Sequential Circuit Analysis

Circuit Diagram to State Diagram
State Diagram to Circuit Diagram
Counters
Application of Counters

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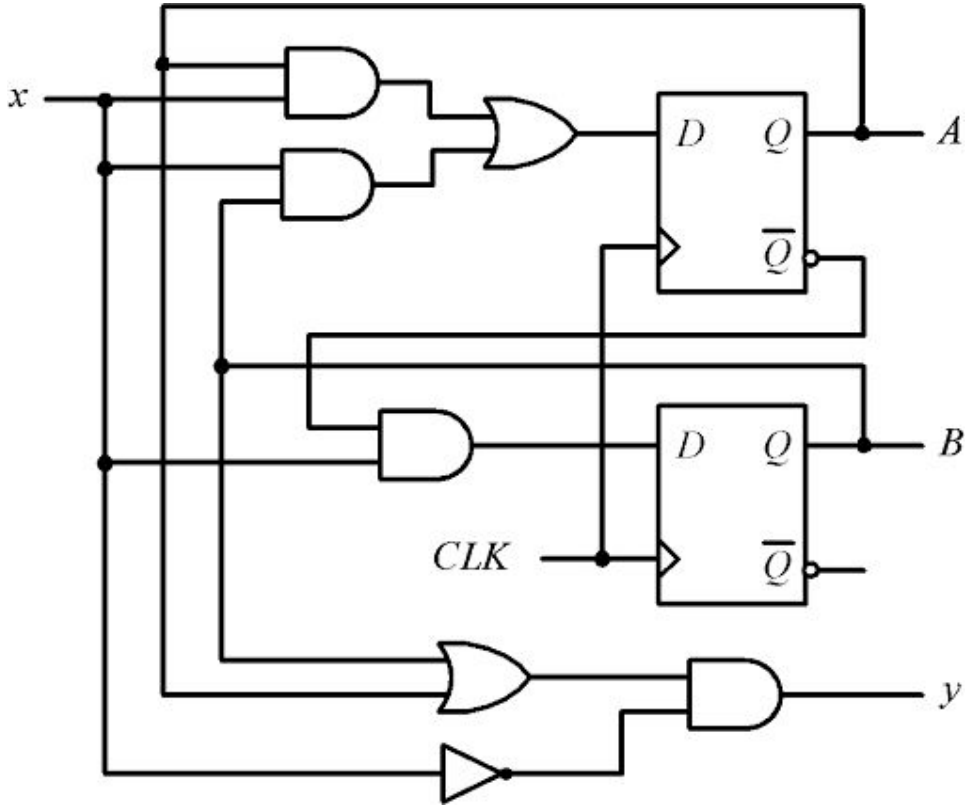
What is sequential logic

- Sequential circuit has additional dimension which is time
- Combinational logic only depends on current input
- Sequential circuit output depends on previous input other than current input
- More powerful than combination logic
- Able to model condition which can’t be modeled by combinational logic

Circuit Diagram
to
State Diagram

Example 1: Draw the state diagram for the given circuit

Sequential circuit using D flip flop



Present States:	**Next States:**
A and B	A+ and B+
Input:	**Output:**
DA, DB, x	y

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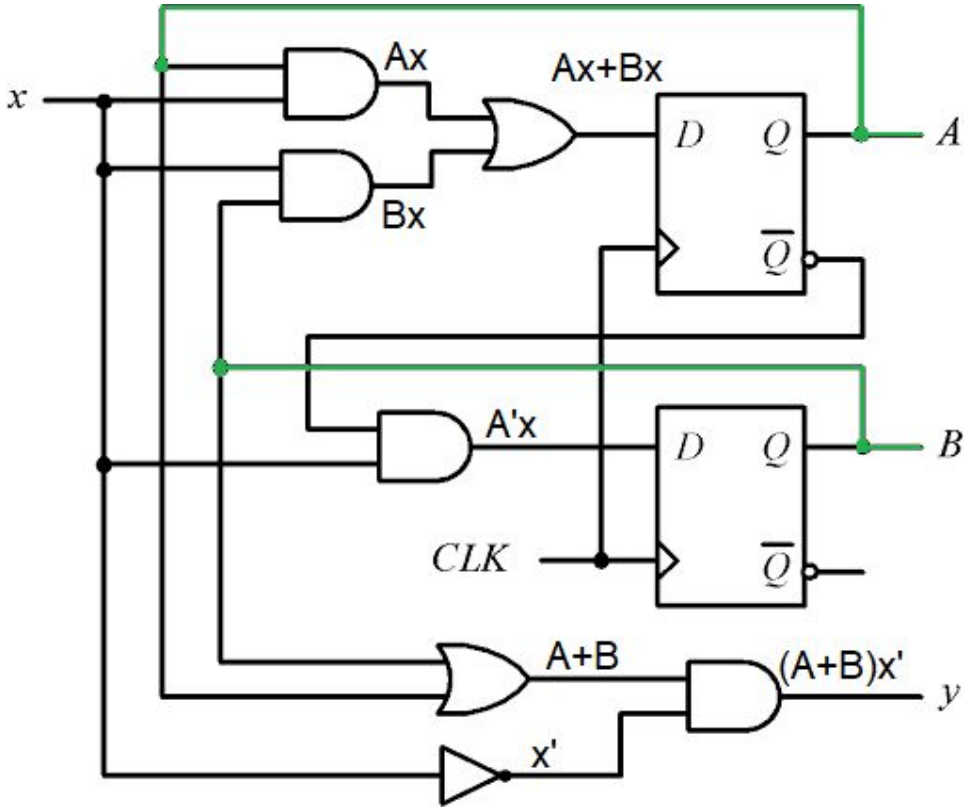
Example 1: Draw the state diagram for the given circuit

1. Find out the input equations of the flip flop and the output equation

DA = Ax + Bx

DB = A'x

y = (A+B)x'



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Example 1: Draw the state diagram for the given circuit

2. Construct the state table according to the flip flop input and output equations

A	B	x	DA	DB	y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0

DA = Ax + Bx

DB = A'x

y = (A+B)x'

When A=0, B=0, x=0,

DA = 0.0 + 0.0 = 0

DB = 0'.0 = 0

y = (0+0).0' = 0

When A=0, B=0, x=1,

DA = 0.1 + 0.1 = 0

DB = 0'.1 = 1

y = (0+0).1' = 0

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Example 1: Draw the state diagram for the given circuit

3. Find out the next states from the flip flop input values

A	B	x	DA	DB	y	A+	B+
0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	1
0	1	0	0	0	1	0	0
0	1	1	1	1	0	1	1
1	0	0	0	0	1	0	0
1	0	1	1	0	0	1	0
1	1	0	0	0	1	0	0
1	1	1	1	0	0	1	0

