Probability

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Probability

Classical Theory

P(state) = No. of desired possible outcomes / No. of all equally possible outcomes

E.g. 1. If we throw a dice then the probability of getting a 4 is

$P(4) = \frac{1}{6}$

2. If we get toss a coin then the probability of getting a Heads is

$P(H) = \frac{1}{2}$

Probability

Statistical Theory

Create a table and collect data accordingly

Eg. 1. What is the probability of getting a 6 if we throw a dice ?

Throw the dice 600 times and then calculate the average of the no. of times 6 appear

1	2	3	4	5	6
95	105	110	94	97	99

Probability

Axiomatic Theory

For any event A it’s probability will be $0 \leq P(A) \leq 1$

Sum of all events probability is 1.

Equations

1. $P(A') = 1 - P(A)$
2. $P(A \vee B) = P(A) + P(B) - P(A \wedge B)$
3. $P(A \mid B) = P(A \wedge B) / P(B)$
4. $P(A \wedge B) = P(A) * P(B)$ if A and B are independent.
5. $P(A|B) = (P(B|A)*P(A)) / P(B)$ Bayes' Theorem

Data to Probability

A survey is conducted on 500 people both male and female about which tv shows they liked. From the 500 people we got the following response.

	Male	Female	Total
GOT	80	120	200
TBBT	100	25	125
Others	50	125	175
Total	230	270	500

	Male	Female	Total
GOT	0.16	0.24	0.40
TBBT	0.2	0.05	0.25
Others	0.1	0.25	0.35
Total	0.46	0.54	1

Marginal Probability

- Also known as **Simple Probability.**
- Represents the probability of a particular event.

Q1. What is the marginal probability of people watching TBBT ?

Ans:

P(TBBT) = 0.25

	Male	Female	Total
GOT	0.16	0.24	0.40
TBBT	0.2	0.05	0.25
Others	0.1	0.25	0.35
Total	0.46	0.54	1

Marginal Probability Distribution

- All the probability that occurs in the margin for a particular variable/event can be summed up to create **Marginal Probability Distribution**.
- Sum of MPD is 1

	Male	Female	Total
GOT	0.16	0.24	0.40
TBBT	0.2	0.05	0.25
Others	0.1	0.25	0.35
Total	0.46	0.54	1

Joint Probability

- Probability of two events occurring at the same time.
- P (A ^ B)

Q1. What is the joint probability of a person being Female and liking TBBT ?
Ans:

P(T ^ F) = 0.05

	Male	Female	Total
GOT	0.16	0.24	0.40
TBBT	0.2	0.05	0.25
Others	0.1	0.25	0.35
Total	0.46	0.54	1

Joint Probability Distribution

- All the probability that occur jointly can be summed up to create **Joint Probability Distribution**.
- Sum of JPD is 1

	Male	Female	Total
GOT	0.16	0.24	0.40
TBBT	0.2	0.05	0.25
Others	0.1	0.25	0.35
Total	0.46	0.54	1

Conditional Probability

- Impact of one variable on another variable.
- $P(A|B) = P(A \wedge B) / P(B)$
- $P(A \wedge B) = P(A|B) * P(B)$

Q1. What is the probability of a person liking GOT given that person is male?

Ans:

$$P(G|M) = P(G \wedge M)/P(M)$$

$$P(G|M) = 0.16/0.46 = 0.347$$

	Male	Female	Total
GOT	0.16	0.24	0.40
TBBT	0.2	0.05	0.25
Others	0.1	0.25	0.35
Total	0.46	0.54	1

Conditional Probability

- $P(A|B) = P(A \wedge B) / P(B)$
- $P(B|A) = P(A \wedge B) / P(A)$

Hence, $P(A|B)*P(B) = P(B|A)*P(A)$

Then, $P(A|B) = (P(B|A)*P(A)) / P(B)$

Which is called **Bayes' Theorem**

Independence

- No effect of one variable on another variable.
- $P(A|B) = P(A)$
- $P(A \wedge B) = P(A|B) * P(B)$ [Conditional Probability]

Hence, $P(A \wedge B) = P(A) * P(B)$

Or. $P(B \wedge A) = P(B) * P(A)$

Q1. Are Male viewers and GOT independent?

Ans:

$P(M \wedge GOT) = P(M) * P(GOT)$

	Male	Female	Total
GOT	**0.16**	0.24	0.40
TBBT	**0.2**	0.05	0.25
Others	**0.1**	0.25	0.35
Total	**0.46**	0.54	1

Independence

Q1. Are Male viewers and GOT independent?

