Assignment 02

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CSE330: Numerical Methods

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Ans to the ques no-01

ⓐ

nodes = 3

$\therefore$ degree = 2

$\therefore P_1(t) = a_0 + a_1 t + a_2 t^2$

According to given data,

$P_1(10) = a_0 + a_1 \cdot 10 + a_2 \cdot 10^2$

$\Rightarrow 24 = a_0 + 10a_1 + 100a_2$ —— (I)

$P_1(15) = a_0 + a_1 \cdot 15 + a_2 \cdot 15^2$

$\Rightarrow 31 = a_0 + 15a_1 + 225a_2$ —— (II)

$P_1(18) = a_0 + a_1 \cdot 18 + a_2 \cdot 18^2$

$\Rightarrow 43 = a_0 + 18a_1 + 324a_2$ —— (III)

Converting eq (I), (II) & (III) into Vandermonde Matrix,

$$A = \begin{bmatrix} 1 & 10 & 100 \\ 1 & 15 & 225 \\ 1 & 18 & 324 \end{bmatrix} \quad x = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} \quad b = \begin{bmatrix} 24 \\ 31 \\ 43 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 27/4 & -12 & 25/4 \\ -33/40 & 28/15 & -25/24 \\ 1/40 & -1/15 & 1/24 \end{bmatrix}$$

$$\therefore A^{-1} b = \begin{bmatrix} 58.75 \\ -6.725 \\ 0.325 \end{bmatrix}$$

Now,

$x = A^{-1}.b$

$$\Rightarrow \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 58.75 \\ -6.725 \\ 0.325 \end{bmatrix}$$

$\therefore P_1(x) = 58.75 - 6.725x + 0.325x^2$

$\therefore P_1'(x) = 0 - 6.725 + 2 \times 0.325x$

$\therefore P_1(17) = \cancel{38.35}$ ~~(Ans)~~

$4.325 \ ms^{-2}$

(b)

$deg = 2$

$\therefore P_1(x) = l_0(x) f(x_0) + l_1(x) f(x_1) + l_2(x) f(x_2)$

Here,

$$l_0(x) = \frac{x - x_1}{x_0 - x_1} \times \frac{x - x_2}{x_0 - x_2}$$

$$= \frac{x-15}{-5} \times \frac{x-18}{-8} = \frac{1}{40}(x-5)(x-18)$$

Again,

$$l_1(x) = \frac{x - x_0}{x_1 - x_0} \times \frac{x - x_2}{x_1 - x_2}$$

$$= \frac{x-10}{5} \times \frac{x-18}{-3} = -\frac{1}{15}(x-10)(x-18)$$

Again,

$$l_2(x) = \frac{x - x_0}{x_2 - x_0} \times \frac{x - x_1}{x_2 - x_1}$$

$$= \frac{x-10}{8} \times \frac{x-15}{3} = \frac{1}{24}(x-10)(x-15)$$

$$\therefore P_1(x) = \frac{1}{40}(x-15)(x-18)\cdot f(x_0) - \frac{1}{15}(x-10)(x-18)\cdot f(x_1) + \frac{1}{24}(x-10)(x-15)\cdot f(x_2)$$

$$= \frac{3}{5}(x-15)(x-18) - \frac{31}{15}(x-10)(x-18) + \frac{43}{24}(x-10)(x-15)$$

(Ans)

Ans to the ques no-02

(a)

Given, $f(x) = -x^2 \cdot \cos(x)$

x	$f(x)$
$-\pi/4$	-0.43618
0	0
$\pi/4$	-0.43618

$$P_n = f[x_0] + f[x_0, x_1](x-x_0) + f[x_0, x_1, x_2](x-x_0)(x-x_1) \quad \text{--- (1)}$$

$f[x_0] = -0.43618$

$f[x_1] = 0$

$f[x_2] = -0.43618$

$$f[x_0, x_1] = \frac{f[x_1] - f[x_0]}{x_1 - x_0} = \frac{0-(-0.43618)}{0-(-\pi/4)} = 0.5554$$

$$f[x_1, x_2] = \frac{f[x_2] - f[x_1]}{x_2 - x_1} = \frac{-0.43618 - 0}{\pi/4 - 0} = -0.5554$$

$$f[x_0, x_1], \; f[x_1, x_2] \Rightarrow f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0}$$

$$= -0.70718$$

Placing the values in eq ①,

$$P_n(x) = -0.43618 + 0.5554(x + \pi/4) - 0.70718\,(x + \pi/4)\cdot x$$ (Ans)

