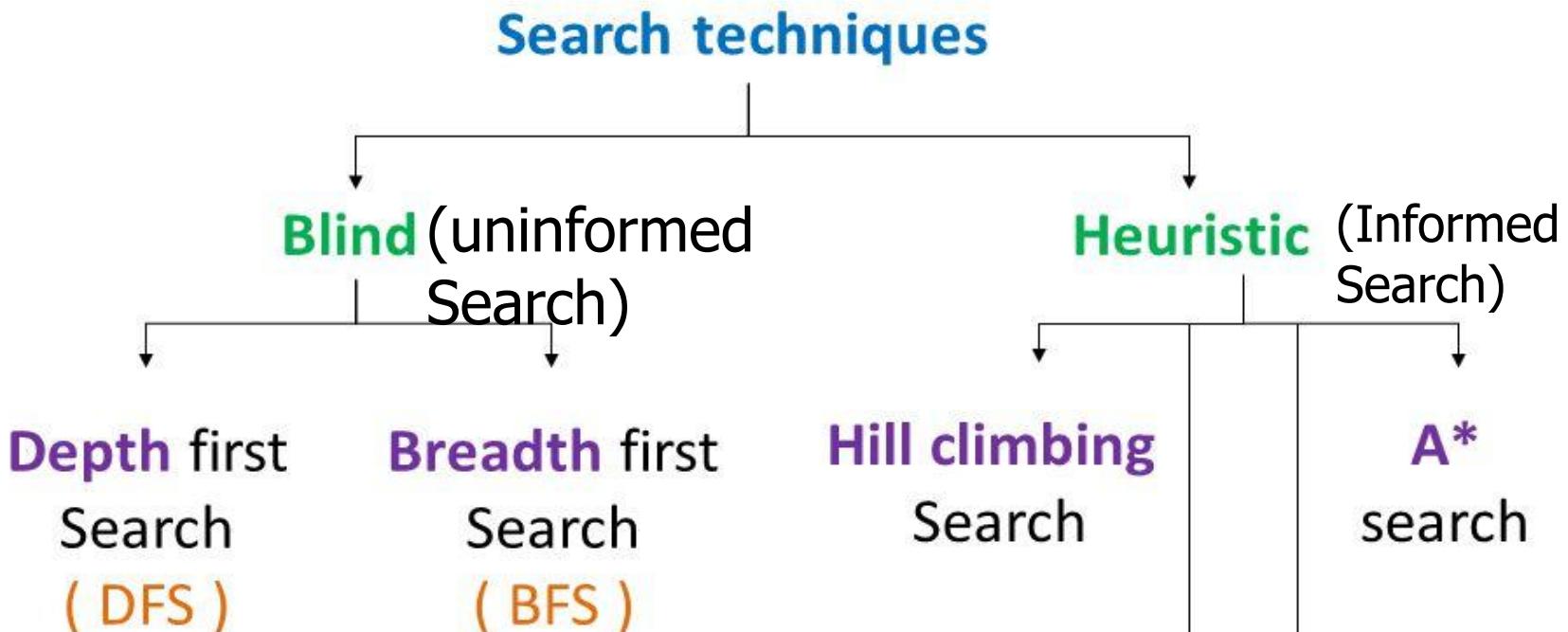


Artificial Intelligence

Informed Search

SEARCH TECHNIQUES



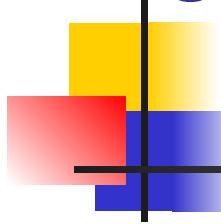
Other blind search strategies are:

- Depth-limited search (Extended DFS)
- Iterative-deepening search (Extended DFS)
- Uniform cost search
- Bi-directional search

Best-First
Search

Greedy
Search

Uninformed Vs Informed Search



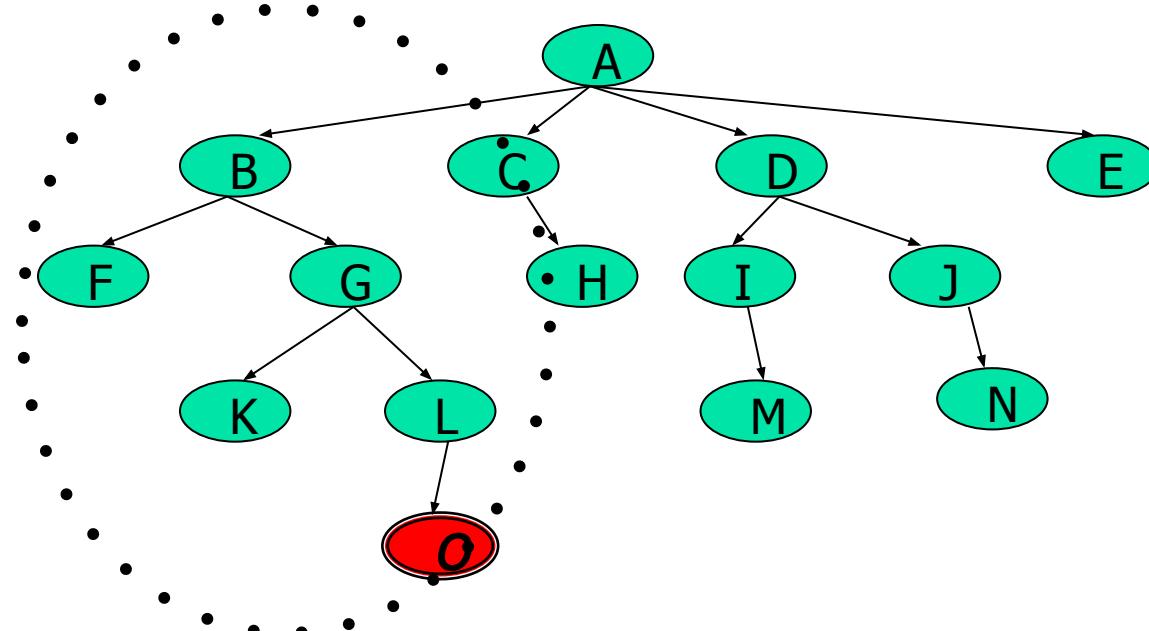
Uninformed search: Use only the information available in the problem definition. Example: breadth-first, depth-first, depth limited, iterative deepening, uniform cost and bidirectional search

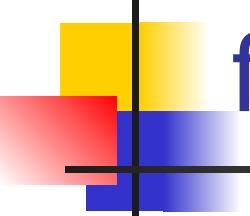
Informed search: Use domain knowledge or heuristic to choose the best move. Example. Greedy best-first, A*, IDA*, and beam search

Using problem specific knowledge to aid searching

- With knowledge, one can search the state space as if he was given “hints” when exploring a maze.
 - Heuristic information in search = Hints
- Leads to dramatic speed up in efficiency.

Search only
in this
subtree!!

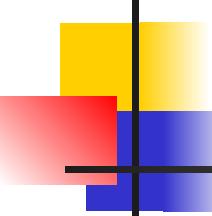




More formally, why heuristic functions work?

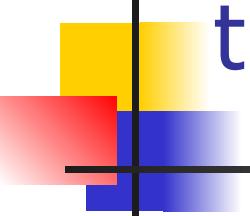
- In any search problem where there are at most b choices at each node and a depth of d at the goal node, a naive search algorithm would have to, in the worst case, search around $O(b^d)$ nodes before finding a solution (Exponential Time Complexity).
- Heuristics improve the efficiency of search algorithms by reducing the effective branching factor from b to (ideally) a low constant b^* such that
 - $1 \leq b^* \ll b$

| Criterion | Breadth-First | Uniform-Cost | Depth-First | Depth-Limited | Iterative Deepening |
|-----------|---------------|-------------------------------------|-------------|---------------|---------------------|
| Complete? | Yes | Yes | No | No | Yes |
| Time | $O(b^{d+1})$ | $O(b^{\lceil C^*/\epsilon \rceil})$ | $O(b^m)$ | $O(b^l)$ | $O(b^d)$ |
| Space | $O(b^{d+1})$ | $O(b^{\lceil C^*/\epsilon \rceil})$ | $O(bm)$ | $O(bl)$ | $O(bd)$ |
| Optimal? | Yes | Yes | No | No | Yes |



Heuristic Functions

- A heuristic function is a function $h(n)$ that gives an estimation on the “cost” of getting from node n to the goal state – so that the node with the least cost among all possible choices can be selected for expansion first. An evaluation function $f(n)$ can use $h(n)$ to define the goodness of a state
- Three approaches to defining f :
 - f measures the value of the current state (its “goodness”)
 - f measures the estimated cost of getting to the goal from the current state:
 - $f(n) = h(n)$ where $h(n)$ = an estimate of the cost to get from n to a goal
 - f measures the estimated cost of getting to the goal state from the *current state* and the cost of the existing path to it. Often, in this case, we decompose f :
 - $f(n) = g(n) + h(n)$ where $g(n)$ = the cost to get to n (from initial state)



Approach 1: f Measures the Value of the Current State

- Usually the case when solving optimization problems
 - Finding a state such that the value of the metric f is optimized
- Often, in these cases, f could be a weighted sum of a set of component values:
 - N-Queens
 - Example: the number of queens under attack ...
 - Data mining
 - Example: the “predictive-ness” (a.k.a. accuracy) of a rule discovered

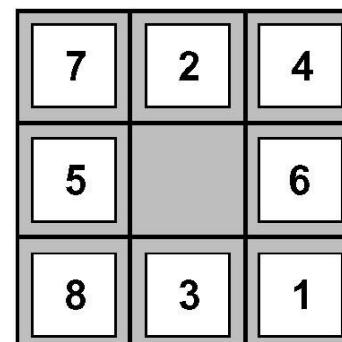
Approach 2: f Measures the Cost to the Goal

A state X would be better than a state Y if the estimated cost of getting from X to the goal is lower than that of Y – because X would be closer to the goal than Y

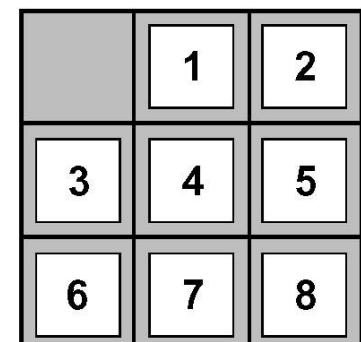
- 8–Puzzle

\mathbf{h}_1 : The number of misplaced tiles (squares with number).

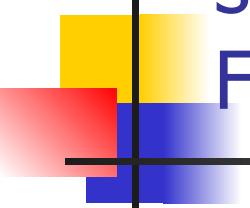
\mathbf{h}_2 : The sum of the distances of the tiles from their goal positions.



Start State



Goal State

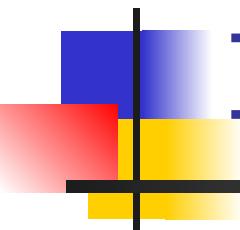


Approach 3: f measures the total cost of the solution path (With Admissible Heuristic Functions)

- A heuristic function is admissible if $h(n)$ **never** overestimates the cost to reach the goal.
 - Admissible heuristics are “optimistic”: “the cost is not that much ...”
- However, $g(n)$ is the exact cost to reach node n from the initial state.
- Therefore, $f(n)$ never over-estimate the true cost to reach the goal state through node n .
- Theorem: A search is optimal if $h(n)$ is admissible.
 - I.e. The search using $h(n)$ returns an optimal solution.
- Given $h_2(n) > h_1(n)$ for all n , it's always more efficient to use $h_2(n)$.
 - h_2 is more realistic than h_1 (*more informed*), though both are optimistic.

Traditional informed search strategies

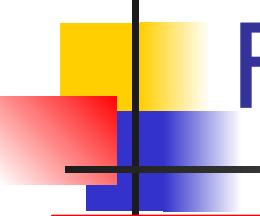
- Greedy Best first search
 - “Always chooses the successor node with the best f value” where $f(n) = h(n)$
 - We choose the one that is nearest to the final state among all possible choices
- A* search
 - Best first search using an evaluation function f that takes into account the “admissible” heuristic function h and the current cost g
 - Always returns the optimal solution path



Informed Search Strategies

Best First Search

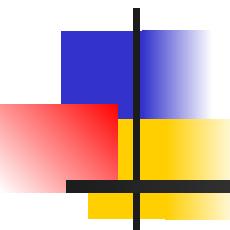
An implementation of Best First Search



```
function BEST-FIRST-SEARCH (problem, eval-fn)  
    returns a solution sequence, or failure
```

queuing-fn = a function that sorts nodes by *eval-fn*

```
return GENERIC-SEARCH (problem,queuing-fn)
```

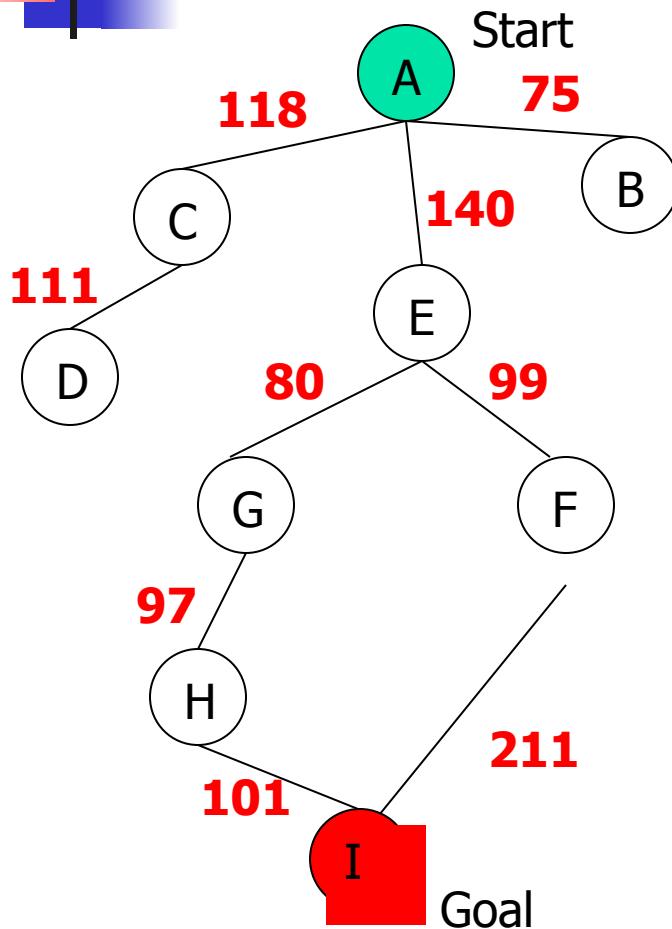


Informed Search Strategies

Greedy Search

$$\textit{eval-fn}: f(n) = h(n)$$

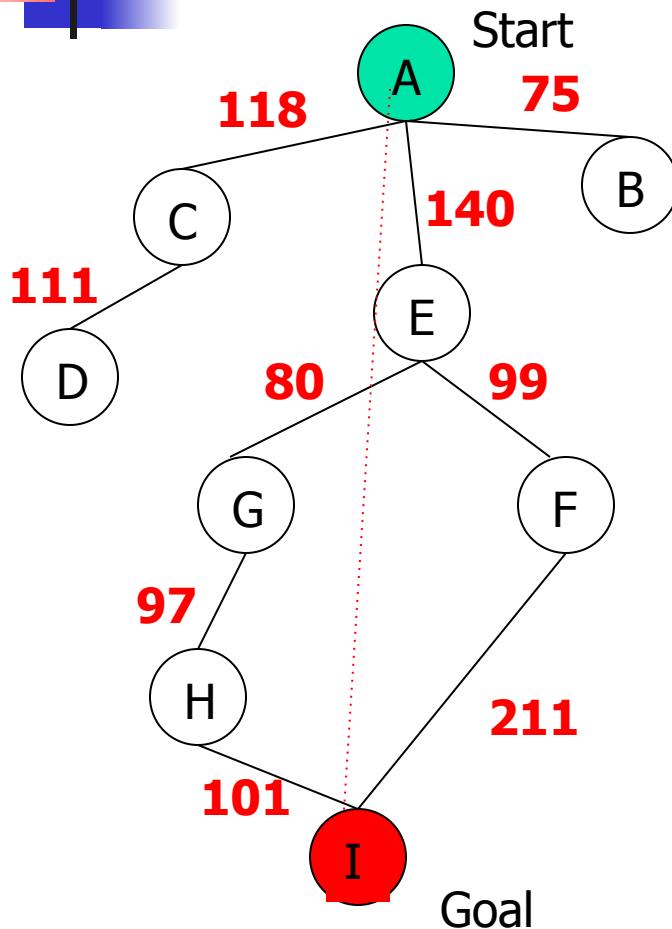
Greedy Search



| State | Heuristic: $h(n)$ |
|-------|-------------------|
| A | 366 |
| B | 374 |
| C | 329 |
| D | 244 |
| E | 253 |
| F | 178 |
| G | 193 |
| H | 98 |
| I | 0 |

