

Practice Problems Chapter 4: Non-Linear Equations

1. Consider a fixed point function $g(x) = (9x - 1)^{\frac{1}{3}}$. The corresponding nonlinear function $f(x)$ has a solution $x_* \in \mathbb{R}$.
 - a. Show that $g(x)$ would lead to linear convergence if $x > \frac{1}{9}(1 + \sqrt{27})$.**
*hint: use the condition for linear convergence ***
 - b. Starting from $x_0 = 2.5$, find the value of x_* after 5 iterations while keeping up to 5 significant figures.
2. Use Secant method to estimate the root of the following function $f(x) = x^3 - 3x^2 + x$ with initial the initial values of $x_0 = 0.35$ & $x_1 = 0.3$. Show your result with the error in a tabular format for the first 5 iterations.
3. Consider the following function $f(x) = x^2 \sqrt{2x + 3}$. Now, find the solution of $f(x) = 0$ by doing 5 iterations using the Newton's Raphson method starting with $x_0 = 1.5$. Keep your answers up to 3 significant figures.
4. Consider the function, $f(x) = x^3 - x^2 - 9x + 9$. Answer the following:
 - a. State the exact roots of $f(x)$.
 - b. Construct three different fixed point functions $g(x)$ such that $f(x) = 0$. (Make sure that one of the $g(x)$'s that you constructed converges to at least a root).
 - c. Find the convergence rate/ratio for each $g(x)$ constructed in the previous part and also find which root it is converging to.
 - d. Find the approximate root, x^* , of the above function using fixed point iterations up to 4 significant figures within the error bound of 1×10^{-3} using $x_0 = 0$ and any fixed point function $g(x)$ from part(b) that converges to the root(s).
5. Use Newton's method to find the root, x_* , of the equation, $f(x) = x^2 e^{-x} - 0.6$, up to machine epsilon of 1×10^{-4} starting with $x_0 = 0.2$.
6. Let $f(x) = x^3 + 4x^2 - 10 = 0$, which can be written as $g(x) = x$ for some function $g(x)$.
 - a. By manipulating $f(x) = 0$, find at least three expressions for $g(x)$ such that $g(x) = x$.
 - b. The given function, $f(x)$, has one real root which is $x_* = 1.36523$. The other two roots are complex and ignore those two roots. Now evaluate the rate λ for the three function, $g(x)$, you evaluated in the previous part using the real root. Are these three function, $g(x)$, converging or diverging?
7. Use Newton's method to find the solutions for (i) $f(x) = \sqrt{x} - \cos(x)$ and (ii) $f(x) = x^2 - 2xe^{-x} + e^{2x}$ starting with $x_0 = 2.0000$ within 10^{-5} .
8. Consider the nonlinear equation, $f(x) = x^3 - 7x^2 + 4x + 12$. Answer the following:
 - a. Find the roots of the given function. Note all three distinct roots are real in this case.
 - b. Construct two different fixed point functions for the given function.
 - c. Find out if the fixed point functions you evaluated in the previous part are converging or diverging. If converging, which root it is converging to.
 - d. Construct a superlinear converging function $g(x)$ for the given function and computer six iterations starting from (i) $x_0 = 4$ and (ii) $x_0 = 0$. Which root the $g(x)$ seems to be converging to?
9. A function is given by $f(x) = x^3 + 2x^2 - x - 2$. Answer the following:
 - a. By manipulating $f(x) = 0$, find at least three expressions for $g(x)$ such that $g(x) = x$.
 - b. The three roots of $f(x)$ are ± 1 and -2 . For all your $g(x)$'s, compute the rate λ to find if it is converging to any of the roots.
10. Use Newton's method to find the solution of $f(x) = 1 - 4x \cos(x) + 2x^2 + \cos(2x) = 0$ within 10^{-5} for $0 \leq x \leq 1$ starting with $x_0 = 0.25$.
11. Use Newton's method to find the root, x_* , of the function $f(x) = x^2 e^{-x} - 0.5$ up to machine epsilon 1.0×10^{-4} starting with $x_0 = 0.2$.
12. A function is given by: $f(x) = x^6 - x^3 - 2$ which has two real roots and the other roots are

complex. Answer the following:

- a. Construct two fixed point function $g(x)$ such that $f(x) = 0$.
 - b. Compute the rate λ for the fixed point functions constructed above, and which root it is converging to or diverging
 - c. Starting from $x_0 = 60$, and the converging fixed point function $g(x)$ that you constructed in the previous part to find the root of the above function accurate up to 3 decimal places.
13. Find the root of the equation, $f(x) = xe^x - 1$ using fixed point iteration accurate up to machine epsilon of 1.0×10^{-5} . Use the fixed point function $g(x) = e^{-x}$ and start with $x_0 = 0$.
14. Use a secant method to find the root of the equation, $f(x) = 2x^3 + 7x^2 - 14x + 5$. Find the root accurate up to 4 decimal places starting with $x_0 = -5.5$ and $x_1 = -4.5$.