Practice Problems: Chapter-5 (Linear Equations)

1. Solve the following system using Gaussian Elimination:

$$x + y + z = 6$$
$$2x + 3y + z = 14$$
$$x + 2y + 3z = 14$$

   a. Write down the Augmented matrix, Aug(A), from the given linear system, and evaluate the upper triangular matrix U. Note that you have to show the row multipliers m_{ij} for each step as necessary.U
   b. Using the upper triangular matrix found in the previous question, compute the solution of the given linear system by Gaussian elimination method.

2. Solve using Gaussian Elimination:

$$y + z = 3$$
$$x + y + z = 6$$
$$2x + y + 3z = 10$$

   a. Examine if the matrix A has any pivoting problem? Explain why or why not?
   b. Write down the Augmented matrix, Aug(A), from the given linear system, and evaluate the upper triangular matrix U. Note that you have to show the row multipliers m_{ij} for each step as necessary.
   c. Using the upper triangular matrix found in the previous question, compute the solution of the given linear system by Gaussian elimination method.

3. Determine if the system has no solution, infinitely many, or one unique solution:

$$x + 2y + 3z = 6$$
$$2x + 4y + 6z = 12$$
$$3x + 6y + 9z = 18.$$

4. The height of a ball (in meters) above the ground is measured at three different times after it is thrown upward:

Time((Seconds)	Height(meters)
1	18.2
3	30.5
6	25.1

   The height is approximated by a quadratic polynomial:

$$h(t) = a_1 t^2 + a_2 t + a_3$$

   a. Use the Gaussian Elimination method to determine the values of a_1, a_2 and a_3.
   b. Use the resulting polynomial to estimate the height of the ball at $t = 4$ seconds.

5. Given, Matrix A is,

$$\begin{bmatrix} 2 & 1 & 1 \\ 4 & -6 & 0 \\ -2 & 7 & 2 \end{bmatrix}$$

   a. Construct the Frobenius matrices $F^{(1)}$ *and* $F^{(2)}$ from the system.
   b. Compute the unit lower triangular matrix L.

6. Given, Matrix A is,

$$\begin{bmatrix} 0 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix}$$

Decompose the matrix A into L and U.

7. Given,

$$x + y + z = 6$$
$$2x + 3y + z = 14$$
$$x + y + 2z = 10$$

   a. From the given linear equations, identify the matrices A, x and b such that the linear system can be expressed as a matrix equation.
   b. Construct the Frobenius matrices $F^{(1)}$ and $F^{(2)}$ from the system.
   c. Compute the unit lower triangular matrix L.
   d. Now find the solution of the linear system using the LU decomposition method. Use the unit lower triangular matrix found in the previous question.

8. A linear system is described by the following linear equations

$$x_1 + x_2 + 3x_4 = 4$$
$$2x_1 + x_2 - x_3 + x_4 = 1$$
$$3x_1 - x_2 - x_3 + 2x_4 = -3$$
$$\text{And} \quad -x_1 + 2x_2 + 3x_3 - x_4 = 4$$

Solve the above system by Gaussian elimination method and also by LU-decomposition method.

9. A linear system is defined by the following equations,

$$-x + y - z = -1$$
$$2x + 6y - z = 3$$
$$6x + 5y + 3z = 8.$$

   Now, answer the following:

   a. Does the system have a unique solution or infinite solution? Explain or show calculation with reasoning.
   b. Find the augmented matrix for the linear system, and solve the system using Gaussian elimination method.

10. A linear system is described by the following equations:

$$2x - 2y + z = -3,$$
$$x + 3y - 2z = 1,$$
$$3x - y - z = 2.$$

   a. From the given linear equations, identify the matrices A, b, and x such that the system can be expressed as a matrix equation.
   b. Does this system have any unique solution? Explain or show calculation.
   c. Evaluate the upper triangular matrix $U$by Gaussian elimination method. Note that you have to show the row multipliers $m_{ij}$for each step as necessary.
   d. Using the upper triangular matrix found in the previous question, compute the solution of the given linear system by Gaussian elimination method.

11. A linear system is described by the following equations:

$$x + 2y - z = 0,$$

$$2x - y + z = 1,$$

$$-x + y + 2z = 2.$$

a. From the given linear equations, identify the matrix A. Examine if the matrix A has any pivoting problem? Explain why or why not?
b. State how many Frobenius matrices, $F^{(i)}, i = 1,2, \cdots$ can be computed, and evaluate them for the given system.
c. Evaluate the unit lower triangular matrix L, and the upper triangular matrix U.
d. Now compute the solution of the given linear system using the LU-decomposition method. Use the matrices L and U found in the previous question. Show your works.

12. A linear system is described by the following equations.

$$2a - 2b + c = -3$$

$$a + 3b - 2c = 1$$

$$3a - b - c = 2.$$

(a) From the given linear equations, identify the matrices A, x and b such that the linear system can be expressed as a matrix equation.

(b) Does this system have any unique solution? Explain.

(c) Write down $Aug(A)$, and evaluate the upper triangular matrix U. Note that you have to show the row multipliers m_{ij} for each step as necessary.

(d) Using the upper triangular matrix found in the previous question, compute the solution of the given linear system by Gaussian elimination method.

13. A linear system is described by the following linear equations

$$4x - y + z = 8,$$

$$2x + 5y + 2z = 3,$$

$$x + 2y + 4z = 11.$$

Answer the following:

a. Show that the linear system has one unique solution.
b. Solve the above linear system by the Gauussian elimination method.
c. Also solve the same system by the LU-decomposition method.