$f(n) = h(n)$ = straight-line distance heuristic

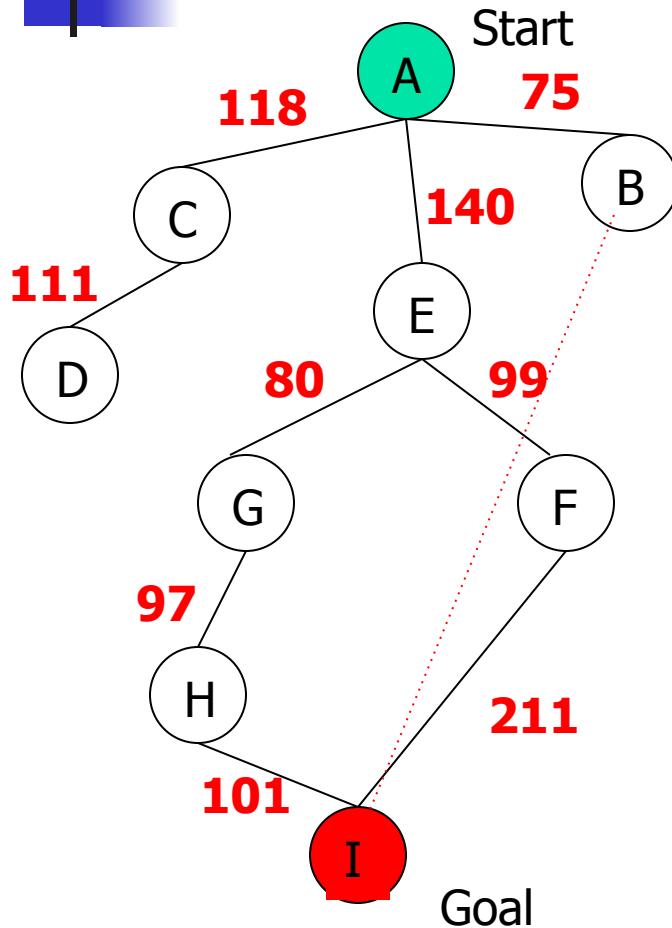
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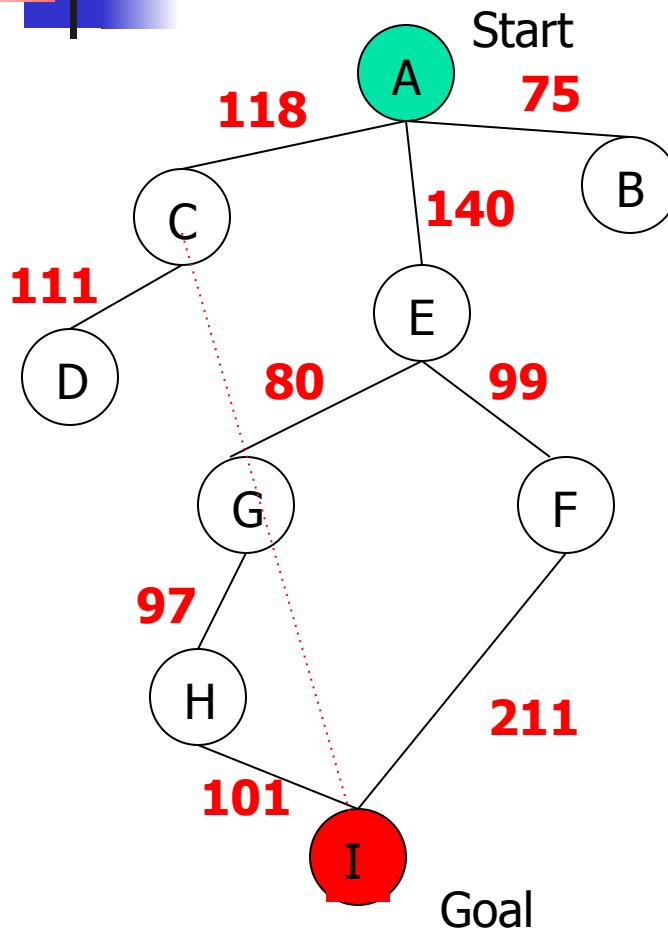
Greedy Search



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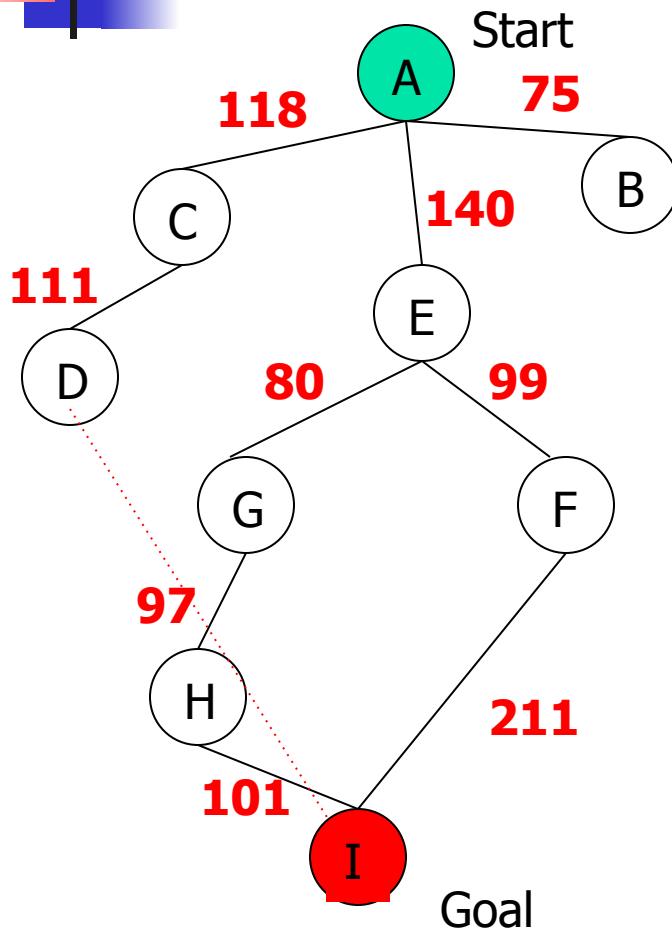
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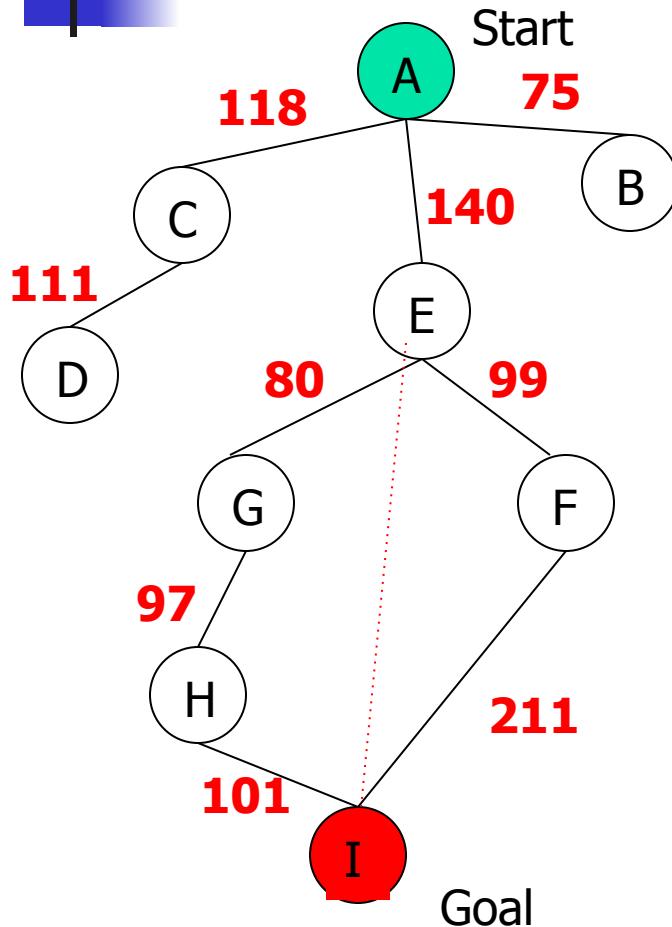
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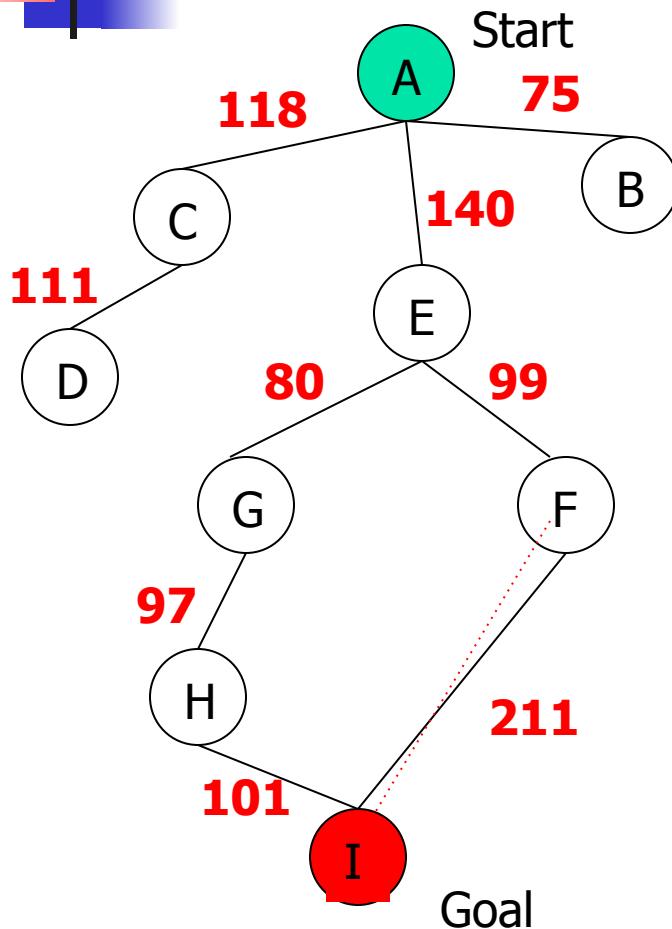
Greedy Search



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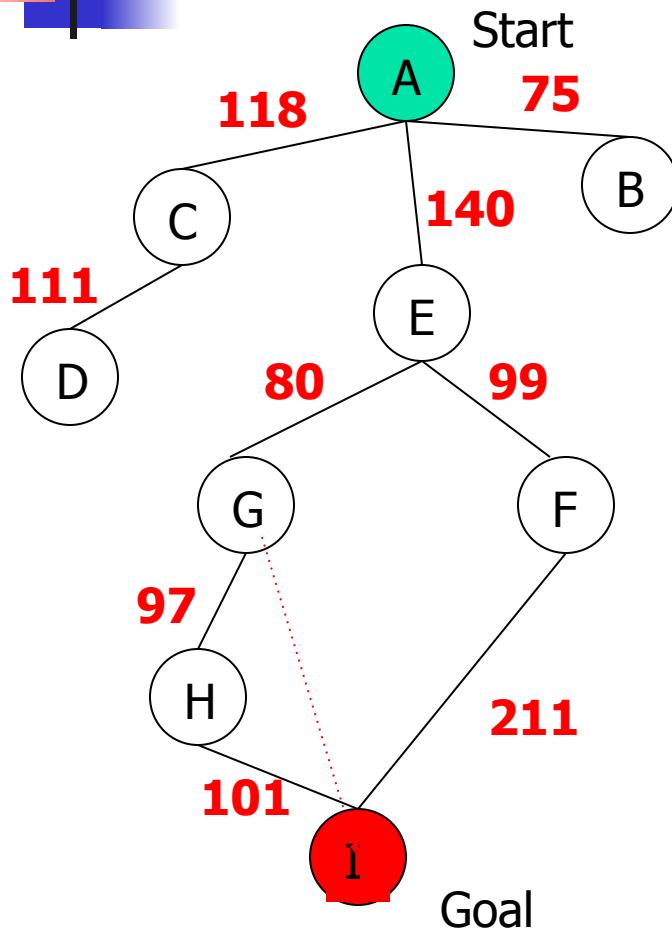
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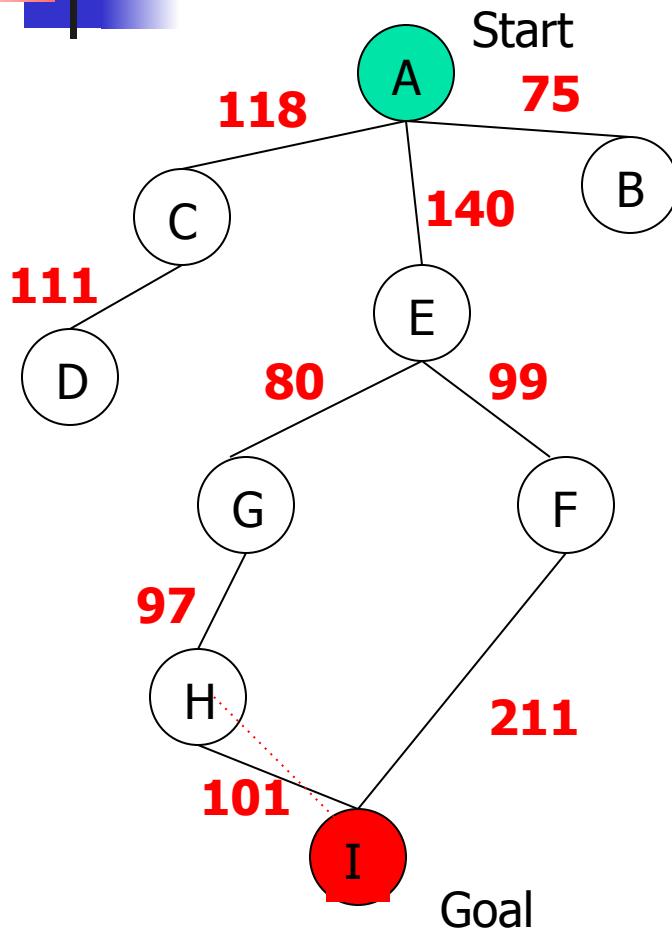
Greedy Search



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$f(n) = h(n)$ (n) = straight-line distance heuristic

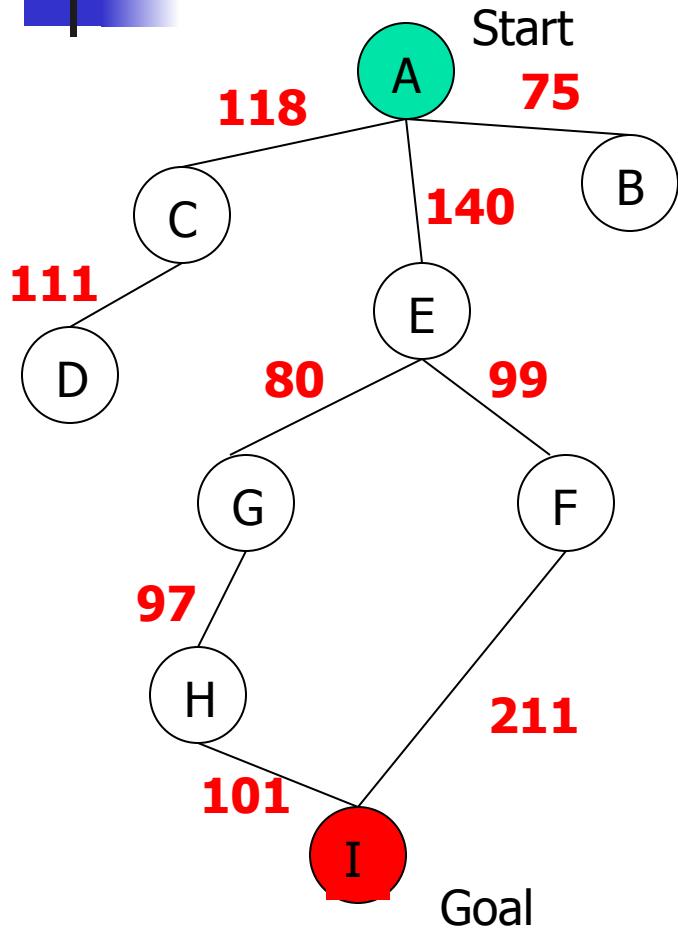
Greedy Search



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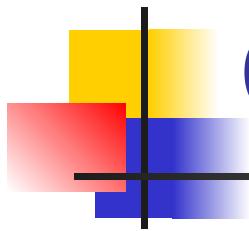
$f(n) = h(n)$ (n) = straight-line distance heuristic