D Flip-flop Truth table

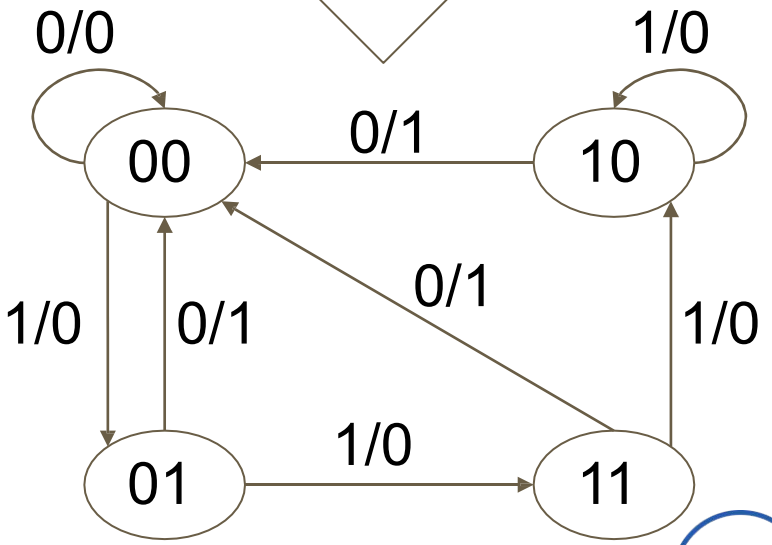
D	Q
0	0
1	1

$\therefore$ DA = A+
$\therefore$ DB = B+

Example 1: Draw the state diagram for the given circuit

A	B	x	A+	B+	y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0

4. Draw the state diagram from the state table



State Diagram

Example 2: Draw the state diagram for the given circuit

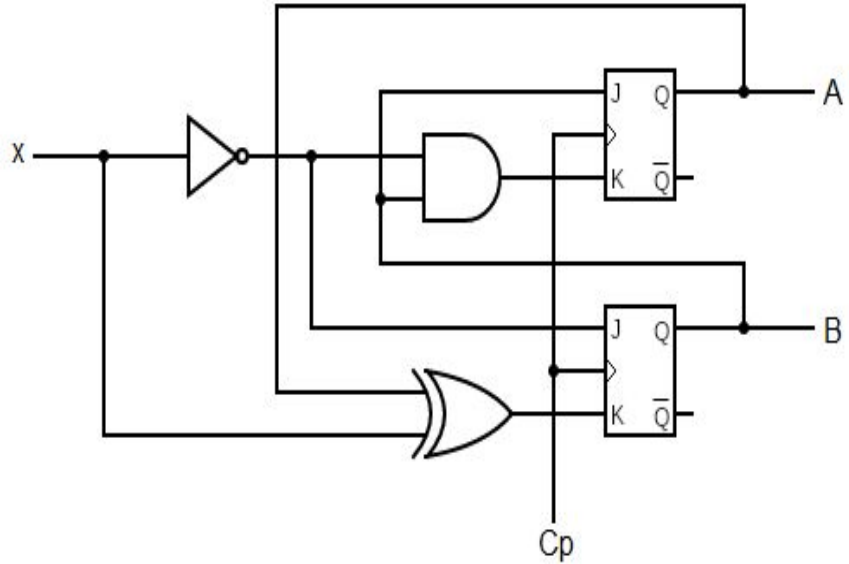
Sequential circuit using JK flip flop

Sequential circuit using JK flip flop

Present States:
A and B

Next States:
A+ and B+

Input:
JA, KA, JB, KB, x



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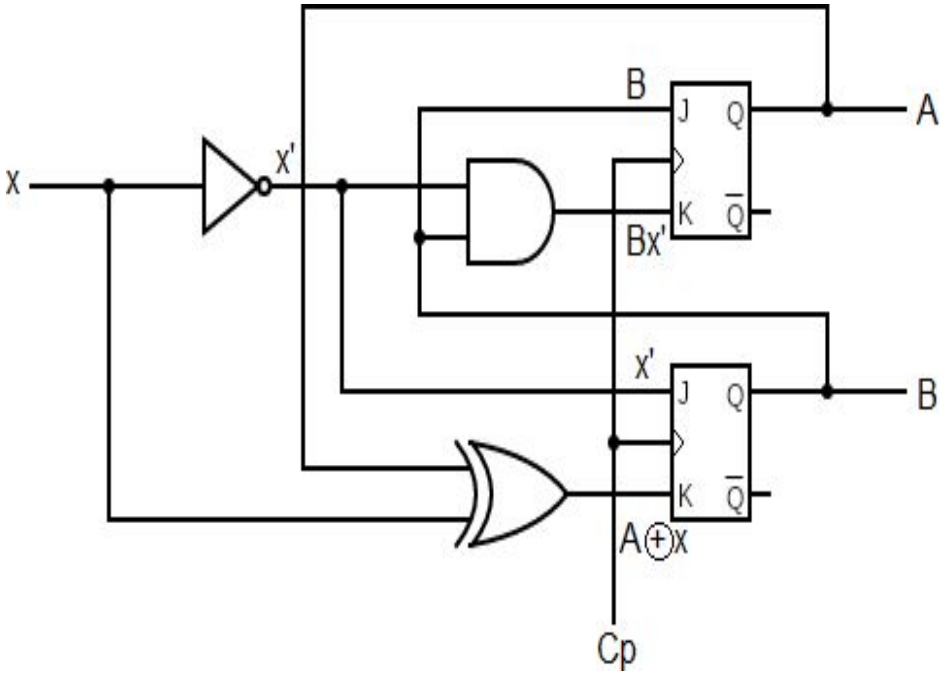
Example 2: Draw the state diagram for the given circuit

1. Find out the input equations of the flip flop and the output equation

JA = B
KA = Bx'
JB = x'
KB = A xor X



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Example 2: Draw the state diagram for the given circuit

2. Construct the state table according to the flip flop input and output equations

A	B	x	JA	KA	JB	KB
0	0	0	0	0	1	0
0	0	1	0	0	0	1
0	1	0	1	1	1	0
0	1	1	1	0	0	1
1	0	0	0	0	1	1
1	0	1	0	0	0	0
1	1	0	1	1	1	1
1	1	1	1	0	0	0

