Data Communications and Networking Fourth Edition

Forouzan

Chapter 10

Error Detection and Correction

Note

Data can be corrupted during transmission.

Some applications require that errors be detected and corrected.

10-1 INTRODUCTION

Let us first discuss some issues related, directly or indirectly, to error detection and correction.

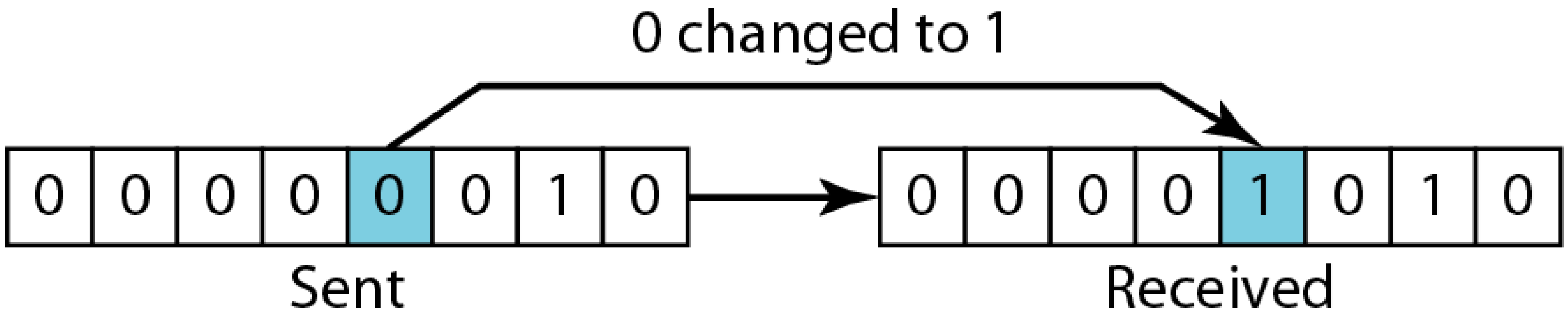
Topics discussed in this section:

Types of Errors
Redundancy
Detection Versus Correction
Forward Error Correction Versus Retransmission
Coding
Modular Arithmetic

Note

In a single-bit error, only 1 bit in the data unit has changed.

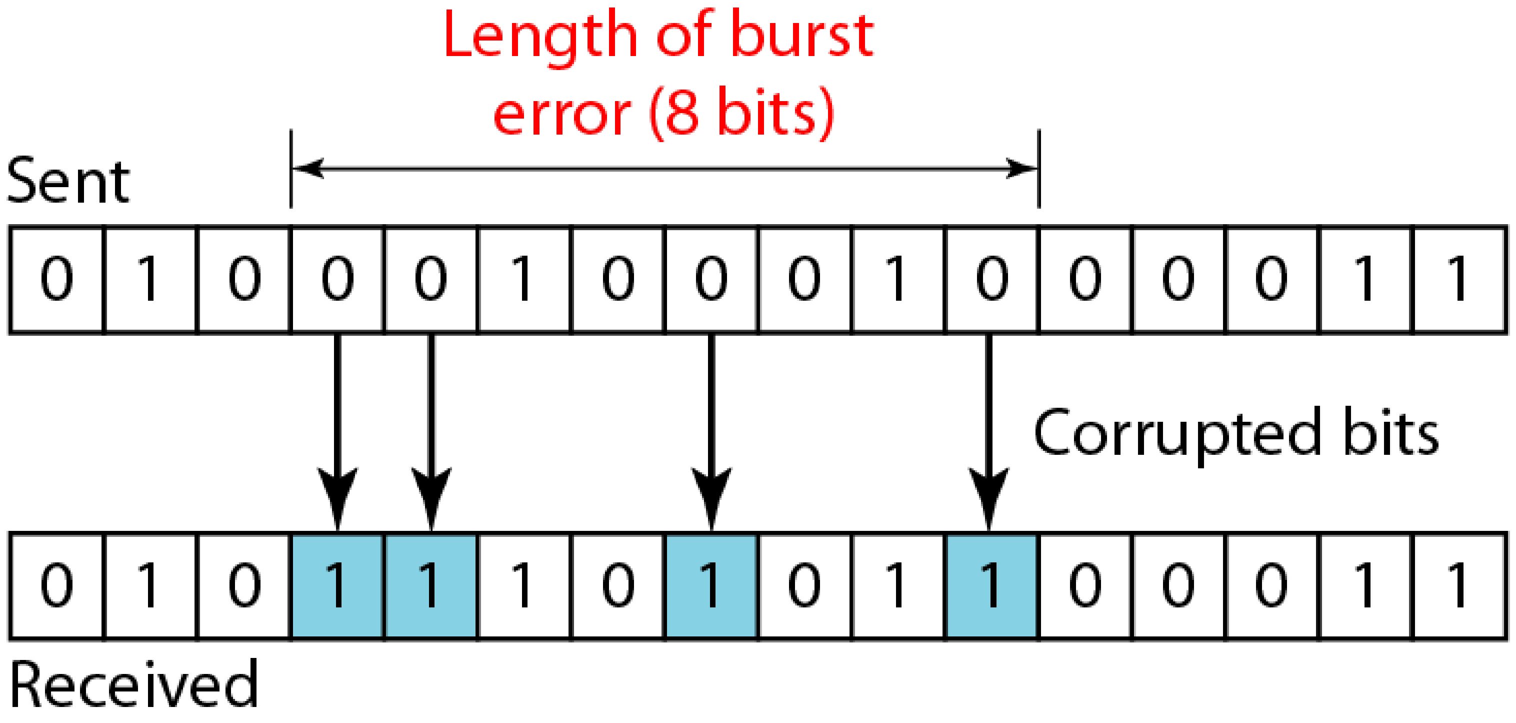
Figure 10.1 *Single-bit error*



Note

A burst error means that 2 or more bits in the data unit have changed.