ⓑ From ⓐ we get,

$$P_n(x) = -0.43618 + 0.5554\,(x + \pi/4) - 0.70718\,(x + \pi/4)\cdot x$$

$$\therefore P(\pi/3) = -0.775498225\,7$$

And, $f(x) = -x^2 \cdot \cos x$

$$\Rightarrow f(\pi/3) = -0.5483113558$$

$$\therefore \text{Relative Error Percentage} = \left| \frac{f(x) - P_n(x)}{f(x)} \right| \times 100\,\%$$

$$= 41.43\%$$ (Ans)

(c)

from (a) we get,

$$P_n(x) = f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_2)$$

After adding new node,

x	$f(x)$
π	9.8696

$$P_{n_1}(x) = f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_2) + f[x_0, x_1, x_2, x_3](x - x_0)(x - x_2)(x - x_3)$$

$$= P_n(x) + f[x_0, x_1, x_2, x_3](x - x_0)(x - x_2)(x - x_3)$$

$f[x_0]$, $f[x_1]$, $f[x_2]$, $f[x_3]$

$f[x_0, x_1]$, $f[x_1, x_2]$

$$f[x_0, x_1, x_2] = 0.70718$$

$$f[x_3, x_2] = \frac{f[x_3] - f[x_2]}{x_3 - x_2} = \frac{9.8696 + 0.43618}{x_3 - x_2}$$

$$= 4.3739$$

$$f[x_1, x_2, x_3] = \frac{f[x_3, x_2] - f[x_1, x_2]}{x_3 - x_1}$$

$$= 1.569044$$

$$\therefore f[x_0, x_1, x_2, x_3] = \frac{f[x_1, x_2, x_3] - f[x_0, x_1, x_2]}{x_3 - x_0}$$

$$= 0.5796356819$$

Placing the values,

$P_{n_1}(x) = -0.43618 + 0.5554(x+\pi/4) - 0.70718(x+\pi/4)(x) + 0.5797(x+\pi/4) \cdot x \cdot (x-\pi/4)$ (Ans)

Ans to the ques no-03

ⓐ $f(x) = e^{3x} - e^{-3x}$

$\therefore f'(x) = 3e^{3x} + 3e^{-3x}$

$\therefore f''(x) = 9 \cdot e^{3x} - 9e^{-3x}$

$\therefore f'''(x) = 27(e^{3x} + e^{-3x})$

$\therefore$ Maximum Error Bound, $|f(x) - P_n(x)| = \frac{f^3(\xi)}{3!}(x+2) \cdot x \cdot (x-2)$

Let, $v(x) = \frac{f^3(\xi)}{3!}$ and $w(x) = (x+2) \cdot x \cdot (x-2)$

Maximizind $v(x)$, where, $x = 1$

$\frac{f^3(\xi)}{3!} = \frac{27(e^{3x} + e^{-3x})}{6} = 90.60895796$

where, $x = -5$

$\frac{f^3(\xi)}{3!} = 14710578.18$

Maximized

And, Maximizing $w(x)$,

$w(x) = (x+2)(x-2) \cdot x = x^3 - 4x$

$\therefore w'(x) = 3x^2 - 4$

Let, $w'(x) = 0$

$\Rightarrow 3x^2 - 4 = 0$

$\Rightarrow x = \pm \frac{2}{\sqrt{3}}$

x	$\lvert w(x) \rvert$
$+2/\sqrt{3}$	3.0792
$-2/\sqrt{3}$	3.0792
-5	105
1	-3

$\therefore$ Maximum Error Bound, $\lvert f(x) - P_n(x) \rvert = \overset{14710578.18}{\cancel{90.6089576}} \times 105$

$\cancel{= 9513.940586}$

$\cancel{= 9513.9405}$ ~~[upto SF=8]~~

$= \underbrace{1544610}7.08$

$= 15446107$ [upto SF = 8]