JA = B
KA = Bx'
JB = x'
KB = A xor X

When A=0, B=0, x=0,
JA = 0
KA = 0.0' = 0
JB = 0' = 1
KB = 0 xor 0 = 0

When A=0, B=0, x=1,
JA = 0
KA = 0.1' = 0
JB = 1' = 0
KB = 0 xor 0 = 0

Example 2: Draw the state diagram for the given circuit

3. Find out the next states from the flip flop input values

JK Flip-flop Truth table

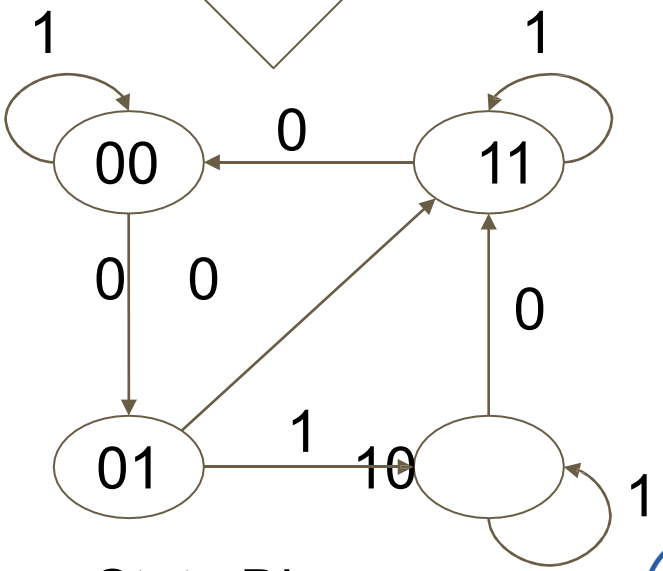
J	K	Q
0	0	Memory/ No change
0	1	0
1	0	1
1	1	Toggle

A	B	x	JA	KA	JB	KB	A+	B+
0	0	0	0	0	1	0	0	1
0	0	1	0	0	0	1	0	0
0	1	0	1	1	1	0	1	1
0	1	1	1	0	0	1	1	0
1	0	0	0	0	1	1	1	1
1	0	1	0	0	0	0	1	0
1	1	0	1	1	1	1	0	0
1	1	1	1	0	0	0	1	1

Example 2: Draw the state diagram for the given circuit

A	B	x	A+	B+
0	0	0	0	1
0	0	1	0	0
0	1	0	1	1
0	1	1	1	0
1	0	0	1	1
1	0	1	1	0
1	1	0	0	0
1	1	1	1	1

4. Draw the state diagram from the state table

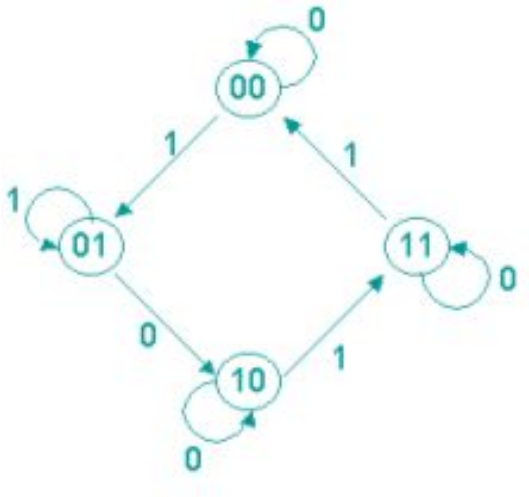


State Diagram

State Diagram
to
Circuit Diagram

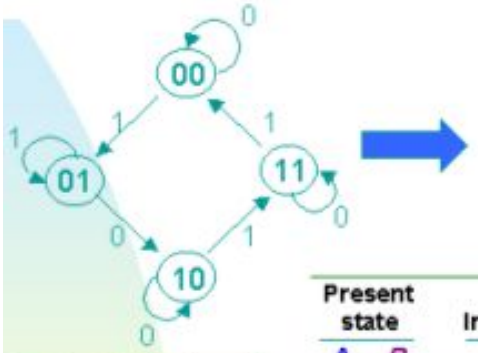
Design: Example 1

Given state diagram as follows, get the sequential circuit using JK flip-flop



Design: Example 1

State/excitation table using JK flip-flop



Present State	Next State x=0	Next State x=1
AB	A^+B^+	A^+B^+
00	00	01
01	10	01
10	10	11
11	11	00

Q	Q^+	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

JK Flip-flop's excitation table.

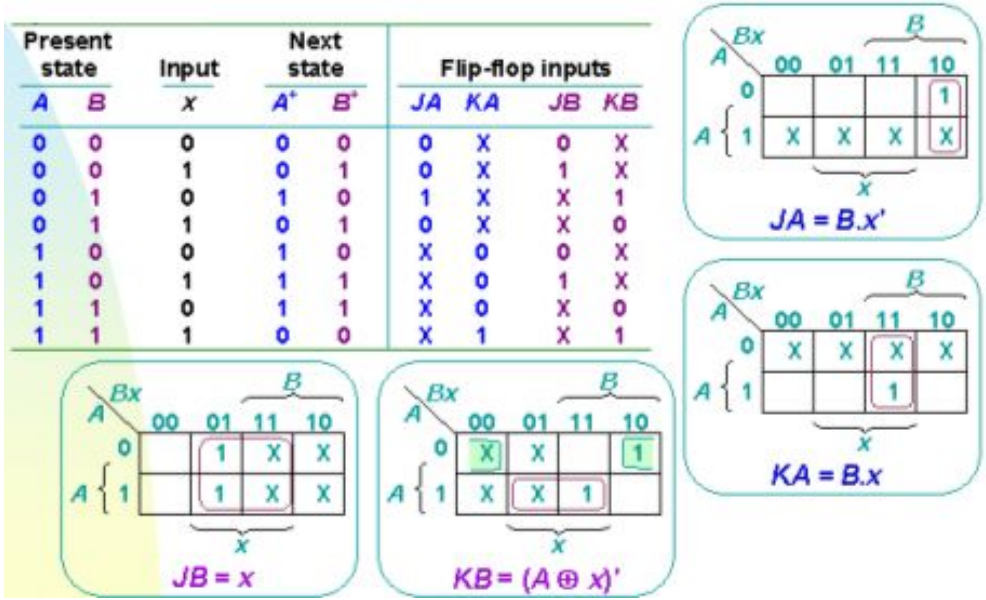
Present state A	Present state B	Input x	Next state A^+	Next state B^+	Flip-flop inputs JA	KA	JB	KB
0	0	0	0	0	0	X	0	X
0	0	1	0	1	0	X	1	X
0	1	0	1	0	1	X	X	1
0	1	1	0	1	0	X	X	0
1	0	0	1	0	X	0	0	X
1	0	1	1	1	X	0	1	X
1	1	0	1	1	X	0	X	0
1	1	1	0	0	X	1	X	1

Note:
Step 1: Build state table
Step 2: Find no. of Flip-Flop *(i.e. 2^m states means m flip-flips)*
Step 3: Build Excitation table
Step 4: Use K-map to find input functions
Step 5: Design logic diagram