Figure 10.2 *Burst error of length 8*



Redundancy

- The central concept in error detection and correction

- The redundant bits are added by the sender and removed by the receiver

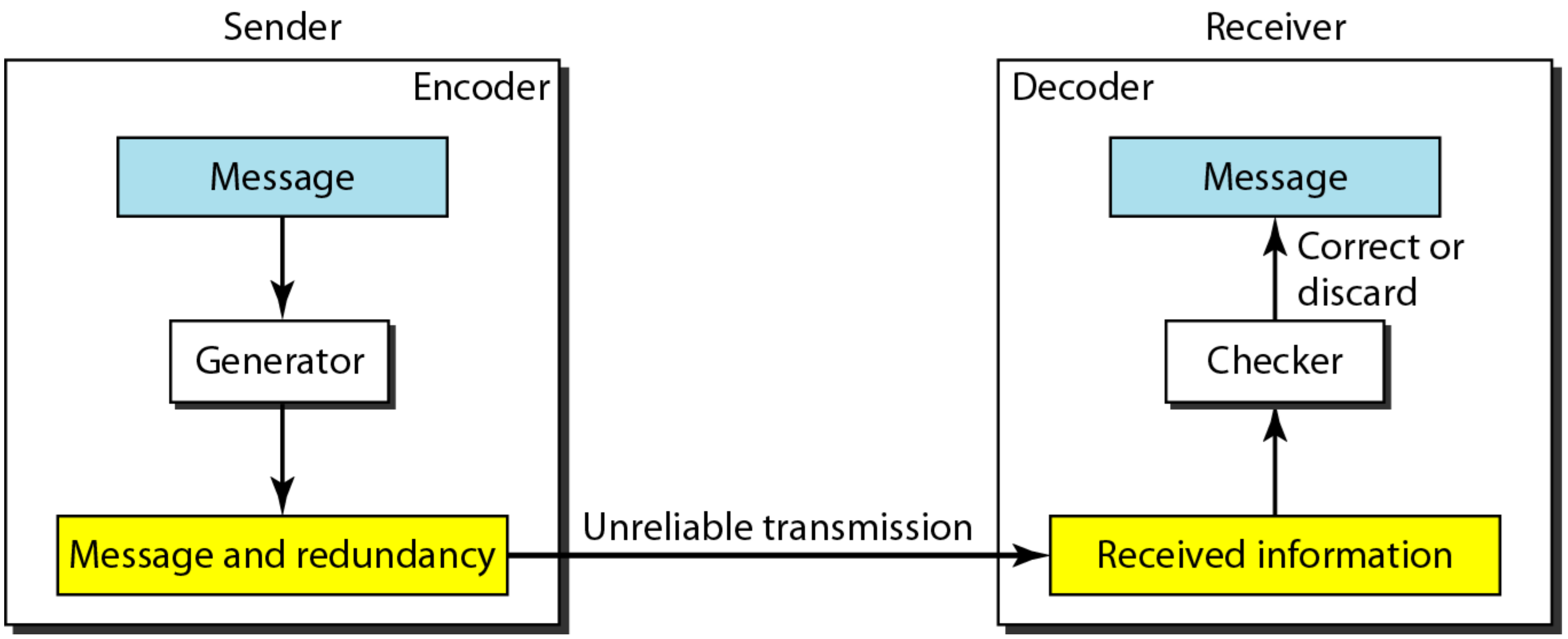
- Their presence helps the receiver to detect and correct corrupted bits

Redundancy

Note

To detect or correct errors, we need to send extra (redundant) bits with data.

Figure 10.3 *The structure of encoder and decoder*



Coding

- Redundancy is achieved through various coding schemes
- The coding schemes are divided into two broad categories
- Block coding
- Convolution coding

Note

In this book, we concentrate on block codes; we leave convolution codes to advanced texts.

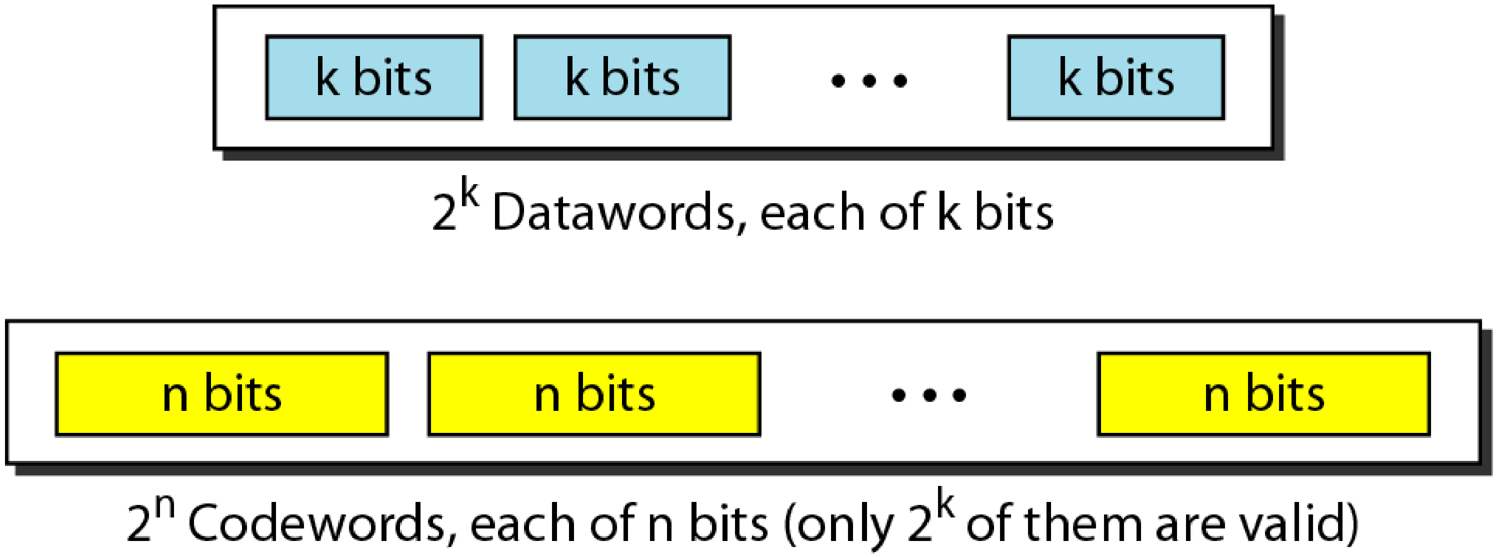
10-2 BLOCK CODING

In block coding, we divide our message into blocks, each of k bits, called datawords. We add r redundant bits to each block to make the length n = k + r. The resulting n-bit blocks are called codewords.

Topics discussed in this section:

Error Detection
Error Correction
Hamming Distance
Minimum Hamming Distance

Figure 10.5 *Datawords and codewords in block coding*


k bits
k bits
• • •
k bits
2^k Datawords, each of k bits
n bits
n bits
• • •
n bits
2^n Codewords, each of n bits (only 2^k of them are valid)

Example 10.1

The 4B/5B block coding discussed in Chapter 4 is a good example of this type of coding. In this coding scheme, k = 4 and n = 5. As we saw, we have 2^k = 16 datawords and 2^n = 32 codewords. We saw that 16 out of 32 codewords are used for message transfer and the rest are either used for other purposes or unused.

