								
convolution	3×3/1	56×56×192	2		64	192				112K	360M
max pool	3×3/2	28×28×192	0								
inception (3a)		28×28×256	2	64	96	128	16	32	32	159K	128M
inception (3b)		28×28×480	2	128	128	192	32	96	64	380K	304M
max pool	3×3/2	14×14×480	0								
inception (4a)		14×14×512	2	192	96	208	16	48	64	364K	73M
inception (4b)		14×14×512	2	160	112	224	24	64	64	437K	88M
inception (4c)		14×14×512	2	128	128	256	24	64	64	463K	100M
inception (4d)		14×14×528	2	112	144	288	32	64	64	580K	119M
inception (4e)		14×14×832	2	256	160	320	32	128	128	840K	170M
max pool	3×3/2	7×7×832	0								
inception (5a)		7×7×832	2	256	160	320	32	128	128	1072K	54M
inception (5b)		7×7×1024	2	384	192	384	48	128	128	1388K	71M
avg pool	7×7/1	1×1×1024	0								
dropout (40%)		1×1×1024	0								
linear		1×1×1000	1							1000K	1M
softmax		1×1×1000	0								

Table 1: GoogLeNet incarnation of the Inception architecture.

Batch Normalization: Accelerating Deep Network Training by Reducing Internal Covariate Shift


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statistics of layer's inputs change during training

$$z = g(Wu + b)$$

replace

$$z = g(\underbrace{\text{BN}(Wu)}_x)$$

Training

$$\text{BN}(x; \gamma, \beta, \underbrace{x_1, x_2, \dots, x_m}_{\text{mini-batch}}) = \gamma \frac{x - \overbrace{\mu(x_1, \dots, x_m)}^{\text{mean}}}{\sqrt{\overbrace{\sigma^2(x_1, \dots, x_m)}^{\text{variance}}} + \epsilon} + \beta$$

$$x \in \{x_1, x_2, \dots, x_m\}$$

$$x_i, \gamma, \beta, \mu, \sigma \in \mathbb{R}^d$$

All operations are element-wise (dimension-wise)

The gradients flow through μ and σ^2

Inference

$$\text{BN}(x; \gamma, \beta) = \gamma \frac{x - \mu}{\sqrt{\sigma^2 + \epsilon}} + \beta$$

$$\mu = \mathbb{E}_{x_1, \dots, x_m} [\mu(x_1, \dots, x_m)]$$

$$\sigma^2 = \frac{m}{m-1} \mathbb{E}_{x_1, \dots, x_m} [\sigma^2(x_1, \dots, x_m)]$$

Convolution

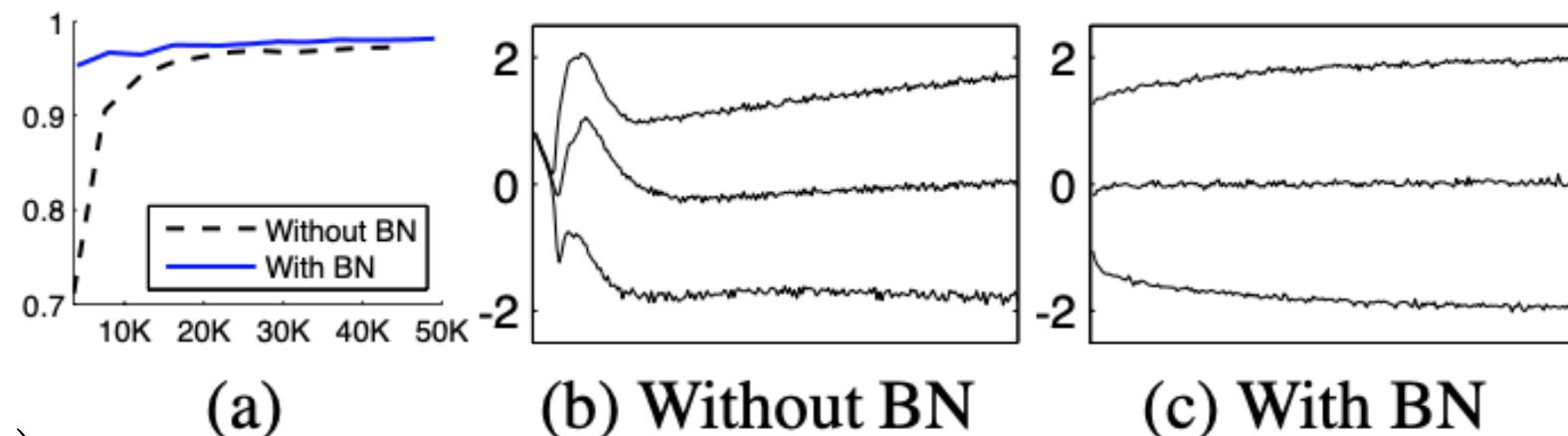
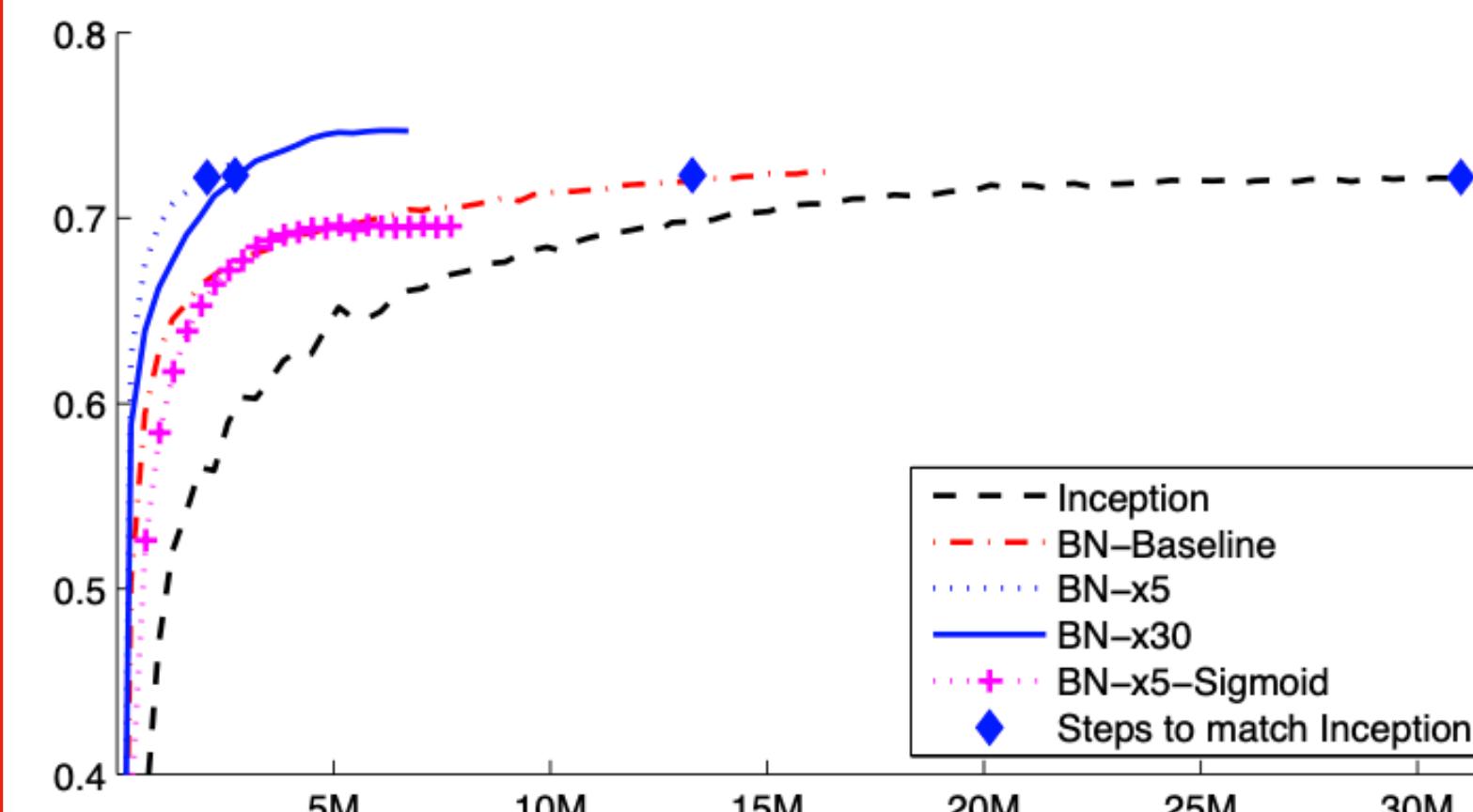
$$X \in \mathbb{R}^{p \times q \times d}$$

$$m' = mpq \rightarrow \text{effective mini-batch size}$$

Batch-norm is done per feature map

Benefits of batch normalization:

1. higher learning rate
2. less sensitive to initialization
3. less sensitive to activation functions
4. regularization effects
5. preserve gradient magnitudes (maybe)



Evolution of input distributions to a typical sigmoid, over the course of training, shown as {15, 50, 85}th percentiles.

Remove Dropout

Increase learning rate

Accelerate the learning rate decay

Reduce the L2 weight regularization

Remove Local Response Normalization

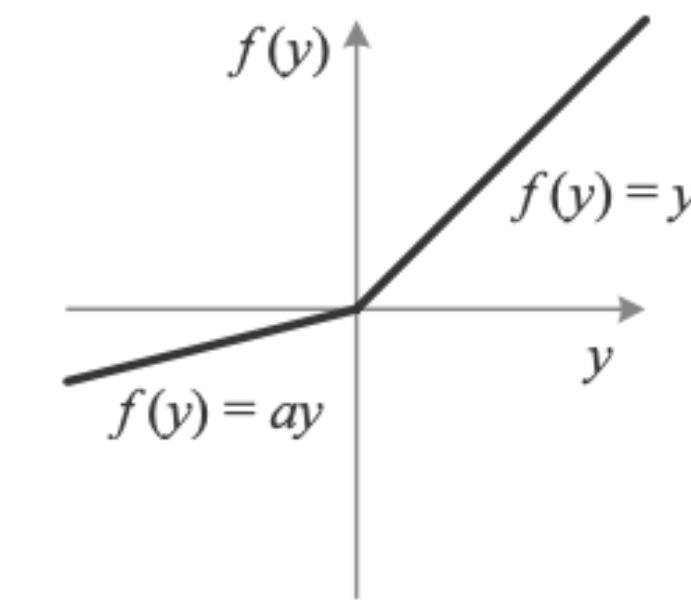
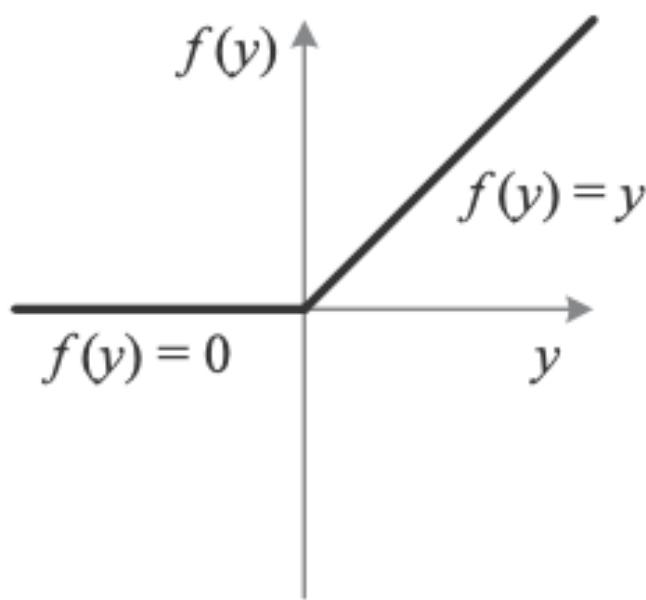
Shuffle training examples more thoroughly

Model	Resolution	Crops	Models	Top-1 error	Top-5 error
GoogLeNet ensemble	224	144	7	-	6.67%
Deep Image low-res	256	-	1	-	7.96%
Deep Image high-res	512	-	1	24.88	7.42%
Deep Image ensemble	variable	-	-	-	5.98%
BN-Inception single crop	224	1	1	25.2%	7.82%
BN-Inception multicrop	224	144	1	21.99%	5.82%
BN-Inception ensemble	224	144	6	20.1%	4.9%*

Delving Deep into Rectifiers: Surpassing Human-Level Performance on ImageNet Classification


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Parametric Rectifiers



$f(y_i) = \max(0, y_i) + a_i \min(0, y_i) \rightarrow$ Parametric ReLU

$a_i = 0 \rightarrow$ ReLU

$a_i = 0.01 \rightarrow$ Leaky ReLU

$i \rightarrow$ indicates different channels

$$\Delta a_i \leftarrow \mu \Delta a_i + \epsilon \frac{\partial \mathcal{E}}{\partial a_i} \quad \text{momentum method}$$

	top-1	top-5
ReLU	33.82	13.34
LReLU ($a = 0.25$)	33.80	13.56
PReLU, channel-shared	32.71	12.87
PReLU, channel-wise	32.64	12.75

Initialization of Filter Weights for Rectifiers

Central idea: investigate the variance of the responses in each layer

$$y_\ell = W_\ell x_\ell + b_\ell$$

$x_\ell \in \mathbb{R}^n \rightarrow$ co-located $k \times k$ pixels in c input channels

$k \rightarrow$ spatial filter size of the layer
 $n = k^2 c \rightarrow$ number of connections of a response
 $d \rightarrow$ number of filters
 $W_\ell \in \mathbb{R}^{d \times n} \rightarrow$ weights of d filters
 $b_\ell \in \mathbb{R}^d \rightarrow$ bias
 $y_\ell \rightarrow$ response at a pixel of the output map
 $x_\ell = f(y_{\ell-1})$
 $c_\ell = d_{\ell-1}$

$$\text{Var}[y_\ell] = n_\ell \text{Var}[w_\ell x_\ell] \underset{\mathbb{E}[w_\ell]=0}{=} n_\ell \underset{=\mathbb{E}[w_\ell^2]}{\text{Var}[w_\ell]} \underset{\neq \text{Var}[x_\ell]}{\mathbb{E}[x_\ell^2]}$$

$y_\ell \rightarrow$ random variable of each element in y_ℓ
 $w_\ell \rightarrow$ random variable of each element in W_ℓ
 $x_\ell \rightarrow$ random variable of each element in x_ℓ

$$\mathbb{E}[x_\ell^2] = \frac{1}{2} \text{Var}[y_{\ell-1}] \text{ for ReLU}$$

$$\text{Var}[y_\ell] = \frac{1}{2} n_\ell \text{Var}[y_{\ell-1}] \text{Var}[w_\ell]$$

$$\text{Var}[y_L] = \text{Var}[y_1] \left(\prod_{\ell=2}^L \frac{1}{2} n_\ell \text{Var}[w_\ell] \right)$$

$$\underbrace{\frac{1}{2} n_\ell \text{Var}[w_\ell]}_{=1} \implies w_\ell \sim \mathcal{N}(0, \sqrt{\frac{2}{n_\ell}})$$

Backward Propagation

$$\Delta x_\ell = \hat{W}_\ell \Delta y_\ell$$

$$\Delta x_\ell = \frac{\partial \mathcal{E}}{\partial x_\ell} \in \mathbb{R}^c$$

gradient at a pixel of this layer

$$\Delta y_\ell = \frac{\partial \mathcal{E}}{\partial y_\ell} \in \mathbb{R}^{\hat{n}}$$

$\hat{k} \times k$ pixels in d channels

$$\hat{W}_\ell \in \mathbb{R}^{c \times \hat{n}} \rightarrow$$
 reshaped from W_ℓ

$$\Delta y_\ell = f'(y_\ell) \Delta x_{\ell+1}$$

$$\mathbb{E}[\Delta y_\ell] = \frac{1}{2} \mathbb{E}[\Delta x_{\ell+1}]$$

for ReLU $f'(y_\ell) = 0$ or 1 with equal prob.

$$\mathbb{E}[\Delta y_\ell^2] = \text{Var}[\Delta y_\ell] = \frac{1}{2} \text{Var}[\Delta x_{\ell+1}]$$

$$\text{Var}[\Delta x_\ell] = \hat{n}_\ell \text{Var}[w_\ell] \text{Var}[\Delta y_\ell]$$

$$= \frac{1}{2} \hat{n}_\ell \text{Var}[w_\ell] \text{Var}[\Delta x_{\ell+1}]$$

$$\text{Var}[\Delta x_2] = \text{Var}[\Delta x_{L+1}] \left(\prod_{\ell=2}^L \frac{1}{2} \hat{n}_\ell \text{Var}[w_\ell] \right)$$