Design: Example 1

From state table, get input flip-flop function

Present state		Input	Next state		Flip-flop inputs			
A	B	x	A^+	B^+	JA	KA	JB	KB
0	0	0	0	0	0	X	0	X
0	0	1	0	1	0	X	1	X
0	1	0	1	0	1	X	X	1
0	1	1	0	1	0	X	X	0
1	0	0	1	0	X	0	0	X
1	0	1	1	1	X	0	1	X
1	1	0	1	1	X	0	X	0
1	1	1	0	0	X	1	X	1



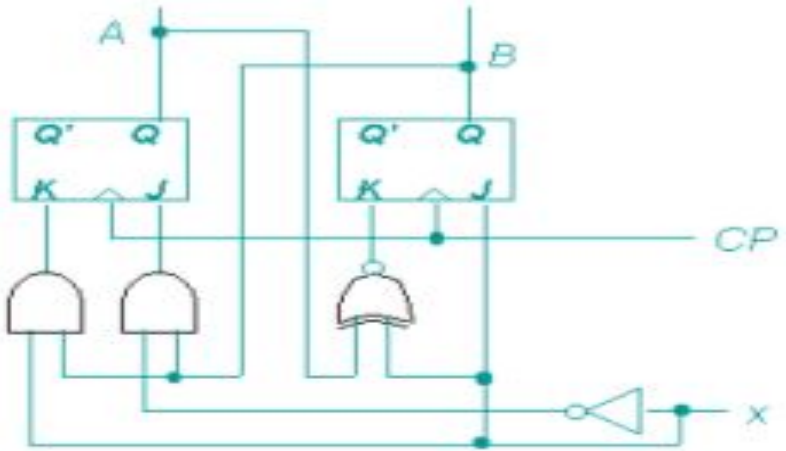
Design: Example 1

Input flip-flop function

$JA = B.x'$ $\quad JB = x$

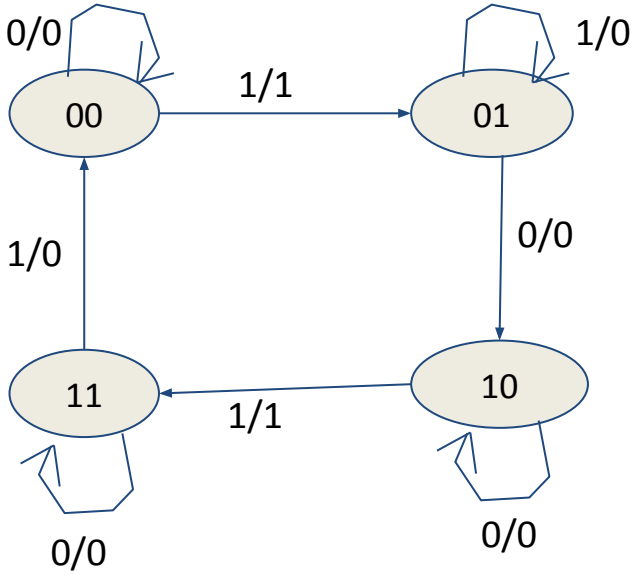
$KA = B.x$ $\quad KB = (A \oplus x)'$

Logic Diagram



Design: Example 2

Design, using D flip-flop, circuit is based on state table below. (Exercise: How if using JK flip-flop)



Present state		Input	Next state		Output
A	B	x	A^+	B^+	y
0	0	0	0	0	0
0	0	1	0	1	1
0	1	0	1	0	0
0	1	1	0	1	0
1	0	0	1	0	0
1	0	1	1	1	1
1	1	0	1	1	0
1	1	1	0	0	0

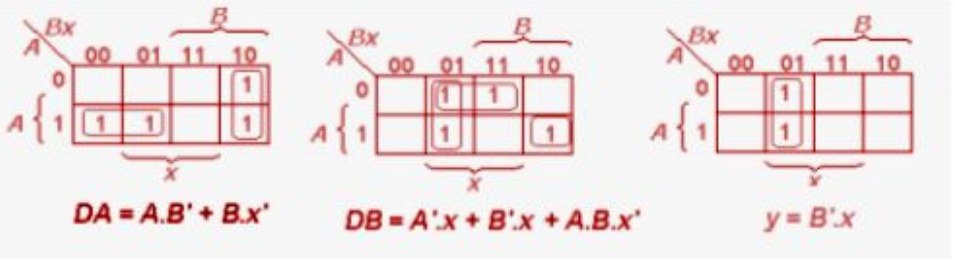
Design: Example 2

Determine input expression for flip-flop and y output

A	B	x	A+	B+	y	DA	DB
0	0	0	0	0	0	0	0
0	0	1	0	1	1	0	1
0	1	0	1	0	0	1	0
0	1	1	0	1	0	0	1
1	0	0	1	0	0	1	0
1	0	1	1	1	1	1	1
1	1	0	1	1	0	1	1
1	1	1	0	0	0	0	0

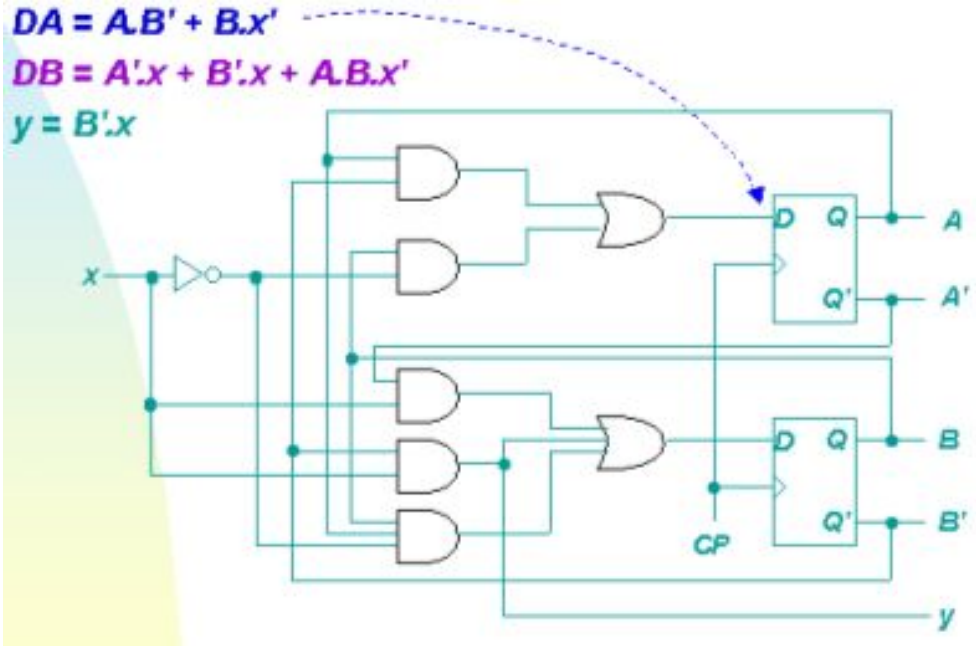
Q	Q^+	D
0	0	0
0	1	1
1	0	0
1	1	1

D Flip-flop



Design: Example 2

From expression built, draw logic diagram



Introduction: Counters

- Counters are circuits that cycle through a specified number of states.
- Two types of counters:
  - synchronous (parallel) counters
  - asynchronous (ripple) counters
- Ripple counters allow some flip-flop outputs to be used as a source of clock for other flip-flops.
- Synchronous counters apply the same clock to all flip-flops.
- **We will cover only synchronous this time**