Figure 10.6 *Process of error detection in block coding*

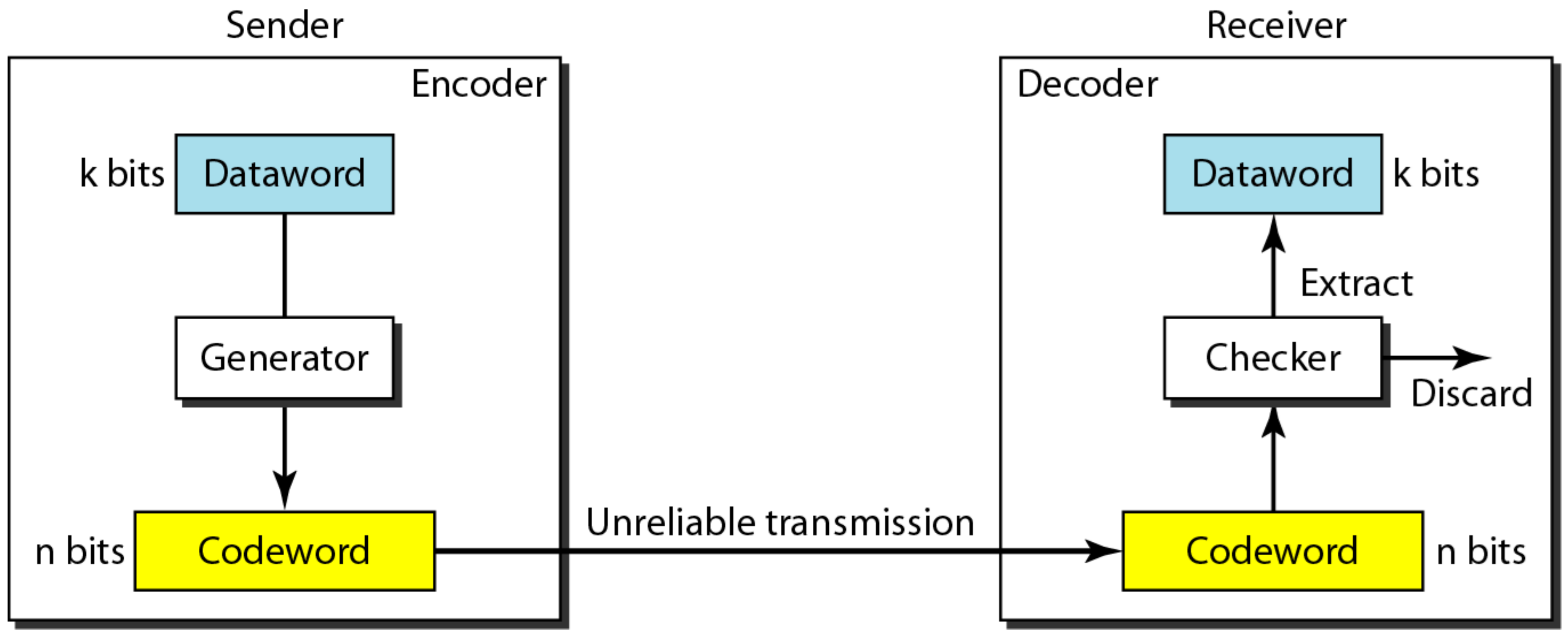


Table 10.1 ***A code for error detection (Example 10.2)***

Datawords	*Codewords*
00	000
01	011
10	101
11	110

Example 10.2

Let us assume that k = 2 and n = 3. Table 10.1 shows the list of datawords and codewords. Later, we will see how to derive a codeword from a dataword.

Assume the sender encodes the dataword 01 as 011 and sends it to the receiver. Consider the following cases:

1. The receiver receives 011. It is a valid codeword. The receiver extracts the dataword 01 from it.

Example 10.2 (continued)

2. ***The codeword is corrupted during transmission, and 111 is received. This is not a valid codeword and is discarded.***

3. ***The codeword is corrupted during transmission, and 000 is received. This is a valid codeword. The receiver incorrectly extracts the dataword 00.***

Thus two corrupted bits have made the error undetectable.

Note

An error-detecting code can detect only the types of errors for which it is designed; other types of errors may remain undetected.

Figure 10.7 *Structure of encoder and decoder in error correction*

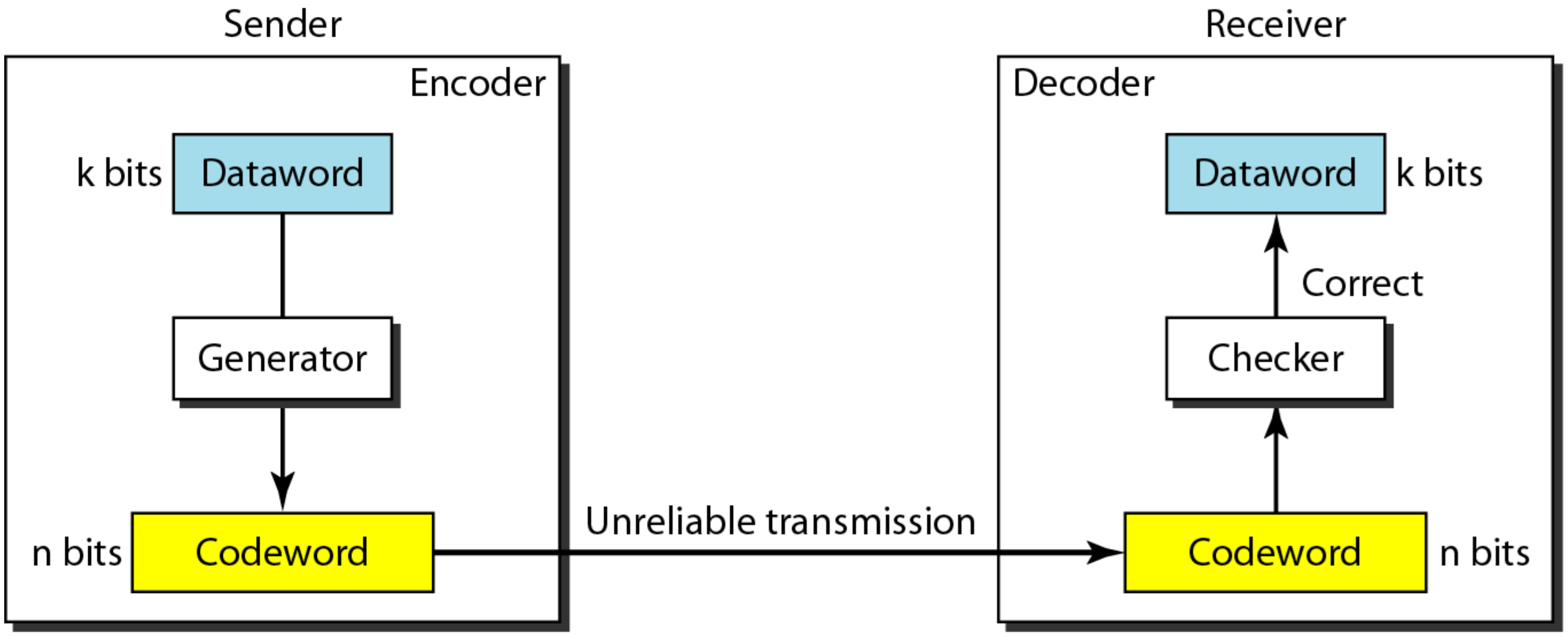


Figure 10.4 *XORing of two single bits or two words*

$0 \oplus 0 = 0 \qquad 1 \oplus 1 = 0$

a. Two bits are the same, the result is 0.

$0 \oplus 1 = 1 \qquad 1 \oplus 0 = 1$

b. Two bits are different, the result is 1.

$$\begin{array}{cccccc} & 1 & 0 & 1 & 1 & 0 \\ \oplus & 1 & 1 & 1 & 0 & 0 \\ \hline & 0 & 1 & 0 & 1 & 0 \end{array}$$

c. Result of XORing two patterns

Note

The Hamming distance between two words is the number of differences between corresponding bits.

Example 10.4

Let us find the Hamming distance between two pairs of words.

1. ***The Hamming distance d(000, 011) is 2 because***

$$\boxed{000 \oplus 011 \text{ is } 011 \text{ (two 1s)}}$$

2. ***The Hamming distance d(10101, 11110) is 3 because***

$$\boxed{10101 \oplus 11110 \text{ is } 01011 \text{ (three 1s)}}$$

Note

The minimum Hamming distance is the smallest Hamming distance between all possible pairs in a set of words.

Table 10.1 *A code for error detection (Example 10.2)*

Datawords	*Codewords*
00	000
01	011
10	101
11	110

- There are four different codewords which can have 6 possible pairs
- (000, 011)
- (000, 101)
- (000, 110)
- (011, 101)
- (011, 110)
- (101.110)

Table 10.2 *A code for error correction (Example 10.3)*

Dataword	*Codeword*
00	00000
01	01011
10	10101
11	11110

Example 10.6

Find the minimum Hamming distance of the coding scheme in Table 10.2.

Solution

We first find all the Hamming distances.