if $\ell = 1, \forall \ell$

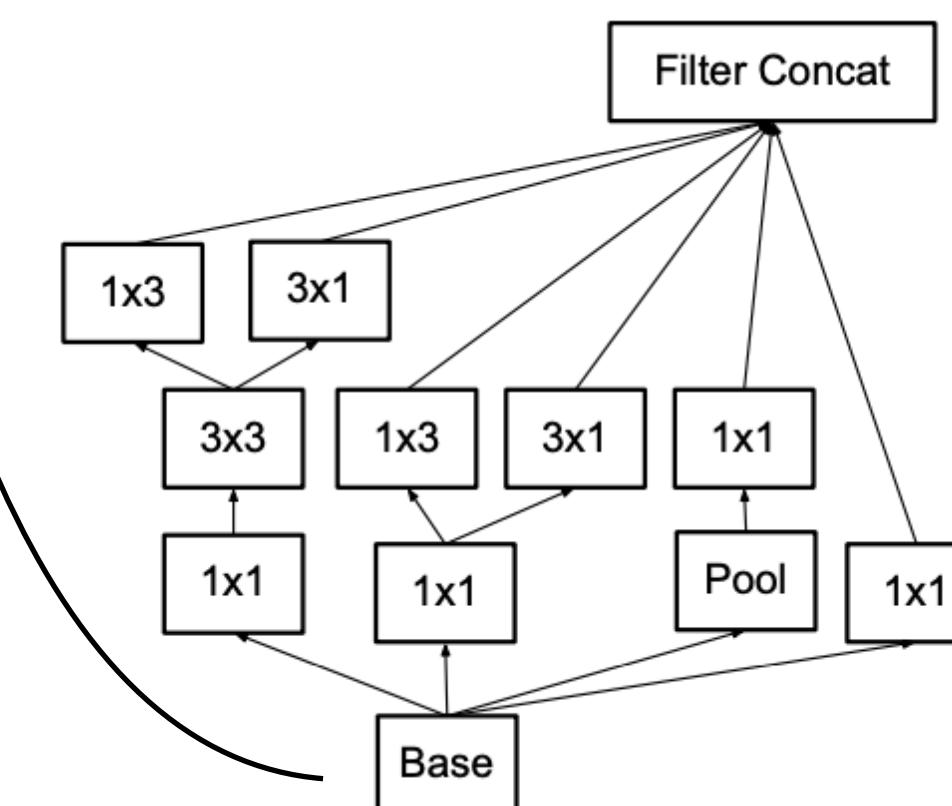


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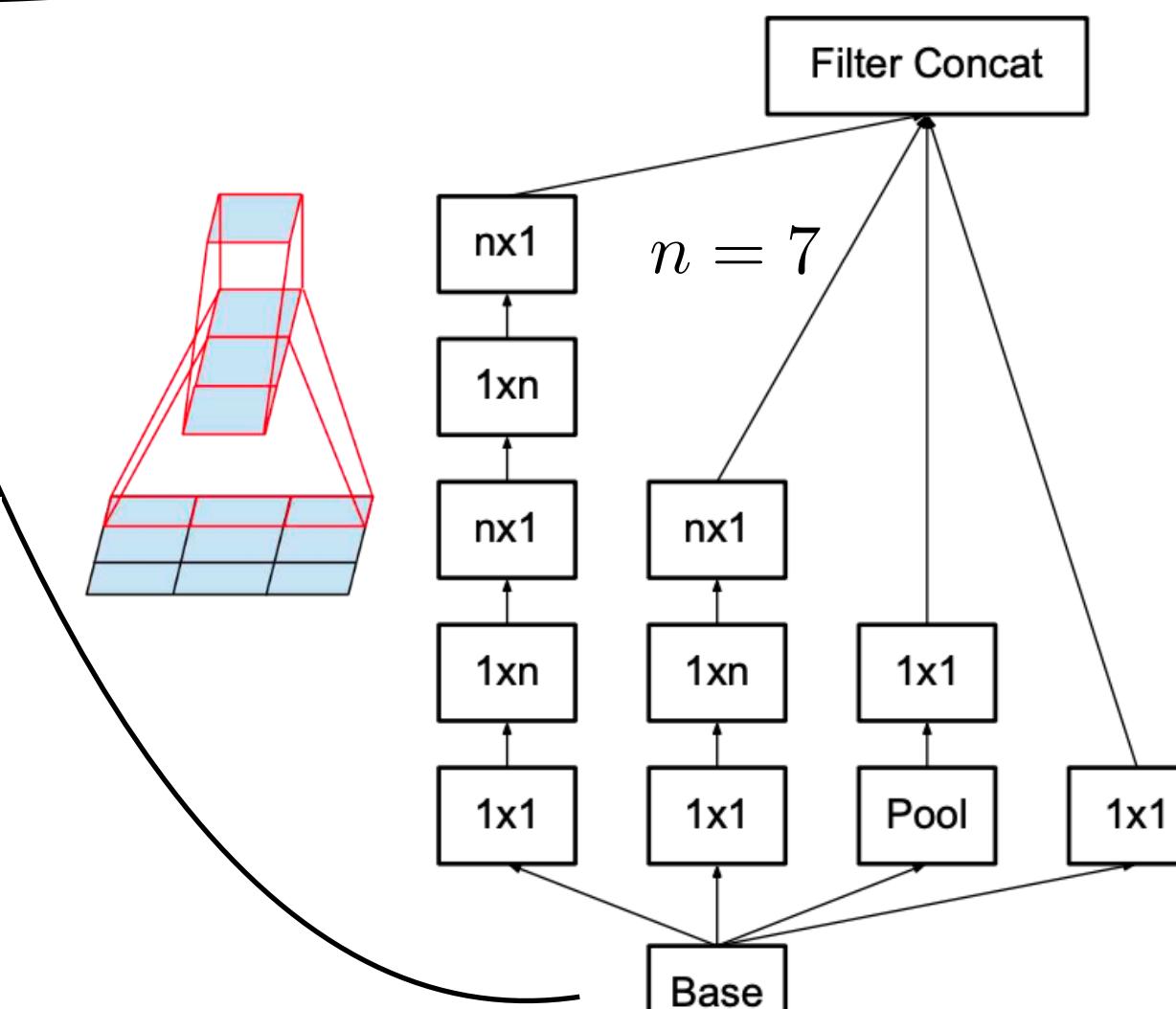
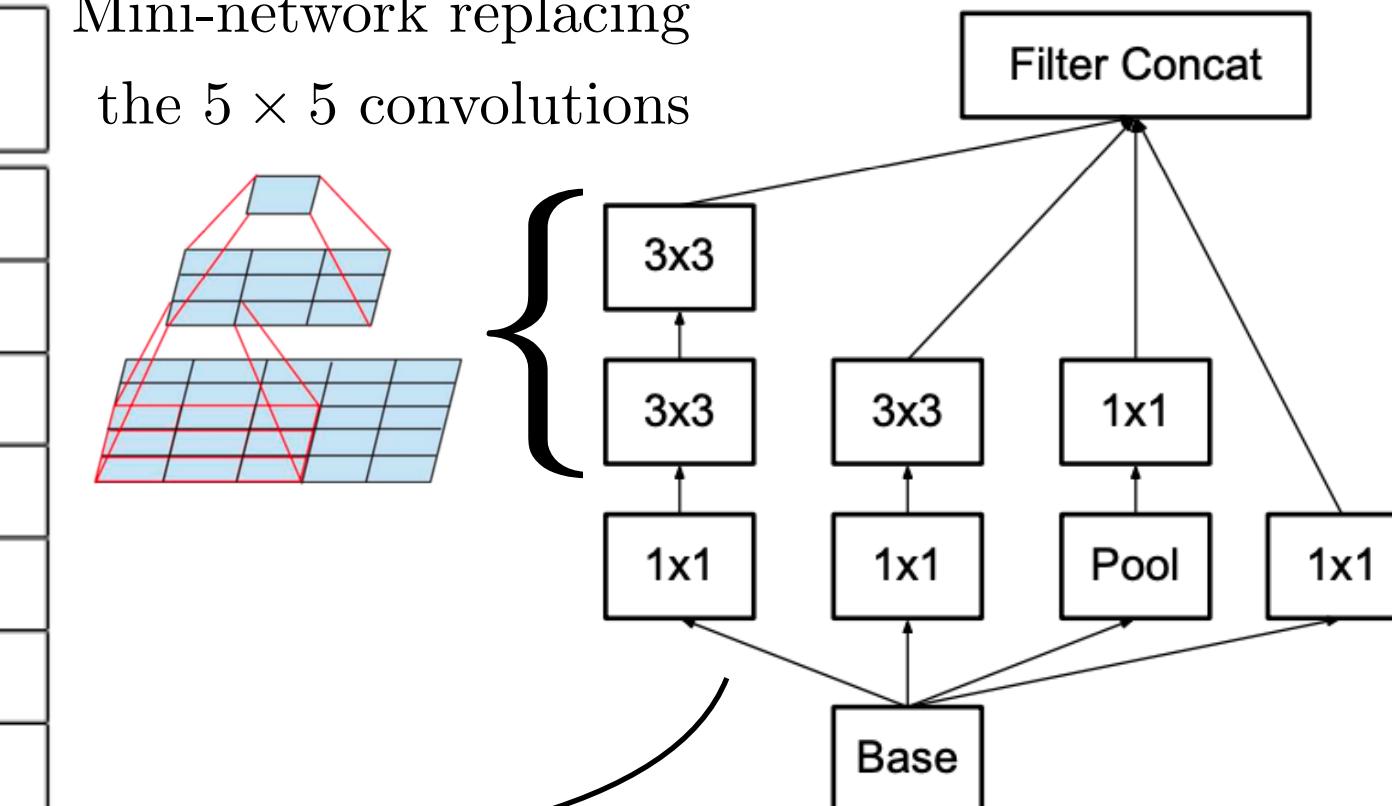
Rethinking the Inception Architecture for Computer Vision

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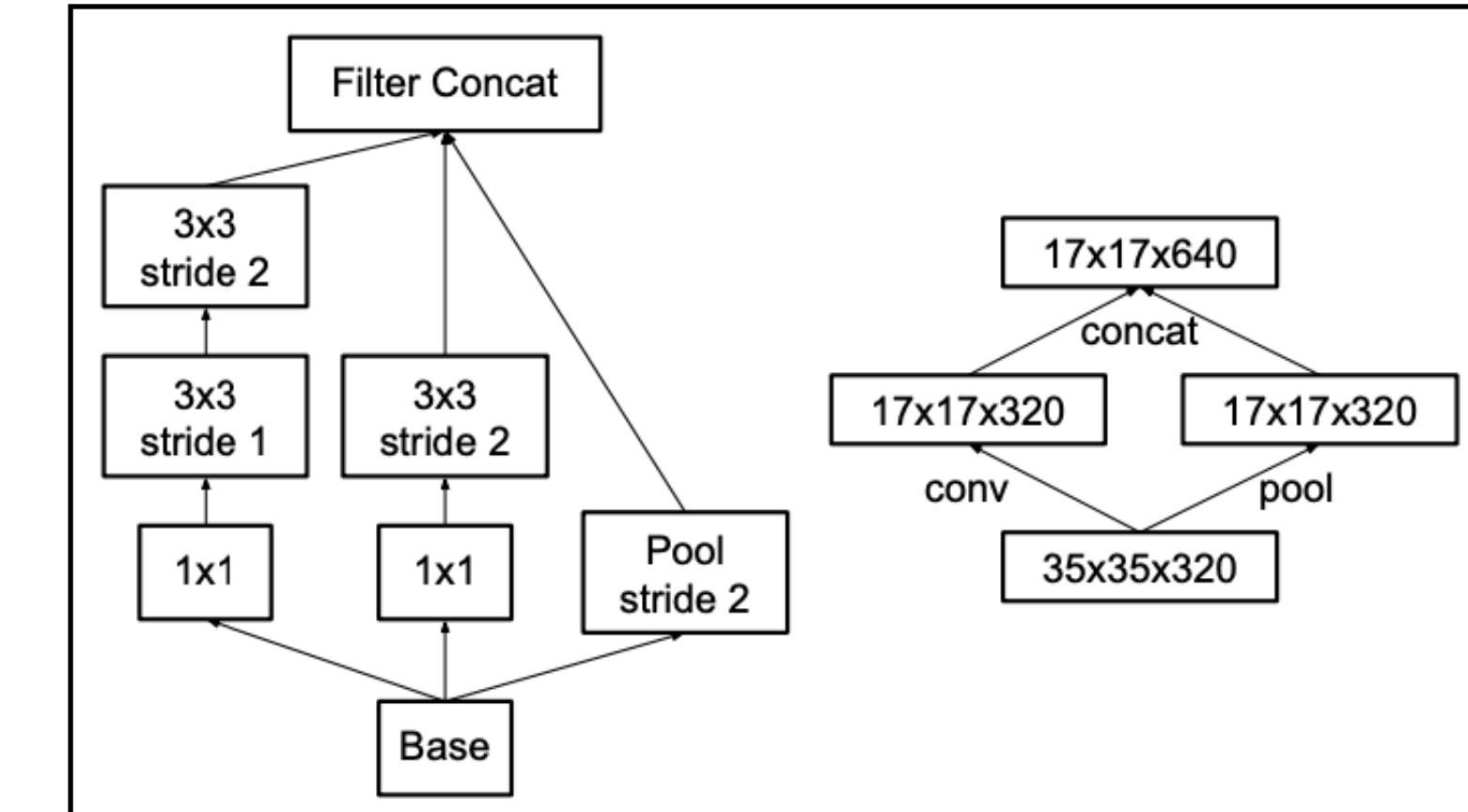
type	patch size/stride or remarks	input size
conv	$3 \times 3/2$	$299 \times 299 \times 3$
conv	$3 \times 3/1$	$149 \times 149 \times 32$
conv padded	$3 \times 3/1$	$147 \times 147 \times 32$
pool	$3 \times 3/2$	$147 \times 147 \times 64$
conv	$3 \times 3/1$	$73 \times 73 \times 64$
conv	$3 \times 3/2$	$71 \times 71 \times 80$
conv	$3 \times 3/1$	$35 \times 35 \times 192$
$3 \times$ Inception	As in figure 5	$35 \times 35 \times 288$
$5 \times$ Inception	As in figure 6	$17 \times 17 \times 768$
$2 \times$ Inception	As in figure 7	$8 \times 8 \times 1280$
pool	8×8	$8 \times 8 \times 2048$
linear	logits	$1 \times 1 \times 2048$
softmax	classifier	$1 \times 1 \times 1000$



Mini-network replacing
the 5×5 convolutions



$$\text{cross-entropy loss} \leftarrow L = - \sum_{k=1}^K q(k|x) \log(p(k|x)) = -\log(p(k^*|x)) \rightarrow \text{negative log-likelihood}$$



Inception module that
reduces the grid-size while
expanding the filter banks

Replace $q(k|x)$ with $q'(k|x) = (1 - \epsilon)q(k|x) + \epsilon u(k)$

label-smoothing regularization

$$H(q', p) = (1 - \epsilon)H(q, p) + \epsilon H(u, p)$$

$$H(u, p) = KL(u||p) + H(u)$$

$$u = 1/K$$

$$p(k|x) = \exp(z_k) / \sum_{i=1}^K \exp(z_i)$$

$z_i \rightarrow$ logits (un-normalized log-probabilities)

$q(k|x) \rightarrow$ ground truth distribution over labels

$k^* \rightarrow$ ground truth label

$$q(k|x) = \delta_{k^*}(k) = \begin{cases} 0 & \text{if } k \neq k^*; \\ 1 & \text{if } k = k^*. \end{cases}$$



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Training Very Deep Networks

Highway Networks

$$y = H(x; W_H) \rightarrow \text{a single layer}$$

$H \rightarrow$ non-linear transformation parametrized by W_H

(e.g., affine transformation followed by a non-linear activation function)

$$y = H(x; W_H) \cdot T(x; W_T) + x \cdot C(x; W_C)$$

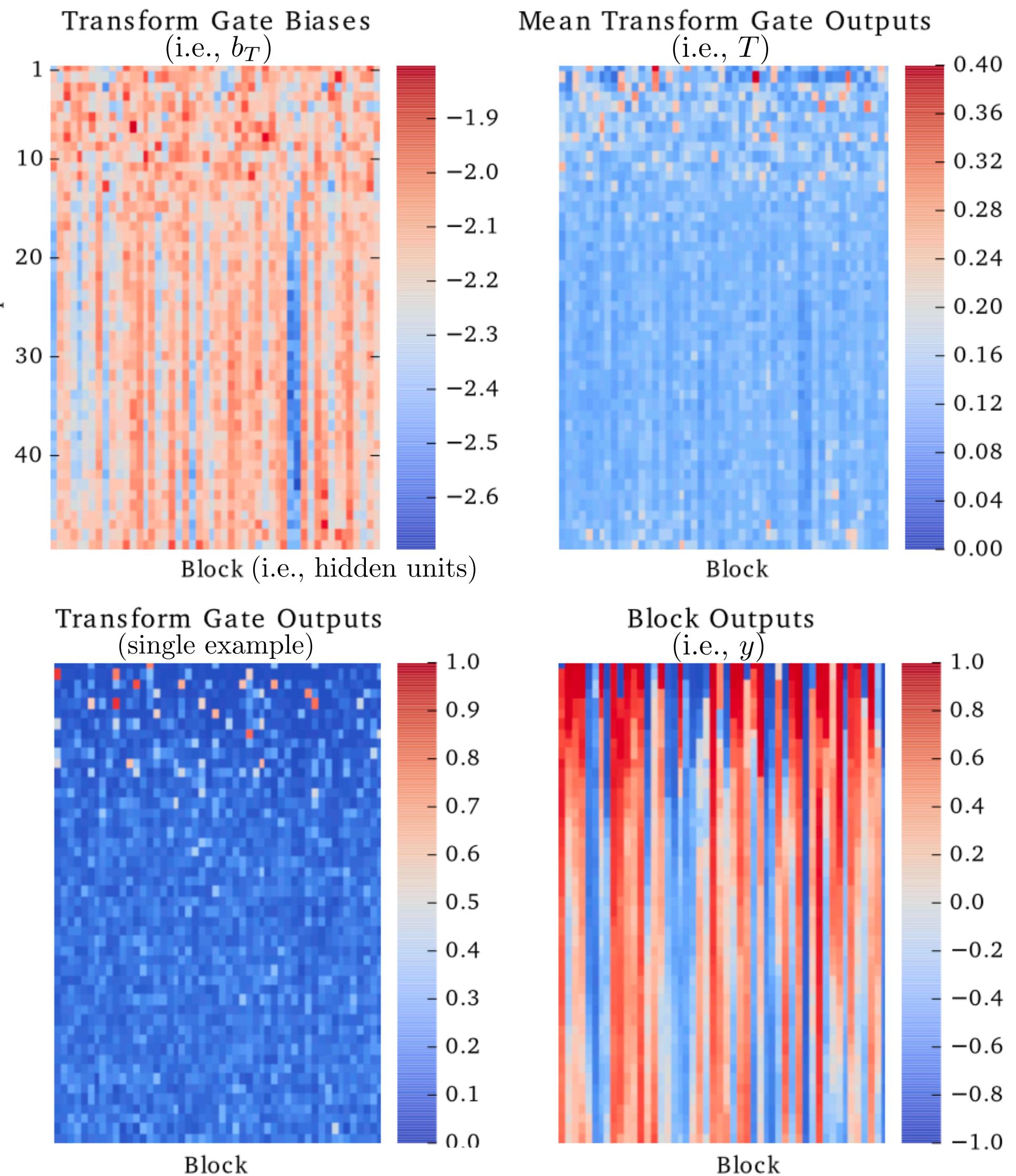
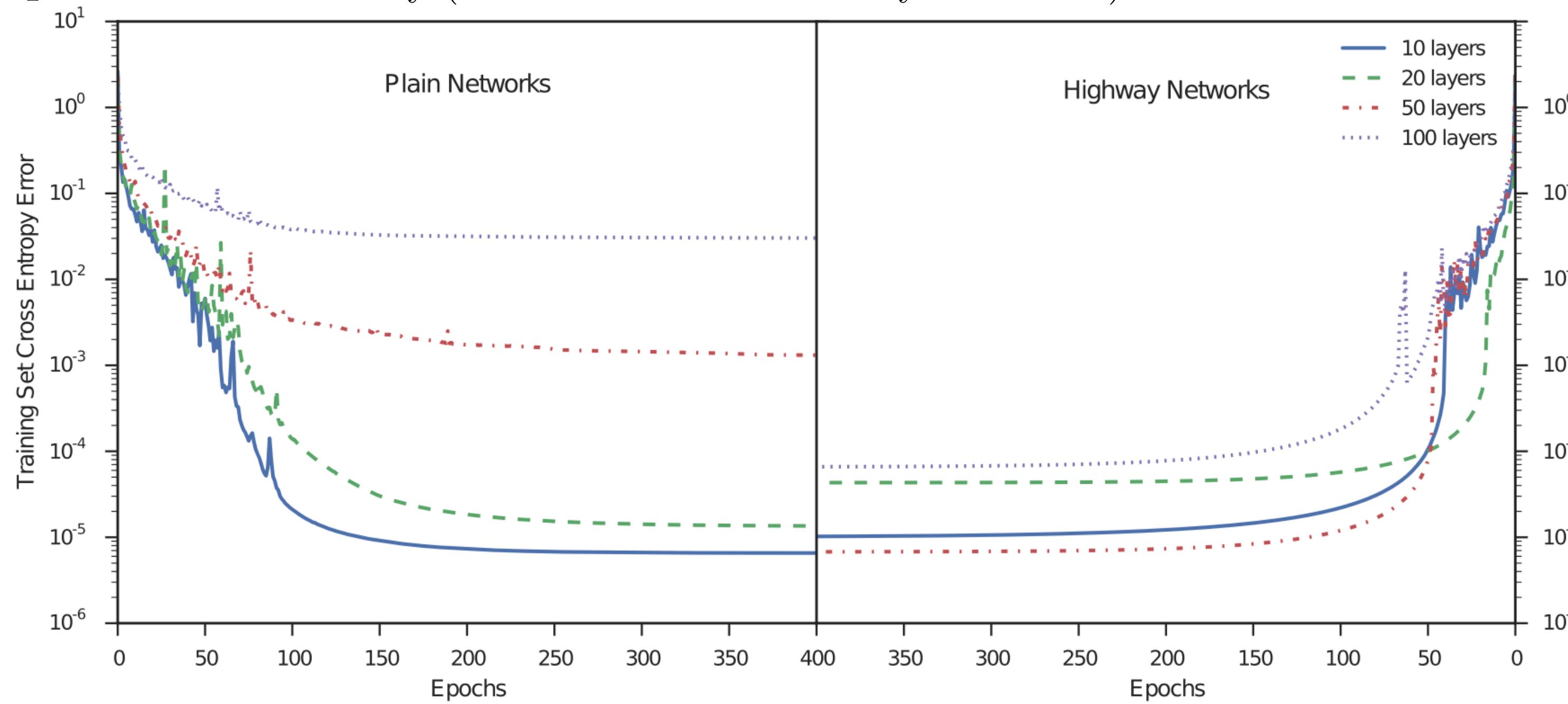
$T \rightarrow$ transform gate $\underbrace{\cdot}_{\text{element-wise multiplication}}$

$C \rightarrow$ carry gate

$$C = 1 - T$$

$$T(x) = \sigma(W_T x + b_T)$$

$b_T = -1$ or -3 initially (biased towards the “carry” behavior)



Srivastava, Rupesh Kumar, Klaus Greff, and Jürgen Schmidhuber. "Training very deep networks." *arXiv preprint arXiv:1507.06228* (2015).

Srivastava, Rupesh Kumar, Klaus Greff, and Jürgen Schmidhuber. "Highway networks." *arXiv preprint arXiv:1505.00387* (2015).