Ans:

P(M^GOT) = 0.16
P(M) = 0.46
P(GOT) = 0.40
P(M)*P(GOT) = 0.46 * 0.40 = 0.184

Since, P(M^GOT) ≠ P(M) * P(GOT) so, not independent.

	Male	Female	Total
GOT	**0.16**	0.24	0.40
TBBT	**0.2**	0.05	0.25
Others	**0.1**	0.25	0.35
Total	**0.46**	0.54	1

Q2. Are Female and TBBT independent ? [Try it]

Conditional Independence

- Conditional probability of A and B given C can be defined as,
  $P(A \wedge B \mid C) = P(A \wedge B \wedge C) / P(C)$

- A and B can be conditionally independent of C if,
  $P(A \wedge B \mid C) = P(A|C) * P(B|C)$
  [Remember $P(A \wedge B) = P(A) * P(B)$ if A and B are independent]

- Joint probability of A, B and C can be defined as,
  $P(A \wedge B \wedge C) = P(A|C) * P(B|C) * P(C)$

Conditional Independence

Q1. Is smart conditionally independent of prepared given study?

Ans:

P(Sm ^ Pr | Sd) = P(Sm | Sd) * P(Pr | Sd)

P(Sm ^ Pr | Sd) = P(Sm ^ Pr ^ Sd) / P (Sd)

= 0.432/0.6

= 0.72

	Smart		Not Smart	
	Study	Not Study	Study	Not Study
Prepared	0.432	0.16	0.084	0.008
Not Prepared	0.048	0.16	0.036	0.072

Conditional Independence

Q1. Is smart conditionally independent of prepared given study?

Ans: (Cont'd)

$P(Sm \mid Sd) * P(Pr \mid Sd)$

$= P(Sm \wedge Sd)/P(Sd) * P(Pr \wedge Sd) / P(Sd)$

$= (0.48/0.6) * (0.512/0.6)$

$= 0.8 * 0.85 = 0.68$

Since, $P(Sm \wedge Pr \mid Sd) \neq P(Sm \mid Sd) * P(Pr \mid Sd)$ So, it isn't conditionally independent.

	Smart		Not Smart	
	Study	Not Study	Study	Not Study
Prepared	0.432	0.16	0.084	0.008
Not Prepared	0.048	0.16	0.036	0.072

Q2. Is study conditionally independent of prepared given smart? [Try it]

Exercise 1

Q1. What is the marginal probability of liking Batman?
Q2. What is the conditional probability of a person liking Spiderman given he is a Male?
Q3. What is the joint probability of a Female liking other superheroes ?
Q4. Are the Male people and people liking Spiderman independent?
Q5. What is the sum of the all joint probabilities?

	Male	Female	Total
Batman	0.25	0.24	0.49
Spiderman	0.24	0.1	0.34
Others	0.1	0.07	0.17
Total	0.59	0.41	1

Exercise 2

Q1. Is Happy conditionally independent of Money given Family?

Q2. Find out the probability of P(Not Happy | No Money)

Q3. Find the value of P(Happy)

Q4. Find the value of P(Happy ^ Not Money | Not Family)

	Family		No Family	
	Money	No Money	Money	No Money
Happy	0.432	0.16	0.084	0.008
Not Happy	0.048	0.16	0.036	0.072

Exercise 3

Q1. A person is brought in front of a jury. The jury finds the defendant guilty in 98% of the cases in which he committed a crime and it finds the defendant not guilty 97% of the cases when the defendant has not committed a crime. Only 0.8% of the population has committed a crime.

If a random person is found guilty by the jury what is more likely: criminal or not ?

Ans:	$P(G\|C) = 0.98$	$P(G'\|C') = 0.97$	$P(C) = 0.008$
	$P(G'\|C) = 0.02$	$P(G\|C') = 0.03$	$P(C') = 0.992$

Exercise 3

Ans: (Cont'd)

$$P(G|C) = P(G \wedge C) / P(C)$$

$$P(G \wedge C) = P(G|C) * P(C) = 0.008 * 0.98 = 0.0078$$

$$\begin{aligned} P(C|G) &= P(C \wedge G) / P(G) \\ &= P(G \wedge C) / P(G) \\ &= 0.0078 / P(G) \end{aligned}$$

$$\begin{aligned} P(C'|G) &= P(C' \wedge G) / P(G) \\ &= P(G \wedge C') / P(G) \\ &= (P(G|C') * P(C')) / P(G) \\ &= (0.03 * 0.992) / P(G) \\ &= 0.029 / P(G) \end{aligned}$$

$$P(C|G) > P(C'|G) \text{ ????}$$