$d(00000, 01011) = 3$	$d(00000, 10101) = 3$	$d(00000, 11110) = 4$
$d(01011, 10101) = 4$	$d(01011, 11110) = 3$	$d(10101, 11110) = 3$

The d_{min} in this case is 3.

Note

To guarantee the detection of up to s errors in all cases, the minimum Hamming distance in a block code must be $d_{min} = s + 1$.

Note

To guarantee the correction of up to t errors in all cases, the minimum Hamming distance in a block code must be $d_{min} = 2t + 1$.

Example 10.9

A code scheme has a Hamming distance d_{min} = 4. What is the error detection and correction capability of this scheme?

Solution

This code guarantees the detection of up to three errors (s = 3), but it can correct up to one error. In other words, if this code is used for error correction, part of its capability is wasted. Error correction codes need to have an odd minimum distance (3, 5, 7, . . .).

10-4 CYCLIC CODES

***Cyclic codes* are special linear block codes with one extra property. In a cyclic code, if a codeword is cyclically shifted (rotated), the result is another codeword.**

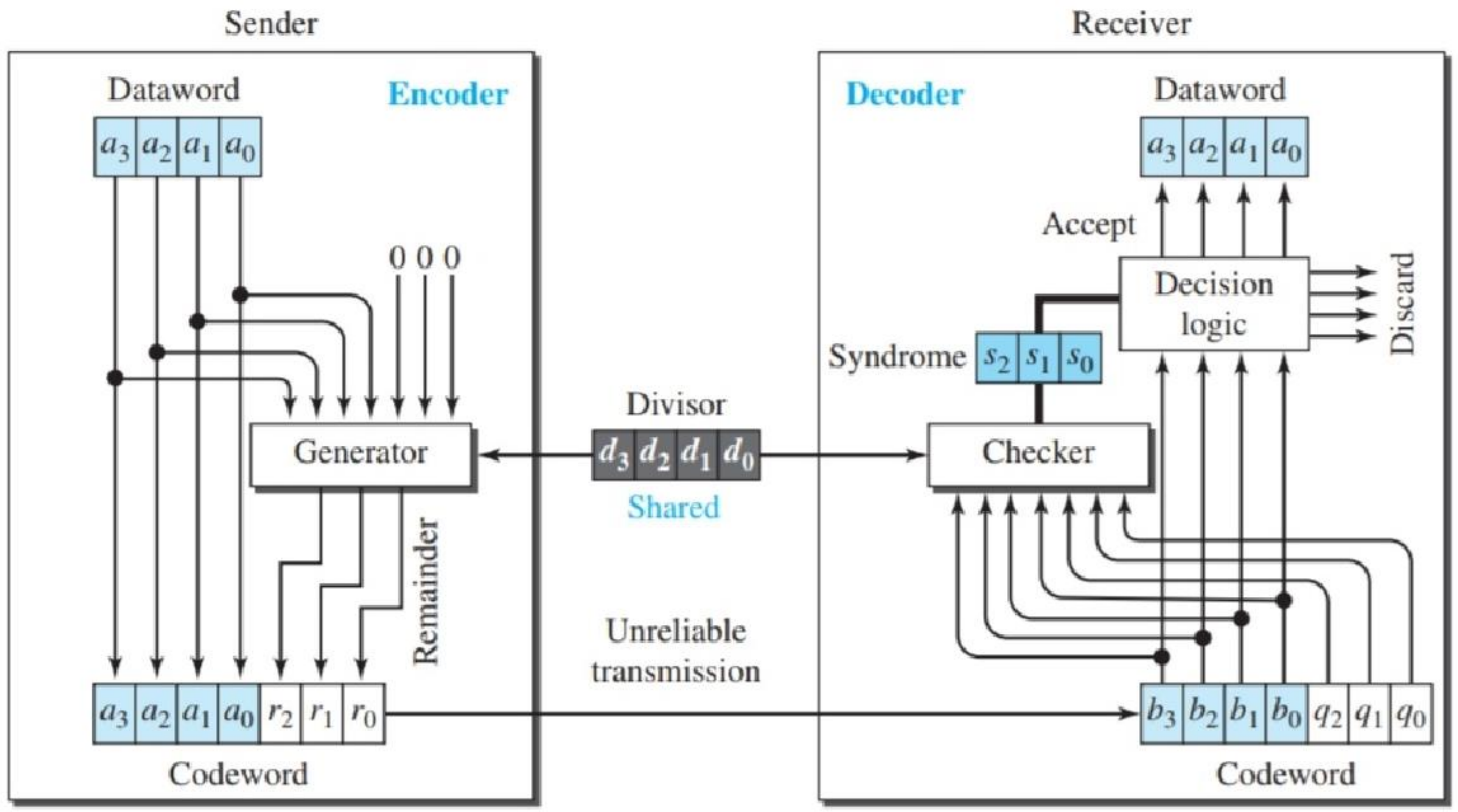
Topics discussed in this section:

Cyclic Redundancy Check
Hardware Implementation
Polynomials
Cyclic Code Analysis
Advantages of Cyclic Codes
Other Cyclic Codes

Table 10.6 *A CRC code with C(7, 4)*

Dataword	*Codeword*	*Dataword*	*Codeword*
0000	0000000	1000	1000101
0001	0001011	1001	1001110
0010	0010110	1010	1010011
0011	0011101	1011	1011000
0100	0100111	1100	1100010
0101	0101100	1101	1101001
0110	0110001	1110	1110100
0111	0111010	1111	1111111

Figure 10.5 *CRC encoder and decoder*



In the encoder, the dataword has k bits (4 here); the codeword has n bits (7 here). The size of the dataword is augmented by adding $n - k$ (3 here) 0s to the right-hand side of the word. The n-bit result is fed into the generator. The generator uses a divisor of size $n - k + 1$ (4 here), predefined and agreed upon. The generator divides the augmented dataword by the divisor (modulo-2 division). The quotient of the division is discarded; the remainder ($r_2r_1r_0$) is appended to the dataword to create the codeword.

The decoder receives the codeword (possibly corrupted in transition). A copy of all n bits is fed to the checker, which is a replica of the generator. The remainder produced by the checker is a syndrome of $n - k$ (3 here) bits, which is fed to the decision logic analyzer. The analyzer has a simple function. If the syndrome bits are all 0s, the 4 left-most bits of the codeword are accepted as the dataword (interpreted as no error); otherwise, the 4 bits are discarded (error).