Greedy Search



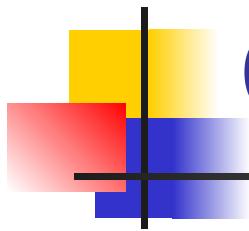
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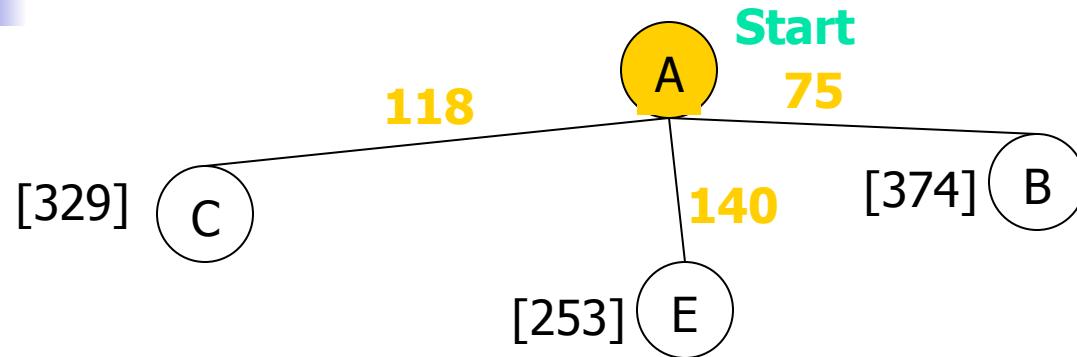


Greedy Search: Tree Search

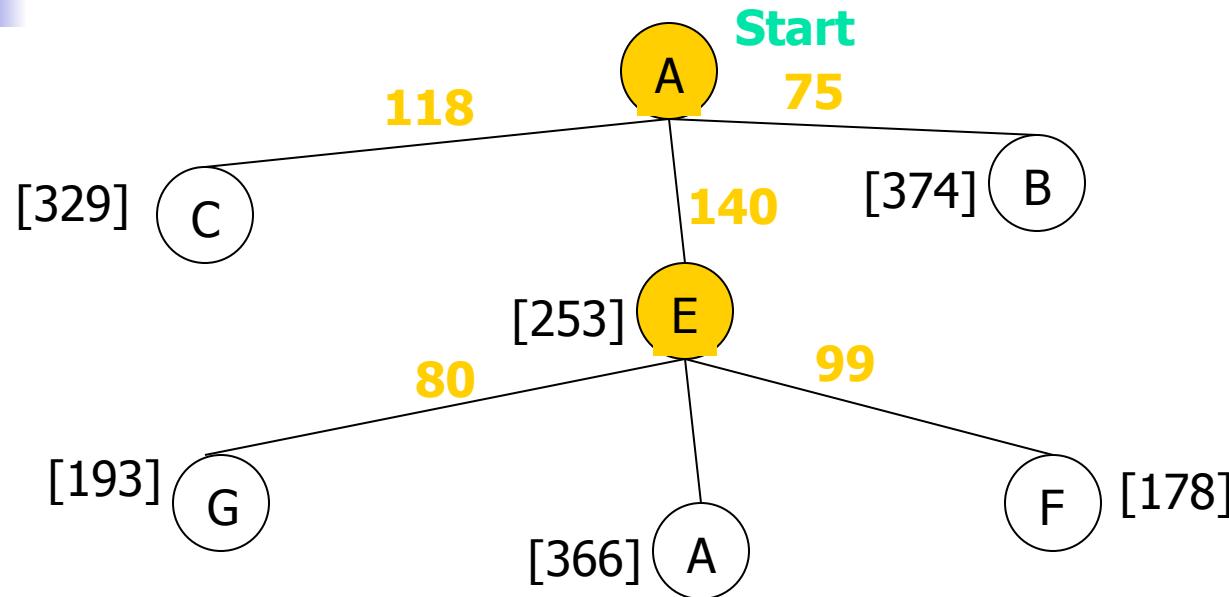




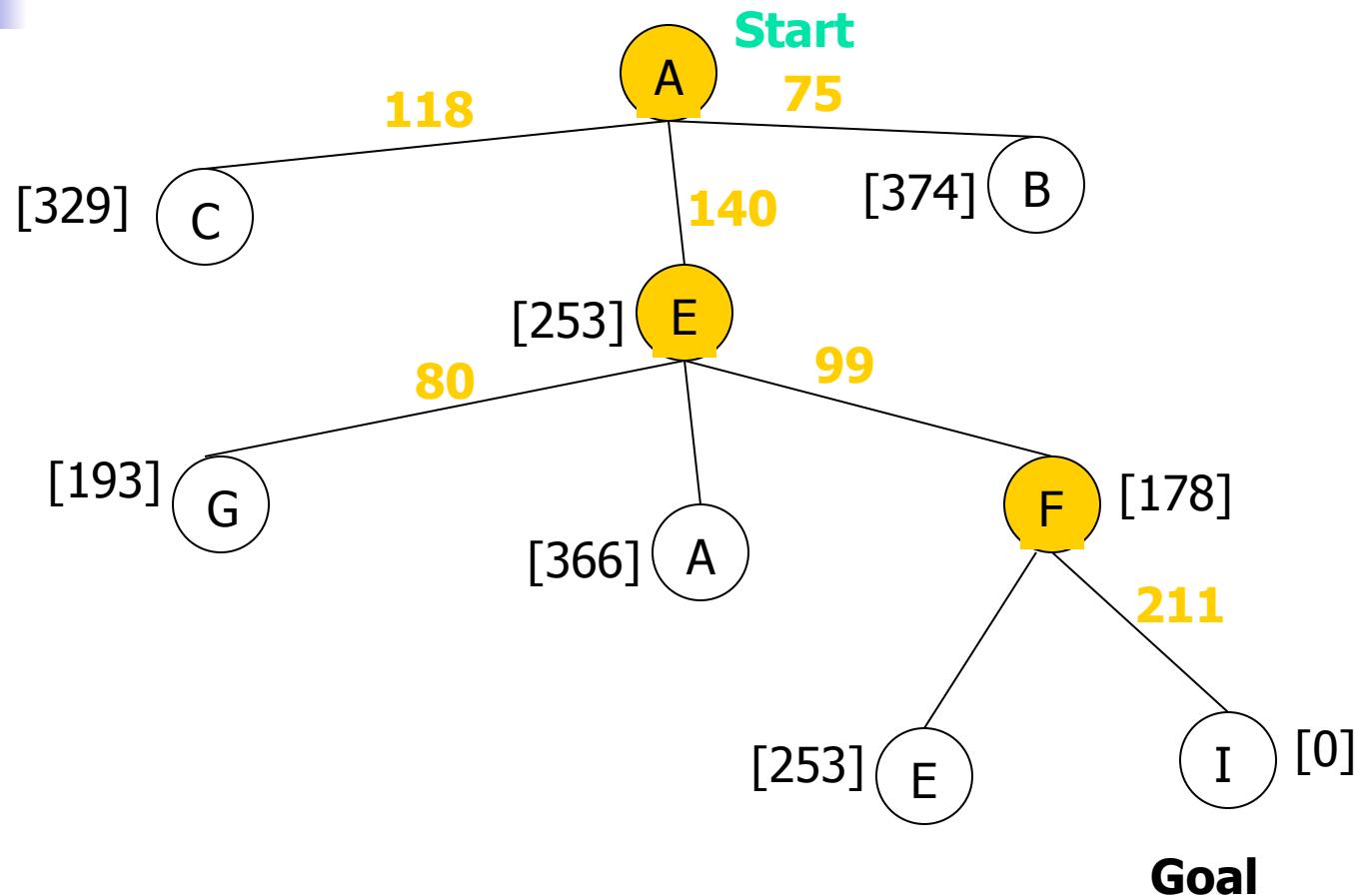
Greedy Search: Tree Search



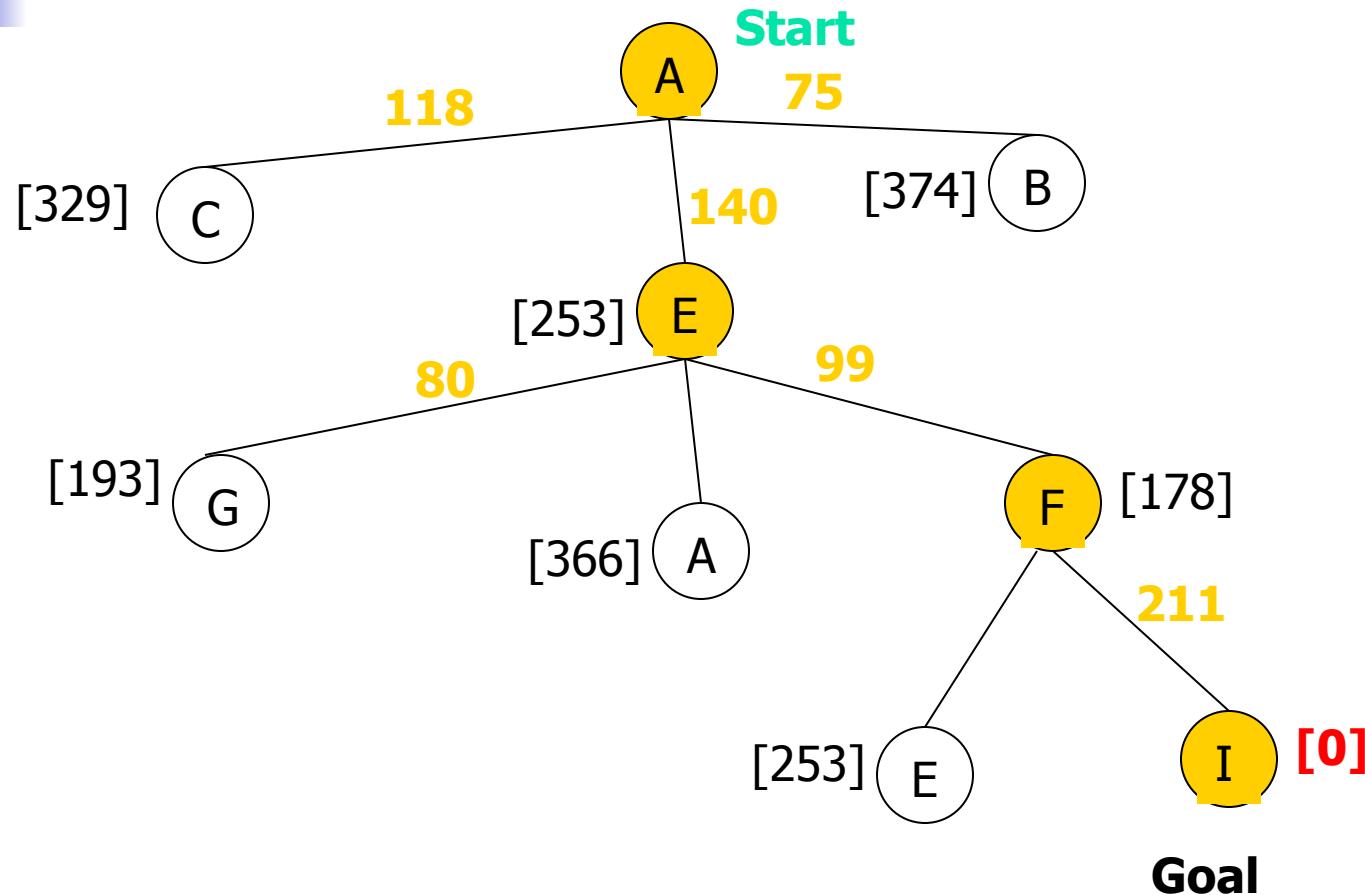
Greedy Search: Tree Search



Greedy Search: Tree Search



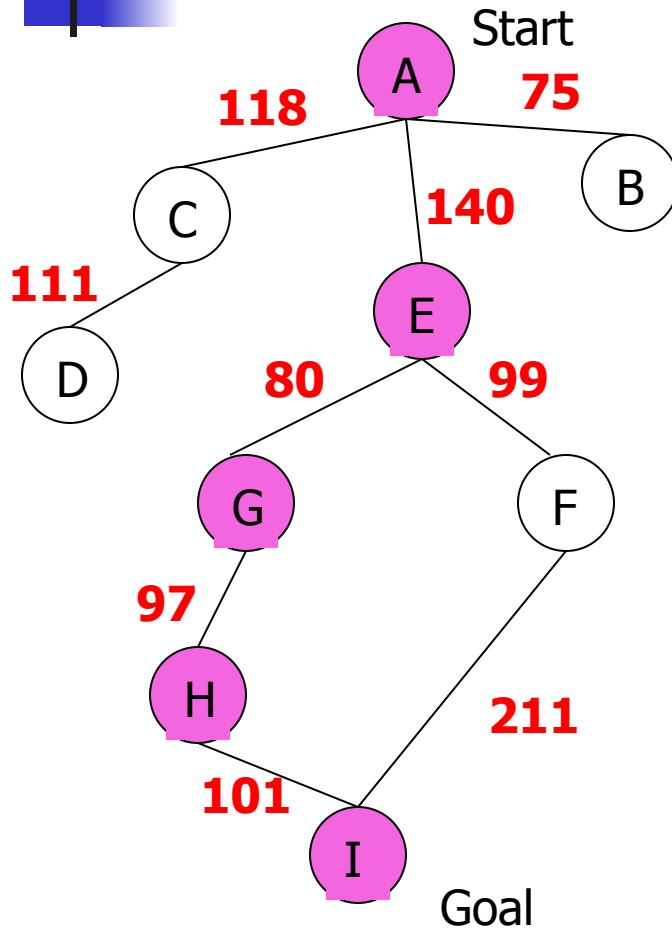
Greedy Search: Tree Search



$$\text{Path cost}(A-E-F-I) = 253 + 178 + 0 = \mathbf{431}$$

$$\text{dist}(A-E-F-I) = 140 + 99 + 211 = \mathbf{450}$$

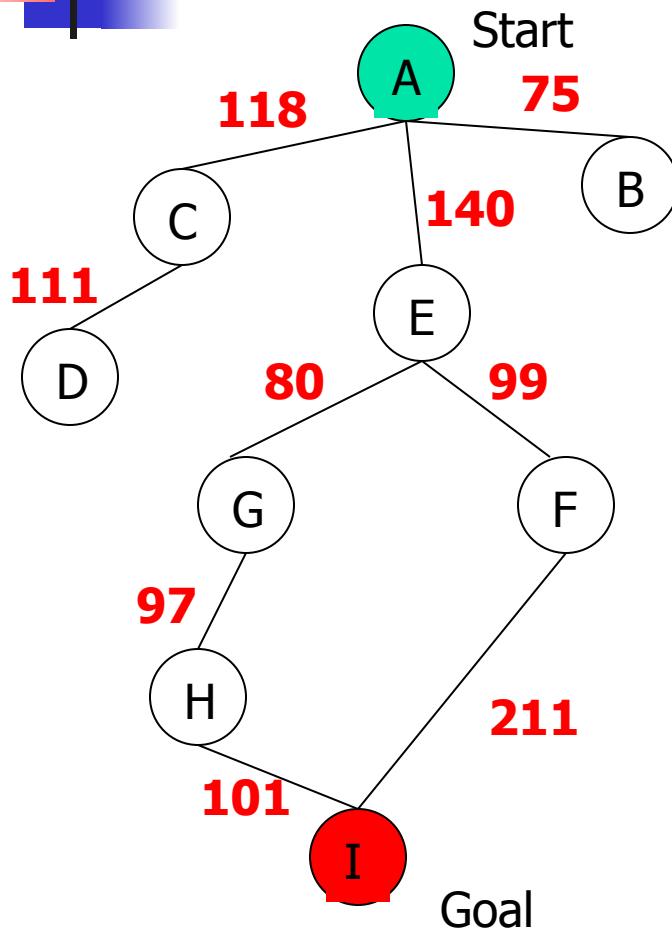
Greedy Search: Optimal ?



| State | Heuristic: $h(n)$ |
|-------|-------------------|
| A | 366 |
| B | 374 |
| C | 329 |
| D | 244 |
| E | 253 |
| F | 178 |
| G | 193 |
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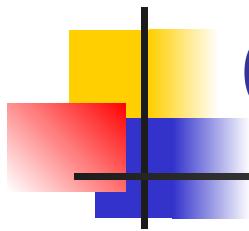
$f(n) = h(n)$ = straight-line distance heuristic
 $\text{dist}(A-E-G-H-I) = 140 + 80 + 97 + 101 = 418$

Greedy Search: Complete ?