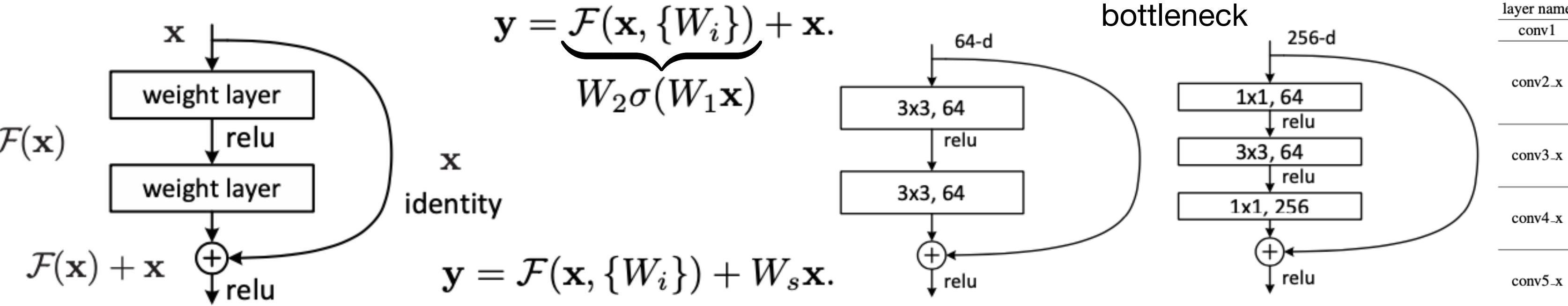


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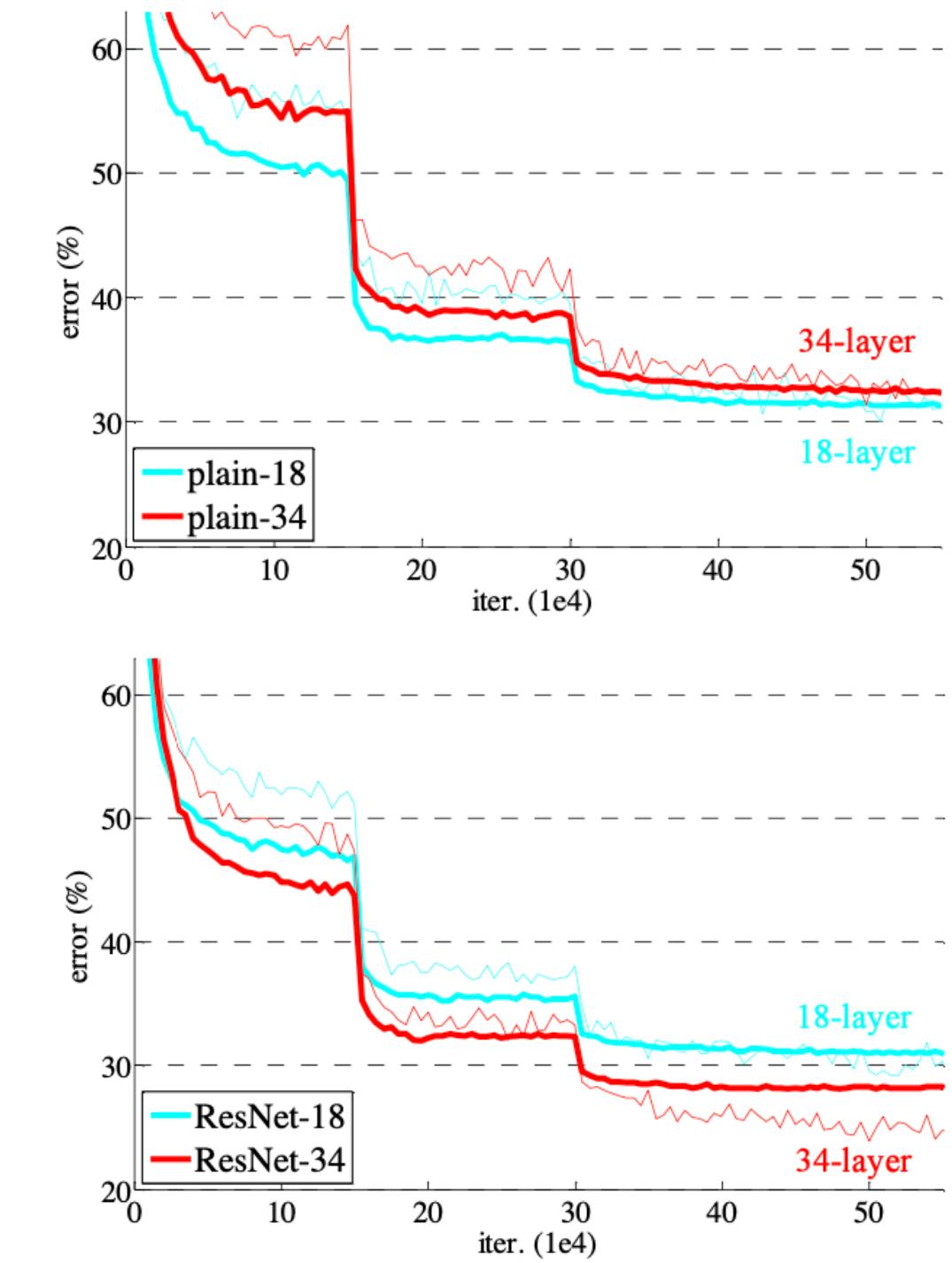
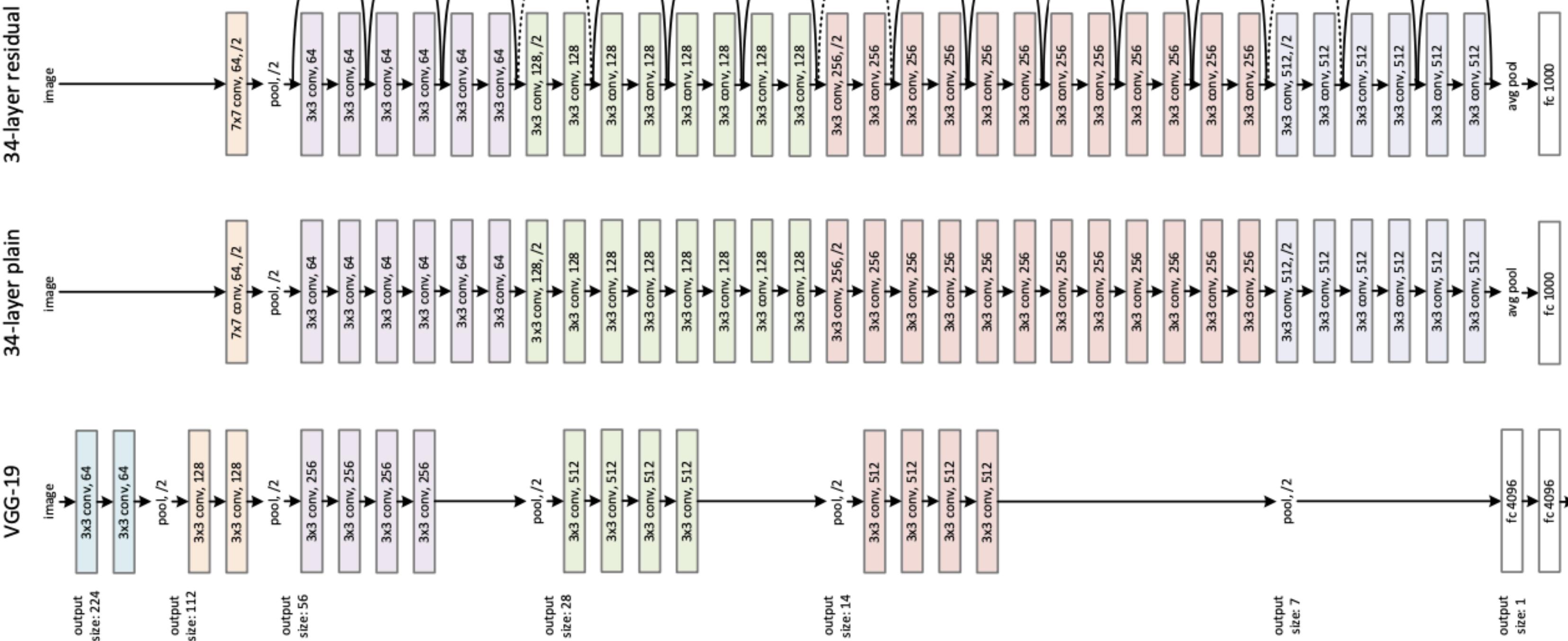


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Deep Residual Learning for Image Recognition



layer name	output size	18-layer	34-layer	50-layer	101-layer	152-layer
conv1	112x112					
conv2_x	56x56					
conv3_x	28x28					
conv4_x	14x14					
conv5_x	7x7					
		average pool, 1000-d fc, softmax				
	FLOPs	1.8×10^9	3.6×10^9	3.8×10^9	7.6×10^9	11.3×10^9



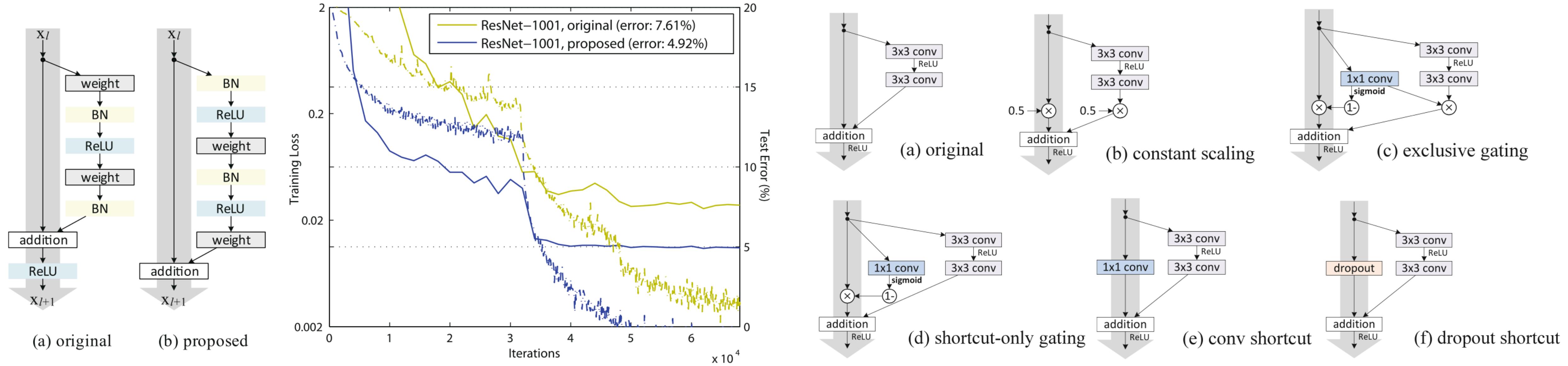


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Identity Mappings in Deep Residual Networks



$$\mathbf{y}_l = h(\mathbf{x}_l) + \mathcal{F}(\mathbf{x}_l, \mathcal{W}_l), \\ \mathbf{x}_{l+1} = f(\mathbf{y}_l).$$

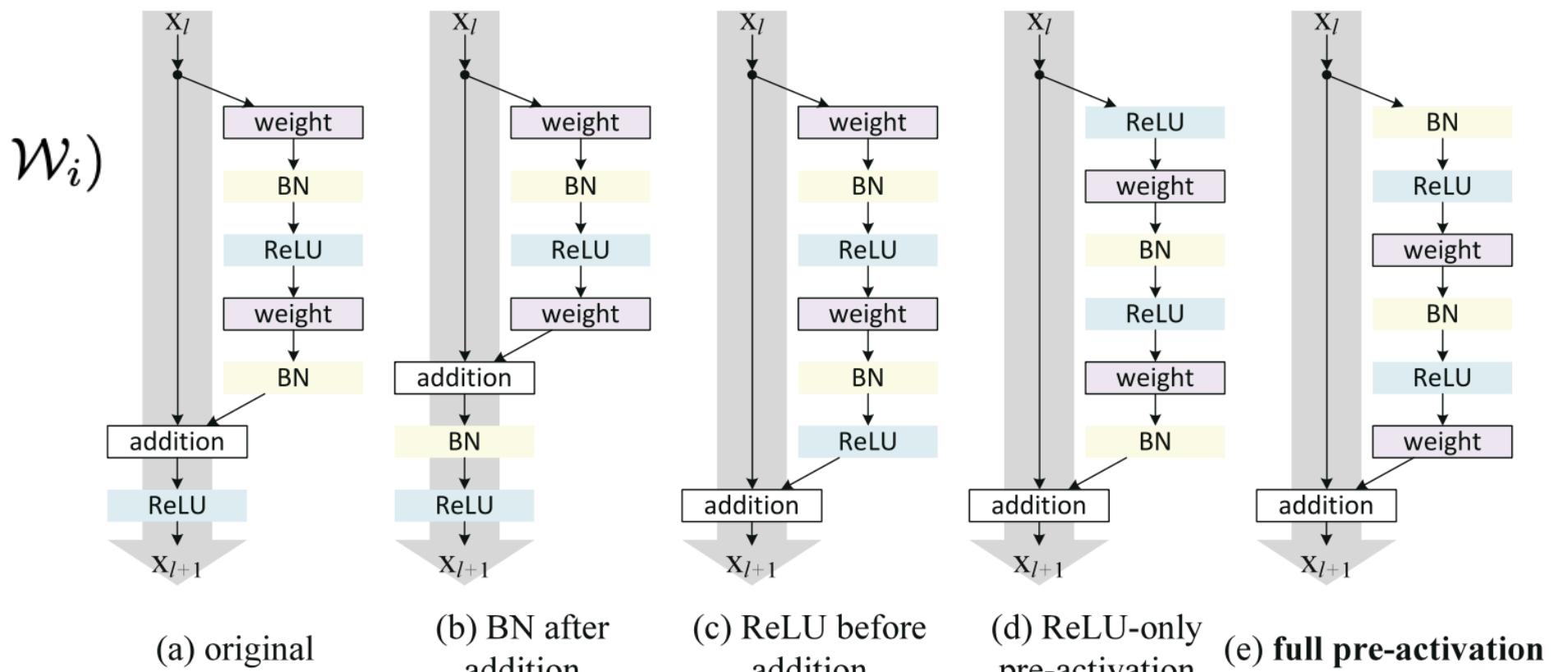
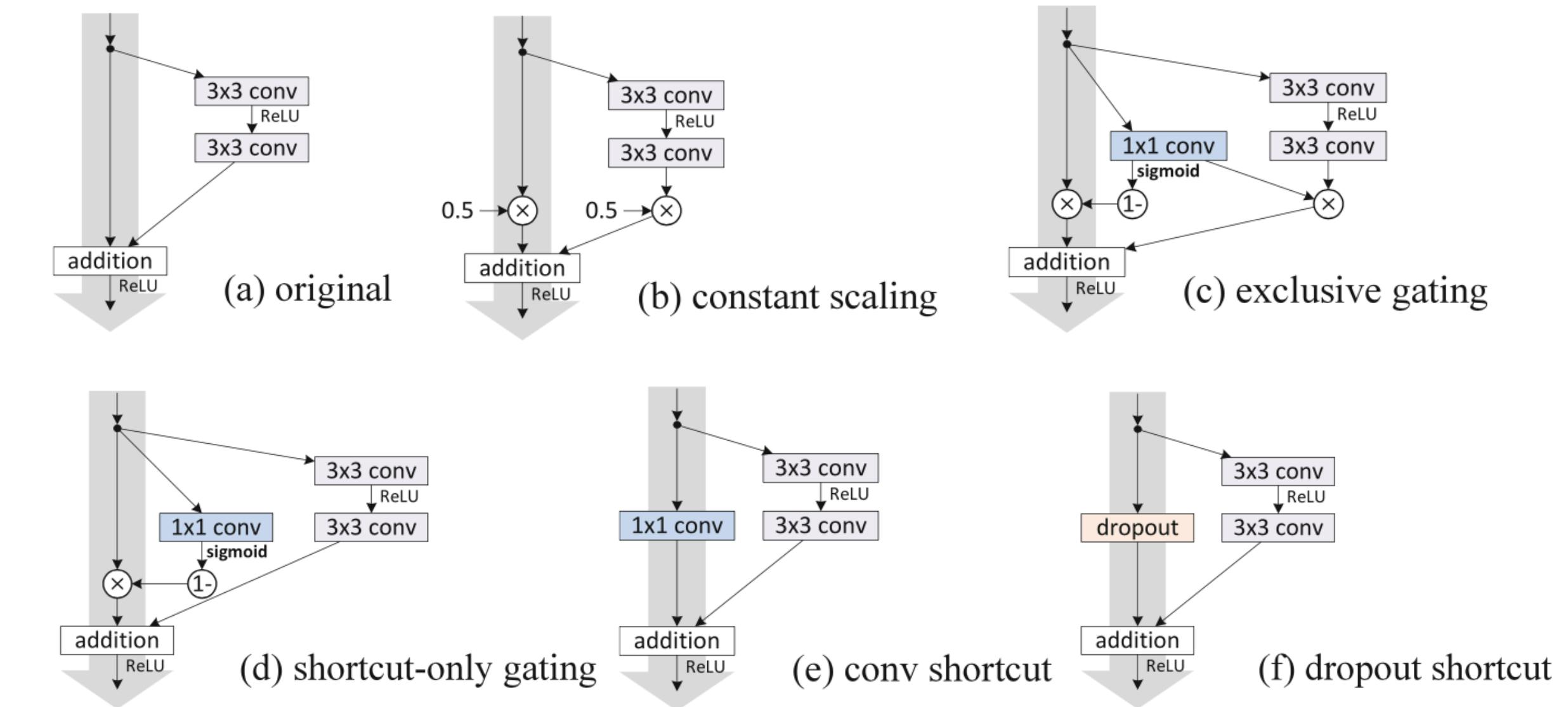
$$h(\mathbf{x}_l) = \mathbf{x}_l \\ f \text{ is ReLU} \quad \left. \right\} \text{Original}$$

$$\mathbf{x}_{l+1} = \mathbf{x}_l + \mathcal{F}(\mathbf{x}_l, \mathcal{W}_l) \rightarrow \text{if } f \text{ and } h \text{ are both identity}$$

$$\mathbf{x}_L = \mathbf{x}_l + \sum_{i=l}^{L-1} \mathcal{F}(\mathbf{x}_i, \mathcal{W}_i),$$

$$\frac{\partial \mathcal{E}}{\partial \mathbf{x}_l} = \frac{\partial \mathcal{E}}{\partial \mathbf{x}_L} \frac{\partial \mathbf{x}_L}{\partial \mathbf{x}_l} = \frac{\partial \mathcal{E}}{\partial \mathbf{x}_L} \left(1 + \frac{\partial}{\partial \mathbf{x}_l} \sum_{i=l}^{L-1} \mathcal{F}(\mathbf{x}_i, \mathcal{W}_i) \right)$$

$$h(\mathbf{x}_l) = \lambda_l \mathbf{x}_l \\ \mathbf{x}_{l+1} = \lambda_l \mathbf{x}_l + \mathcal{F}(\mathbf{x}_l, \mathcal{W}_l) \\ \mathbf{x}_L = (\prod_{i=l}^{L-1} \lambda_i) \mathbf{x}_l + \sum_{i=l}^{L-1} (\prod_{j=i+1}^{L-1} \lambda_j) \mathcal{F}(\mathbf{x}_i, \mathcal{W}_i) \\ \mathbf{x}_L = (\prod_{i=l}^{L-1} \lambda_i) \mathbf{x}_l + \sum_{i=l}^{L-1} \hat{\mathcal{F}}(\mathbf{x}_i, \mathcal{W}_i) \\ \frac{\partial \mathcal{E}}{\partial \mathbf{x}_l} = \frac{\partial \mathcal{E}}{\partial \mathbf{x}_L} \left((\underbrace{\prod_{i=l}^{L-1} \lambda_i}_{\prod_{i=l}^{L-1} h'_i}) + \frac{\partial}{\partial \mathbf{x}_l} \sum_{i=l}^{L-1} \hat{\mathcal{F}}(\mathbf{x}_i, \mathcal{W}_i) \right)$$





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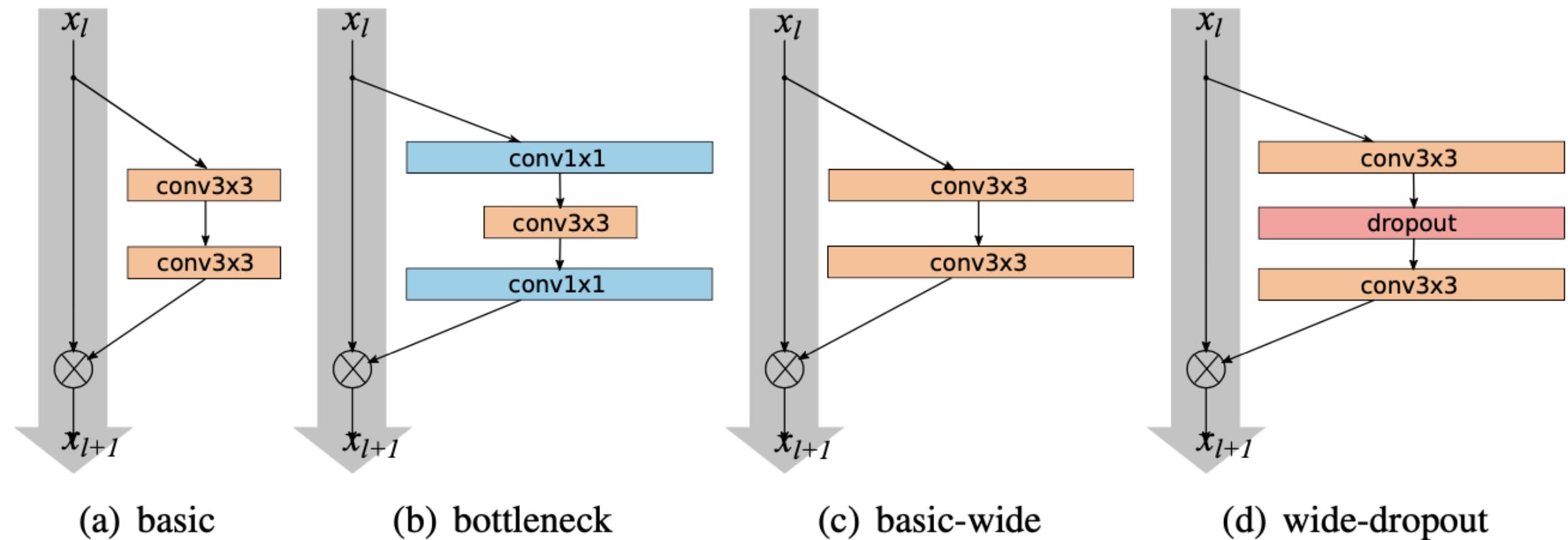