Figure 10.15 *Division in CRC encoder*

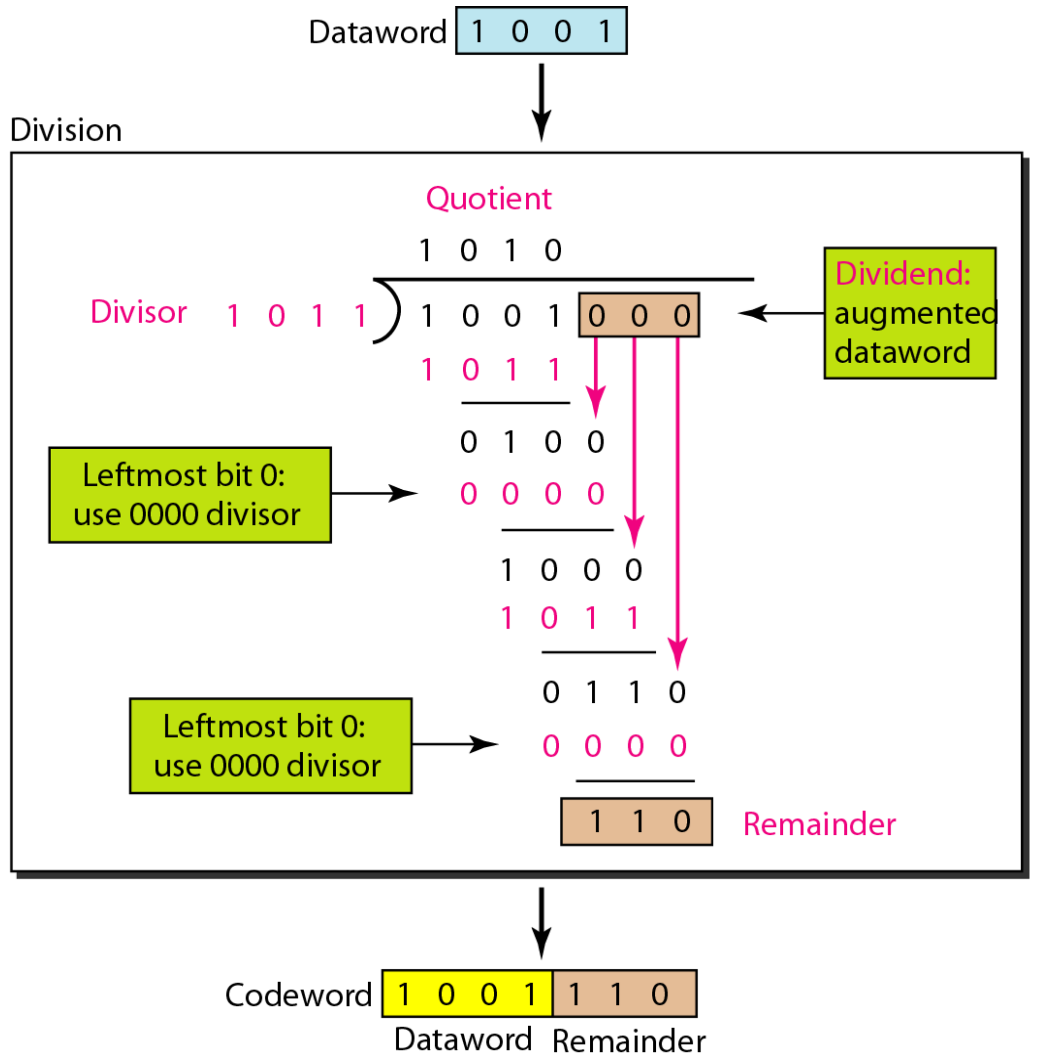


Figure 10.16 *Division in the CRC decoder for two cases*

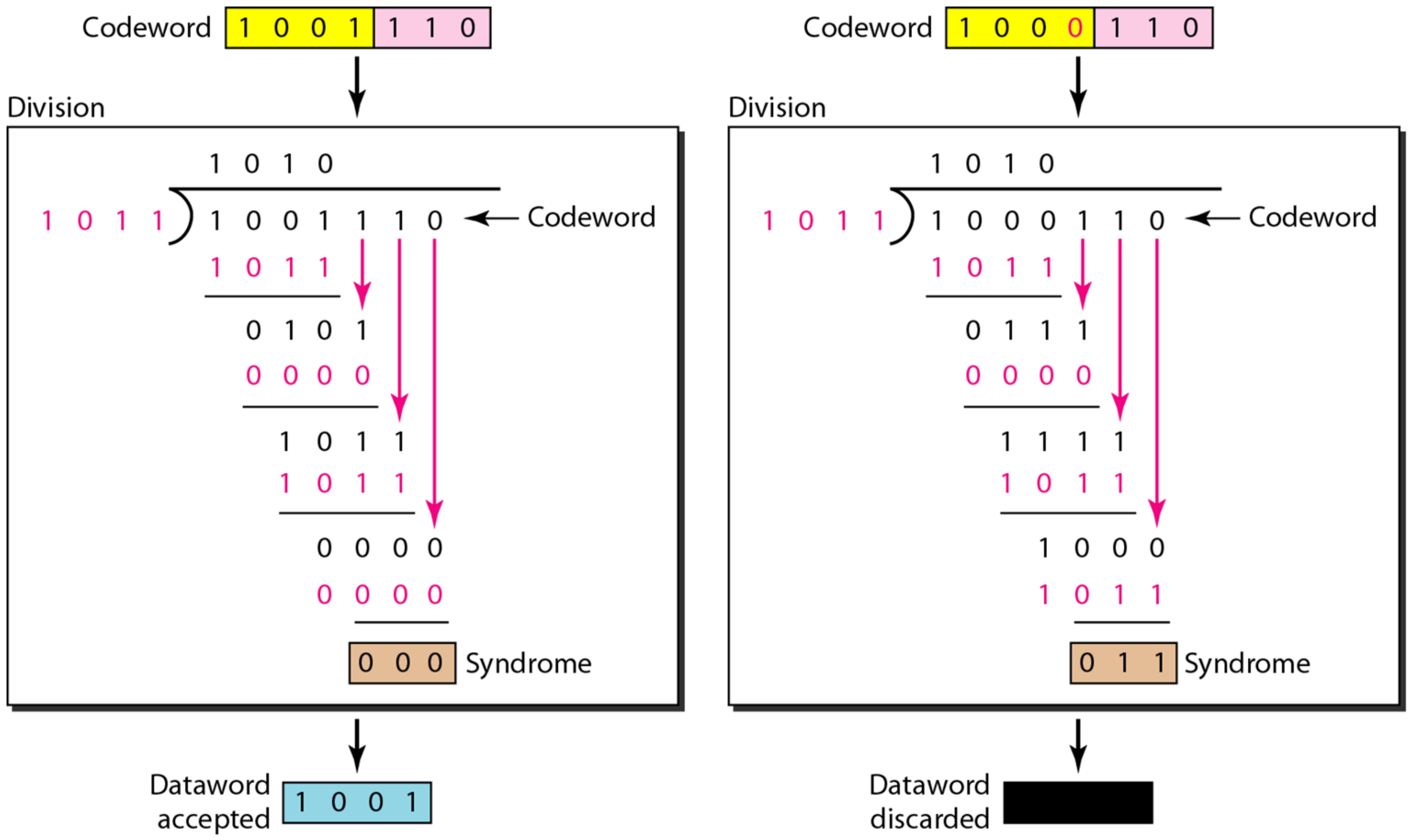
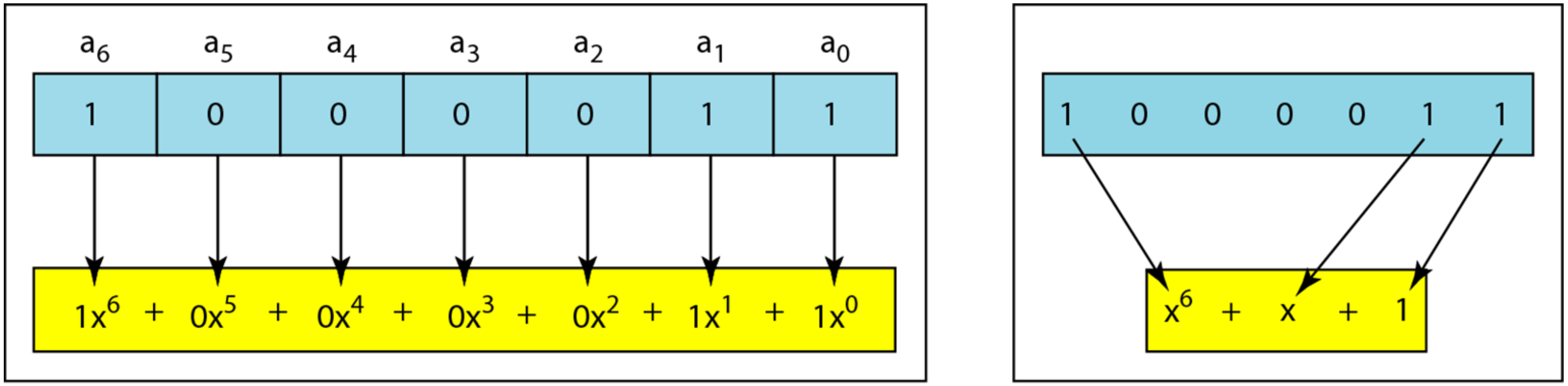


