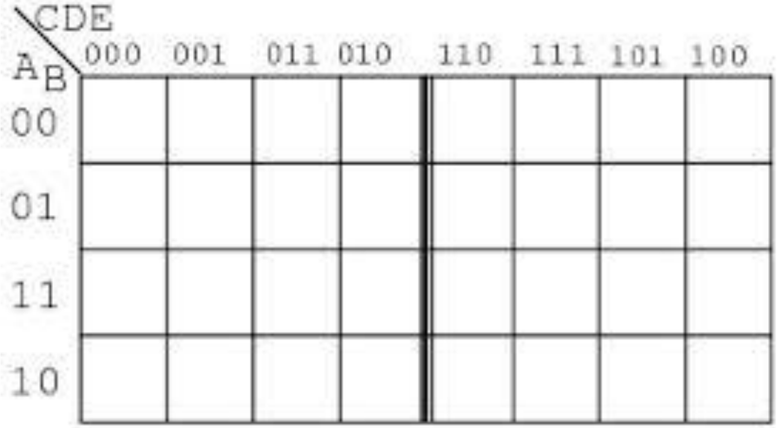
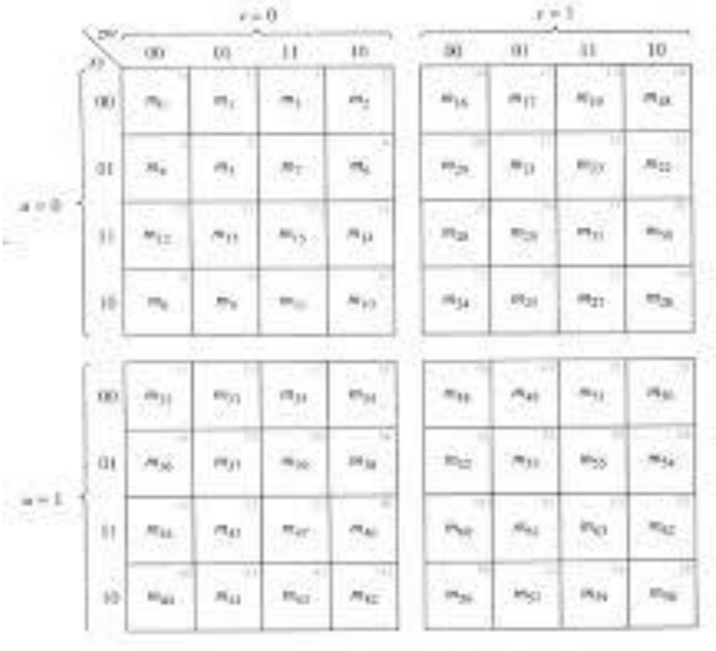
Tabulation Method

The Quine-McCluskey Method

K-map going big!



5- variable Karnaugh map (Gray code)



(a) Map organization

FIGURE 4.11
Six-variable map

6 variable k-map

- K-map works well upto 4 variable, but when number of variable is more than that trouble starts as it gets difficult to recognize patters leading to wrong selection
- Quine McCluskey method or the tabulation method is more systematic and works well for any number of variable

The starting point of the tabulation method is the list of minterms that specify the function. The first tabular operation is to find the prime implicants by using a matching process. This process compares each minterm with every other minterm. If two minterms differ in only one variable, that variable is removed and a term with one less literal is found. This process is repeated for every minterm until the exhaustive search is completed. The matching-process cycle is repeated for those new terms just found. Third and further cycles are continued until a single pass through a cycle yields no further elimination of literals. The remaining terms and all the terms that did not match during the process comprise the prime implicants. This tabulation method is illustrated by the following example.

Part 1:Determination of the prime implicant

Forming the tabulation method

Example

$F(w,x,y,z)=\sum(4,1,7,6,9,8,11,15,10)$

Step 1: group the minterms based on the no. of ‘1’s in them
Group A: single ‘1’: {0001,0100,1000}
Group B: two ‘1’s: {0110,1001,1010}
Group C: three ‘1’s: {0111,1011}
Group D:four ‘1’s: {1111}

	Step 1				
1	0001				
4	0100				
8	1000				
6	0110				
9	1001				
10	1010				
7	0111				
11	1011				
15	1111				

- **Step 2:** Any 2 minterm that differ by 1 variable is combined and the different variable is removed.The minterms of a group is compared with minterms of immediate next group (this is because 2 term differing by more than 1 bit cannot match). If 2 matching minterms are found, a '√' (tick) is place besides them

	Step 1	Step 2			
1	0001	(1,9)	_001		
4	0100	(4,6)	01_ 0		
8	1000	(8,9)	100_		
		(8,10)	10_0		
6	0110	(6,7)	011_		
9	1001	(9,11)	10_1		
10	1010	(10,11)	101_		
7	0111	(7,15)	_111		
11	1011	(11,15)	1_11		
15	1111				

- Step 3:

Terms from step 2 has 3 options. ‘1’ means variable is unprimed, ‘0’ means variable is primed and ‘_’ means variable is not included in the term.