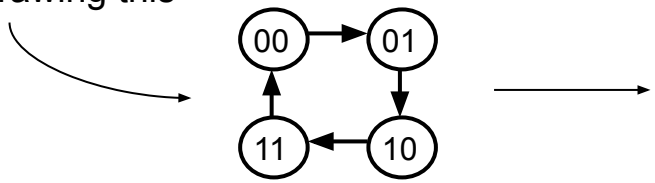
Synchronous (Parallel) Counters

- Synchronous (parallel) counters: the flip-flops are clocked at the same time by a common clock pulse.
- We can design these counters using the sequential logic design process.
- Example: 2-bit synchronous binary counter (using T flip-flops, or JK flip-flops with identical J,K inputs).

Synchronous (Parallel) Counters

Question: Design a 2-bit synchronous binary counter (using T flip-flops).

Start by drawing this



Present state		Next state		Flip-flop inputs	
A_1	A_0	A_1^+	A_0^+	TA_1	TA_0
0	0	0	1	0	1
0	1	1	0	1	1
1	0	1	1	0	1
1	1	0	0	1	1

Q_n	Q_{n+1}	T
0	0	0
0	1	1
1	0	1
1	1	0

Excitation table of T flip flop

Synchronous (Parallel) Counters

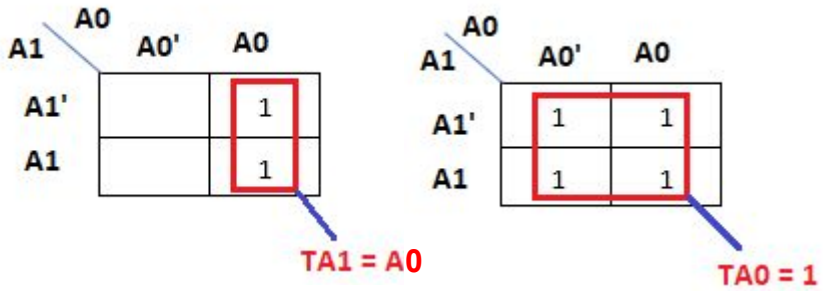
Present state		Next state		Flip-flop inputs	
A_1	A_0	A_1^+	A_0^+	TA_1	TA_0
0	0	0	1	0	1
0	1	1	0	1	1
1	0	1	1	0	1
1	1	0	0	1	1

Next, find out the equations for the flip-flop inputs.

$\mathbf{TA_1} = A_1'A_0 + A_1A_0 = A_0(A_1'+A_1) = A_0$

$\mathbf{TA_0} = 1$

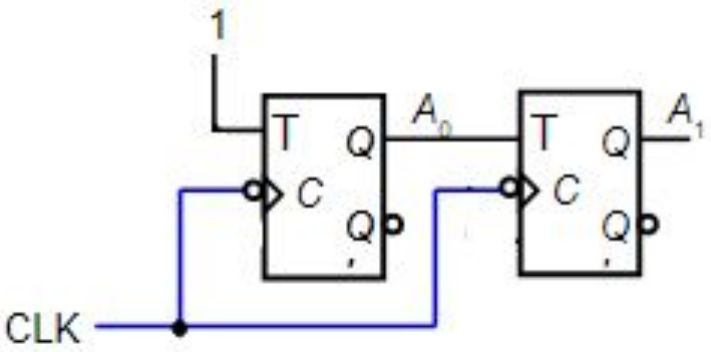
FYI, these can also be calculated using K-Maps



Synchronous (Parallel) Counters

$\mathbf{TA_1} = A_1'A_0 + A_1A_0 = A_0(A_1'+A_1) = A_0$
$\mathbf{TA_0} = 1$

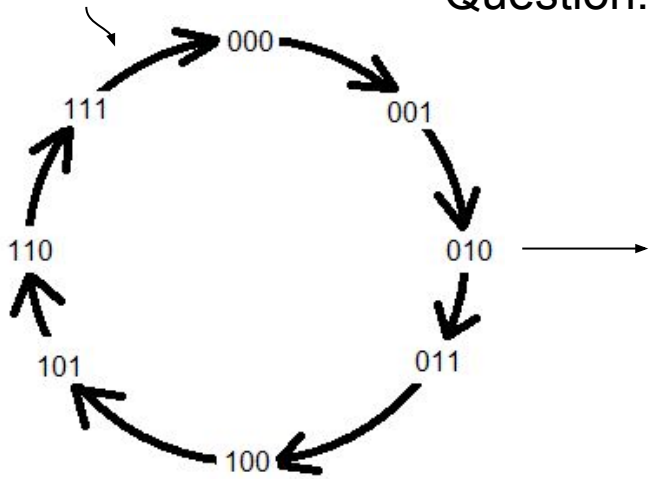
Finally draw the circuit diagram



Synchronous (Parallel) Counters

Start by drawing this

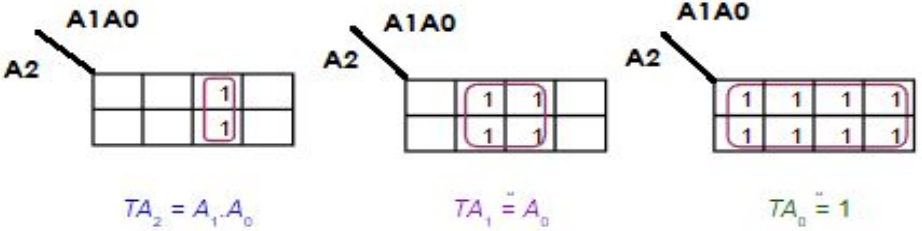
- Question: Design a 3-bit synchronous binary counter (using T).