Figure 10.21 *A polynomial to represent a binary word*

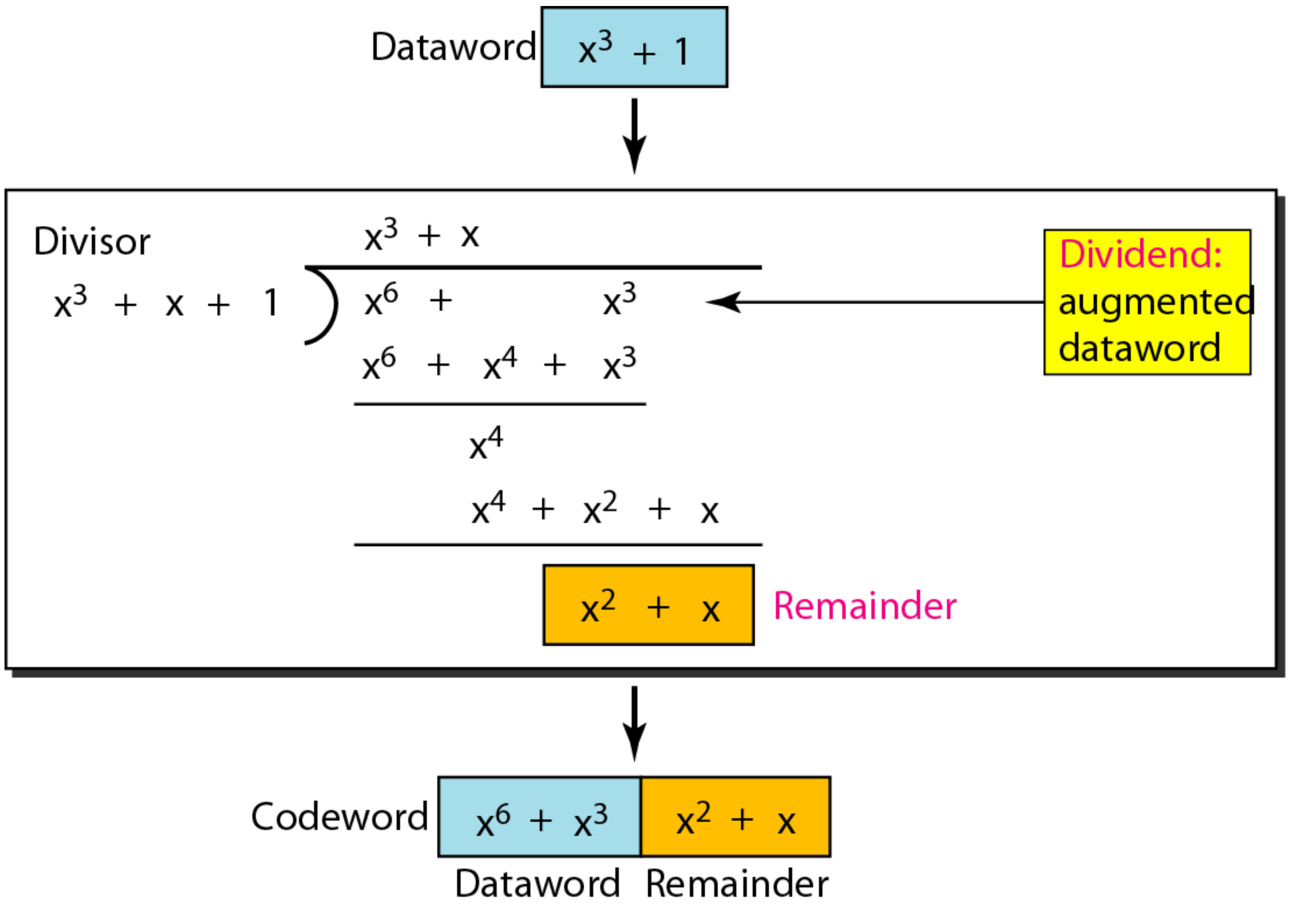


a_6 a_5 a_4 a_3 a_2 a_1 a_0
1 0 0 0 0 1 1
$1x^6 + 0x^5 + 0x^4 + 0x^3 + 0x^2 + 1x^1 + 1x^0$
1 0 0 0 0 1 1
$x^6 + x + 1$

a. Binary pattern and polynomial

b. Short form

Figure 10.22 *CRC division using polynomials*



Note

The divisor in a cyclic code is normally called the generator polynomial or simply the generator.

- Dataword: $d(x)$
- Syndrome: $s(x)$
- Codeword: $c(x)$
- Error: $e(x)$
- Generator: $g(x)$

Note

In a cyclic code,
If $s(x) \neq 0$, one or more bits is corrupted.
If $s(x) = 0$, either

a. No bit is corrupted. or
b. Some bits are corrupted, but the decoder failed to detect them.

Note

In a cyclic code, those *e*(x) errors that are divisible by *g*(x) are not caught.

Table 10.7 *Standard polynomials*

Name	*Polynomial*	*Application*
CRC-8	$x^8 + x^2 + x + 1$	ATM header
CRC-10	$x^{10} + x^9 + x^5 + x^4 + x^2 + 1$	ATM AAL
CRC-16	$x^{16} + x^{12} + x^5 + 1$	HDLC
CRC-32	$x^{32} + x^{26} + x^{23} + x^{22} + x^{16} + x^{12} + x^{11} + x^{10} + x^8 + x^7 + x^5 + x^4 + x^2 + x + 1$	LANs

10-5 CHECKSUM

The last error detection method we discuss here is called the checksum. The checksum is used in the Internet by several protocols although not at the data link layer. However, we briefly discuss it here to complete our discussion on error checking

Topics discussed in this section:

Idea
One's Complement
Internet Checksum

Example 10.18

Suppose our data is a list of five 4-bit numbers that we want to send to a destination. In addition to sending these numbers, we send the sum of the numbers. For example, if the set of numbers is (7, 11, 12, 0, 6), we send (7, 11, 12, 0, 6, 36), where 36 is the sum of the original numbers. The receiver adds the five numbers and compares the result with the sum. If the two are the same, the receiver assumes no error, accepts the five numbers, and discards the sum. Otherwise, there is an error somewhere and the data are not accepted.

Example 10.19

We can make the job of the receiver easier if we send the negative (complement) of the sum, called the checksum. In this case, we send (7, 11, 12, 0, 6, −36). The receiver can add all the numbers received (including the checksum). If the result is 0, it assumes no error; otherwise, there is an error.