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Wide Residual Networks

$$\mathbf{x}_{l+1} = \mathbf{x}_l + \mathcal{F}(\mathbf{x}_l, \mathcal{W}_l)$$

Conv-BN-ReLU → BN-ReLU-Conv
ResNet Identity Mapping



$n \rightarrow$ total number of convolutional layers

$k \rightarrow$ widening factor (multiplies the number of features in conv layers)

WRN- $n-k$

1. $B(3, 3)$ - original «basic» block
2. $B(3, 1, 3)$ - with one extra 1×1 layer
3. $B(1, 3, 1)$ - with the same dimensionality of all convolutions, «straightened» bottleneck
4. $B(1, 3)$ - the network has alternating 1×1 - 3×3 convolutions everywhere
5. $B(3, 1)$ - similar idea to the previous block
6. $B(3, 1, 1)$ - Network-in-Network style block

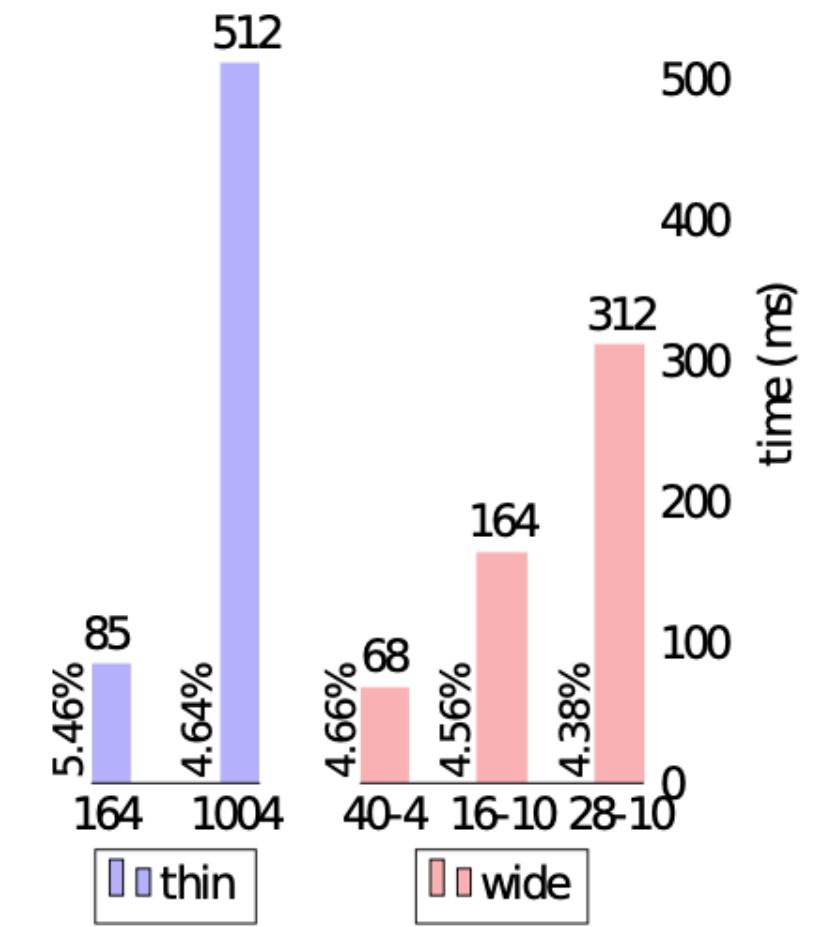
$\ell \rightarrow$ deepening factor (number of convs in a block)

l	CIFAR-10
1	6.69
2	5.43 ←
3	5.65
4	5.93

group name	output size	block type = $B(3, 3)$	depth	k	# params	CIFAR-10	CIFAR-100
conv1	32×32	$[3 \times 3, 16]$	40	1	0.6M	6.85	30.89
conv2	32×32	$[3 \times 3, 16 \times k]$ $[3 \times 3, 16 \times k] \times N$	40	2	2.2M	5.33	26.04
conv3	16×16	$[3 \times 3, 32 \times k]$ $[3 \times 3, 32 \times k] \times N$	40	4	8.9M	4.97	22.89
conv4	8×8	$[3 \times 3, 64 \times k]$ $[3 \times 3, 64 \times k] \times N$	40	8	35.7M	4.66	-
avg-pool	1×1	$[8 \times 8]$	28	10	36.5M	4.17	20.50
			28	12	52.5M	4.33	20.43
			22	8	17.2M	4.38	21.22
			22	10	26.8M	4.44	20.75
			16	8	11.0M	4.81	22.07
			16	10	17.1M	4.56	21.59

block type	depth	# params	time,s	CIFAR-10	depth	k	dropout	CIFAR-10	CIFAR-100	SVHN
$B(1, 3, 1)$	40	1.4M	85.8	6.06	16	4		5.02	24.03	1.85
$B(3, 1)$	40	1.2M	67.5	5.78	16	4	✓	5.24	23.91	1.64
$B(1, 3)$	40	1.3M	72.2	6.42	28	10		4.00	19.25	-
$B(3, 1, 1)$	40	1.3M	82.2	5.86	28	10	✓	3.89	18.85	-
$B(3, 3)$	28	1.5M	67.5	5.73 ←	52	1		6.43	29.89	2.08
$B(3, 1, 3)$	22	1.1M	59.9	5.78	52	1	✓	6.28	29.78	1.70

	depth- k	# params	CIFAR-10	CIFAR-100
NIN [20]			8.81	35.67
DSN [19]			8.22	34.57
FitNet [24]			8.39	35.04
Highway [28]			7.72	32.39
ELU [5]			6.55	24.28
original-ResNet[11]	110	1.7M	6.43	25.16
	1202	10.2M	7.93	27.82
stoc-depth[14]	110	1.7M	5.23	24.58
	1202	10.2M	4.91	-
	110	1.7M	6.37	-
pre-act-ResNet[13]	164	1.7M	5.46	24.33
	1001	10.2M	4.92(4.64)	22.71
WRN	40-4	8.9M	4.53	21.18
	16-8	11.0M	4.27	20.43
	28-10	36.5M	4.00	19.25



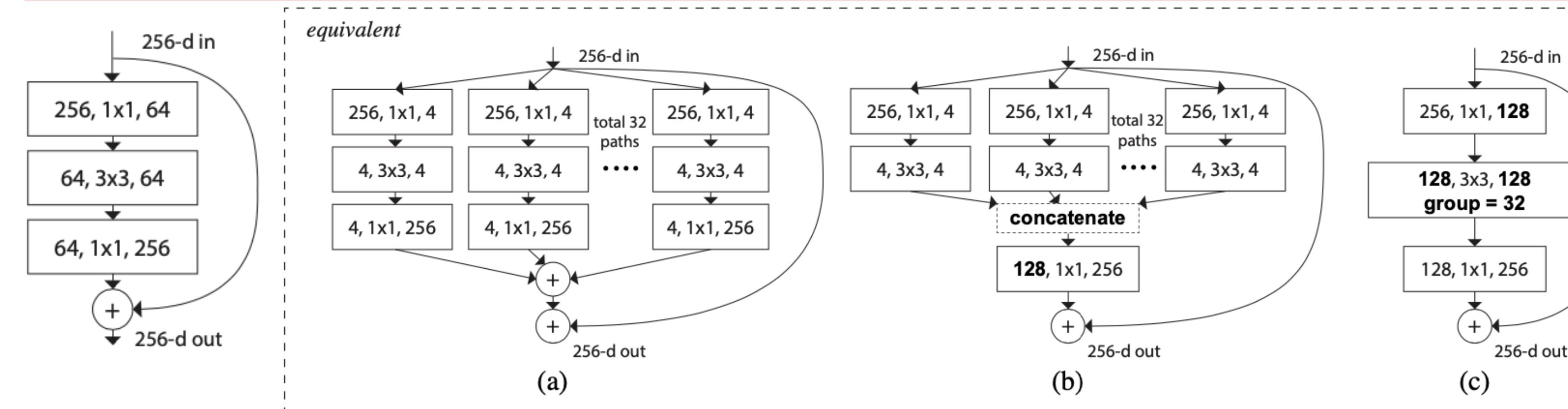


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Aggregated Residual Transformations for Deep Neural Networks



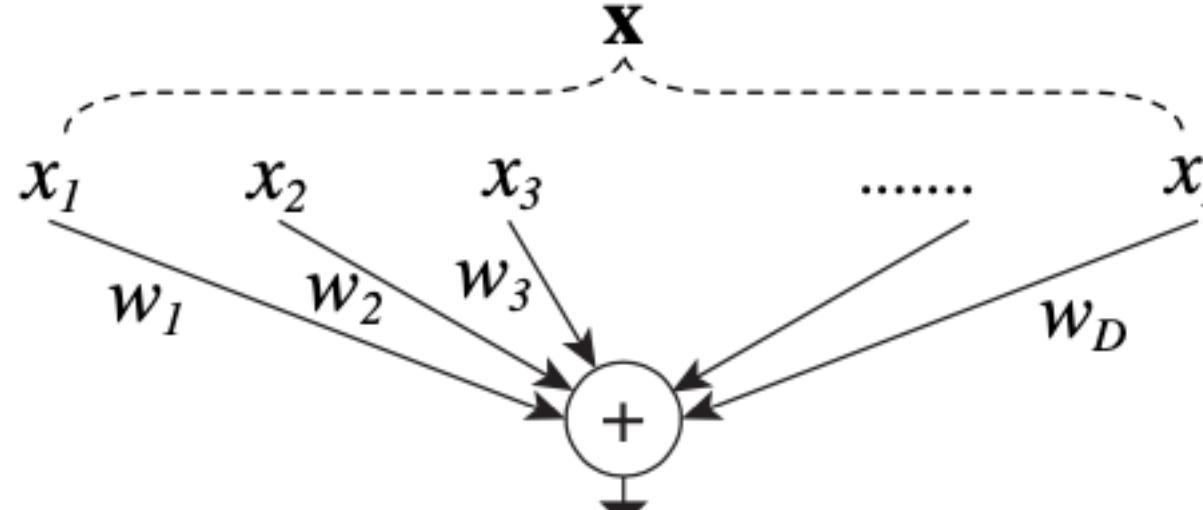
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$$\sum_{i=1}^{32} \underbrace{W_i}_{\in \mathbb{R}^{256 \times 4}} \underbrace{x_i}_{\in \mathbb{R}^4} = \underbrace{[W_1, \dots, W_{32}]}_{\in \mathbb{R}^{256 \times 128}} \underbrace{[x_1; \dots; x_{32}]}_{\in \mathbb{R}^{128}}$$

split — transform — merge strategy → inception
 1×1 conv $3 \times 3, 5 \times 5$, etc. conv concatenation

cardinality: size of the set of transformations



A simple neuron that performs inner product.

$$\mathcal{F}(x) = \sum_{i=1}^C \mathcal{T}_i(x) \rightarrow \text{"Network in Neuron"}$$

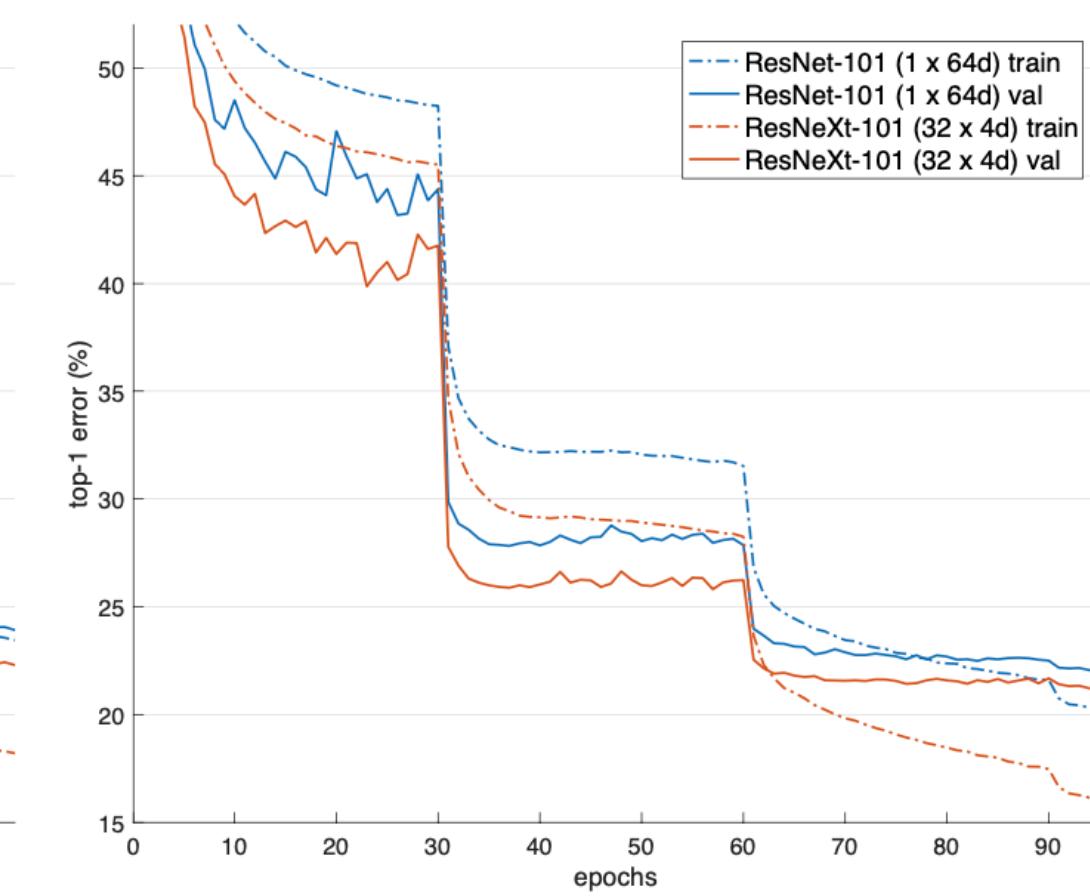
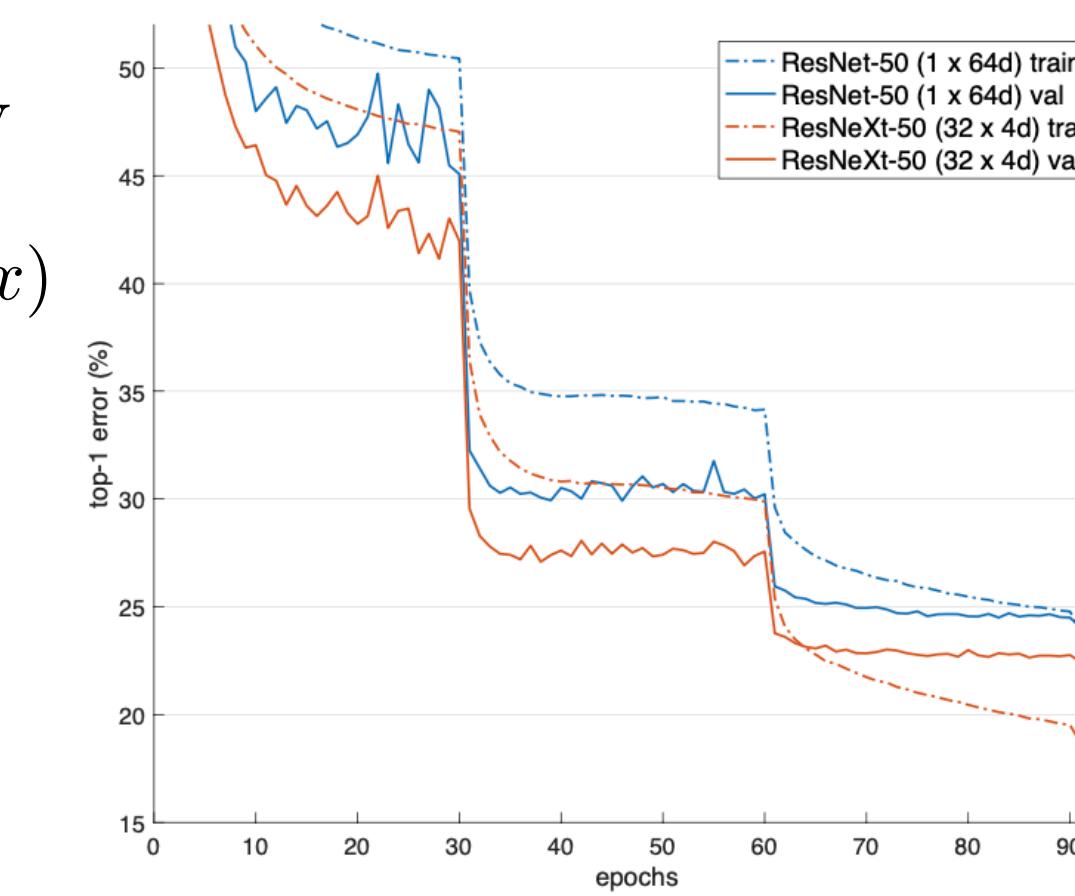
$C \rightarrow$ cardinality

$$y = x + \sum_{i=1}^C \mathcal{T}_i(x)$$

split: $x \rightarrow x_i$

transform: $x_i w_i$

$$\text{aggregate: } \sum_{i=1}^D w_i x_i$$



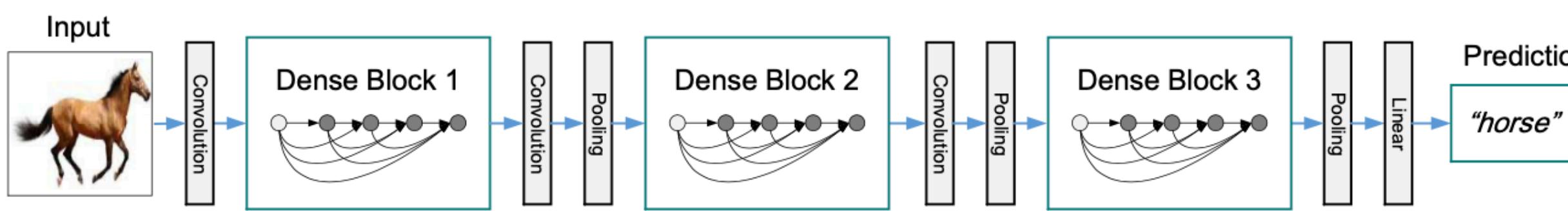
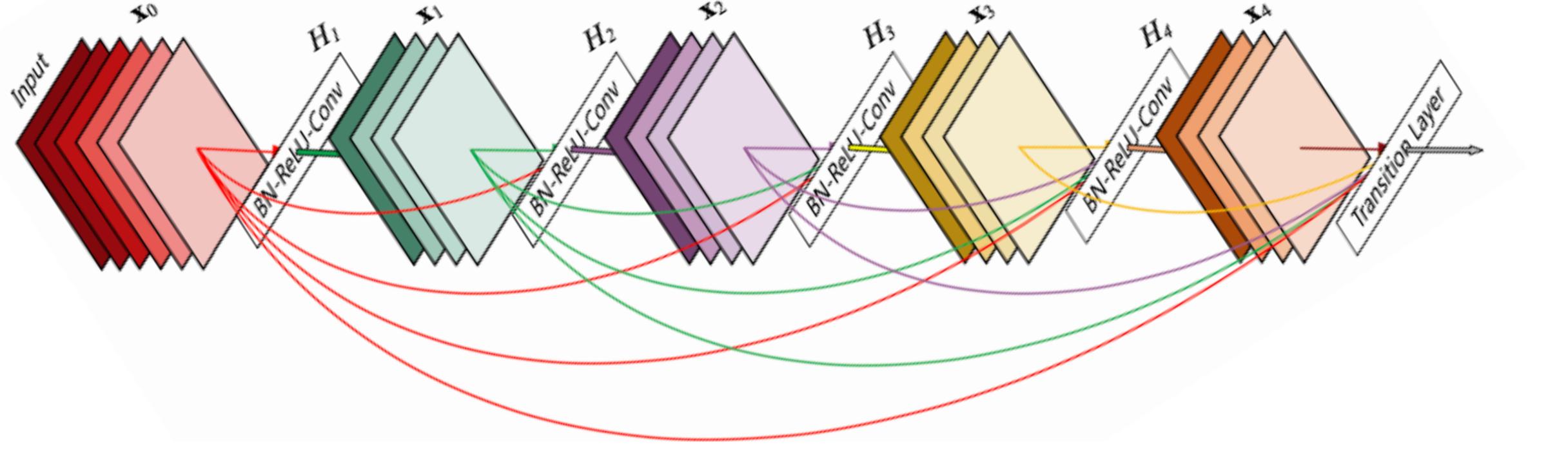