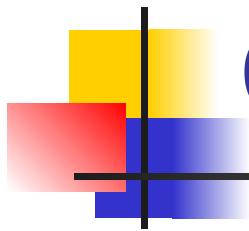
| State | Heuristic: $h(n)$ |
|-------|-------------------|
| A | 366 |
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| ** C | 250 |
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| E | 253 |
| F | 178 |
| G | 193 |
| H | 98 |
| I | 0 |

$f(n) = h(n)$ (n) = straight-line distance heuristic

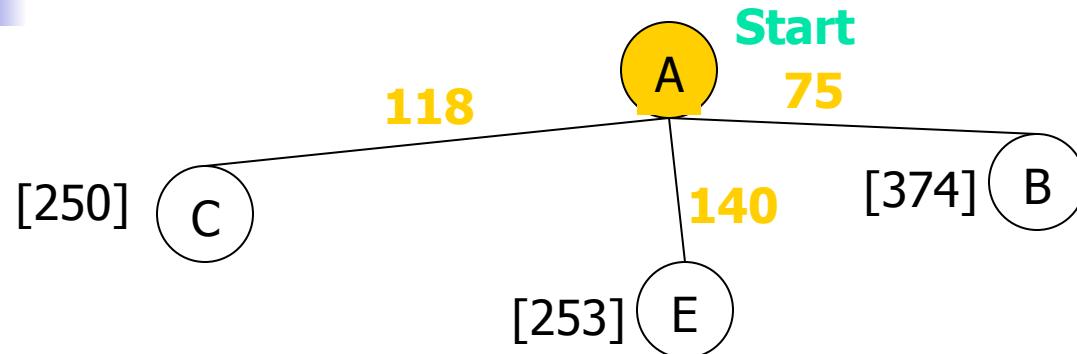


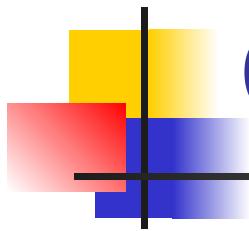
Greedy Search: Tree Search



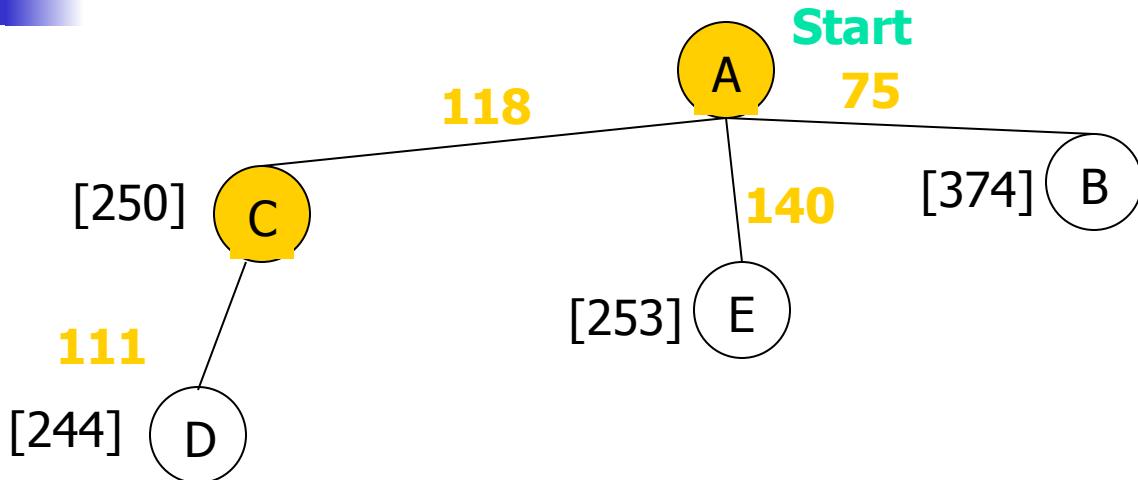


Greedy Search: Tree Search

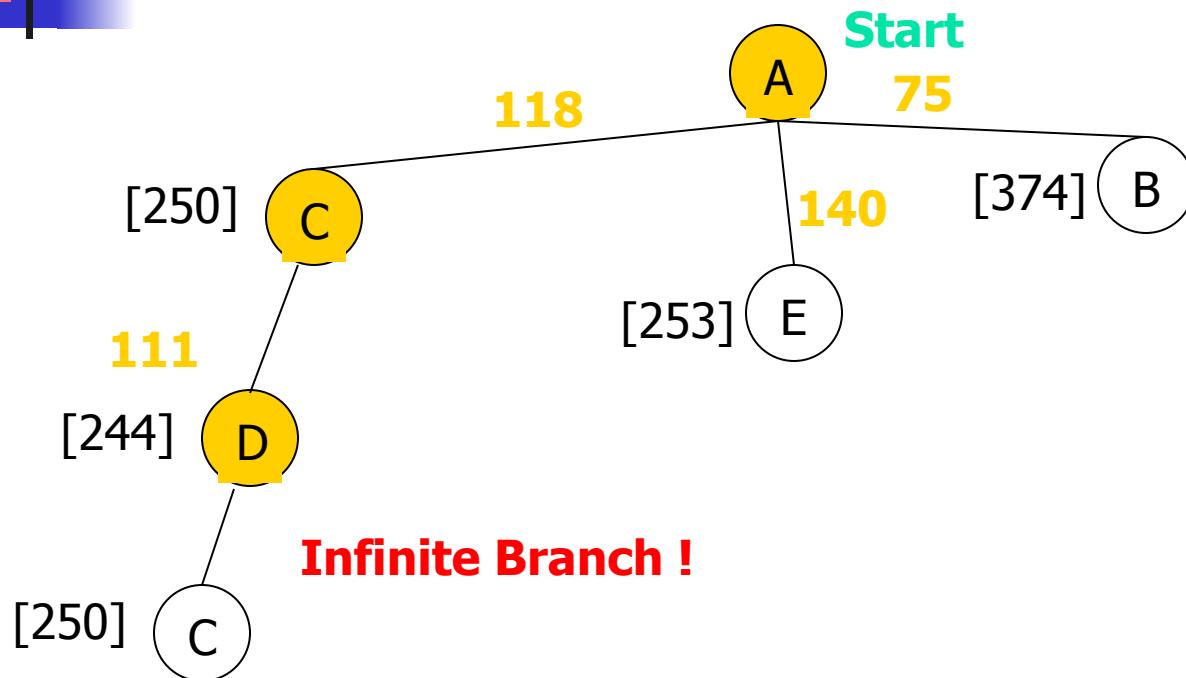


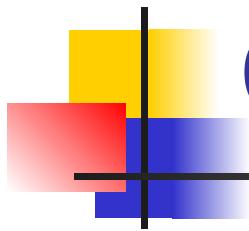


Greedy Search: Tree Search

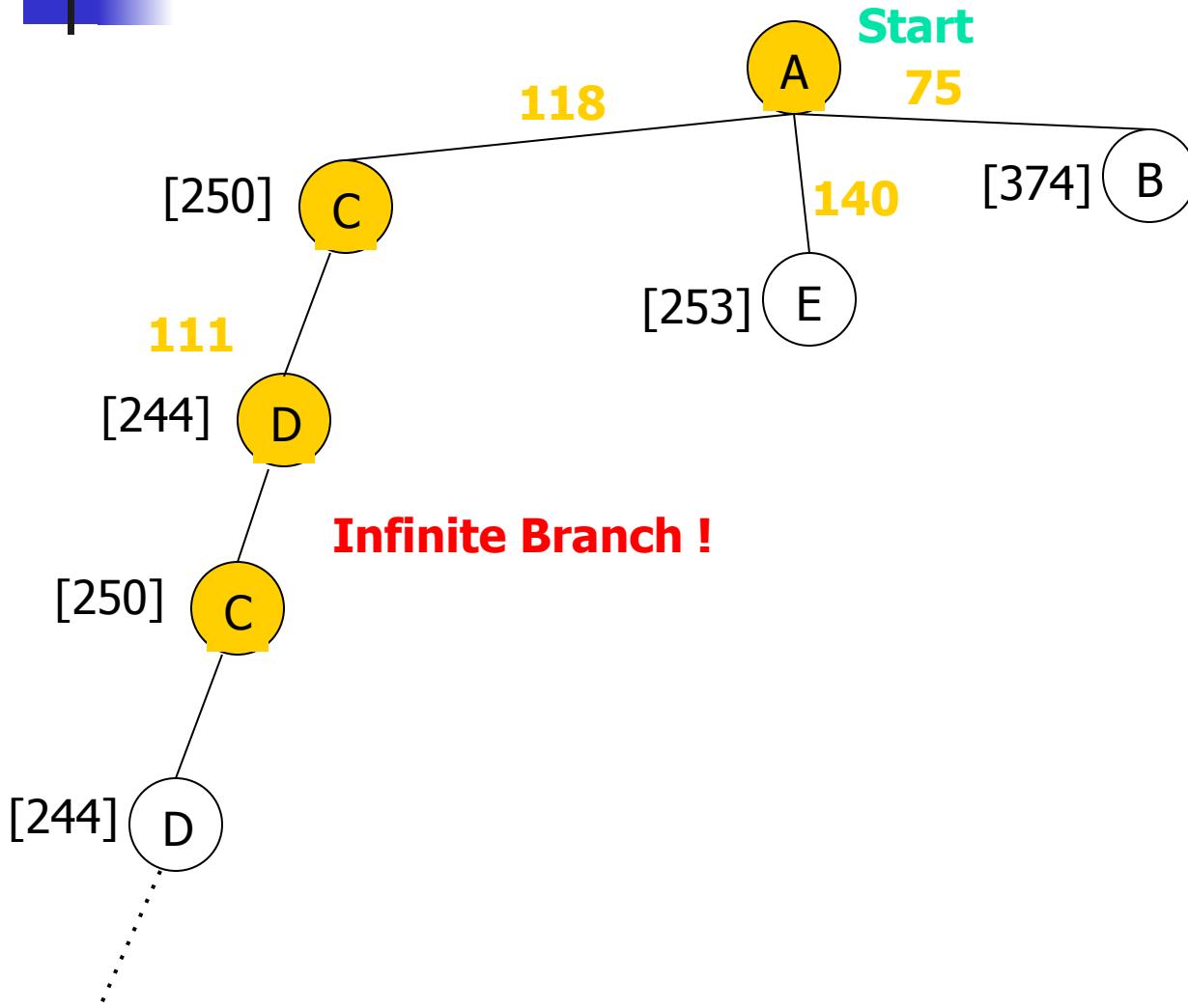


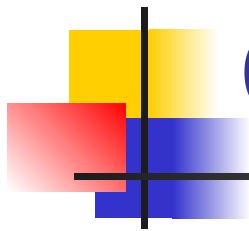
Greedy Search: Tree Search



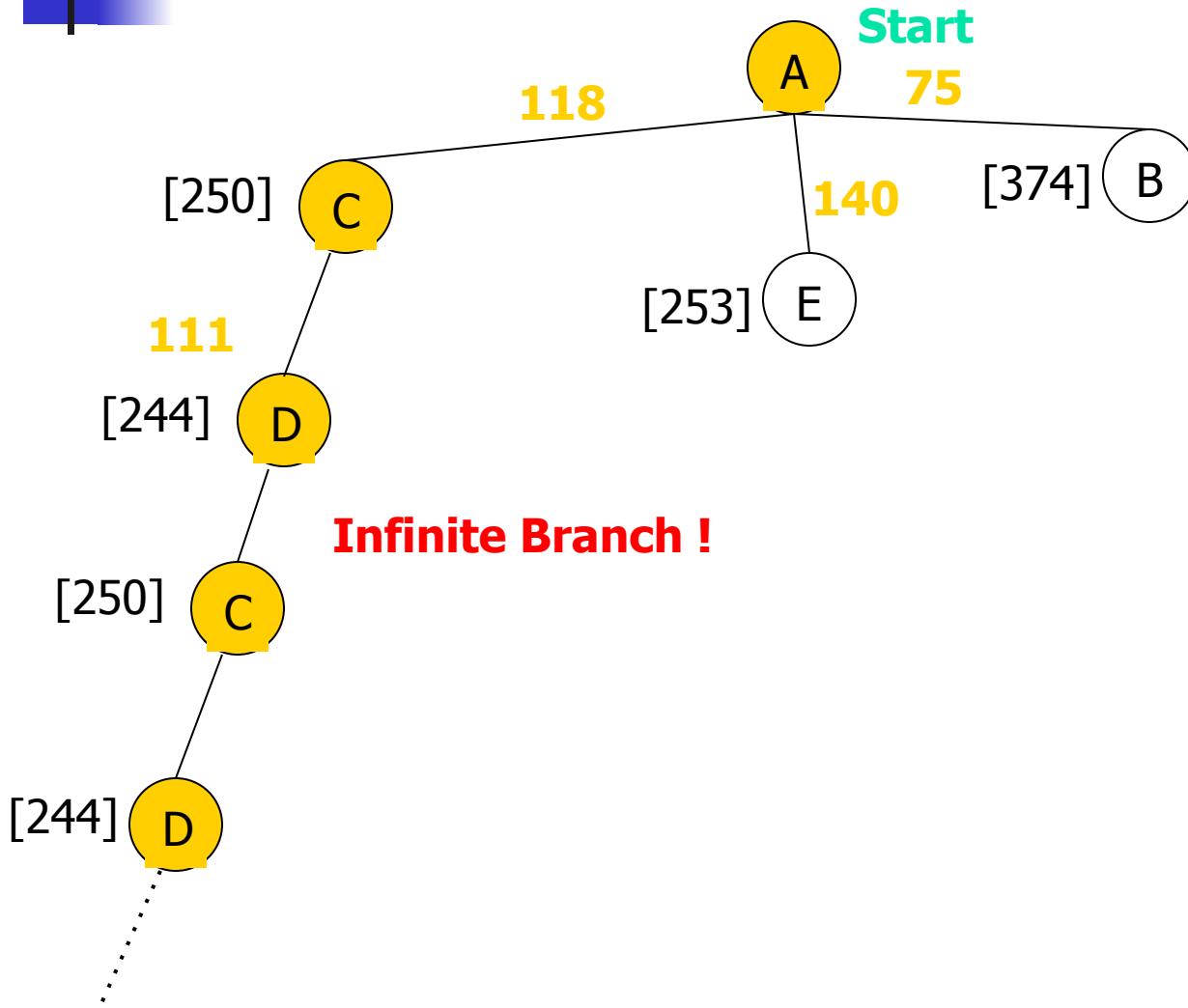


Greedy Search: Tree Search

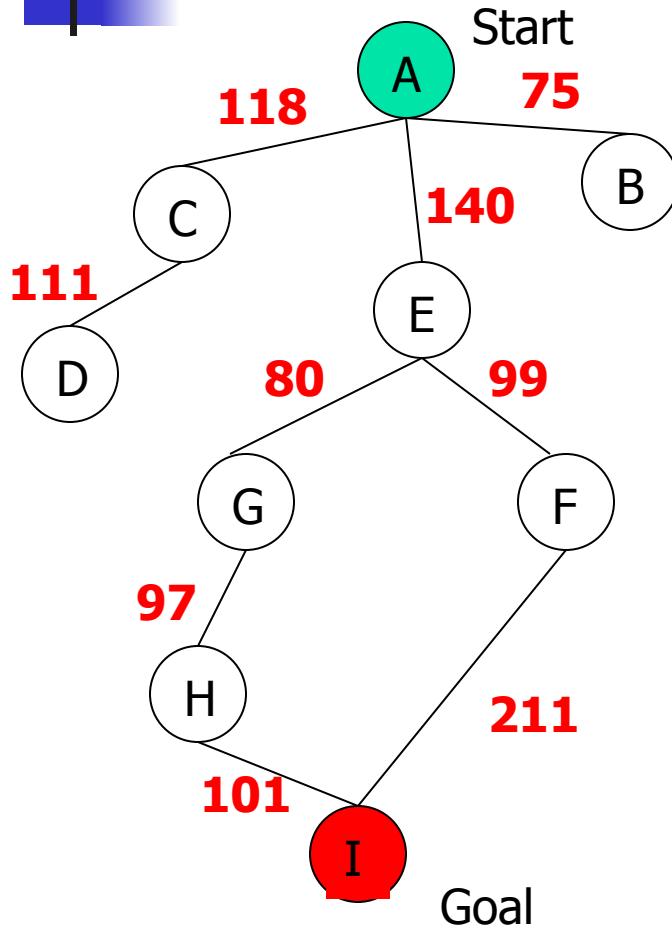




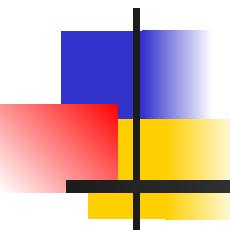
Greedy Search: Tree Search



Greedy Search: Time and Space Complexity ?



