CSE260
Digital Logic Design
SOP & POS

SOP & POS

- **Sum-of-Products (SOP) Expression**: a product term or a logical sum (OR) of several product terms.

  Examples: x+yz', xy'+x'yz, AB+A'B’

  Product-of-Sums (POS) Expression: a sum term or a logical product (AND) of several sum terms.

  Examples: x(y+z'), (x+y')(x'+y+z), (A+B)(A'+B’)

- Every boolean expression can either be expressed as sum-of-products or product-of-sums expression.

  Examples:

SOP:	$x'y + xy' + xyz$
POS:	$(x + y')(x' + y)(x' + z')$

MIN & MAX TERM

MIN & MAX TERM

Minterms are sum terms.

For Boolean functions, the minterms of a function are the terms for which the result is 1.

Boolean functions can be expressed as sum- of-Minterms.

Maxterms are Product terms.

For Boolean functions, the maxterms of a function are the terms for which the result is 0.

Boolean functions can be expressed as Products-of-Maxterms.

MIN and MAX

	A	B	C	F
0	0	0	0	1
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	0
7	1	1	1	0

Min Terms : 0,1,4,5
[000,001,100,101] $F=\sum(0,1,4,5)$

Max Terms : 2,3,6,7
[010,011,110,111] $F=\Pi(2,3,6,7)$

MIN-SOP and MAX-POS

	A	B	C	F
0	0	0	0	1
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	0
7	1	1	1	0

Min Terms : 0,1,4,5 [000,001,100,101] F=∑(0,1,4,5)

F=A'B'C'+A'B'C+AB'C'+AB'C

Max Terms : 2,3,6,7 [010,011,110,111] F=Π(2,3,6,7)

F=(A+B'+C)(A+B'+C')(A'+B'+C) (A'+B'+C')

Min/SOP

0-Prime, 1-No prime

AND among literals

OR among terms

Max/POS

1-Prime, 0-No prime

OR among literals

AND among terms

Conversion of Function to SOP & POS

How to Convert into SOP:

Check if each term contains all variable, if not then AND (x+x') if x is the missing term

Function, F(A,B,C)=A+B'C

How To Convert into SOP

- F(A,B,C)=A+B'C

=A(B+B')(C+C')+B'C(A+A')
=(AB+AB')(C+C')+B'C(A+A')
=AB(C+C')+AB'(C+C')+B'C(A+A')
=ABC+ABC'+***AB'C***+AB'C'+***B'CA***+B'CA'

Now, removing duplicates and writing the literals in order,

=ABC+ABC'+AB'C+AB'C'+A'B'C
= 111, 110, 101, 100, 001
=∑(1,4,5,6,7)

How to Convert into POS:

1. Often distributive law (x+yz)=(x+y)(x+z) is used
2. If then terms, like x, are missing, OR xx’
3. Each POS is missing a term so OR missing terms, Again apply distributive law

Function, F(A,B,C)=A+B’C

How to Convert into POS:

- F(A,B,C)=A+B'C
  = (A+B')(A+C)
  = (A+B'+CC')(A+C+BB')
  = (A+B'+C)(A+B'+C')(A+C+B)(A+C+B')
  Now, removing duplicates and writing the literals in order,
  = (A+B'+C)(A+B'+C')(A+B+C)
  = 010, 011, 000
  = π (2,3,0)
  = π (0,2,3)

Another Example

Find **SOP** for F(w,x,y,z) =wy+x'z

F(w,x,y,z)
=wy+x'z
=wy[(x+x')(z+z')] + x'z[(w+w')(y+y')]
=wy(xz+xz'+x'z+x'z') + x'z(wy+wy'+w'y+w'y')
=wyxz+wyxz'+wyx'z+wyx'z'+x'zwy+x'zwy'+x'zw'y+x'zw'y'
=wxyz+wxyz'+wx'yz+wx'yz'+wx'y'z+w'x'yz+w'x'y'z
=1111,1110,1011,1010,1001,0011,0001
=∑(15,14,11,10,9,3,1)
=∑(1,3,9,10,11,14,15)

Another Example

Find **POS** for F(w,x,y,z) =wy+x'z

F(w,x,y,z)
= wy+x'z
= (wy+x')(wy+z)
= (x'+w)(x'+y)(w+z)(y+z)
= (x'+w+yy')(x'+y+zz')(w+z+xx')(y+z+xx')
= (x'+w+y)(x'+w+y')(x'+y+z)(x'+y+z')(w+z+x)(w+z+x')(y+z+x)(y+z+x')
= (x'+w+y+zz')(x'+w+y'+zz')(x'+y+z+ww')(x'+y+z'+ww')
(w+z+x+yy')(w+z+x'+yy')(y+z+x+ww')(y+z+x'+ww')
= (w+x'+y+z)(w+x'+y+z')(w+x'+y'+z)(w+x'+y'+z')
(w+x'+y+z)(w'+x'+y+z)(w+x'+y+z')(w'+x'+y+z')
(w+x+y+z)(w+x+y'+z)(w+x'+y+z)(w+x'+y'+z)
(w+x+y+z)(w'+x+y+z)(w+x'+y+z)(w'+x'+y+z)
= (w+x'+y+z)(w+x'+y+z')(w+x'+y'+z)(w+x'+y'+z')(w'+x'+y+z)
(w'+x'+y+z')(w+x+y+z)(w+x+y'+z)(w'+x+y+z)
= 0100,0101,0110,0111,1100,1101,0000,0010,1000
= Π(4,5,6,7,12,13,0,2,8) = Π(0,2,4,5,6,7,8,12,13)

Same thing can be done by taking all missing terms together

F(w,x,y,z)
= wy+x'z
= (wy+x')(wy+z)
= (x'+w)(x'+y)(w+z)(y+z)
= (x'+w+yy'+zz')(x'+y+zz'+ww')(w+z+xx'+yy')(y+z+xx'+ww')
= (x'+w+y+zz')(x'+w+y'+zz')(x'+y+z+ww')(x'+y+z'+ww')
(w+z+x+yy')(w+z+x'+yy')(y+z+x+ww')(y+z+x'+ww')
= (w+x'+y+z)(w+x'+y+z')(w+x'+y'+z)(w+x'+y'+z')
(w+x'+y+z)(w'+x'+y+z)(w+x'+y+z')(w'+x'+y+z')
(w+x+y+z)(w+x+y'+z)(w+x'+y+z)(w+x'+y'+z)
(w+x+y+z)(w'+x+y+z)(w+x'+y+z)(w'+x'+y+z)
= (w+x'+y+z)(w+x'+y+z')(w+x'+y'+z)(w+x'+y'+z')(w'+x'+y+z)
(w'+x'+y+z')(w+x+y+z)(w+x+y'+z)(w'+x+y+z)
= 0100,0101,0110,0111,1100,1101,0000,0010,1000
= Π(4,5,6,7,12,13,0,2,8) = Π(0,2,4,5,6,7,8,12,13)