Present state			Next state			Flip-flop inputs		
A_2	A_1	A_0	A_2^+	A_1^+	A_0^+	TA_2	TA_1	TA_0
0	0	0	0	0	1	0	0	1
0	0	1	0	1	0	0	1	1
0	1	0	0	1	1	0	0	1
0	1	1	1	0	0	1	1	1
1	0	0	1	0	1	0	0	1
1	0	1	1	1	0	0	1	1
1	1	0	1	1	1	0	0	1
1	1	1	0	0	0	1	1	1

Q_n	Q_{n+1}	T
0	0	0
0	1	1
1	0	1
1	1	0

Excitation table of T flip flop

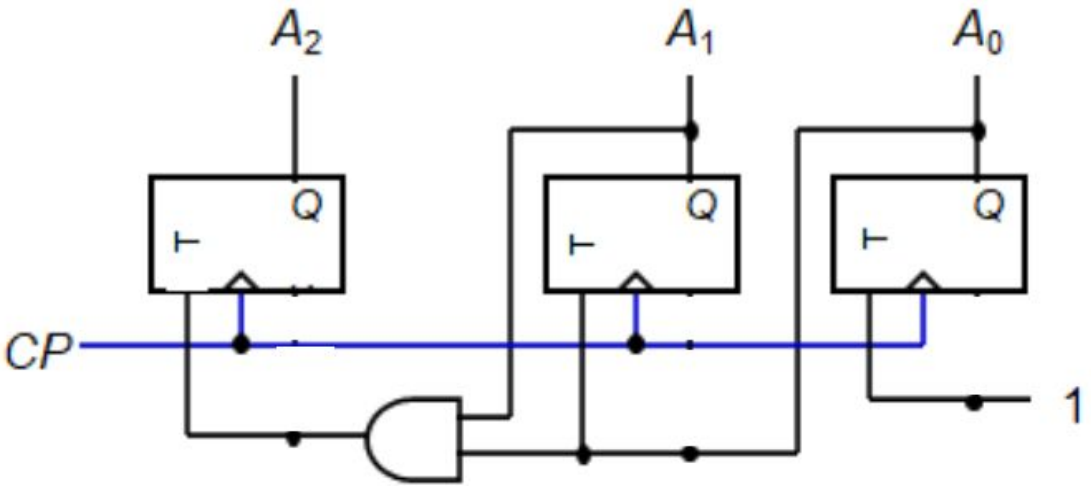


FYI, these can also be calculated using Boolean simplifications

Synchronous (Parallel) Counters

Draw the circuit diagram

$TA_2 = A_1.A_0$ $TA_1 = A_0$ $TA_0 = 1$

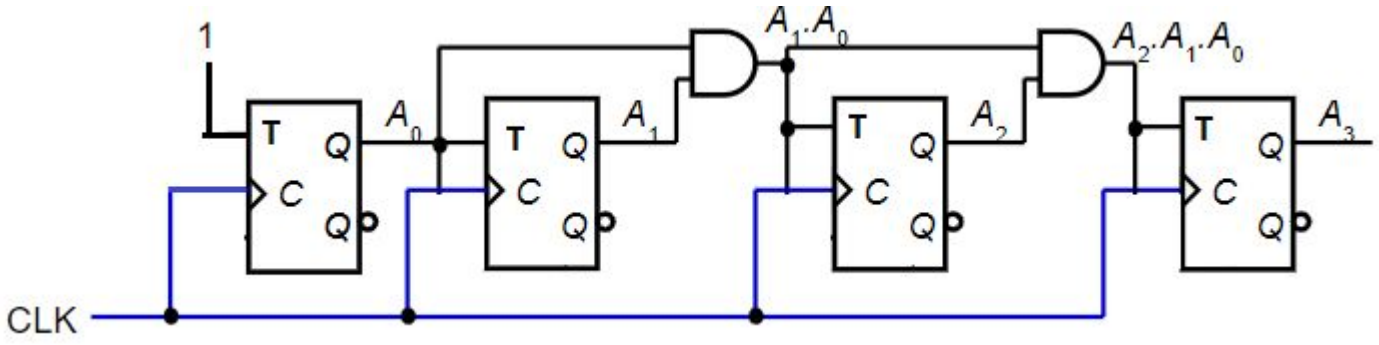


Self-Study

4 bit synchronous up counter

- Equations and diagram will look like the following:

$TA_3 = A_2 \cdot A_1 \cdot A_0$
$TA_2 = A_1 \cdot A_0$
$TA_1 = A_0$
$TA_0 = 1$

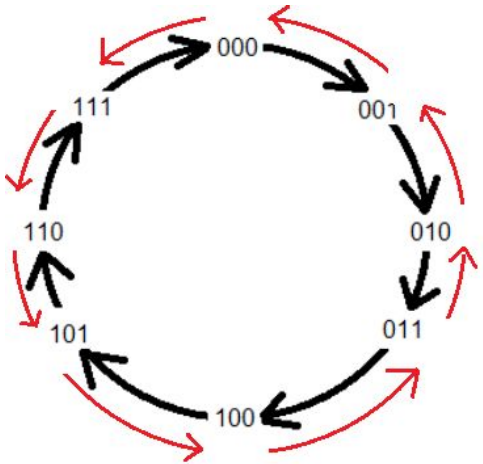


Up/Down Synchronous Counters

- Up/down synchronous counter: a *bidirectional* counter that is capable of counting either up or down.
- An input (control) line *Up*/*Down* (or simply *Up*) specifies the direction of counting.
  - *Up*/$\overline{Down}$ = 0 → Count upward
  - *Up*/*Down* = 1 → Count downward

3 bit Synchronous Up/Down Counter

U/D	Present states			Next States			Flipflop inputs		
M	Q_2	Q_1	Q_0	Q_2+	Q_1+	Q_0+	TQ_2	TQ_1	TQ_0
0	0	0	0	0	0	1	0	0	1
0	0	0	1	0	1	0	0	1	1
0	0	1	0	0	1	1	0	0	1
0	0	1	1	1	0	0	1	1	1
0	1	0	0	1	0	1	0	0	1
0	1	0	1	1	1	0	0	1	1
0	1	1	0	1	1	1	0	0	1
0	1	1	1	0	0	0	1	1	1
1	0	0	0	1	1	1	1	1	1
1	0	0	1	0	0	0	0	0	1
1	0	1	0	0	0	1	0	1	1
1	0	1	1	0	1	0	0	0	1
1	1	0	0	0	1	1	1	1	1
1	1	0	1	1	0	0	0	0	1
1	1	1	0	1	0	1	0	1	1
1	1	1	1	1	1	0	0	0	1



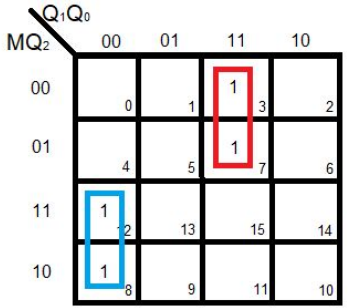
Q_n	Q_{n+1}	T
0	0	0
0	1	1
1	0	1
1	1	0

Excitation table of T flip flop

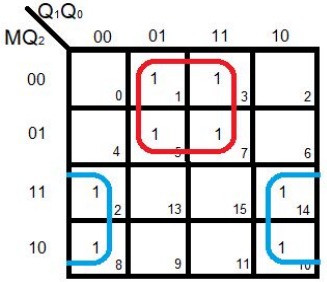
https://www.youtube.com/watch?v=svFUEJkoeVY&list=PLBlnK6fEyqRjMH3mWf6kwqiTbT798eAOm&index=189