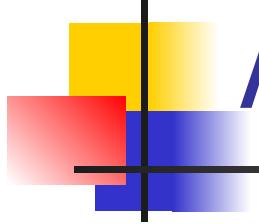
- Greedy search is not optimal.
- Greedy search is incomplete without systematic checking of repeated states.
- In the worst case, the Time and Space Complexity of Greedy Search are both $O(b^m)$ Where b is the branching factor and m the maximum path length



Informed Search Strategies

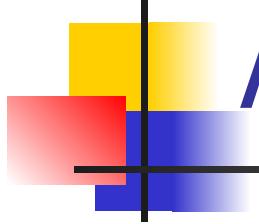
A* Search

eval-fn: $f(n) = g(n) + h(n)$



A* (A Star)

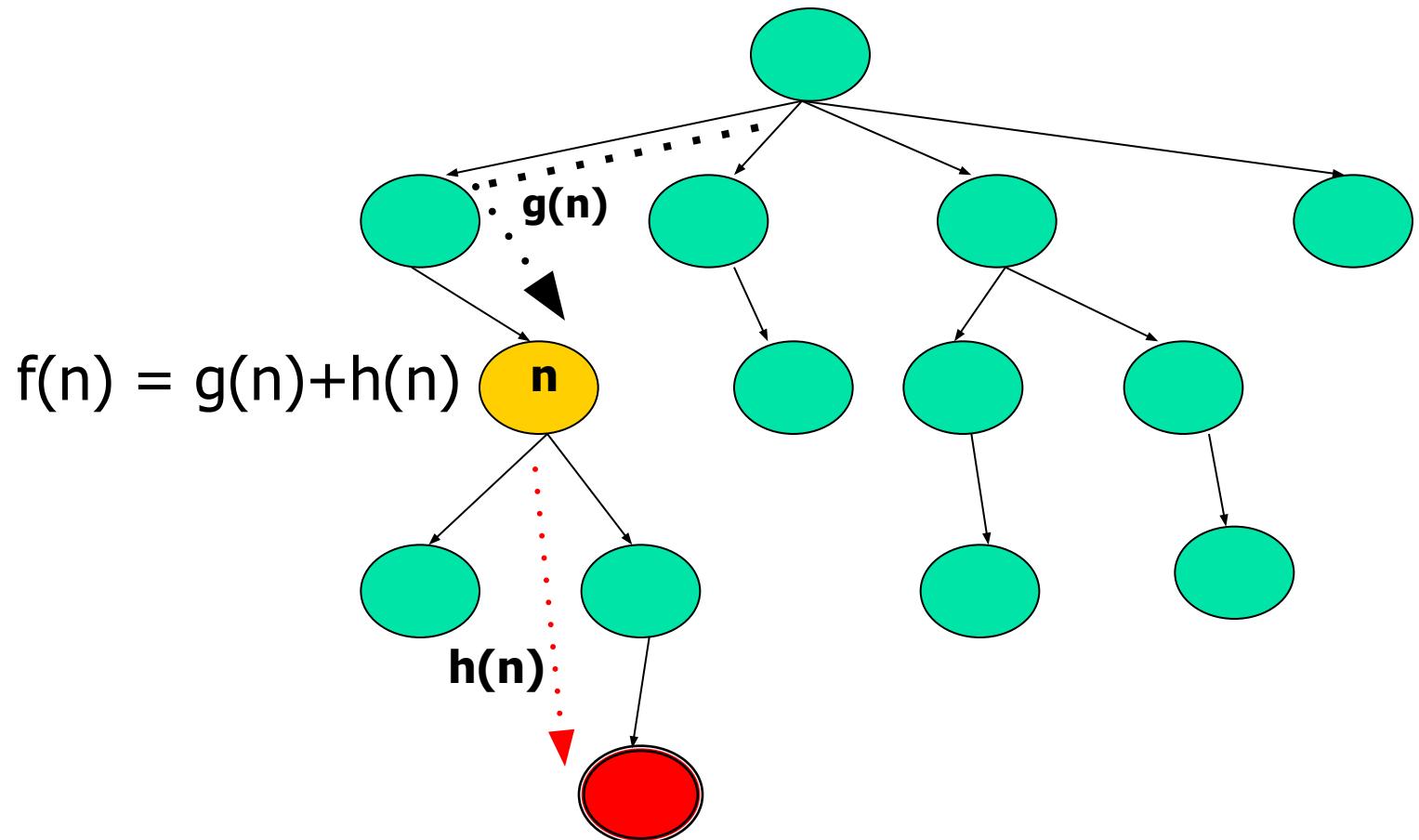
- Greedy Search minimizes a heuristic $h(n)$ which is an estimated cost from a node n to the goal state. Greedy Search is efficient but it is not optimal nor complete.
- Uniform Cost Search minimizes the cost $g(n)$ from the initial state to n . UCS is optimal and complete but not efficient.
- **New Strategy:** Combine Greedy Search and UCS to get an **efficient algorithm** which is **complete and optimal**.



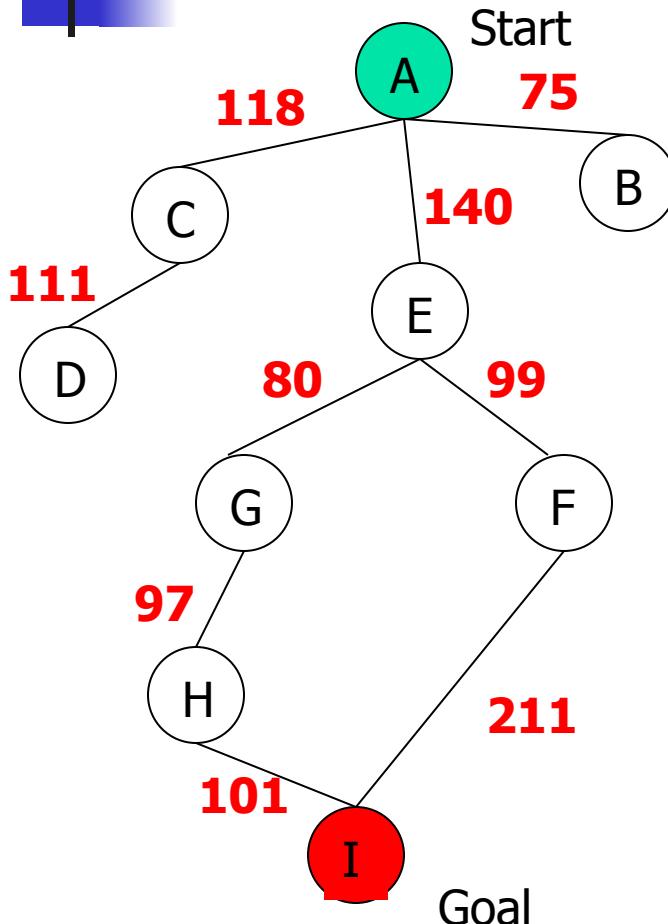
A* (A Star)

- A* uses an evaluation function which combines $g(n)$ and $h(n)$: $f(n) = g(n) + h(n)$
- $g(n)$ is the exact cost to reach node n from the initial state.
- $h(n)$ is an estimation of the remaining cost to reach the goal.

A* (A Star)



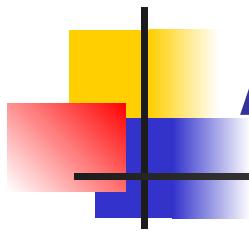
A* Search



| State | Heuristic: h(n) |
|-------|-----------------|
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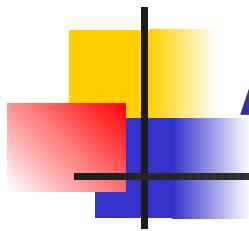
$$f(n) = g(n) + h(n)$$