Now compare terms only if they have ‘_’ in the same position. Once two term match place a ‘$\surd$’ beside each

	Step 1	Step 2			Step 3		
1	0001	√	(1,9)	_001		(8,9,10,11)	10_ _
4	0100	√	(4,6)	01_ 0		(8,9,10,11)	10_ _
8	1000	√	(8,9)	100_	√		
			(8,10)	10_0	√		
6	0110	√	(6,7)	011_			
9	1001	√	(9,11)	10_1	√		
10	1010	√	(10,11)	101_	√		
7	0111	√	(7,15)	_111			
11	1011	√	(11,15)	1_11			
15	1111	√					

Prime Implicant

- The remaining term (those without '√') in Step 2 and the terms from Step 3 are called the prime implcants. These terms are candidates suitable for forming the simplified form of the expression

Here prime implicants are:

(1,9)	_001	X'Y'Z
(4,6)	01_0	W'XZ'
(6,7)	011_	W'XY
(7,15)	_111	XYZ
(11,15)	1_11	WYZ
(8,9,10,11)	10_ _	WX'

Part 2 (Optimization): Selection of prime implicants- the setup

Optimization Phase - Optimizing the tabular method

Optimization Phase

Prime Implicants

	1	4	6	7	8	9	10	11	15
X’Y’Z	X					X			
W’XZ’		X	X						
W’XY			X	X					
XYZ				X					X
WYZ								X	X
WX’					X	X	X	X	
	✓	✓	✓	✓	✓	✓	✓	✓	✓

Now, check the circles and write the corresponding terms in SOP format:

X'Y'Z +
W'XZ' +
XYZ +
WX'

	1	4	6	7	8	9	10	11	15
X'Y'Z	X					X			
W'XZ'		X	X						
W'XY			X	X					
XYZ				X					X
WYZ								X	X
WX'					X	X	X	X	
	✓	✓	✓	✓	✓	✓	✓	✓	✓

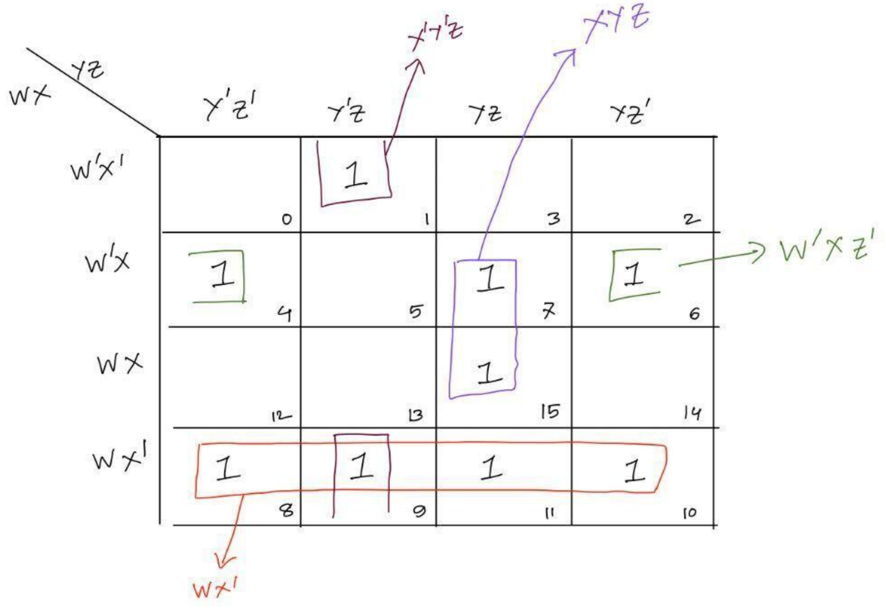
So solution is

$F(W,X,Y,Z)= X'Y'Z+W'XZ'+WX'+XYZ$

This matches the result from K-MAP done below!

K-MAP using prime implicant shows that it can be further simplified to

$F(w,x,y,z)= X'Y'Z+W'XZ'+XYZ+WX'$



Example 2 (with Don’t Cares)

$F(a,b,c,d)=\sum m(2,5,6,11,12,14,15)+ \sum d(0,3,4)$

Note: avoid the don’t care terms in the set-up(optimization) phase.

	Step 1	Step 2				Step 3	
0	0000	√	(0,2)	00-0	√	(0,2,4,6)	0--0
			(0,4)	0-00	√	(0,4,2.6)	0--0
2	0010	√	(2,3)	001-		(4,6,12,14)	-1-0
4	0100	√	(2,6)	0-10	√	(4,12,6,14)	-1-0
			(4,5)	010-			
			(4,6)	01-0	√		
			(4,12)	-100	√		
3	0011	√	(3,11)	-011			
5	0101	√	(6,14)	-110	√		
6	0110	√	(12,14)	11-0	√		
12	1100	√					
11	1011	√	(11,15)	1-11			
14	1110	√	(14,15)	111-			
15	1111	√					

Optimization Phase - Don’t care terms are avoided here (that’s the only difference).

	2	5	6	11	12	14	15
A’D’	X		X				
BD’			X		X	X	
A’B’C	X						
A’BC’		X					
B’CD				X			
ACD				X			X
ABC						X	X

Answer: A’D’ + BD’ + A’BC’ + ACD

Try it yourself

- $F(w,x,y,z)=\sum(0,1,2,8,10,11,14,15)$

Solution (Part 1)

(a)

	w	x	y	z	
0	0	0	0	0	√
1	0	0	0	1	√
2	0	0	1	0	√
8	1	0	0	0	√
10	1	0	1	0	√
11	1	0	1	1	√
14	1	1	1	0	√
15	1	1	1	1	√

(b)

	w	x	y	z	
0, 1	0	0	0	–	
0, 2	0	0	–	0	√
0, 8	–	0	0	0	√
2, 10	–	0	1	0	√
8, 10	1	0	–	0	√
10, 11	1	0	1	–	√
10, 14	1	–	1	–	√
11, 15	1	–	1	1	√
14, 15	1	1	1	–	√

(c)

	w	x	y	z
0, 2, 8, 10	–	0	–	0
0, 8, 2, 10	–	0	–	0
10, 11, 14, 15	1	–	1	–
10, 14, 11, 15	1	–	1	–

Optimization Phase – Do it yourself

Final answer:

$F = w'x'y' + x'z' + wy$