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Densely Connected Convolutional Networks



$x_0 \rightarrow$ single image

$L \rightarrow$ number of layers

$H_\ell(\cdot) \rightarrow$ nonlinear transformation

$\ell = 1, \dots, L \rightarrow$ layer index

$x_\ell \rightarrow$ output of the ℓ -th layer

$x_\ell = H_\ell(x_{\ell-1})$

$x_\ell = H_\ell(x_{\ell-1}) + x_{\ell-1}$

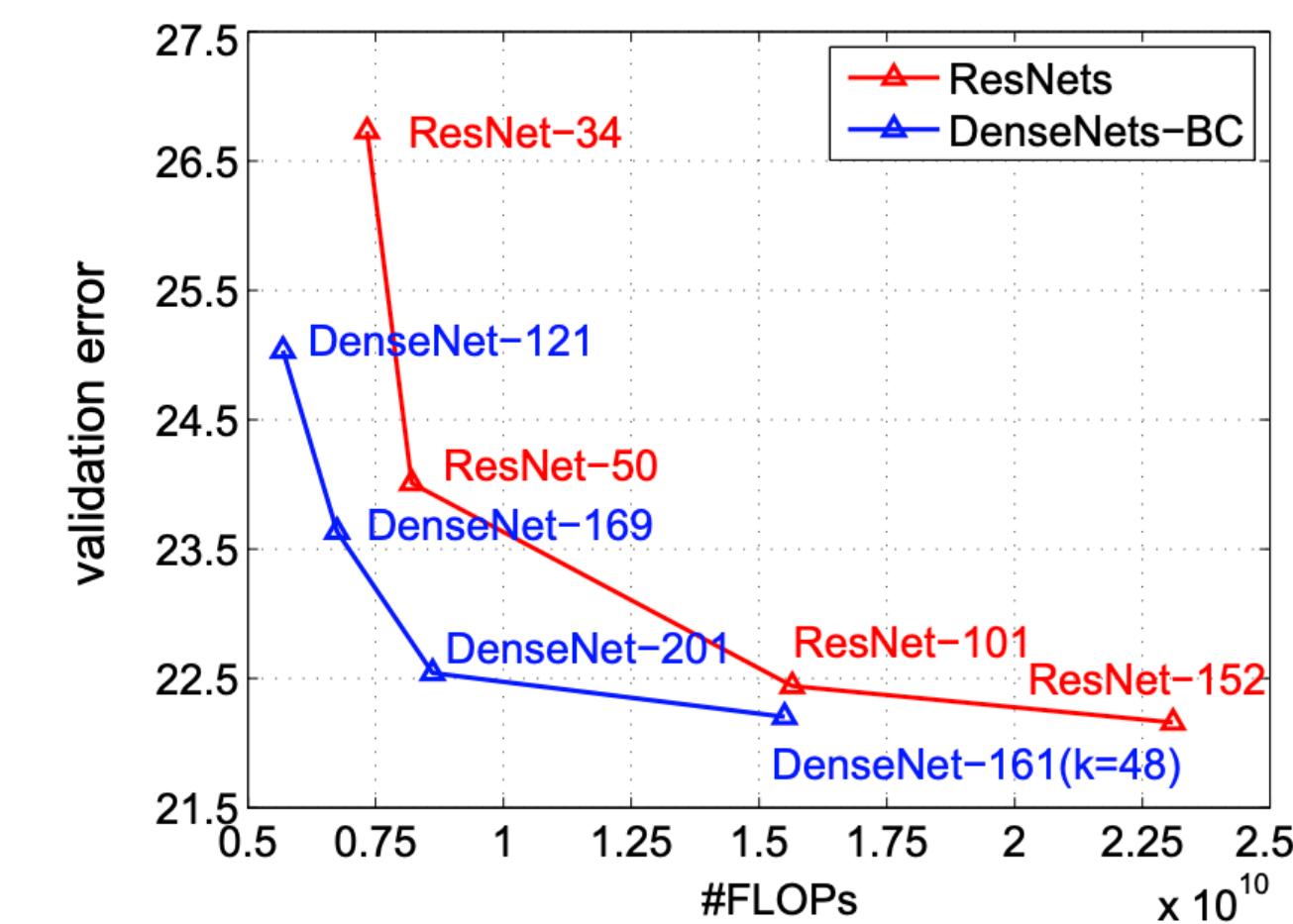
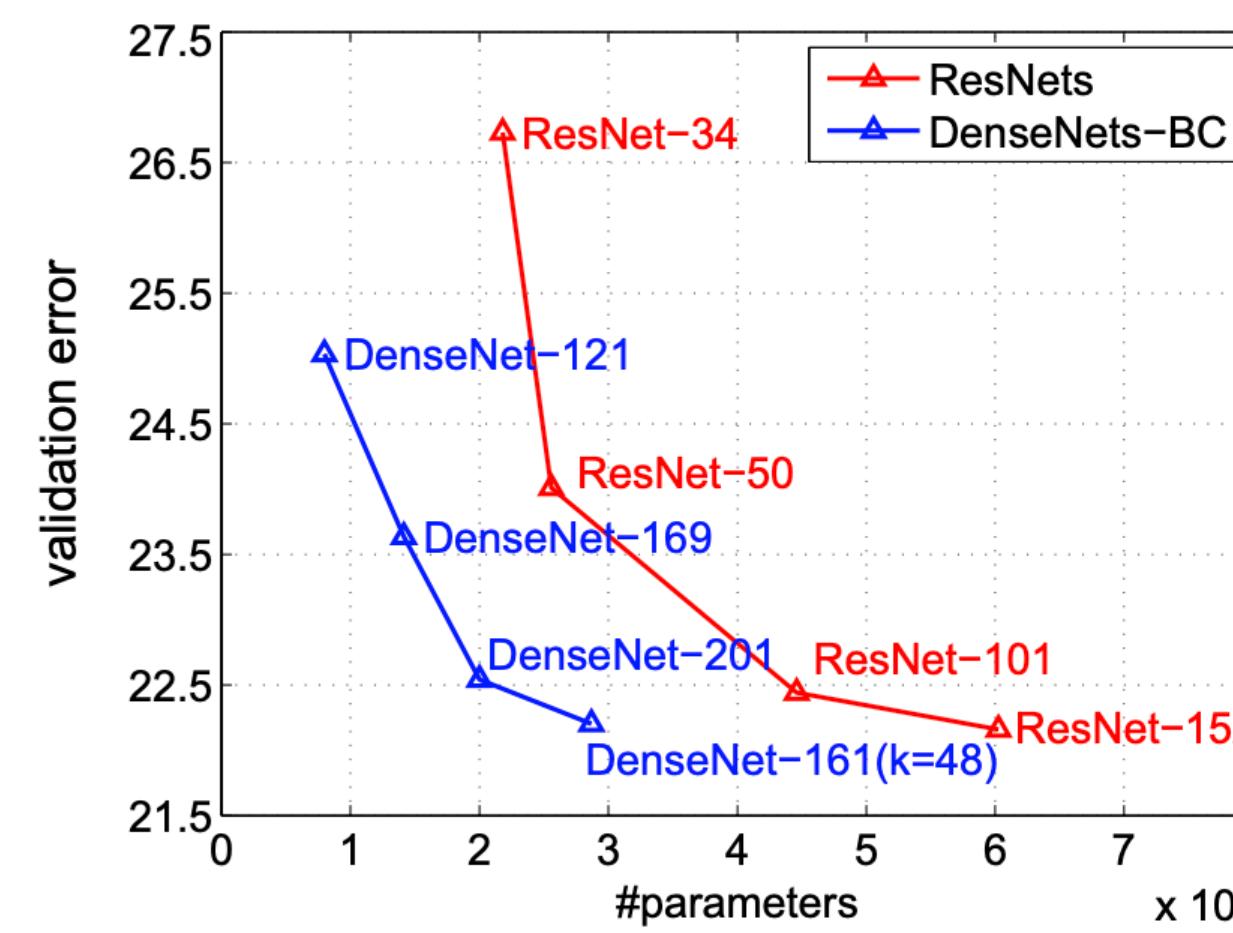
$x_\ell = H_\ell(\underbrace{[x_0, x_1, \dots, x_{\ell-1}]}_{\text{concatenate}})$

BN-ReLU-Conv(1x1)-BN-ReLU-Conv(3x3)
↳ 4k feature maps

growth rate of the network
 $k = 12$

If each function H_ℓ produces k feature-maps,
then the ℓ -th layer has $k_0 + k(\ell - 1)$ input feature maps

Layers	Output Size	DenseNet-121	DenseNet-169	DenseNet-201	DenseNet-265
Convolution	112 × 112				
Pooling	56 × 56				
Dense Block (1)	56 × 56	$[1 \times 1 \text{ conv}, 3 \times 3 \text{ conv}] \times 6$	$[1 \times 1 \text{ conv}, 3 \times 3 \text{ conv}] \times 6$	$[1 \times 1 \text{ conv}, 3 \times 3 \text{ conv}] \times 6$	$[1 \times 1 \text{ conv}, 3 \times 3 \text{ conv}] \times 6$
	56 × 56				
Transition Layer (1)	56 × 56			$1 \times 1 \text{ conv}$	
Dense Block (2)	28 × 28			$2 \times 2 \text{ average pool, stride 2}$	
	28 × 28	$[1 \times 1 \text{ conv}, 3 \times 3 \text{ conv}] \times 12$	$[1 \times 1 \text{ conv}, 3 \times 3 \text{ conv}] \times 12$	$[1 \times 1 \text{ conv}, 3 \times 3 \text{ conv}] \times 12$	$[1 \times 1 \text{ conv}, 3 \times 3 \text{ conv}] \times 12$
Transition Layer (2)	28 × 28			$1 \times 1 \text{ conv}$	
Dense Block (3)	14 × 14			$2 \times 2 \text{ average pool, stride 2}$	
	14 × 14	$[1 \times 1 \text{ conv}, 3 \times 3 \text{ conv}] \times 24$	$[1 \times 1 \text{ conv}, 3 \times 3 \text{ conv}] \times 32$	$[1 \times 1 \text{ conv}, 3 \times 3 \text{ conv}] \times 48$	$[1 \times 1 \text{ conv}, 3 \times 3 \text{ conv}] \times 64$
Transition Layer (3)	14 × 14			$1 \times 1 \text{ conv}$	
Dense Block (4)	7 × 7			$2 \times 2 \text{ average pool, stride 2}$	
	7 × 7	$[1 \times 1 \text{ conv}, 3 \times 3 \text{ conv}] \times 16$	$[1 \times 1 \text{ conv}, 3 \times 3 \text{ conv}] \times 32$	$[1 \times 1 \text{ conv}, 3 \times 3 \text{ conv}] \times 32$	$[1 \times 1 \text{ conv}, 3 \times 3 \text{ conv}] \times 48$
Classification Layer	1 × 1			$7 \times 7 \text{ global average pool}$	1000D fully-connected, softmax



Huang, Gao, et al. "Densely connected convolutional networks." Proceedings of the IEEE conference on computer vision and pattern recognition. 2017.

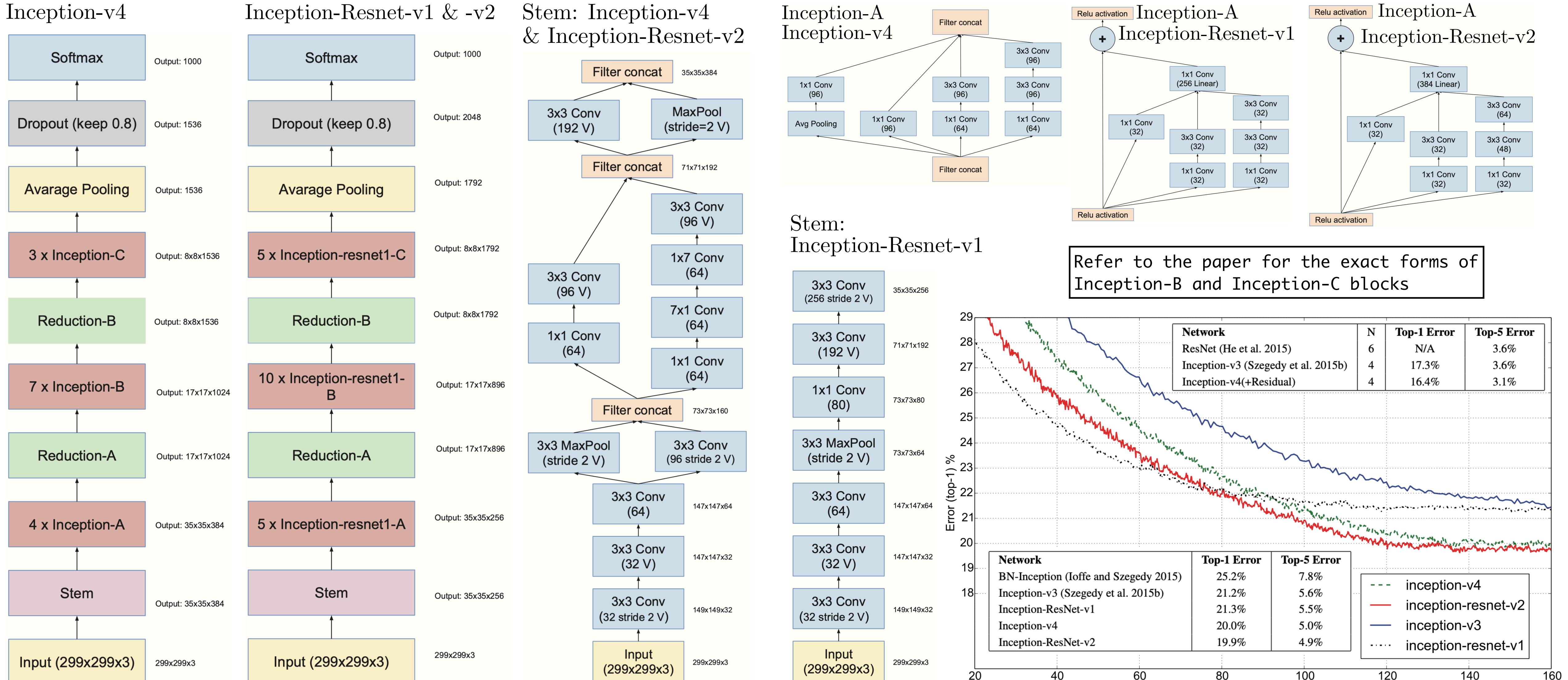
Huang, Gao, et al. "Convolutional networks with dense connectivity." IEEE transactions on pattern analysis and machine intelligence (2019).



Inception-v4, Inception-ResNet and the Impact of Residual Connections on Learning



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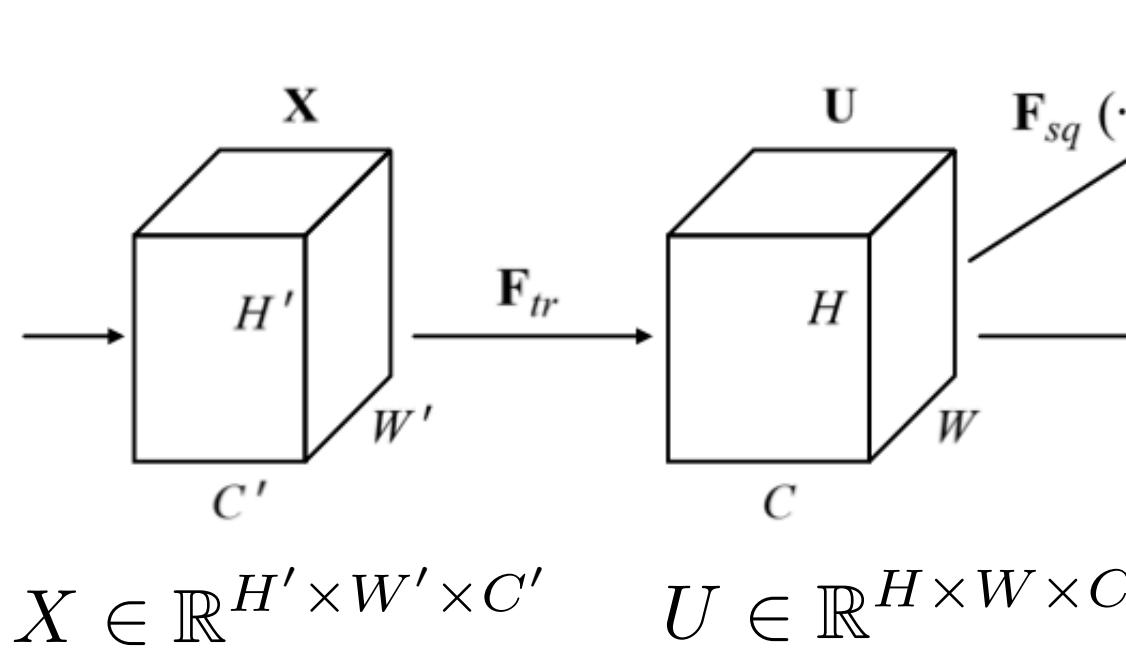


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Squeeze-and-Excitation Networks



$$F_{tr} : X \mapsto U$$

$V = [v_1, \dots, v_C]$ → learned set of filter kernels

v_c → parameters of the c -th filter

$$U = [u_1, \dots, u_C]$$

$$u_c = v_c * X = \sum_{s=1}^{C'} v_c^s * x^s$$

$$v_c = [v_c^1, \dots, v_c^{C'}]$$

$$X = [x^1, \dots, x^{C'}]$$

Squeeze

$$z \in \mathbb{R}^C \quad z_c = F_{sq}(u_c) = \frac{1}{HW} \sum_{i=1}^H \sum_{j=1}^W u_c(i, j)$$