$g(n)$ is the exact cost to reach node n from the initial state

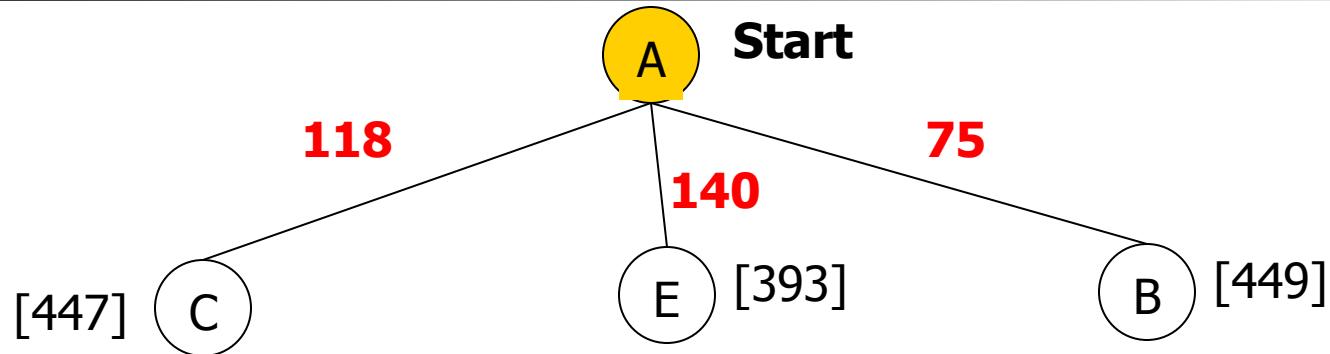


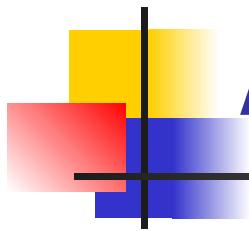
A* Search: Tree Search

(A) Start

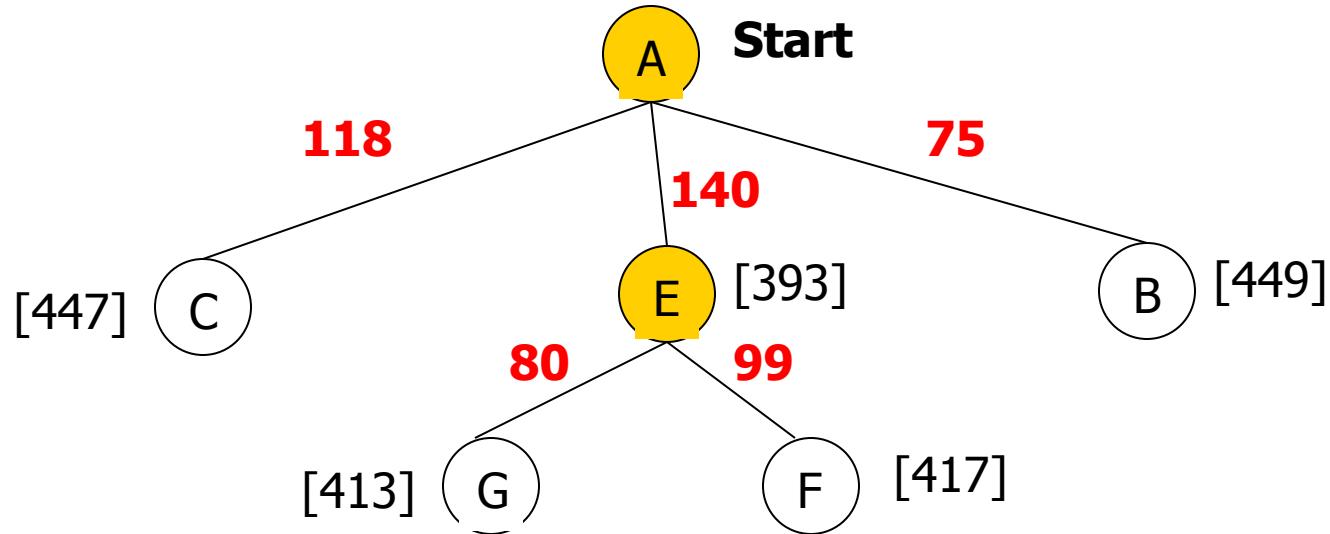


A* Search: Tree Search

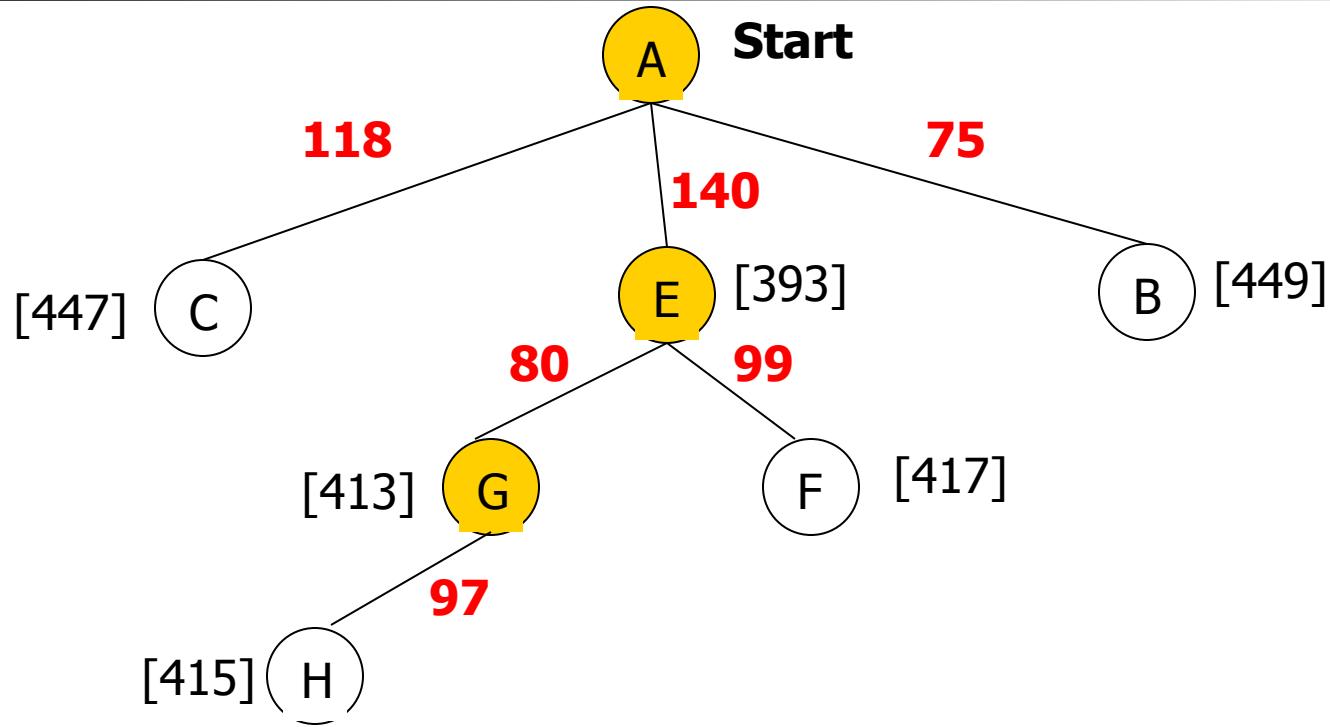




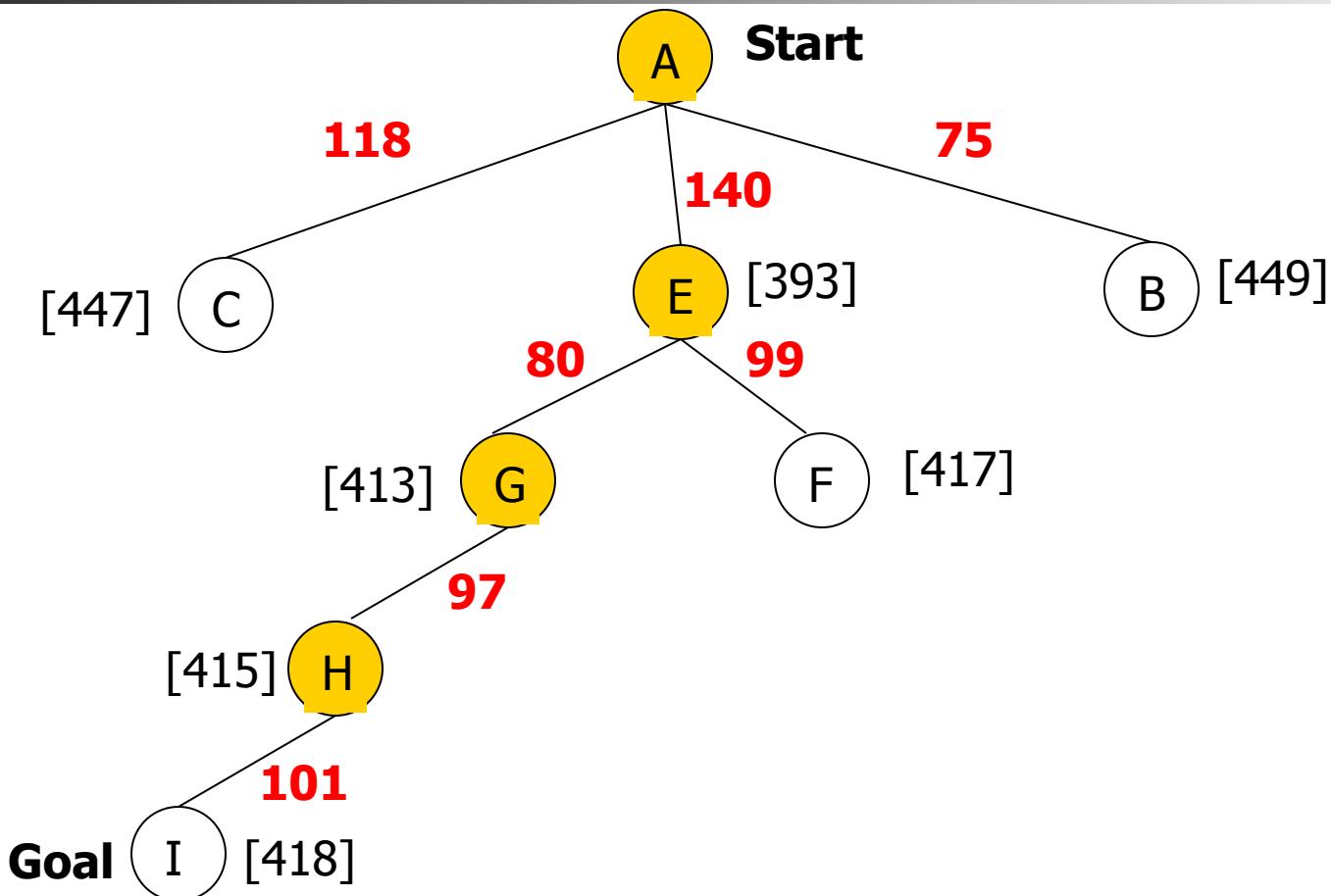
A* Search: Tree Search



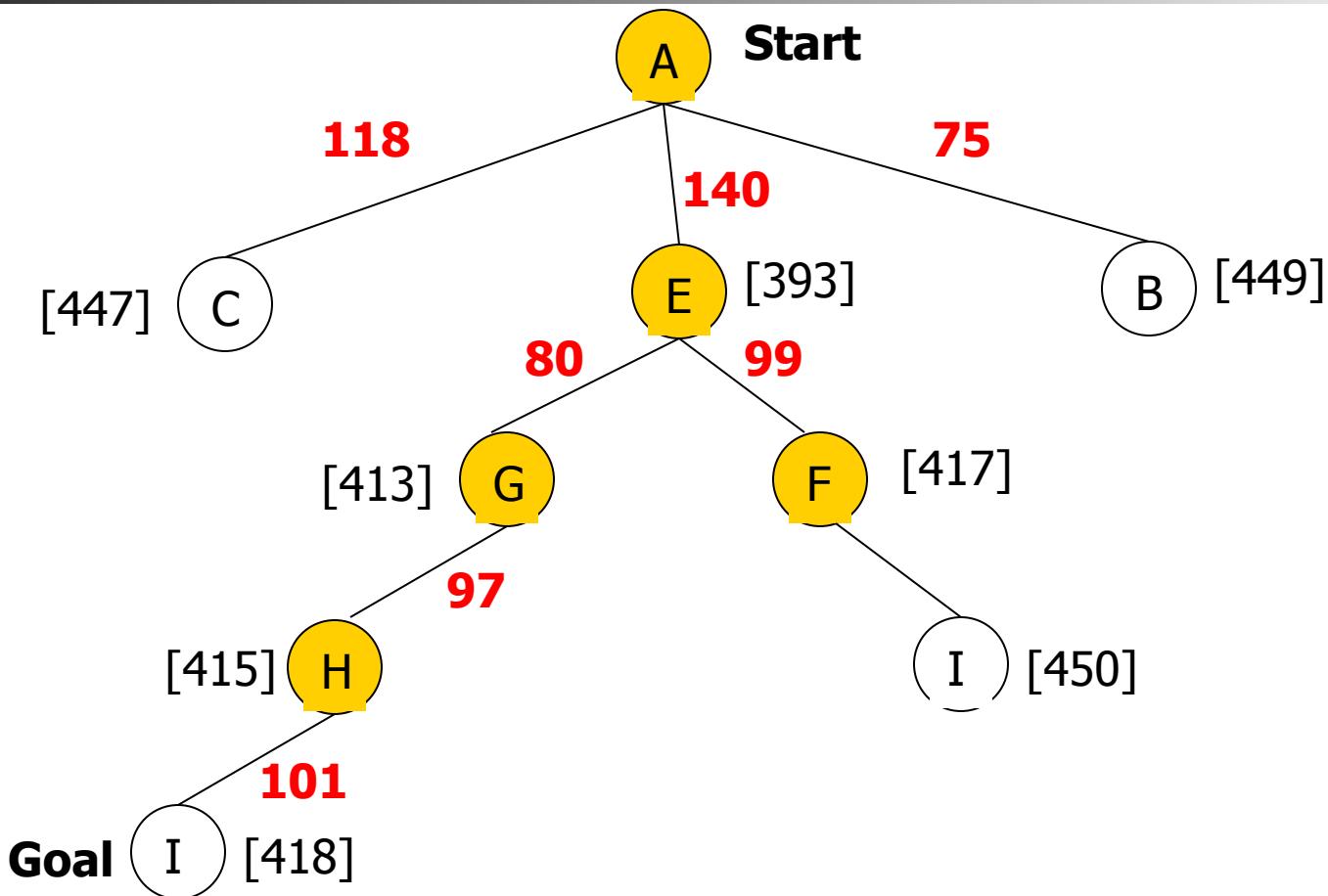
A* Search: Tree Search



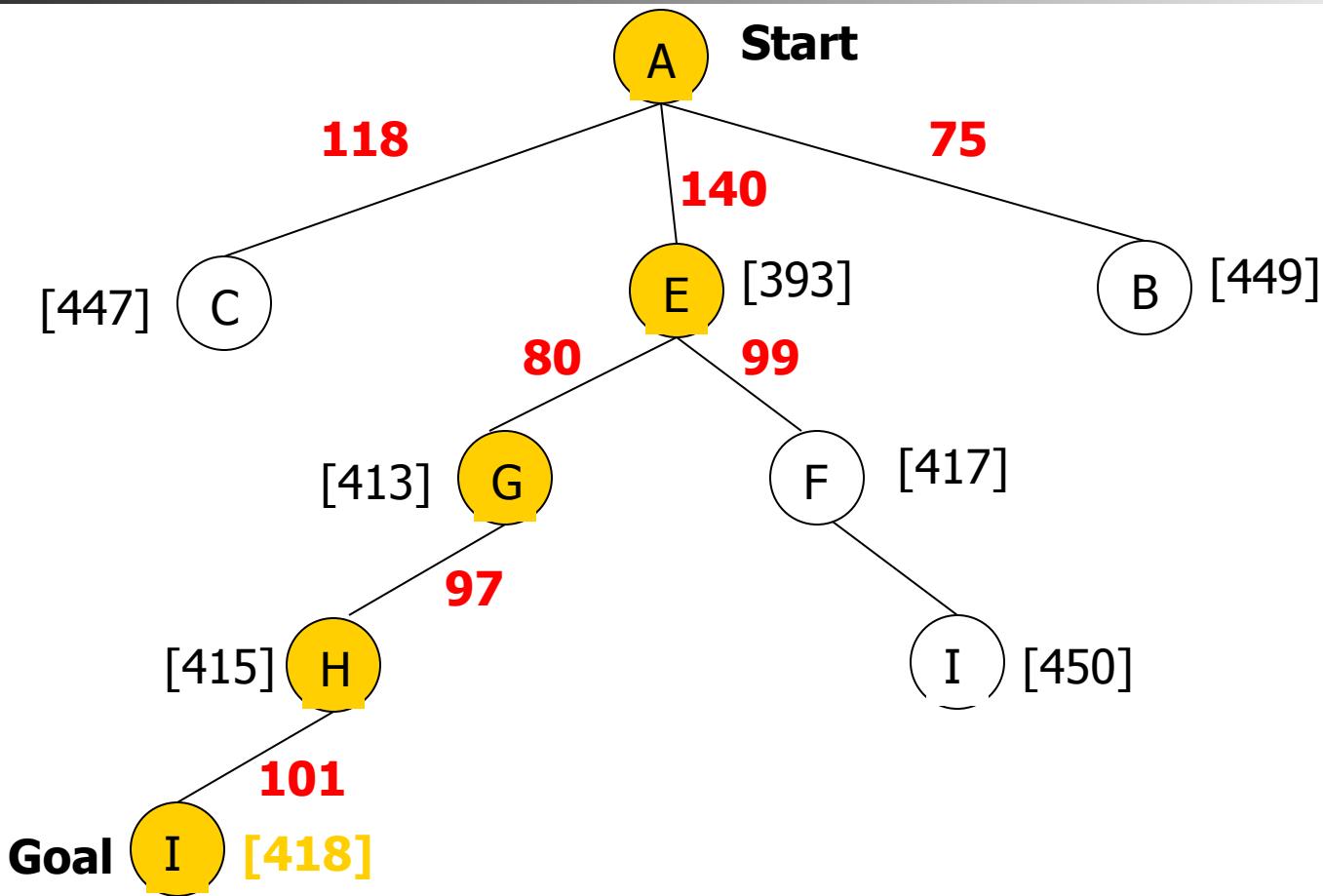
A* Search: Tree Search



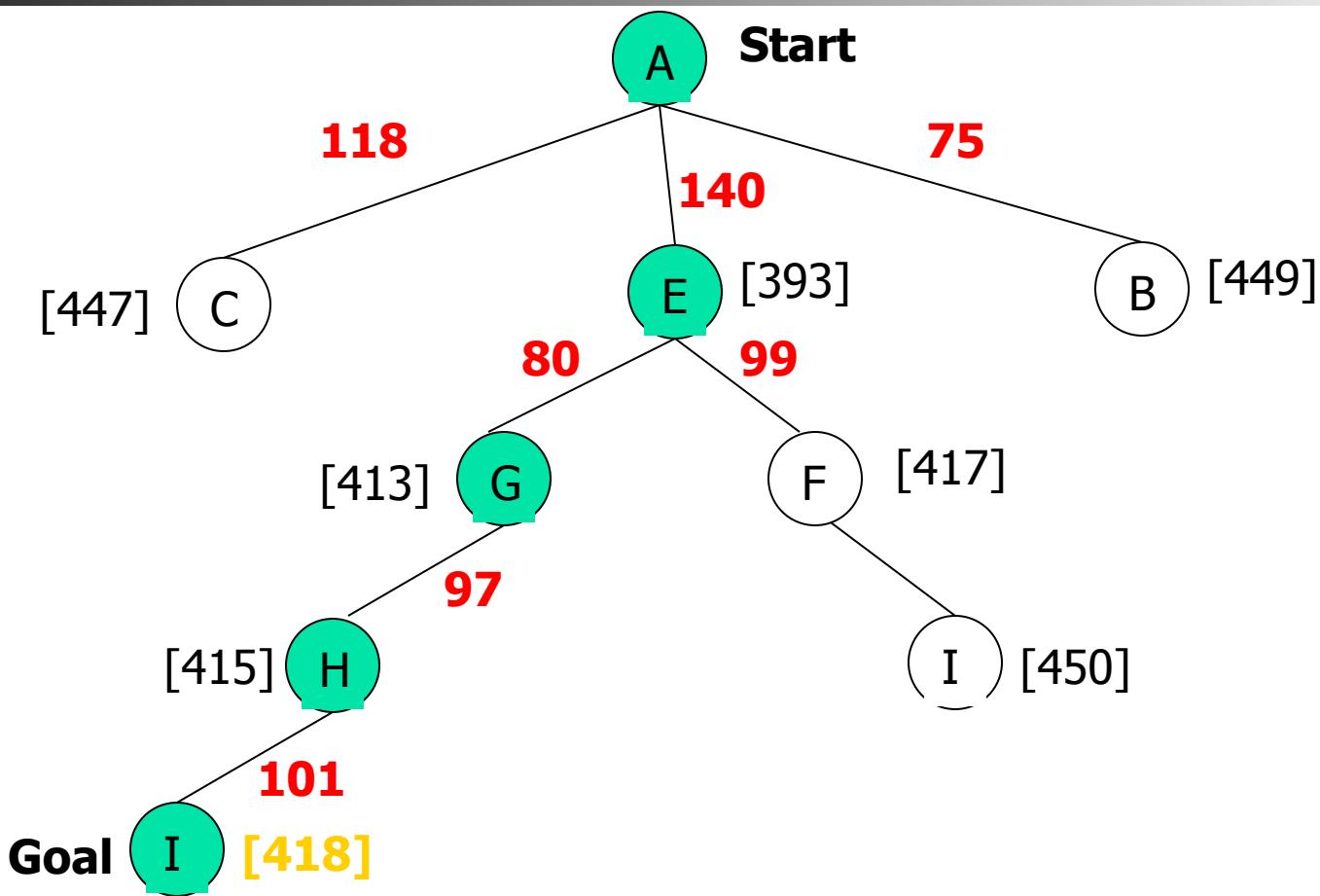
A* Search: Tree Search

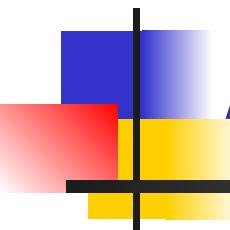


A* Search: Tree Search



A* Search: Tree Search

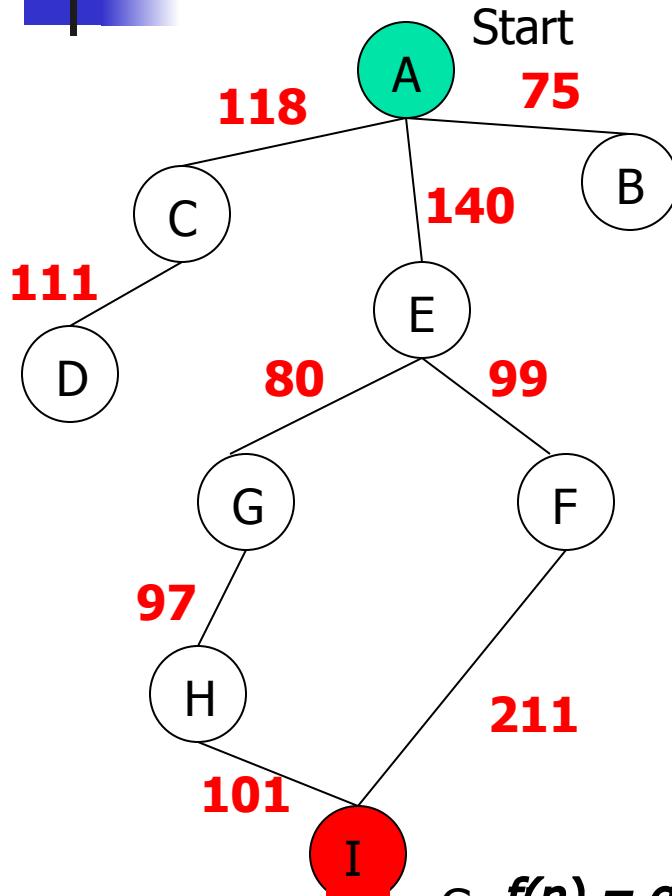




A* with $f()$ not Admissible

$h()$ overestimates the cost to reach the goal state

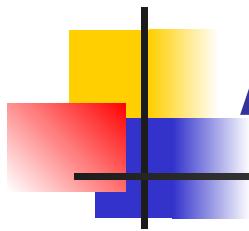
A* Search: h not admissible !