3 bit Synchronous Up/Down Counter

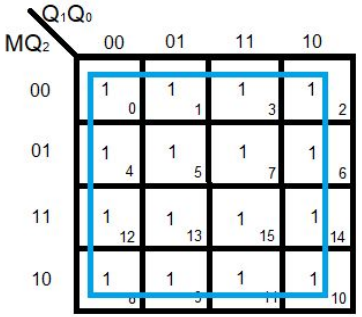
U/D	Present states			Next States			Flipflop inputs		
M	Q_2	Q_1	Q_0	Q_2+	Q_1+	Q_0+	TQ_2	TQ_1	TQ_0
0	0	0	0	0	0	1	0	0	1
0	0	0	1	0	1	0	0	1	1
0	0	1	0	0	1	1	0	0	1
0	0	1	1	1	0	0	1	1	1
0	1	0	0	1	0	1	0	0	1
0	1	0	1	1	1	0	0	1	1
0	1	1	0	1	1	1	0	0	1
0	1	1	1	0	0	0	1	1	1
1	0	0	0	1	1	1	1	1	1
1	0	0	1	0	0	0	0	0	1
1	0	1	0	0	0	1	0	1	1
1	0	1	1	0	1	0	0	0	1
1	1	0	0	0	1	1	1	1	1
1	1	0	1	1	0	0	0	0	1
1	1	1	0	1	0	1	0	1	1
1	1	1	1	1	1	0	0	0	1



$TQ_2 = M'Q_1Q_0 + MQ_1'Q_0'$



$TQ_1 = M'Q_0 + MQ_0'$

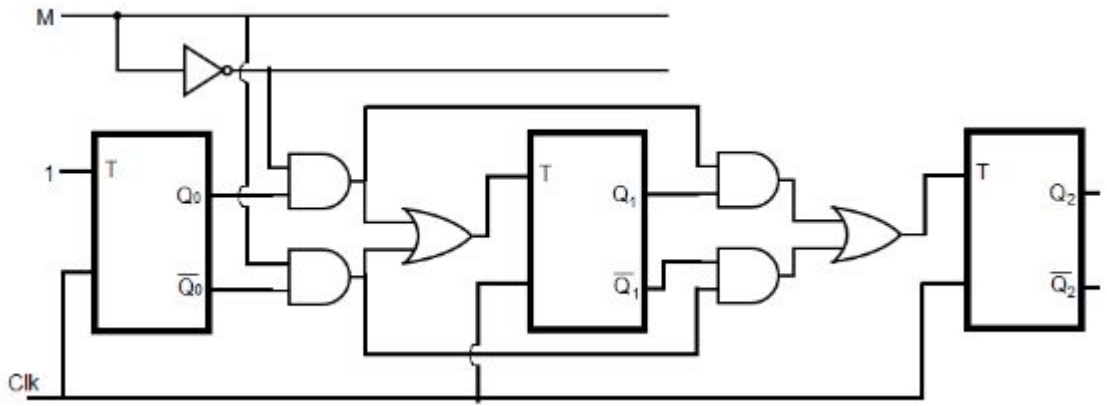


$TQ_0 = 1$

https://www.youtube.com/watch?v=svFUEJkoeVY&list=PLBlnK6fEyqRjMH3mWf6kwqiTbT798eAOm&index=189

3 bit Up/Down Synchronous Counter

Draw the circuit diagram
(The number of present state will be equal to the number of flip-flops in the circuit)



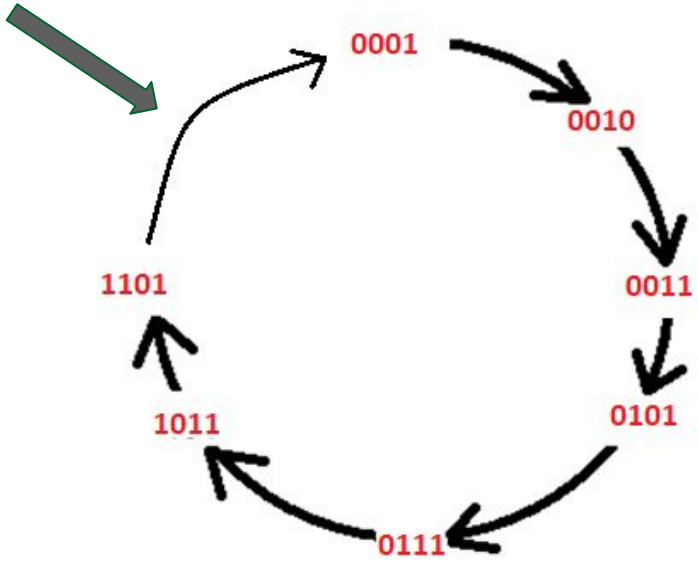
https://www.youtube.com/watch?v=svFUEJkoeVY&list=PLBlnK6fEyqRjMH3mWf6kwqiTbT798eAOm&index=189

Practice Problem using Counters

PRACTICE PROBLEM - 1

- Implement the following counter using T FF: 1->2->3->5->7->11->13->1

Start by drawing this



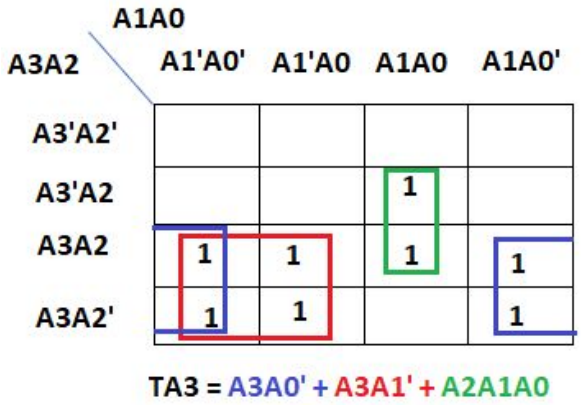
Present states				Next States				Flip-flop inputs			
A_3	A_2	A_1	A_0	A_3+	A_2+	A_1+	A_0+	TA_3	TA_2	TA_1	TA_0
0	0	0	0	0	0	0	1	0	0	0	1
0	0	0	1	0	0	1	0	0	0	1	1
0	0	1	0	0	0	1	1	0	0	0	1
0	0	1	1	0	1	0	1	0	1	1	0
0	1	0	0	0	0	0	1	0	1	0	1
0	1	0	1	0	1	1	1	0	0	1	0
0	1	1	0	0	0	0	1	0	1	1	1
0	1	1	1	1	0	1	1	1	1	0	0
1	0	0	0	0	0	0	1	1	0	0	1
1	0	0	1	0	0	0	1	1	0	0	0
1	0	1	0	0	0	0	1	1	0	1	1
1	0	1	1	1	1	0	1	0	1	1	0
1	1	0	0	0	0	0	1	1	1	0	1
1	1	0	1	0	0	0	1	1	1	0	0
1	1	1	0	0	0	0	1	1	1	1	1
1	1	1	1	0	0	0	1	1	1	1	0