Excitation

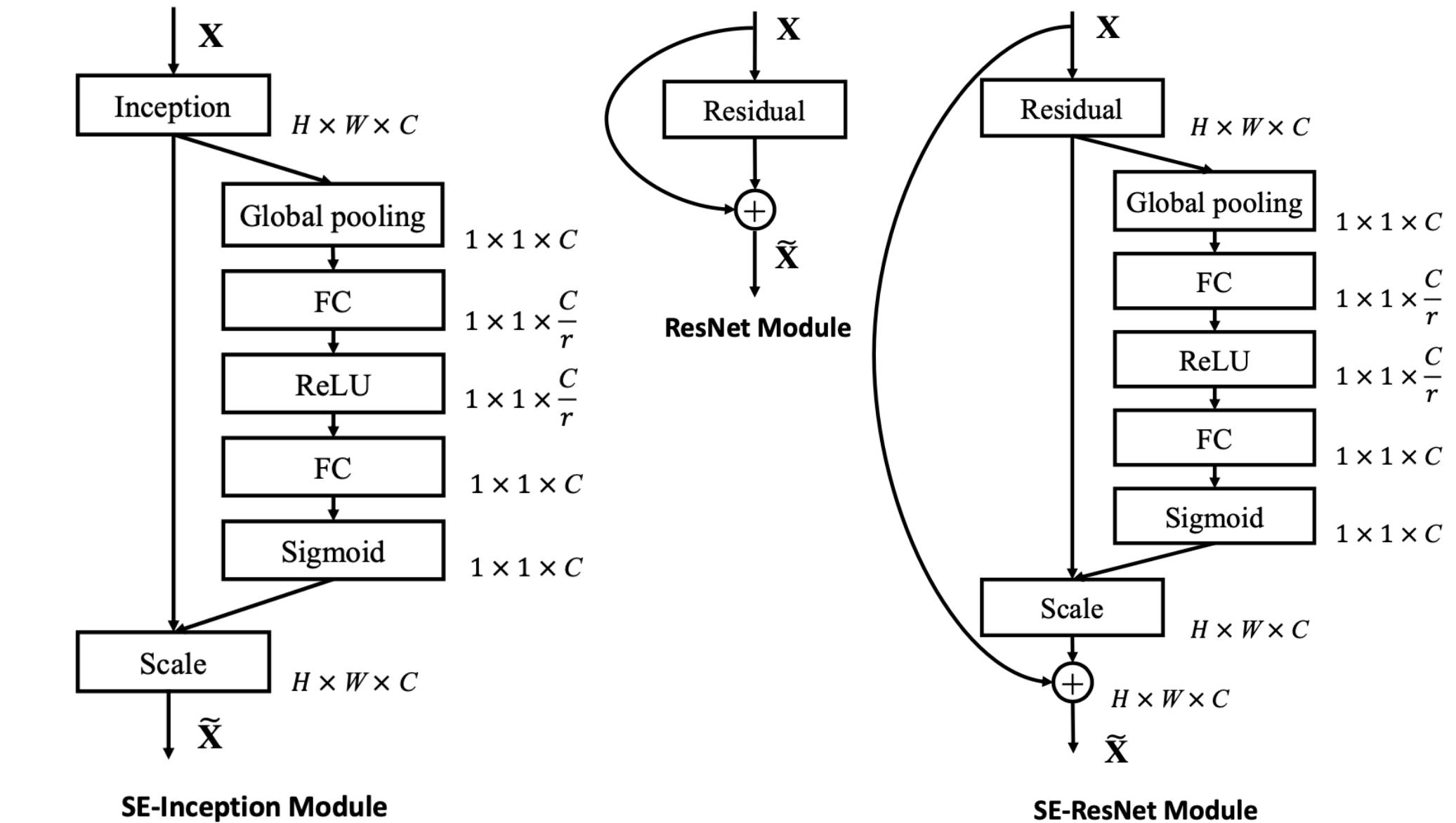
$$s = F_{ex}(z, W) = \sigma(g(z, W)) = \sigma(W_2 \delta(W_1 z))$$

$$W_1 \in \mathbb{R}^{\frac{C}{r} \times C}$$

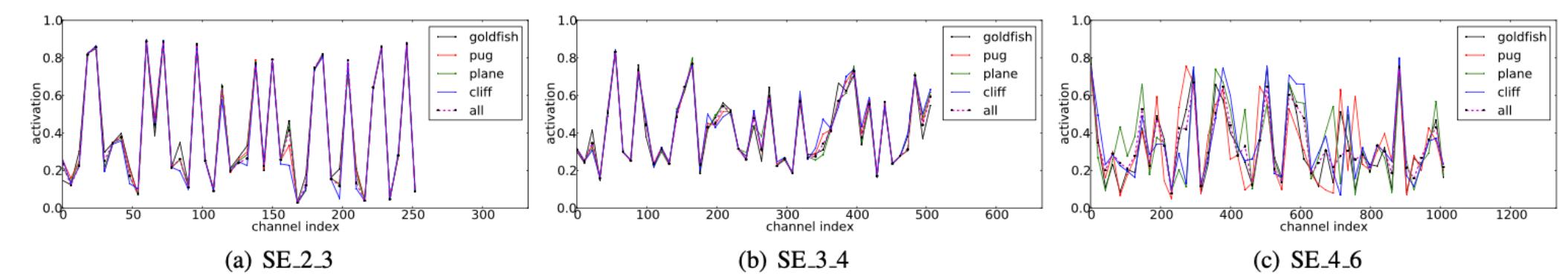
$$W_2 \in \mathbb{R}^{C \times \frac{C}{r}}$$

$$\begin{aligned} \tilde{x}_c &= F_{scale}(u_c, s_c) = s_c u_c \\ u_c &\in \mathbb{R}^{H \times W} \\ \tilde{X} &= [\tilde{x}_1, \dots, \tilde{x}_C] \end{aligned}$$

	224 × 224		320 × 320 / 299 × 299	
	top-1 err.	top-5 err.	top-1 err.	top-5 err.
ResNet-152 [10]	23.0	6.7	21.3	5.5
ResNet-200 [11]	21.7	5.8	20.1	4.8
Inception-v3 [44]	-	-	21.2	5.6
Inception-v4 [42]	-	-	20.0	5.0
Inception-ResNet-v2 [42]	-	-	19.9	4.9
ResNeXt-101 (64 × 4d) [47]	20.4	5.3	19.1	4.4
DenseNet-264 [14]	22.15	6.12	-	-
Attention-92 [46]	-	-	19.5	4.8
Very Deep PolyNet [51] [†]	-	-	18.71	4.25
PyramidNet-200 [8]	20.1	5.4	19.2	4.7
DPN-131 [5]	19.93	5.12	18.55	4.16
SENet-154	18.68	4.47	17.28	3.79
NASNet-A (6@4032) [55] [†]	-	-	17.3 [‡]	3.8 [‡]
SENet-154 (post-challenge)	-	-	16.88[‡]	3.58[‡]



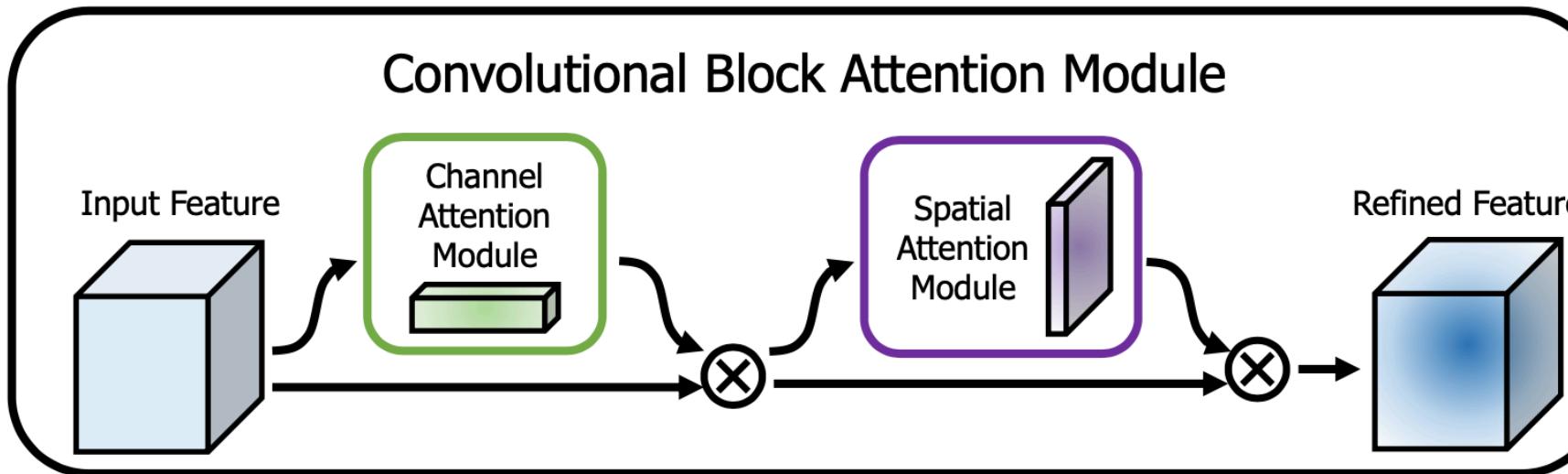
	original		re-implementation			SENet		
	top-1 err.	top-5 err.	top-1 err.	top-5 err.	GFLOPs	top-1 err.	top-5 err.	GFLOPs
ResNet-50 [10]	24.7	7.8	24.80	7.48	3.86	23.29 _(1.51)	6.62 _(0.86)	3.87
ResNet-101 [10]	23.6	7.1	23.17	6.52	7.58	22.38 _(0.79)	6.07 _(0.45)	7.60
ResNet-152 [10]	23.0	6.7	22.42	6.34	11.30	21.57 _(0.85)	5.73 _(0.61)	11.32
ResNeXt-50 [47]	22.2	-	22.11	5.90	4.24	21.10 _(1.01)	5.49 _(0.41)	4.25
ResNeXt-101 [47]	21.2	5.6	21.18	5.57	7.99	20.70 _(0.48)	5.01 _(0.56)	8.00
VGG-16 [39]	-	-	27.02	8.81	15.47	25.22 _(1.80)	7.70 _(1.11)	15.48
BN-Inception [16]	25.2	7.82	25.38	7.89	2.03	24.23 _(1.15)	7.14 _(0.75)	2.04
Inception-ResNet-v2 [42]	19.9 [†]	4.9 [†]	20.37	5.21	11.75	19.80 _(0.57)	4.79 _(0.42)	11.76





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CBAM: Convolutional Block Attention Module



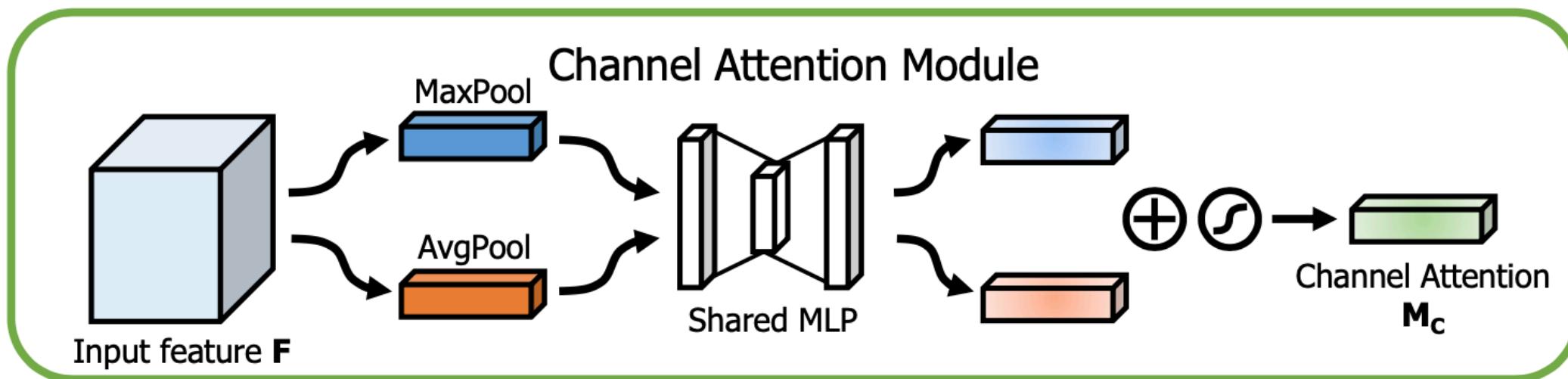
$$F \in \mathbb{R}^{C \times H \times W} \rightarrow \text{intermediate feature map}$$

$$M_c \in \mathbb{R}^{C \times 1 \times 1} \rightarrow \text{1D channel attention map}$$

$$M_s \in \mathbb{R}^{1 \times H \times W} \rightarrow \text{2D spatial attention map}$$

$$F' = M_c(F) \odot F$$

$$F'' = M_s(F') \odot F'$$

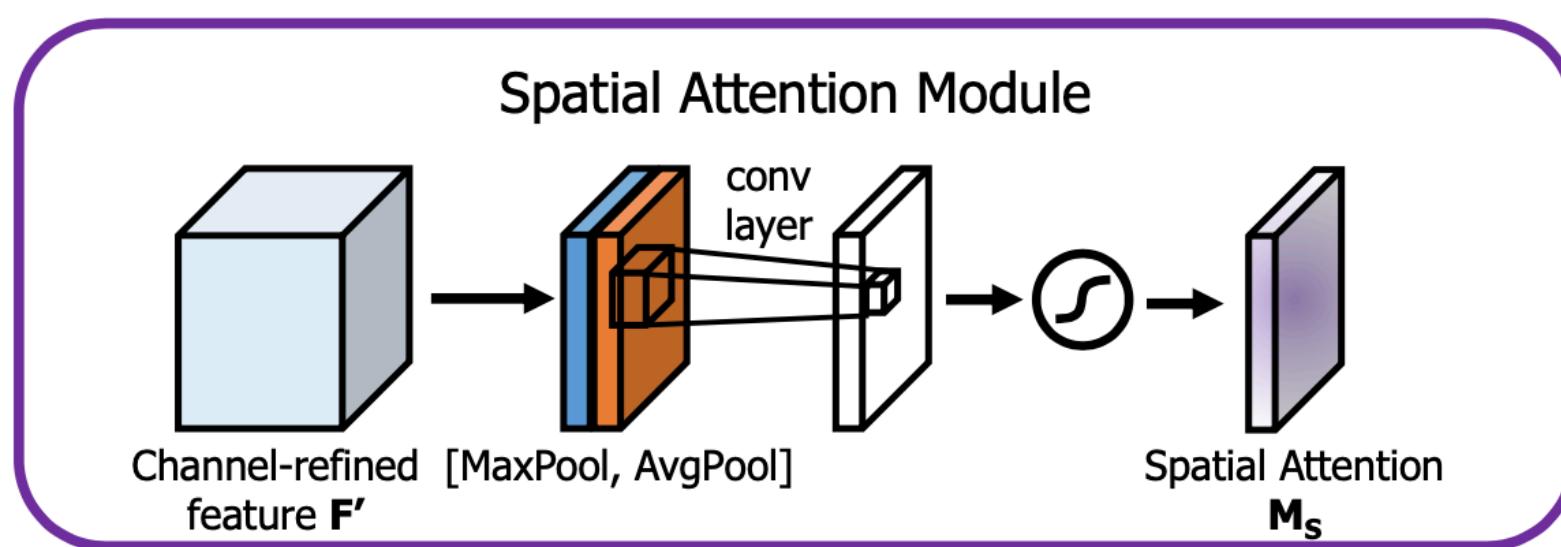


$$M_c(F) = \sigma(MLP(AvgPool(F)) + MLP(MaxPool(F)))$$

$$= \sigma(\mathbf{W}_1(\mathbf{W}_0(F_{avg}^c)) + \mathbf{W}_1(\mathbf{W}_0(F_{max}^c))),$$

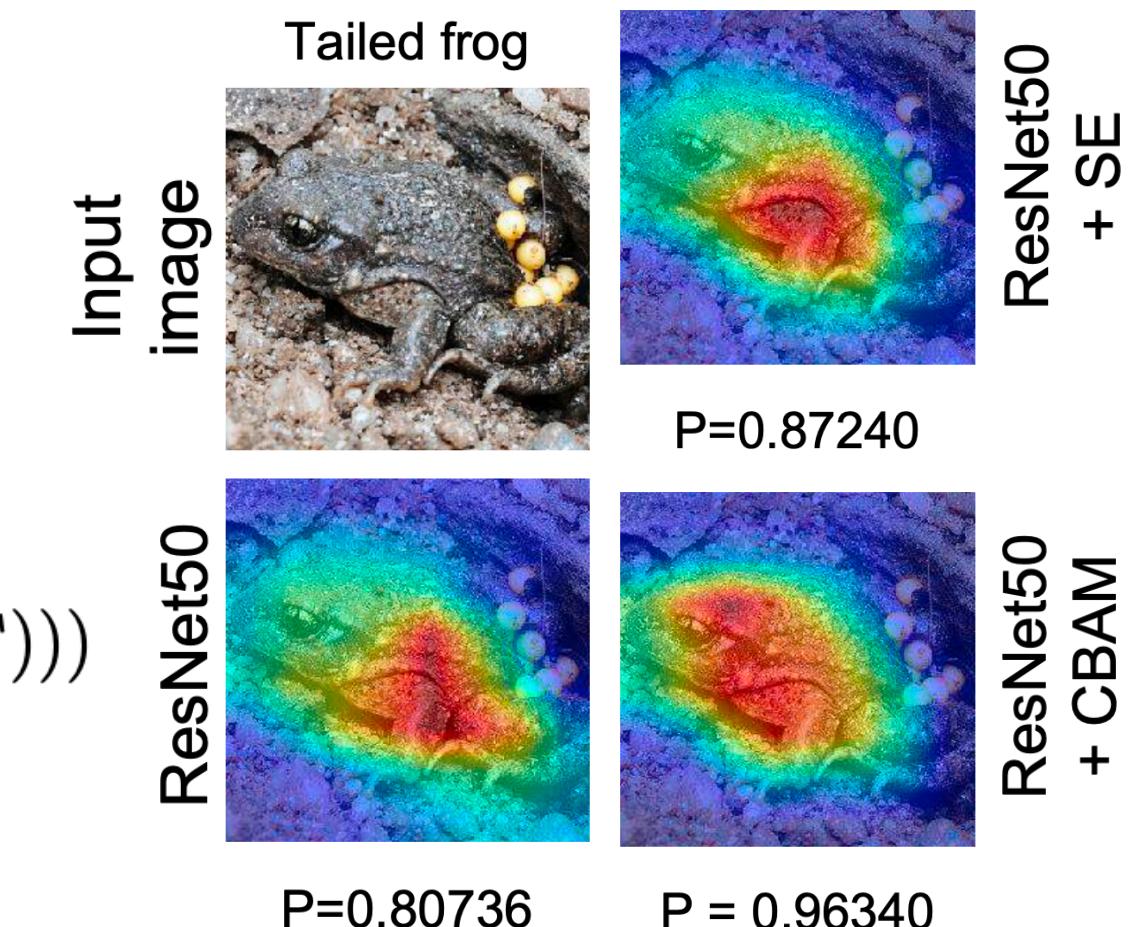
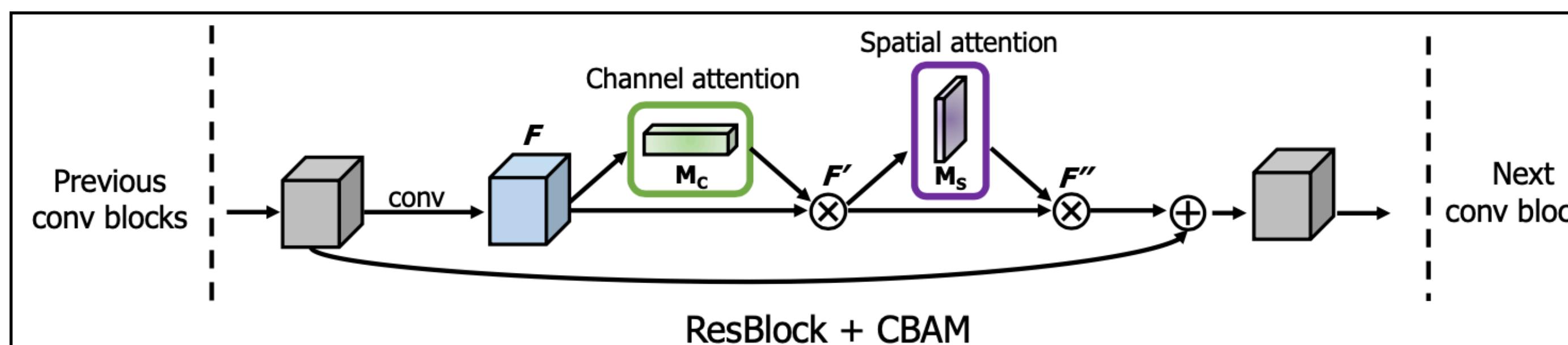
$$\mathbf{W}_0 \in \mathbb{R}^{C/r \times C}, \text{ and } \mathbf{W}_1 \in \mathbb{R}^{C \times C/r}$$

$r \rightarrow$ reduction ratio

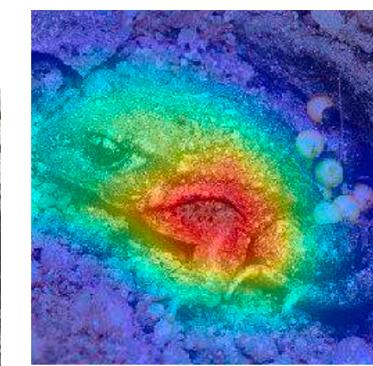


$$M_s(F) = \sigma(f^{7 \times 7}([AvgPool(F); MaxPool(F)]))$$

$$= \sigma(f^{7 \times 7}([F_{avg}^s; F_{max}^s])),$$

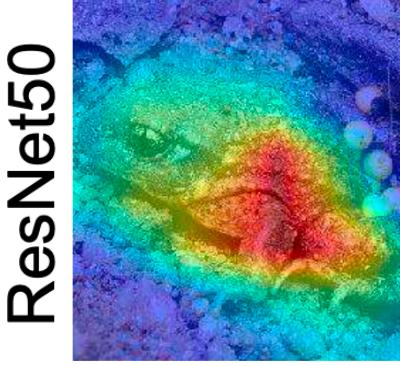


Input image

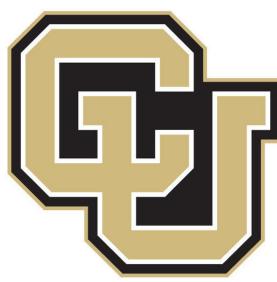


P=0.87240

ResNet50



P=0.96340



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Random Erasing Data Augmentation

Algorithm 1: Random Erasing Procedure

Input : Input image I ;
 Image size W and H ;
 Area of image S ;
 Erasing probability p ;
 Erasing area ratio range s_l and s_h ;
 Erasing aspect ratio range r_1 and r_2 .