| State | Heuristic: $h(n)$ |
|----------|-------------------|
| A | 366 |
| B | 374 |
| C | 329 |
| D | 244 |
| E | 253 |
| F | 178 |
| G | 193 |
| H | 138 |
| I | 0 |

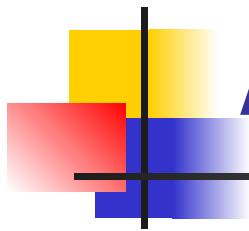
$$f(n) = g(n) + h(n) \quad (n) - (\text{H-I}) \text{ Overestimated}$$

g(n): is the exact cost to reach node n from the initial state. 52

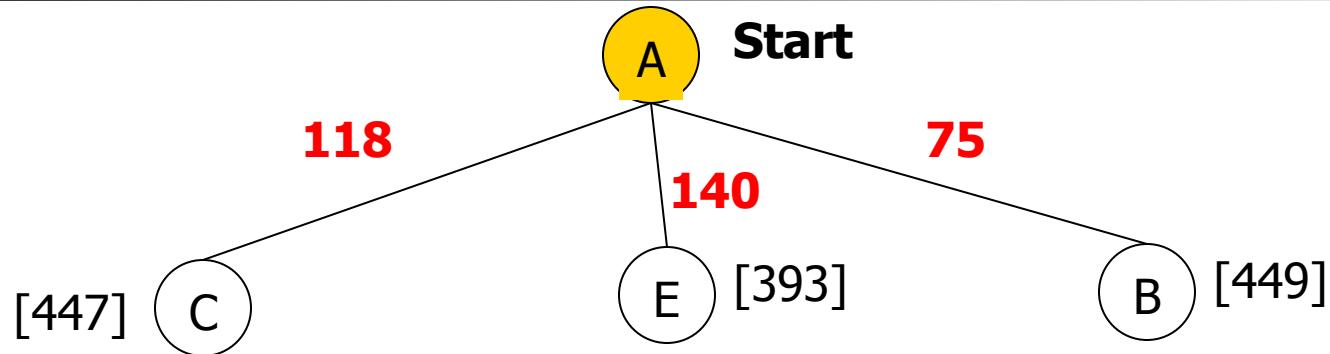


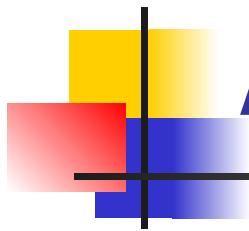
A* Search: Tree Search

(A) Start

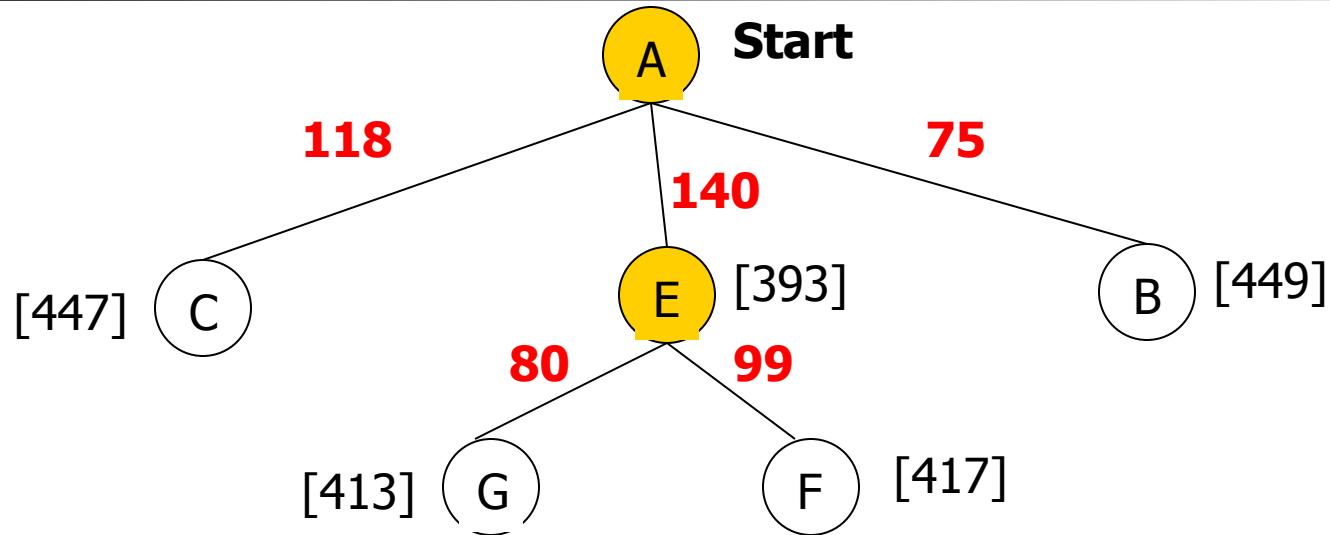


A* Search: Tree Search

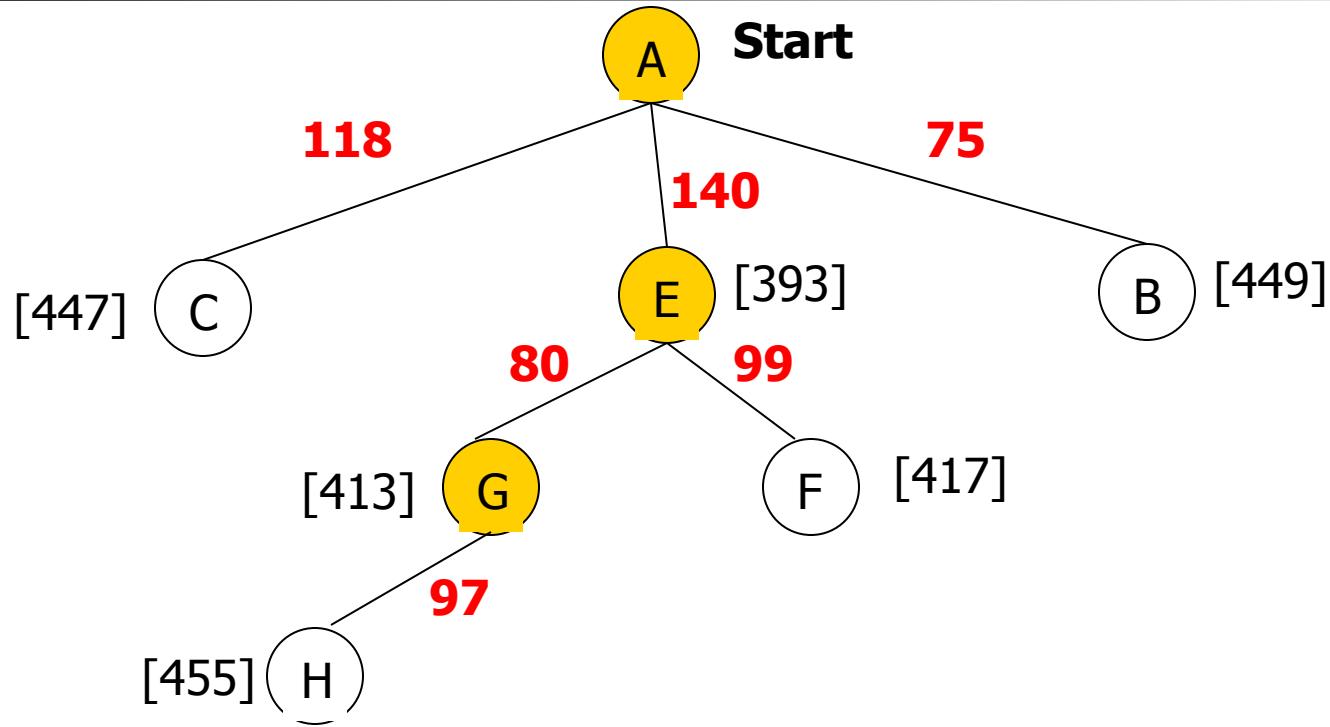




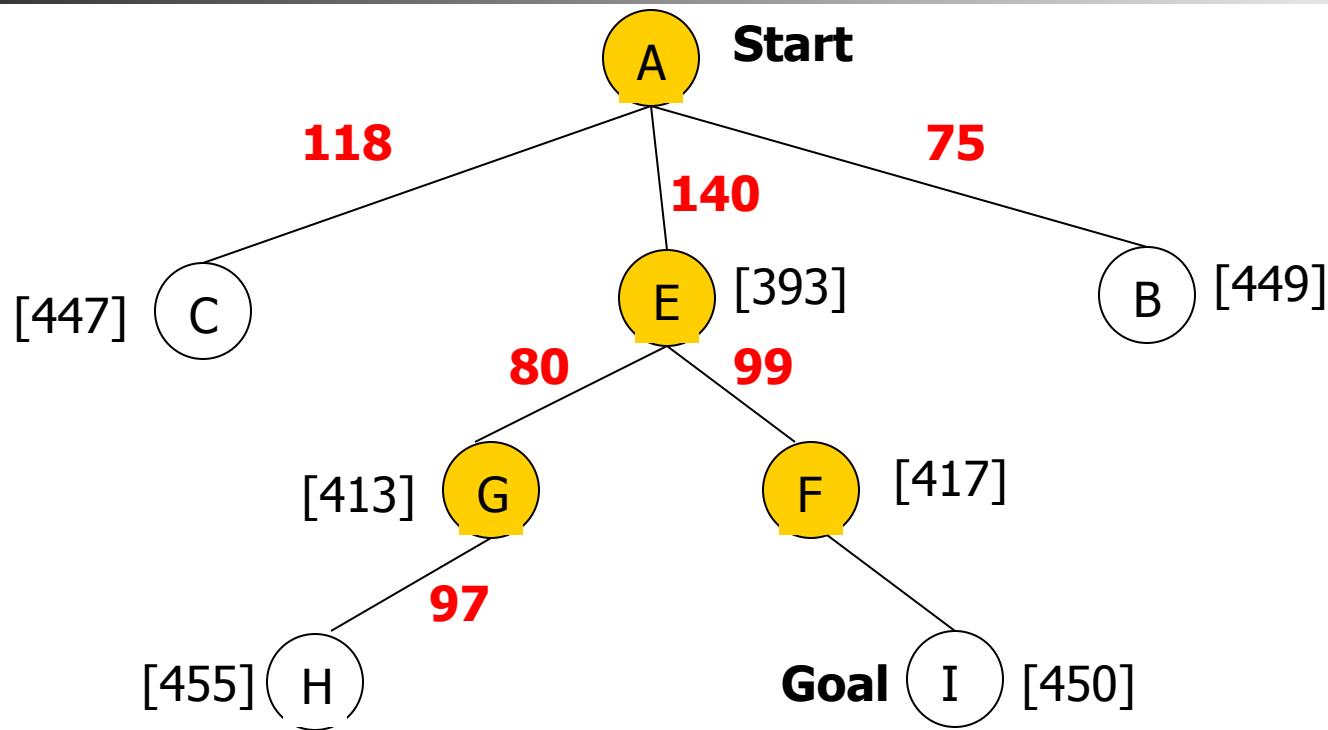
A* Search: Tree Search



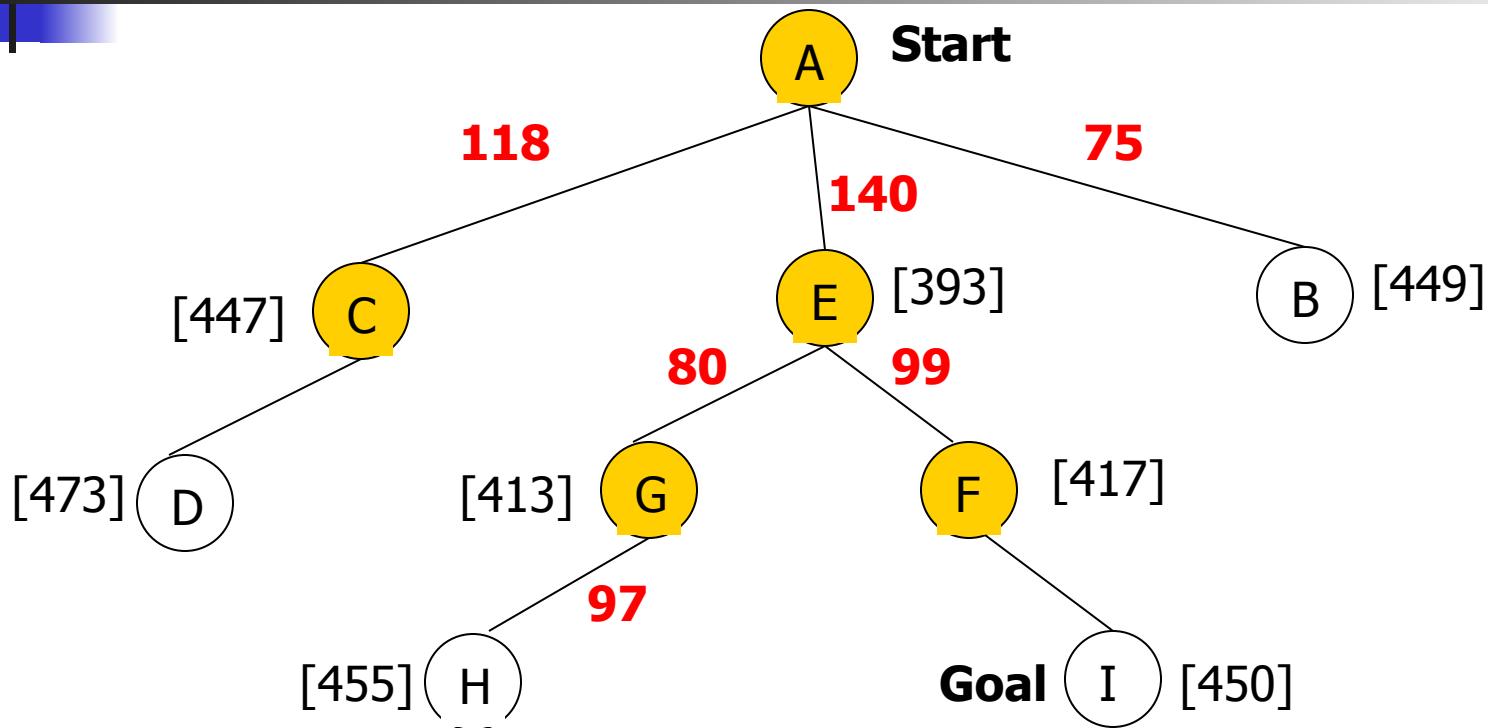
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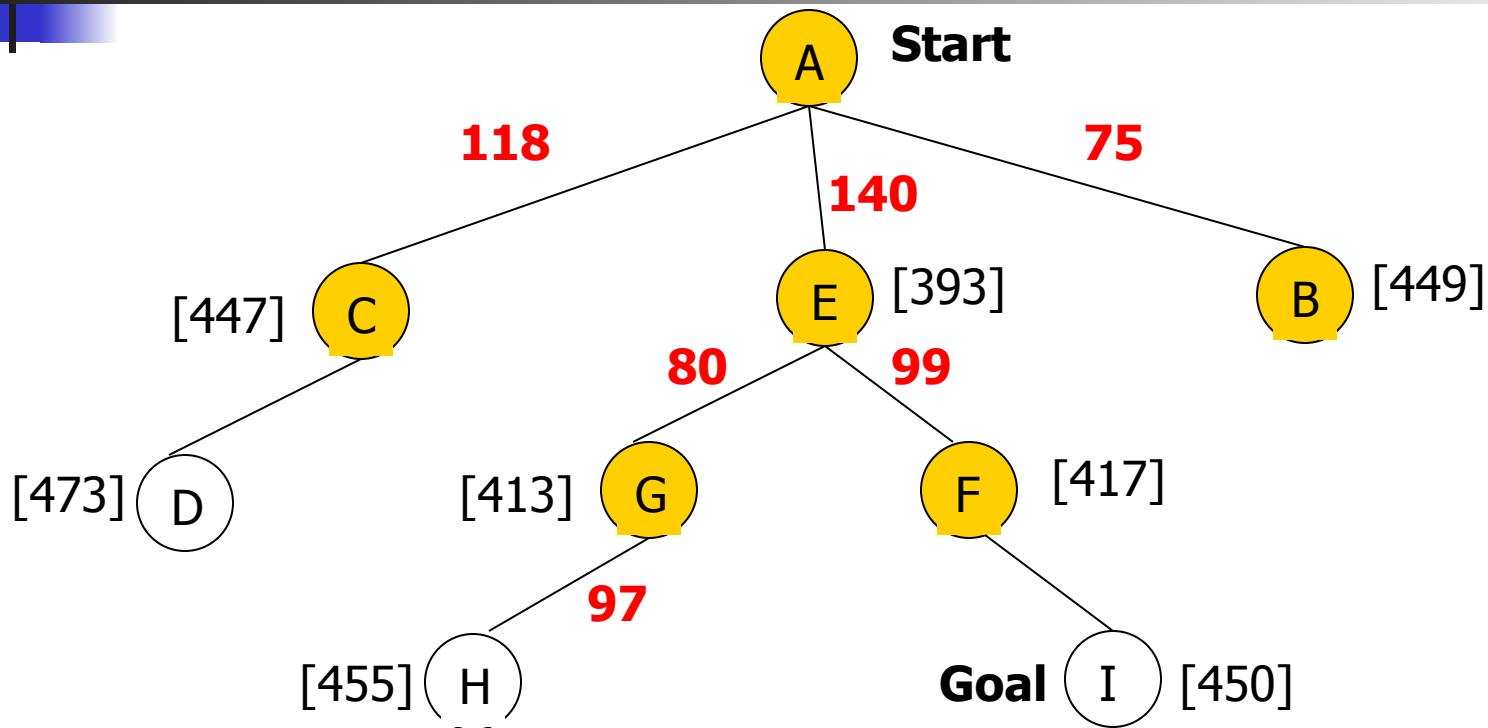
A* Search: Tree Search