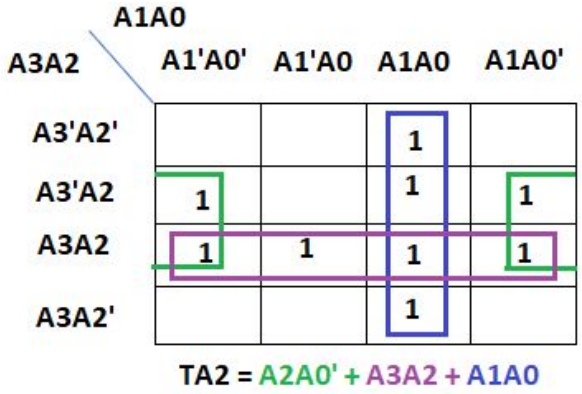
Q_n	Q_{n+1}	T
0	0	0
0	1	1
1	0	1
1	1	0

Excitation table of T flip flop

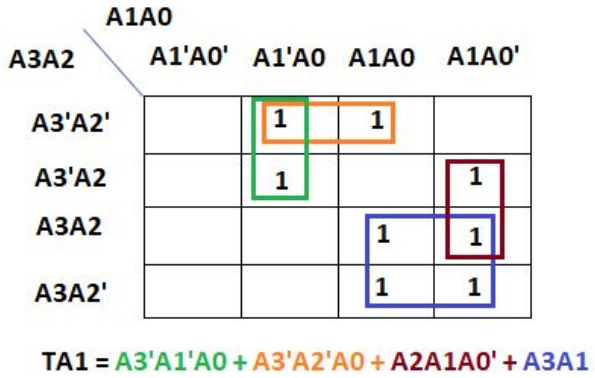
For present states not mentioned in the question, next state will be the INITIAL STATE [0001 in this question]. It's important to show the states that are not mentioned in the question.Otherwise, the circuit will stop working if any of those states show up.

Present states				Next States				Flip-flop inputs			
A_3	A_2	A_1	A_0	A_3+	A_2+	A_1+	A_0+	TA_3	TA_2	TA_1	TA_0
0	0	0	0	0	0	0	1	0	0	0	1
0	0	0	1	0	0	1	0	0	0	1	1
0	0	1	0	0	0	1	1	0	0	0	1
0	0	1	1	0	1	0	1	0	1	1	0
0	1	0	0	0	0	0	1	0	1	0	1
0	1	0	1	0	1	1	1	0	0	1	0
0	1	1	0	0	0	0	1	0	1	1	1
0	1	1	1	1	0	1	1	1	1	0	0
1	0	0	0	0	0	0	1	1	0	0	1
1	0	0	1	0	0	0	1	1	0	0	0
1	0	1	0	0	0	0	1	1	0	1	1
1	0	1	1	1	1	0	1	0	1	1	0
1	1	0	0	0	0	0	1	1	1	0	1
1	1	0	1	0	0	0	1	1	1	0	0
1	1	1	0	0	0	0	1	1	1	1	1
1	1	1	1	0	0	0	1	1	1	1	0





Present states				Next States				Flip-flop inputs			
A_3	A_2	A_1	A_0	A_3+	A_2+	A_1+	A_0+	TA_3	TA_2	TA_1	TA_0
0	0	0	0	0	0	0	1	0	0	0	1
0	0	0	1	0	0	1	0	0	0	1	1
0	0	1	0	0	0	1	1	0	0	0	1
0	0	1	1	0	1	0	1	0	1	1	0
0	1	0	0	0	0	0	1	0	1	0	1
0	1	0	1	0	1	1	1	0	0	1	0
0	1	1	0	0	0	0	1	0	1	1	1
0	1	1	1	1	0	1	1	1	1	0	0
1	0	0	0	0	0	0	1	1	0	0	1
1	0	0	1	0	0	0	1	1	0	0	0
1	0	1	0	0	0	0	1	1	0	1	1
1	0	1	1	1	1	0	1	0	1	1	0
1	1	0	0	0	0	0	1	1	1	0	1
1	1	0	1	0	0	0	1	1	1	0	0
1	1	1	0	0	0	0	1	1	1	1	1
1	1	1	1	0	0	0	1	1	1	1	0



TA1 = A3'A1'A0 + A3'A2'A0 + A2A1A0' + A3A1

TA0 = A0' + A3'A2'A1'

$$TA3 = A3A0' + A3A1' + A2A1A0$$

$$TA2 = A2A0' + A3A2 + A1A0$$

$$TA1 = A3'A1'A0 + A3'A2'A0 + A2A1A0' + A3A1$$

$$TA0 = A0' + A3'A2'A1'$$

Draw the circuit yourself

PRACTICE PROBLEM - 2

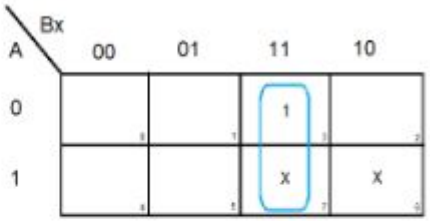
- Implement the following counter using D FF: Green->Yellow->Red->Yellow->Green

Let's assume: Green = 00, Yellow = 01, Red = 10

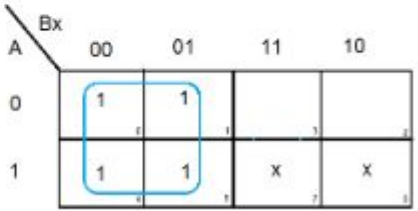
00-> 01->10->01->00

Present states			Next States		Flip-flop inputs	
A	B	x	A+	B+	DA	DB
0	0	0	0	1	0	1
0	0	1	0	1	0	1
0	1	0	0	0	0	0
0	1	1	1	0	1	0
1	0	0	0	1	0	1
1	0	1	0	1	0	1
1	1	0	x	x	x	x
1	1	1	x	x	x	x

Find out the equations for the flip-flop inputs



DA = Bx



DB = B

Q_n	Q_{n+1}	D
0	0	0
0	1	1
1	0	0
1	1	1

Excitation table of D flip flop

Special Note: You may need to use more than 1 external input depending upon the question.

Draw the circuit diagram (The number of present state will be equal to the number of flip-flops in the circuit)