Output: Erased image I^* .

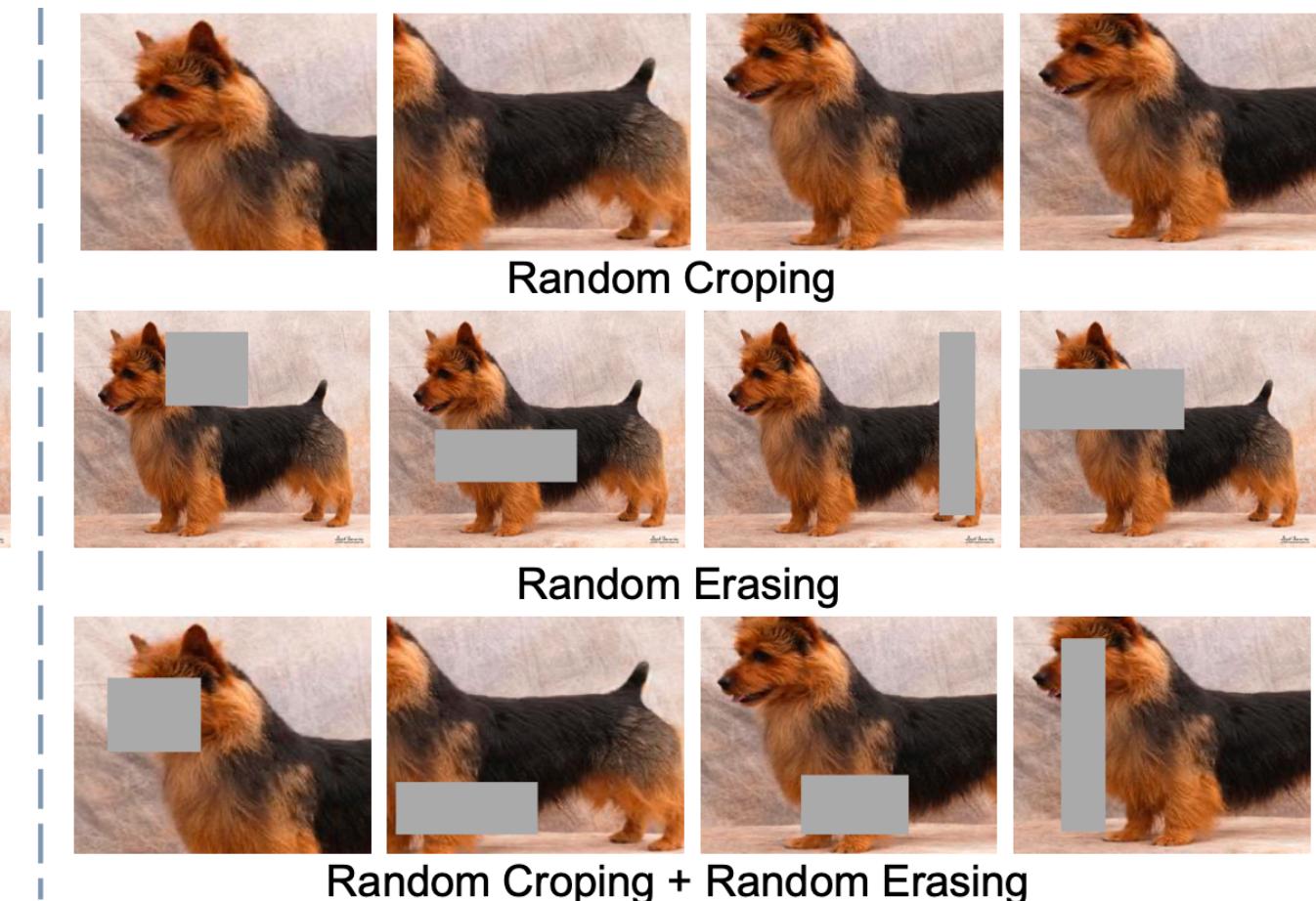
Initialization: $p_1 \leftarrow \text{Rand}(0, 1)$.

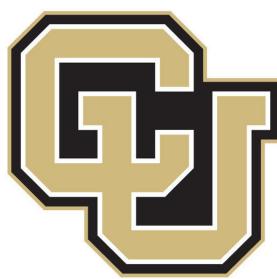
```

1 if  $p_1 \geq p$  then
2    $I^* \leftarrow I$ ;
3   return  $I^*$ .
4 else
5   while True do
6      $S_e \leftarrow \text{Rand}(s_l, s_h) \times S$ ;
7      $r_e \leftarrow \text{Rand}(r_1, r_2)$ ;
8      $H_e \leftarrow \sqrt{S_e \times r_e}$ ,  $W_e \leftarrow \sqrt{\frac{S_e}{r_e}}$ ;
9      $x_e \leftarrow \text{Rand}(0, W)$ ,  $y_e \leftarrow \text{Rand}(0, H)$ ;
10    if  $x_e + W_e \leq W$  and  $y_e + H_e \leq H$  then
11       $I_e \leftarrow (x_e, y_e, x_e + W_e, y_e + H_e)$ ;
12       $I(I_e) \leftarrow \text{Rand}(0, 255)$ ;
13       $I^* \leftarrow I$ ;
14      return  $I^*$ .
15    end
16  end
17 end
  
```



Model	CIFAR-100	
	Baseline	Random Erasing
ResNet-20	30.84 ± 0.19	29.97 ± 0.11
ResNet-32	28.50 ± 0.37	27.18 ± 0.32
ResNet-44	25.27 ± 0.21	24.29 ± 0.16
ResNet-56	24.82 ± 0.27	23.69 ± 0.33
ResNet-110	23.73 ± 0.37	22.10 ± 0.41
ResNet-20-PreAct	30.58 ± 0.16	30.18 ± 0.13
ResNet-32-PreAct	29.04 ± 0.25	27.82 ± 0.28
ResNet-44-PreAct	25.22 ± 0.19	24.10 ± 0.26
ResNet-56-PreAct	24.14 ± 0.25	22.93 ± 0.27
ResNet-110-PreAct	22.11 ± 0.20	20.99 ± 0.11
ResNet-18-PreAct	24.50 ± 0.29	24.03 ± 0.19
WRN-28-10	18.49 ± 0.11	17.73 ± 0.15
ResNeXt-8-64	19.27 ± 0.30	18.84 ± 0.18



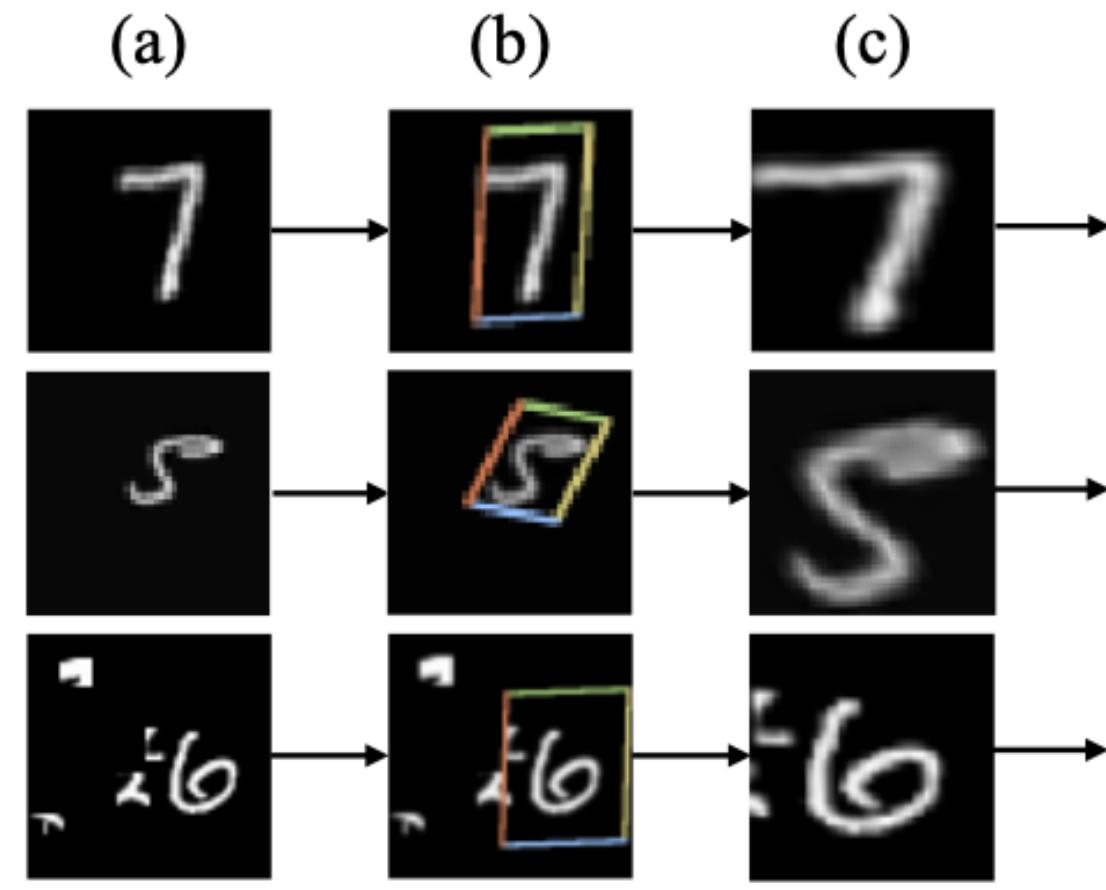


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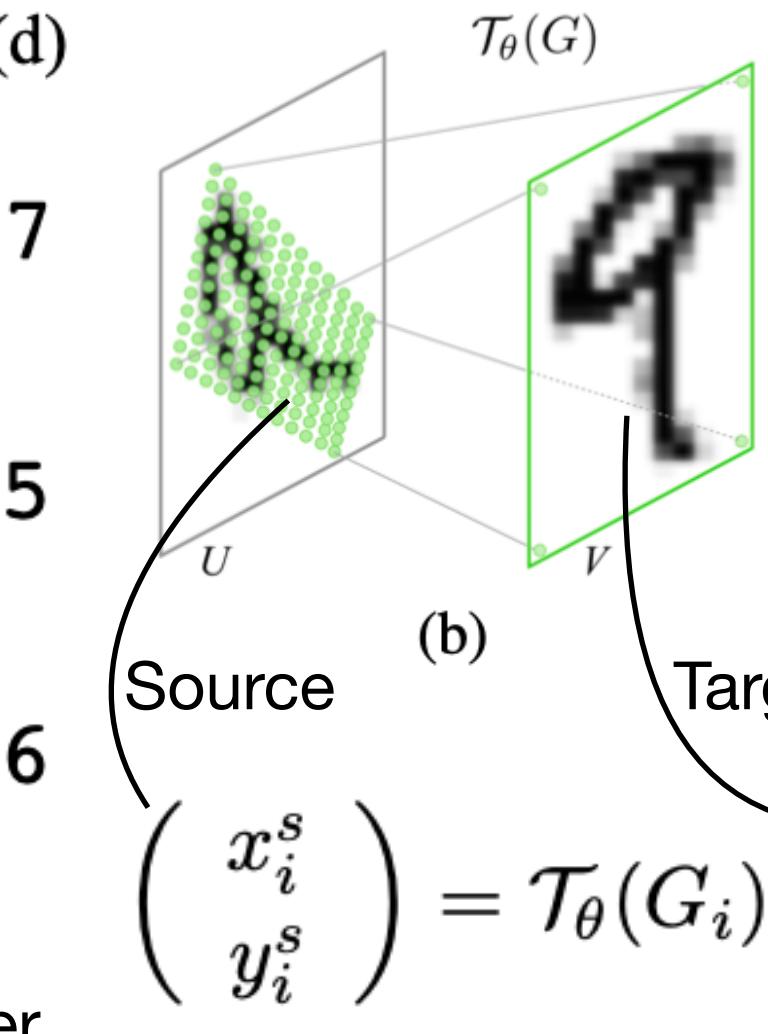
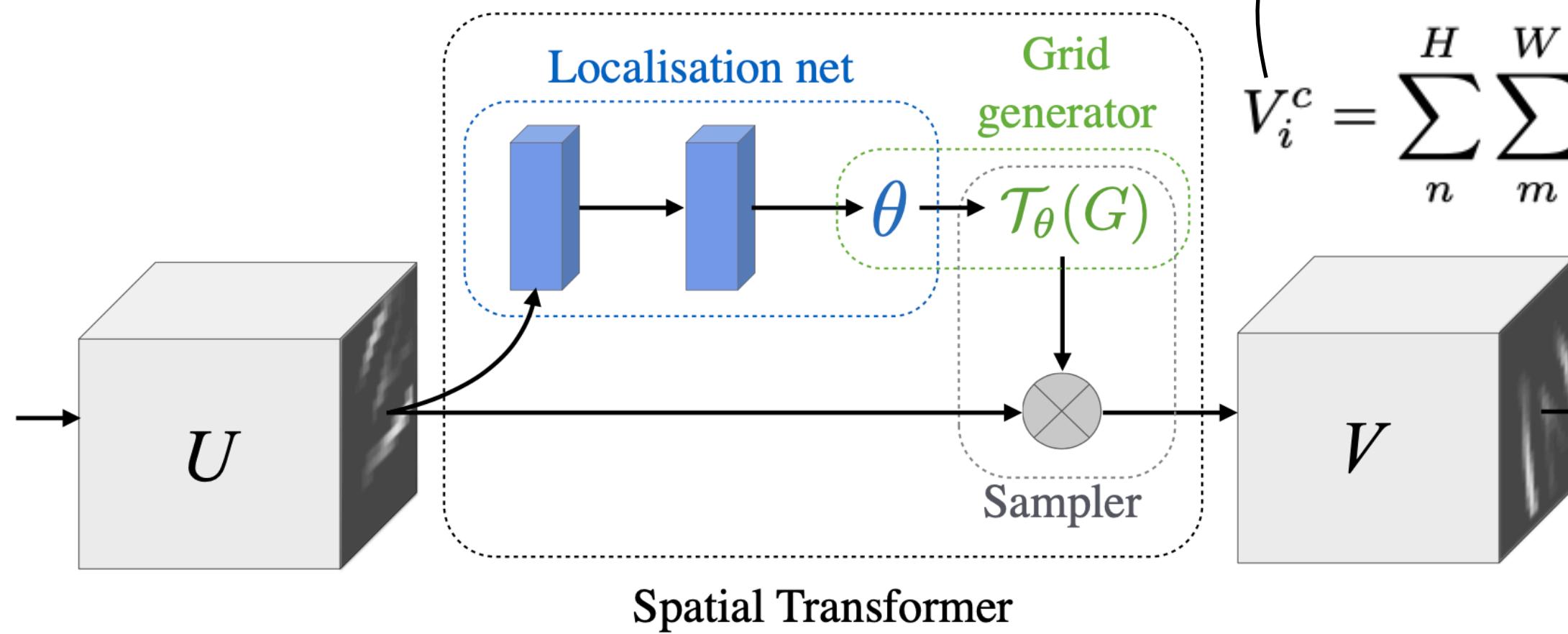


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Spatial Transformer Networks



(a) MNIST distorted with random translation, scale, rotation, and clutter.



Model	Size	
	64px	128px
Maxout CNN [10]	4.0	-
CNN (ours)	4.0	5.6
DRAM* [1]	3.9	4.5
ST-CNN	Single Multi	3.7 3.6 3.9

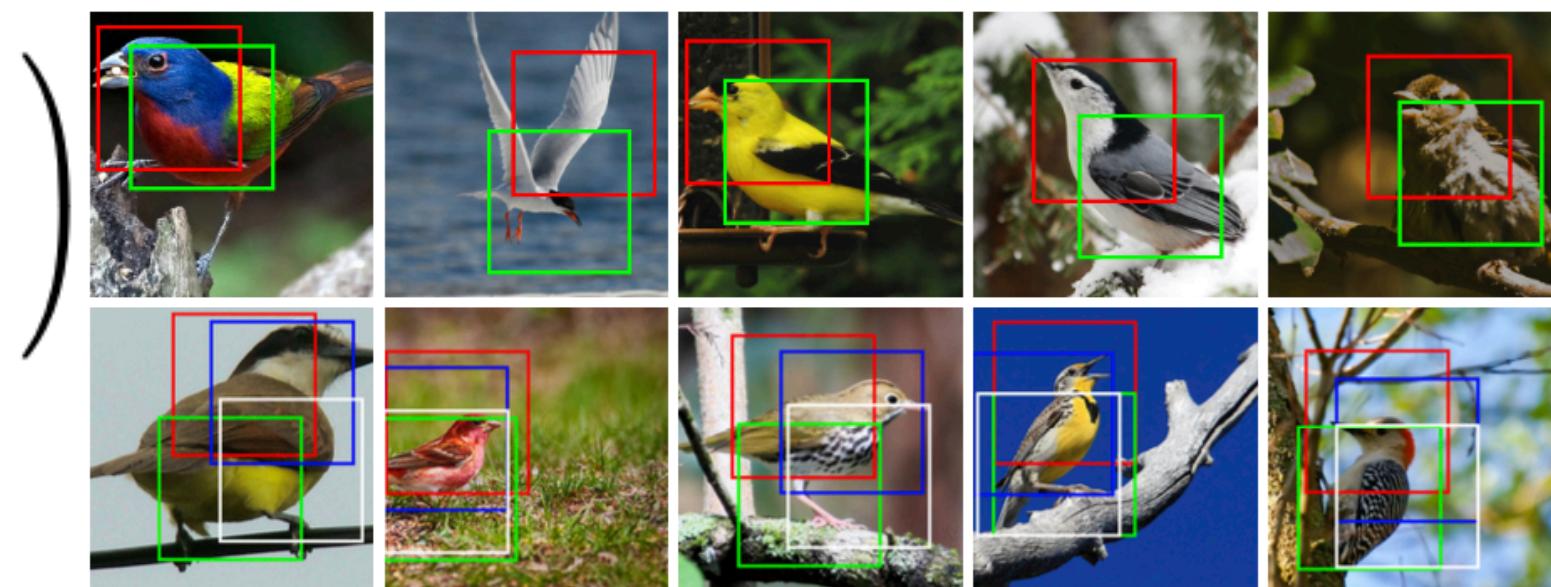
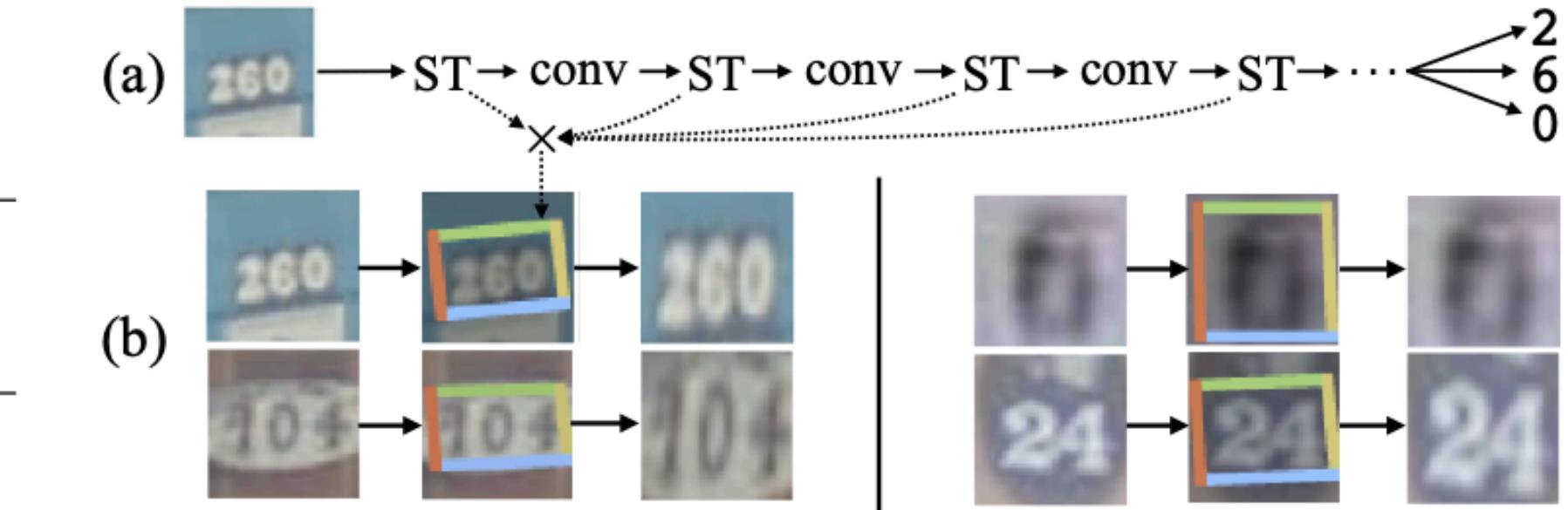
$$\begin{pmatrix} x_i^s \\ y_i^s \end{pmatrix} = \mathcal{T}_\theta(G_i) = \mathbf{A}_\theta \begin{pmatrix} x_i^t \\ y_i^t \\ 1 \end{pmatrix} = \begin{bmatrix} \theta_{11} & \theta_{12} & \theta_{13} \\ \theta_{21} & \theta_{22} & \theta_{23} \end{bmatrix} \begin{pmatrix} x_i^t \\ y_i^t \\ 1 \end{pmatrix}$$