Example 10.20

How can we represent the number 21 in one's complement arithmetic using only four bits?

Solution

The number 21 in binary is 10101 (it needs five bits). We can wrap the leftmost bit and add it to the four rightmost bits. We have (0101 + 1) = 0110 or 6.

Example 10.21

How can we represent the number −6 in one's complement arithmetic using only four bits?

Solution

In one's complement arithmetic, the negative or complement of a number is found by inverting all bits. Positive 6 is 0110; negative 6 is 1001. If we consider only unsigned numbers, this is 9. In other words, the complement of 6 is 9. Another way to find the complement of a number in one's complement arithmetic is to subtract the number from $2^n - 1$ (16 − 1 in this case).

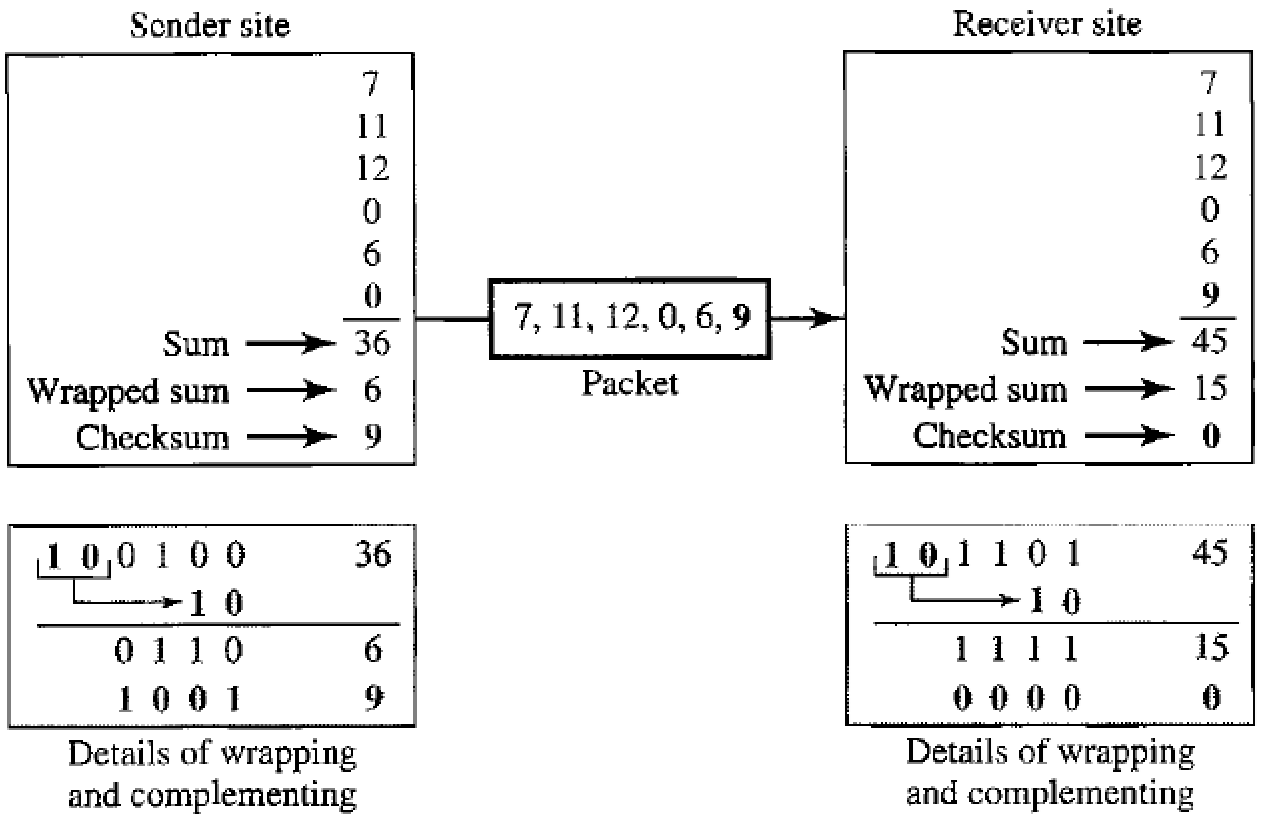
Example 10.22

Let us redo Exercise 10.19 using one's complement arithmetic. Figure 10.24 shows the process at the sender and at the receiver. The sender initializes the checksum to 0 and adds all data items and the checksum (the checksum is considered as one data item and is shown in color). The result is 36. However, 36 cannot be expressed in 4 bits. The extra two bits are wrapped and added with the sum to create the wrapped sum value 6. In the figure, we have shown the details in binary. The sum is then complemented, resulting in the checksum value 9 (15 − 6 = 9). The sender now sends six data items to the receiver including the checksum 9.

Example 10.22 (continued)

The receiver follows the same procedure as the sender. It adds all data items (including the checksum); the result is 45. The sum is wrapped and becomes 15. The wrapped sum is complemented and becomes 0. Since the value of the checksum is 0, this means that the data is not corrupted. The receiver drops the checksum and keeps the other data items. If the checksum is not zero, the entire packet is dropped.

Figure 10.24 *Example 10.22*



Internet Checksum

Note

Sender site:

1. **The message is divided into 16-bit words.**
2. **The value of the checksum word is set to 0.**
3. **All words including the checksum are added using one's complement addition.**
4. **The sum is complemented and becomes the checksum.**
5. **The checksum is sent with the data.**

Internet Checksum

Note

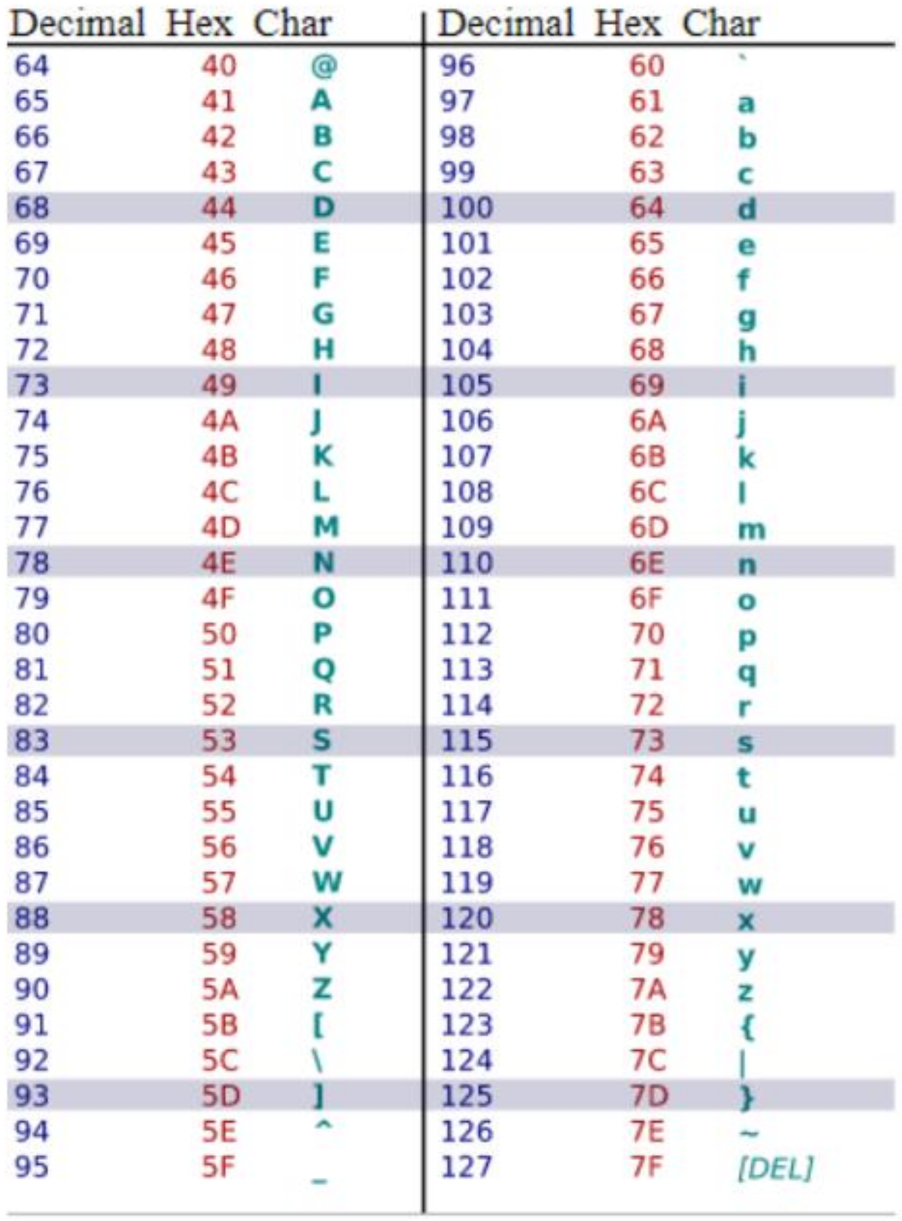
Receiver site:

1. **The message (including checksum) is divided into 16-bit words.**
2. **All words are added using one's complement addition.**
3. **The sum is complemented and becomes the new checksum.**
4. **If the value of checksum is 0, the message is accepted; otherwise, it is rejected.**

Example 10.23

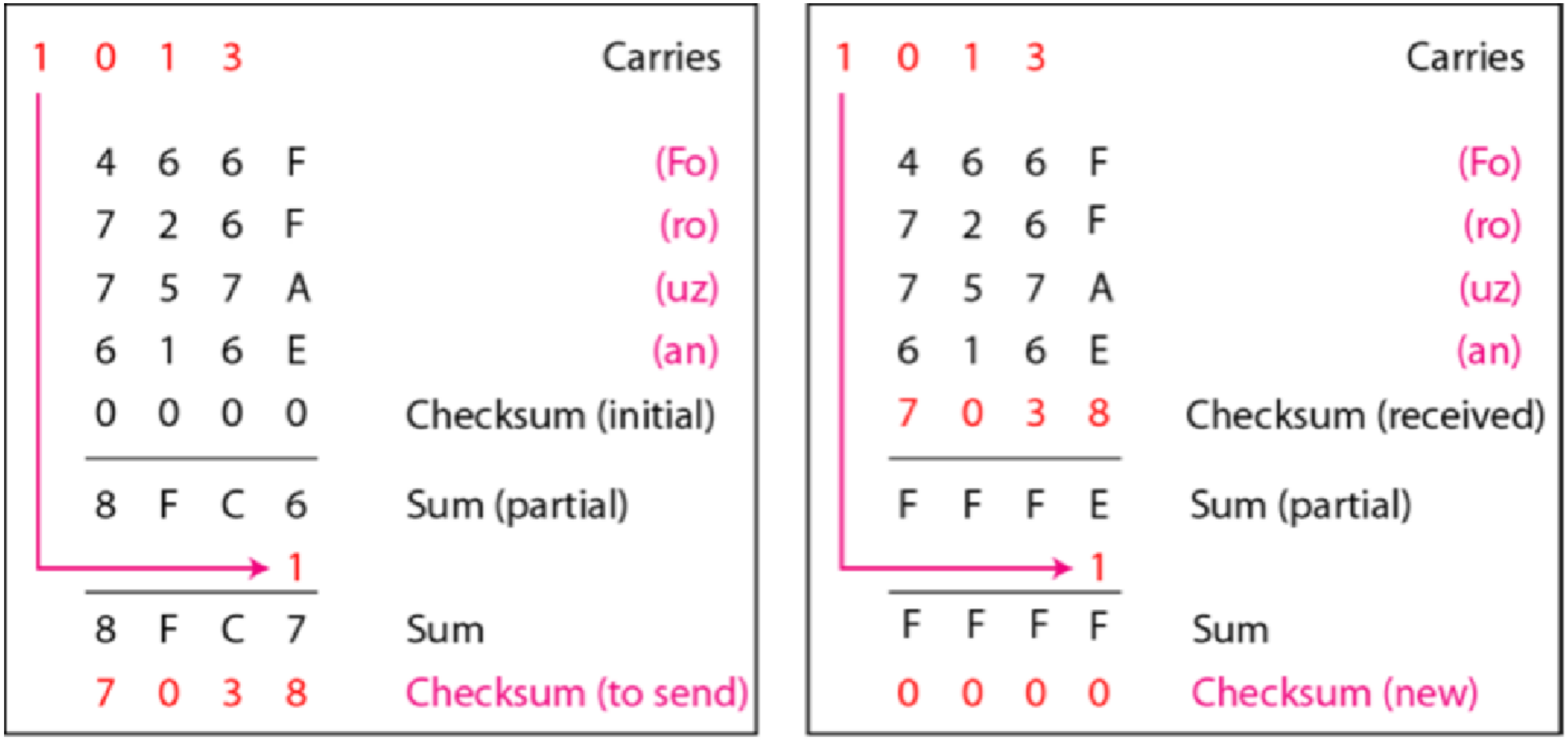
Let us calculate the checksum for a text of 8 characters ("Forouzan"). The text needs to be divided into 2-byte (16-bit) words. We use ASCII (see Appendix A) to change each byte to a 2-digit hexadecimal number. For example, F is represented as 0x46 and o is represented as 0x6F. Figure 10.25 shows how the checksum is calculated at the sender and receiver sites. In part a of the figure, the value of partial sum for the first column is 0x36. We keep the rightmost digit (6) and insert the leftmost digit (3) as the carry in the second column. The process is repeated for each column. Note that if there is any corruption, the checksum recalculated by the receiver is not all 0s. We leave this an exercise.

Appendix A



Decimal	Hex	Char	Decimal	Hex	Char
64	40	@	96	60	`
65	41	A	97	61	a
66	42	B	98	62	b
67	43	C	99	63	c
68	44	D	100	64	d
69	45	E	101	65	e
70	46	F	102	66	f
71	47	G	103	67	g
72	48	H	104	68	h
73	49	I	105	69	i
74	4A	J	106	6A	j
75	4B	K	107	6B	k
76	4C	L	108	6C	l
77	4D	M	109	6D	m
78	4E	N	110	6E	n
79	4F	O	111	6F	o
80	50	P	112	70	p
81	51	Q	113	71	q
82	52	R	114	72	r
83	53	S	115	73	s
84	54	T	116	74	t
85	55	U	117	75	u
86	56	V	118	76	v
87	57	W	119	77	w
88	58	X	120	78	x
89	59	Y	121	79	y
90	5A	Z	122	7A	z
91	5B	[	123	7B	{
92	5C	\	124	7C	|
93	5D	]	125	7D	}
94	5E	^	126	7E	~
95	5F	_	127	7F	*[DEL]*

Figure 10.25 *Example 10.23*



a. Checksum at the sender site

a. Checksum at the receiver site