Bilinear sampling

$$V_i^c = \sum_n^H \sum_m^W U_{nm}^c \max(0, 1 - |x_i^s - m|) \max(0, 1 - |y_i^s - n|)$$

$$\frac{\partial V_i^c}{\partial U_{nm}^c} = \sum_n^H \sum_m^W \max(0, 1 - |x_i^s - m|) \max(0, 1 - |y_i^s - n|)$$

$$\frac{\partial V_i^c}{\partial x_i^s} = \sum_n^H \sum_m^W U_{nm}^c \max(0, 1 - |y_i^s - n|) \begin{cases} 0 & \text{if } |m - x_i^s| \geq 1 \\ 1 & \text{if } m \geq x_i^s \\ -1 & \text{if } m < x_i^s \end{cases}$$



Model	
Cimpoi '15 [4]	66.7
Zhang '14 [30]	74.9
Branson '14 [2]	75.7
Lin '15 [20]	80.9
Simon '15 [24]	81.0
CNN (ours) 224px	82.3
2×ST-CNN 224px	83.1
2×ST-CNN 448px	83.9
4×ST-CNN 448px	84.1



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Dynamic Routing Between Capsules

neuron → $\underbrace{\text{capsule}}$ → layer
group of neurons

s_j → total input to capsule j

v_j → vector output of capsule j

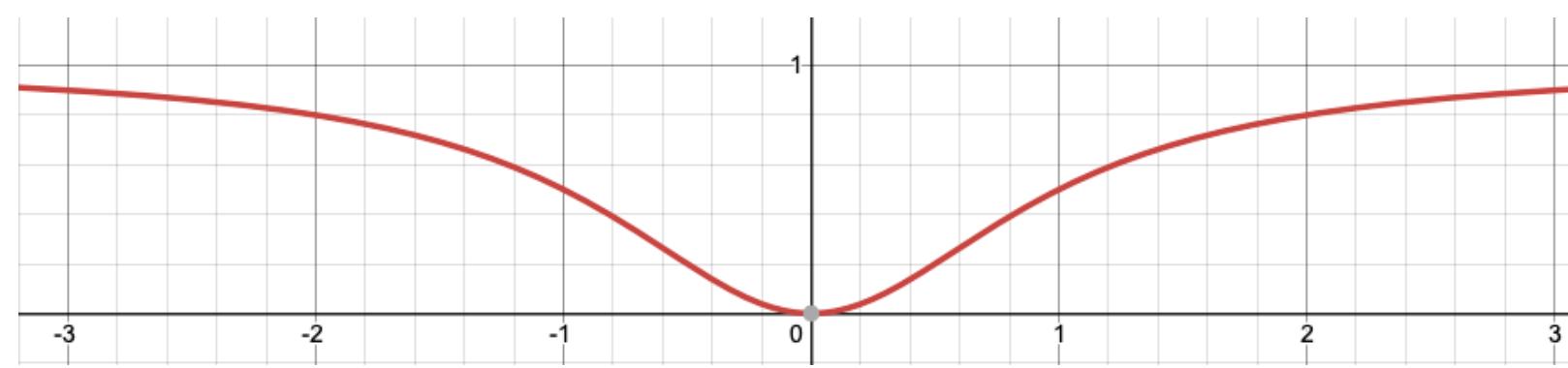
$\|v_j\|$ → probability that the entity represented by the capsule is present in the current input

$\frac{v_j}{\|v_j\|}$ → properties of the entity

entity: an object or an object part

$$v_j = \frac{\|s_j\|^2}{1 + \|s_j\|^2} \frac{s_j}{\|s_j\|} \leftarrow \text{squash}$$

squashing function



$$s_j = \sum_i c_{ij} u_{j|i}$$

c_{ij} → coupling coefficients

$$u_{j|i} = W_{ij} u_i$$

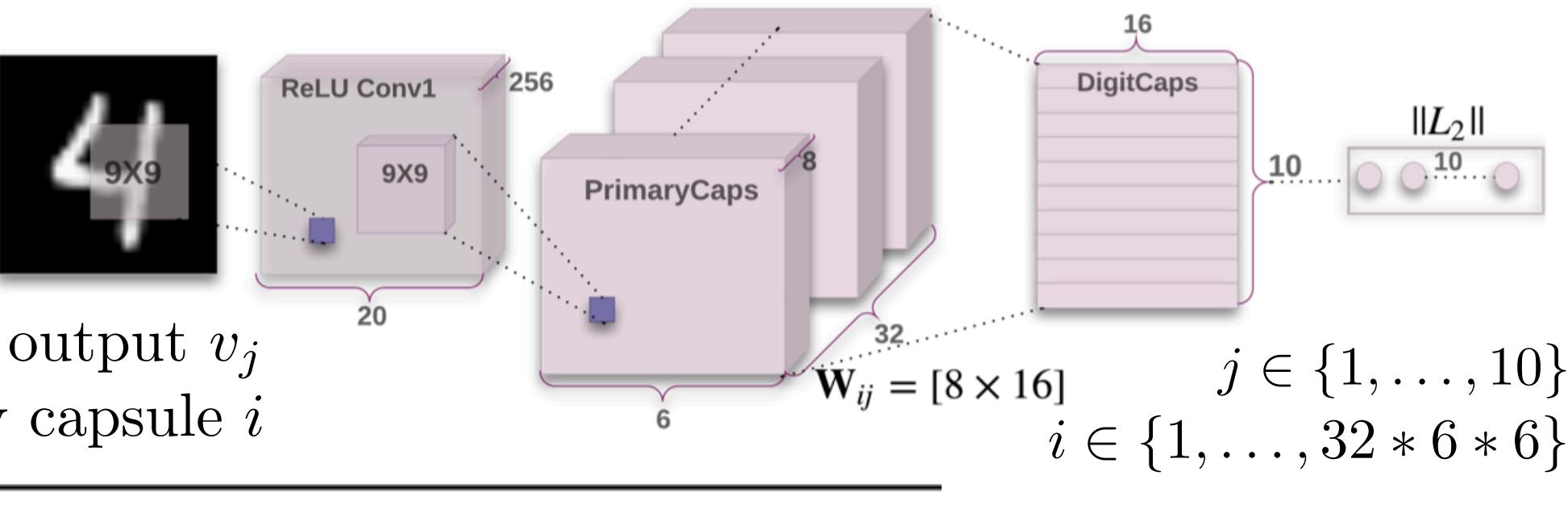
$u_{j|i}$ → prediction vectors from the capsules in the layer below

u_i → output of a capsule in the layer below

$$c_{ij} = \frac{\exp(b_{ij})}{\sum_k \exp(b_{ik})} \leftarrow \text{softmax}$$

b_{ij} → log prob. that capsule i should be coupled to capsule j

$$a_{ij} = v_j \cdot u_{j|i} \rightarrow \text{agreement btw current output } v_j \text{ and the predictions } u_{j|i} \text{ made by capsule } i$$

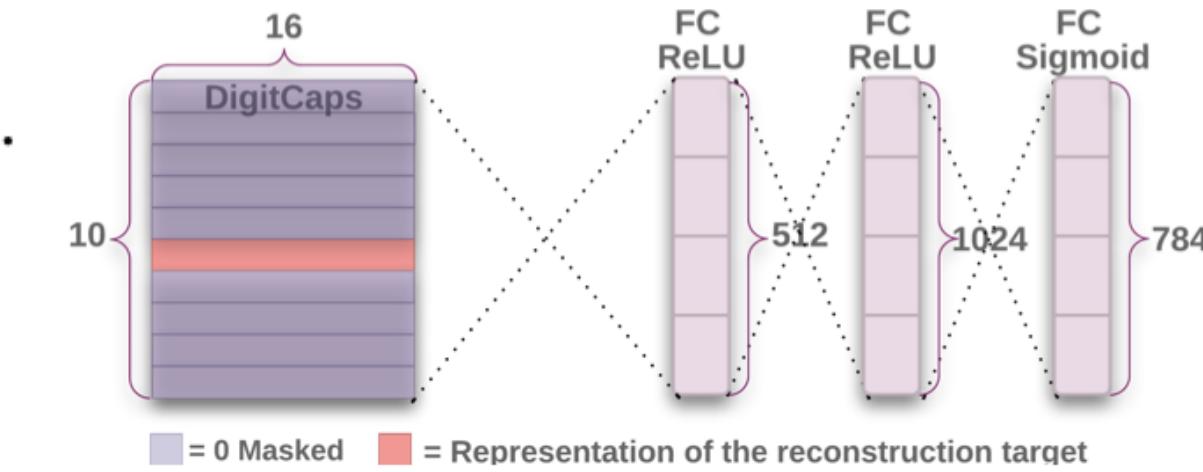


Procedure 1 Routing algorithm.

```

1: procedure ROUTING( $u_{j|i}, r, l$ )
2:   for all capsule  $i$  in layer  $l$  and capsule  $j$  in layer  $(l + 1)$ :  $b_{ij} \leftarrow 0$ .
3:   for  $r$  iterations do
4:     for all capsule  $i$  in layer  $l$ :  $\mathbf{c}_i \leftarrow \text{softmax}(\mathbf{b}_i)$ 
5:     for all capsule  $j$  in layer  $(l + 1)$ :  $\mathbf{s}_j \leftarrow \sum_i c_{ij} \mathbf{u}_{j|i}$ 
6:     for all capsule  $j$  in layer  $(l + 1)$ :  $\mathbf{v}_j \leftarrow \text{squash}(\mathbf{s}_j)$ 
7:     for all capsule  $i$  in layer  $l$  and capsule  $j$  in layer  $(l + 1)$ :  $b_{ij} \leftarrow b_{ij} + \mathbf{u}_{j|i} \cdot \mathbf{v}_j$ 
return  $\mathbf{v}_j$ 

```



Margin loss for digit existence

$$L_k = T_k \max(0, m^+ - \|\mathbf{v}_k\|)^2 + \lambda (1 - T_k) \max(0, \|\mathbf{v}_k\| - m^-)^2$$

$T_k = 1$ iff a digit of class k is present

$$m^+ = 0.9 \text{ and } m^- = 0.1 \quad \lambda = 0.5$$

Method	Routing	Reconstruction	MNIST (%)	MultiMNIST (%)
Baseline	-	-	0.39	8.1
CapsNet	1	no	0.34 ± 0.032	-
CapsNet	1	yes	0.29 ± 0.011	7.5
CapsNet	3	no	0.35 ± 0.036	-
CapsNet	3	yes	0.25 ± 0.005	5.2

Scale and thickness	4 4 4 4 6 6 6 6 6 6
Localized part	6 6 6 6 6 6 6 6 6 6
Stroke thickness	5 5 5 5 5 5 5 5 5 5
Localized skew	4 4 4 4 4 4 4 4 4 4
Width and translation	3 3 3 3 3 3 3 3 3 3
Localized part	2 2 2 2 2 2 2 2 2 2



An Image is Worth 16x16 Words: Transformers for Image Recognition at Scale

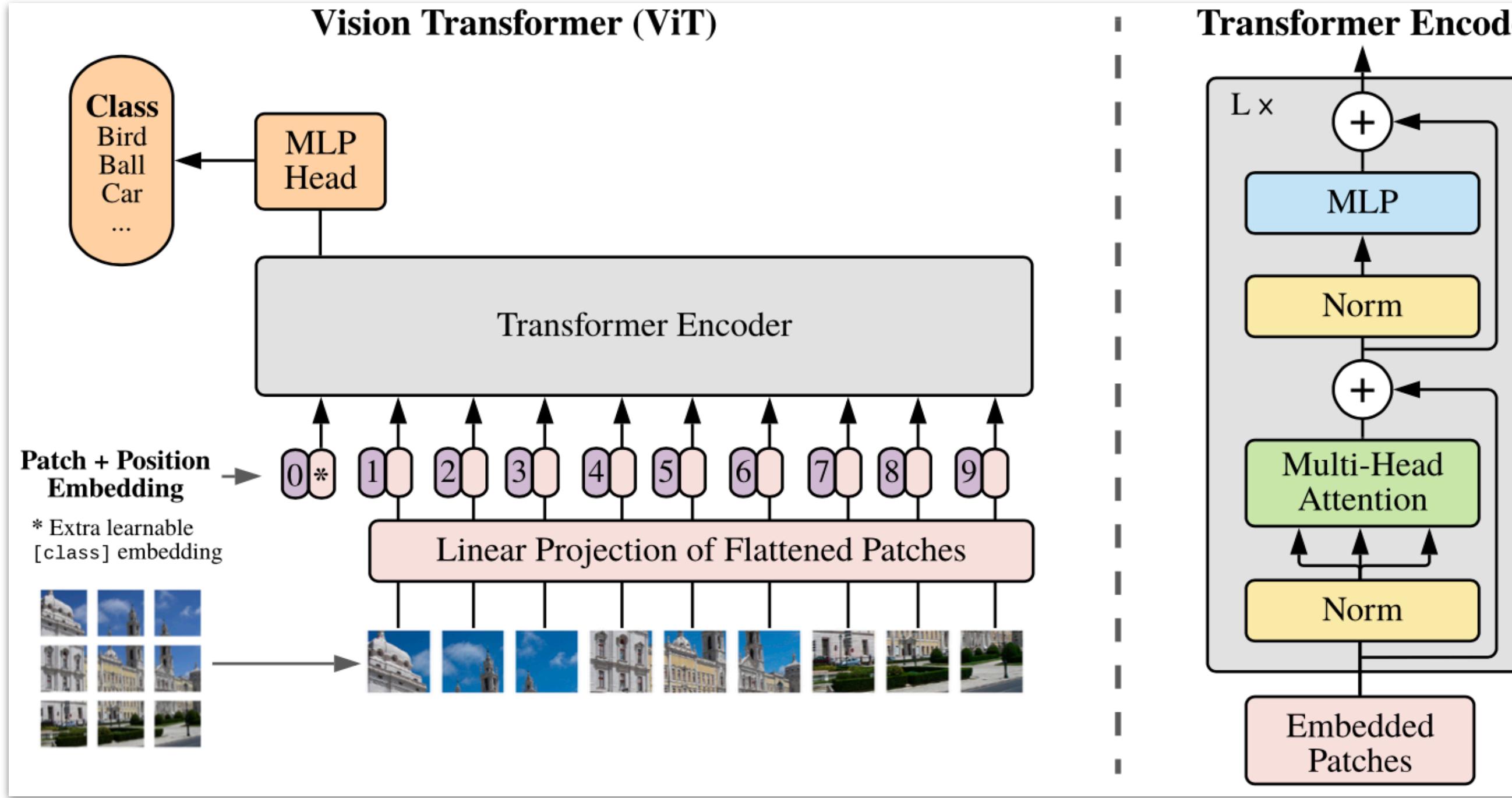


Image Patches \equiv Tokens (Words) in NLP

$x \in \mathbb{R}^{H \times W \times C} \rightarrow$ image

$x_p \in \mathbb{R}^{N \times (P^2 C)} \rightarrow$ sequence of flattened 2D patches (reshape x)

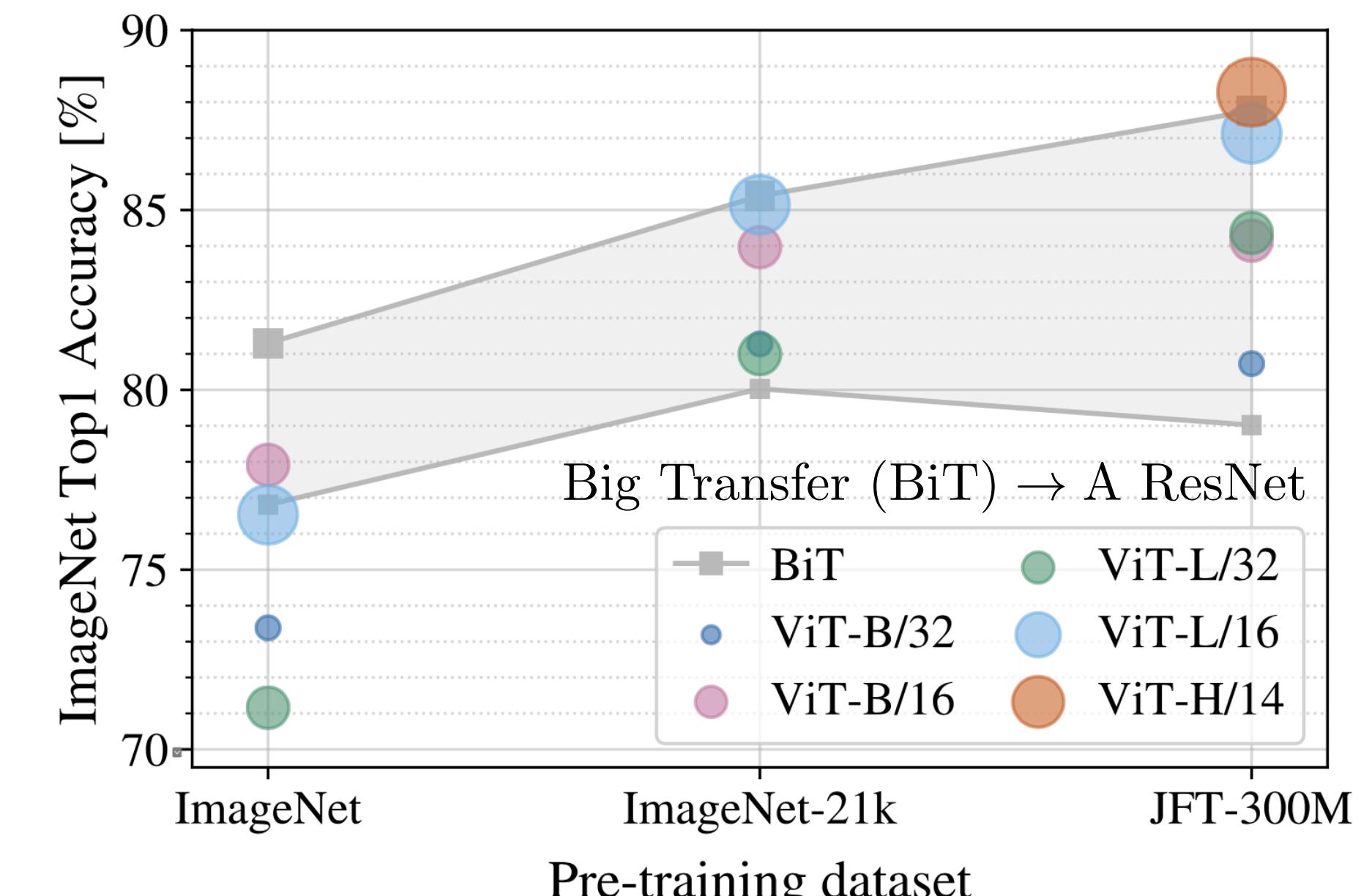
$P \times P \rightarrow$ resolution of each image patch

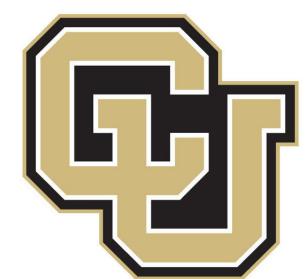
$N = \frac{HW}{P^2} \rightarrow$ resulting number of patches

$D \rightarrow$ latent vector size

$z_0 = [x_{\text{class}}; x_p^1 E; x_p^2 E; \dots; x_p^N E] + E_{\text{pos}}, E \in \mathbb{R}^{(P^2 C) \times D}, E_{\text{pos}} \in \mathbb{R}^{(N+1) \times D}$

$z'_l = \text{MSA}(\text{LN}(z_{l-1}) + z_{l-1}), l = 1, \dots, L \quad z_l = \text{MLP}(\text{LN}(z'_l)) + z'_l, l = 1, \dots, L \quad y = \text{LN}(z_L^0)$





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Questions?

[YouTube Playlist](#)