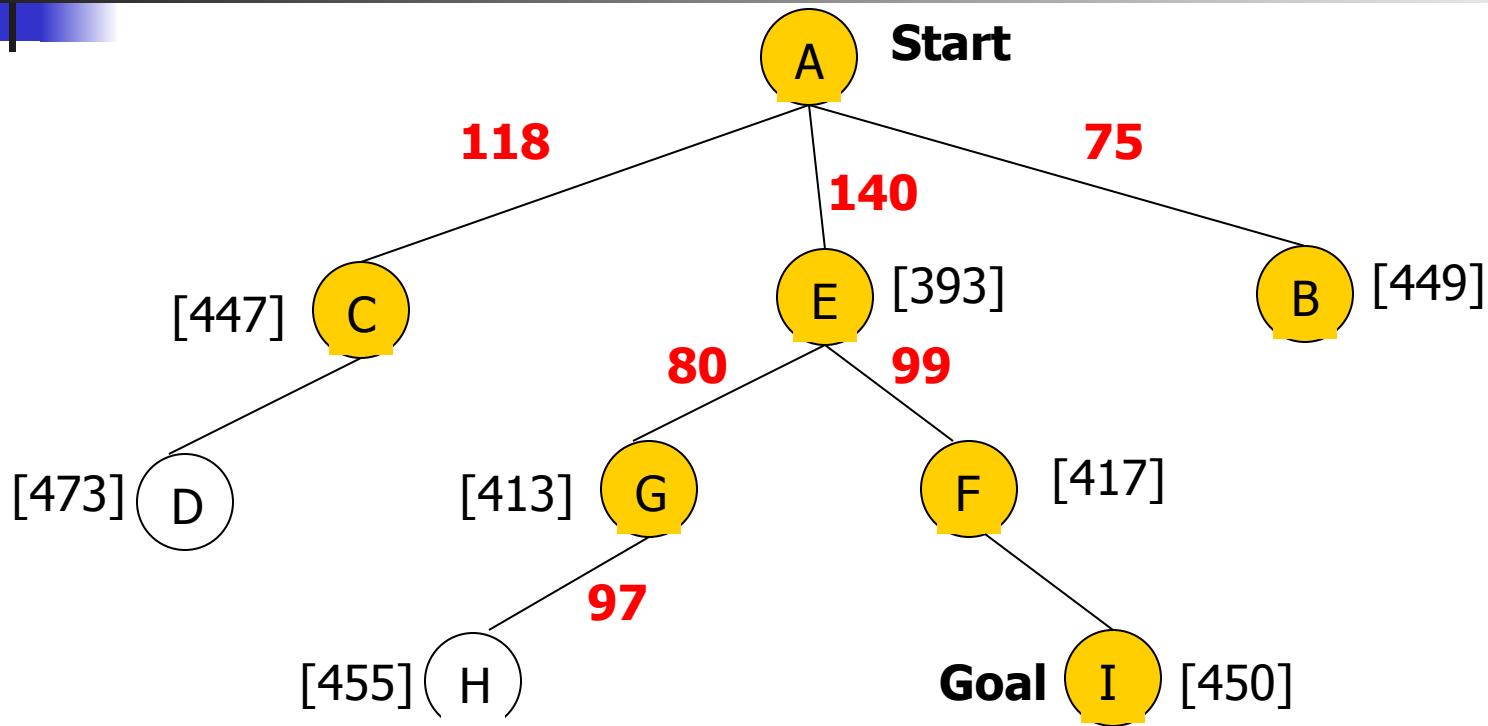
A* Search: Tree Search



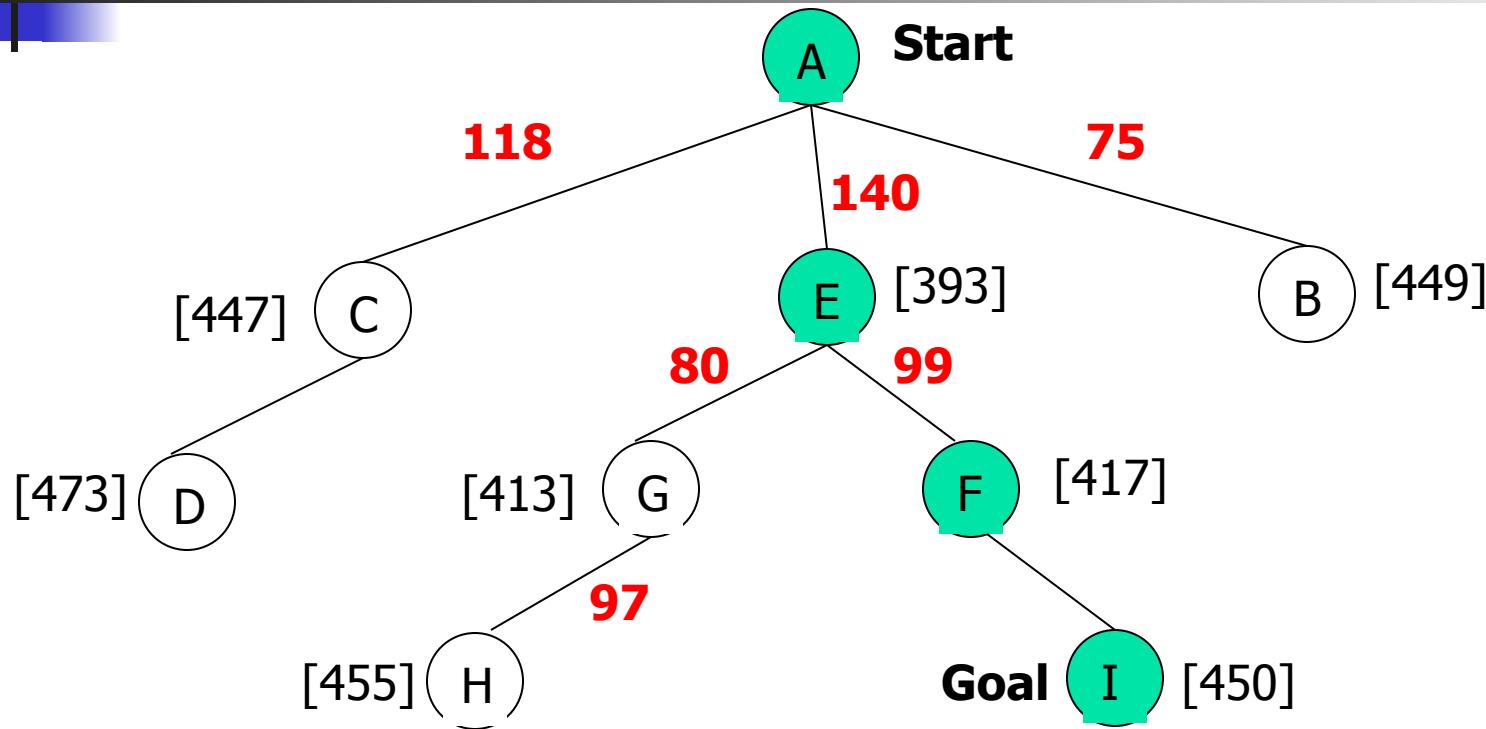
A* Search: Tree Search



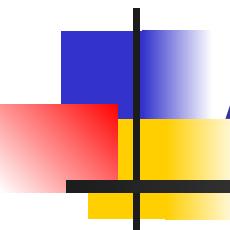
A* Search: Tree Search



A* Search: Tree Search

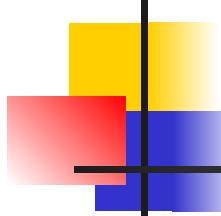


A* not optimal !!!



A* Algorithm

A* with systematic checking for
repeated states ...

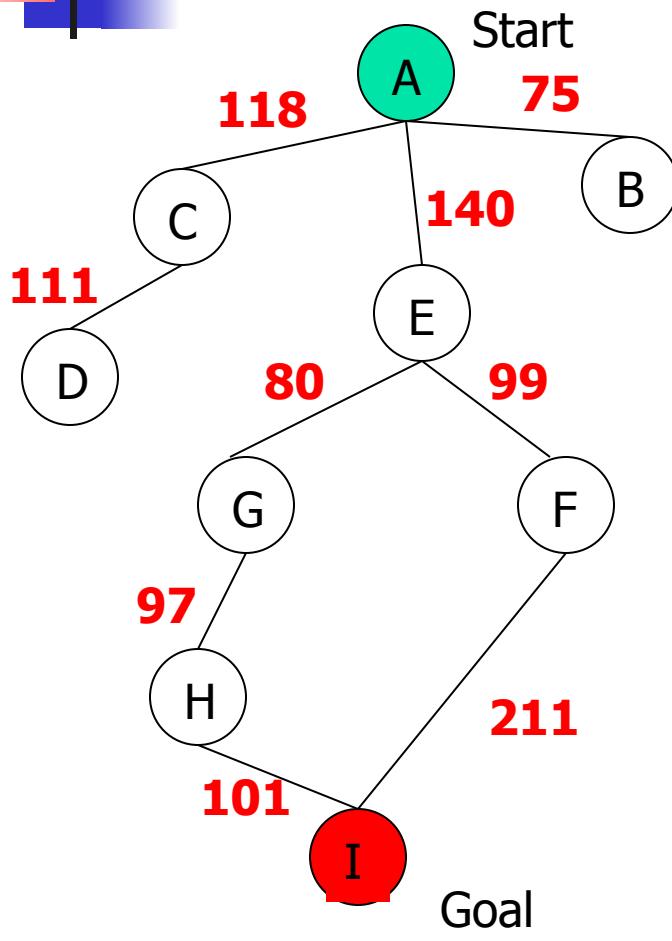


A* Algorithm

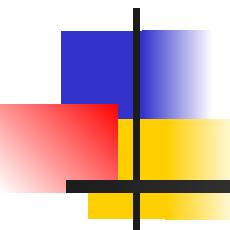
1. Search queue Q is empty.
2. Place the start state s in Q with f value $h(s)$.
3. If Q is empty, return failure.
4. Take node n from Q with lowest f value.
(Keep Q sorted by f values and pick the first element).
5. If n is a goal node, stop and return solution.
6. Generate successors of node n.
7. For each successor n' of n do:
 - a) Compute $f(n') = g(n) + \text{cost}(n,n') + h(n')$.
 - b) If n' is new (never generated before), add n' to Q.
 - c) If node n' is already in Q with a higher f value, replace it with current $f(n')$ and place it in sorted order in Q.

End for
8. Go back to step 3.

A* Search: Analysis

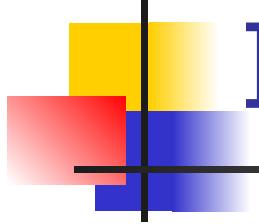


- A* is complete except if there is an infinity of nodes with $f < f(G)$.
- A* is optimal if heuristic h is admissible.
- Time complexity depends on the quality of heuristic but is still exponential.
- For space complexity, A* keeps all nodes in memory. A* has worst case $O(b^d)$ space complexity, but an iterative deepening version is possible (IDA*).



Informed Search Strategies

Iterative Deepening A*



Iterative Deepening A*:IDA*

- Use $f(N) = g(N) + h(N)$ with admissible and consistent h
- Each iteration is depth-first with cutoff on the value of f of expanded nodes

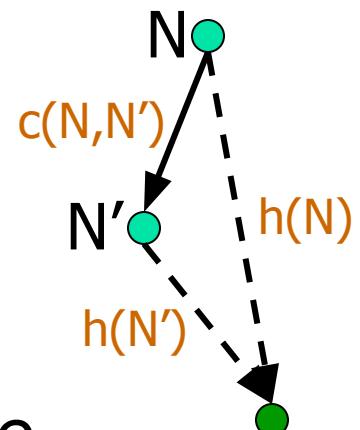
Consistent Heuristic

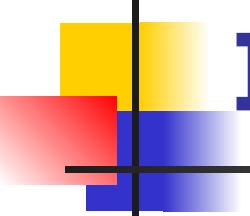
- The admissible heuristic h is **consistent** (or satisfies the **monotone restriction**) if for every node N and every successor N' of N :

$$h(N) \leq c(N, N') + h(N')$$

(triangular inequality)

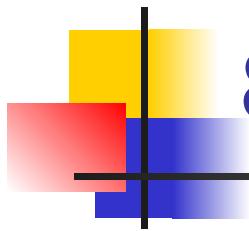
- A consistent heuristic is admissible.





IDA* Algorithm

- In the first iteration, we determine a "**f-cost limit**" – **cut-off value** $f(n_0) = g(n_0) + h(n_0) = h(n_0)$, where n_0 is the start node.
- We expand nodes using the **depth-first algorithm** and backtrack whenever $f(n)$ for an expanded node n exceeds the cut-off value.
- If this search does not succeed, determine the **lowest f-value** among the nodes that were visited but not expanded.
- Use this f-value as the **new limit value** – **cut-off value** and do another depth-first search.
- Repeat this procedure until a goal node is found.

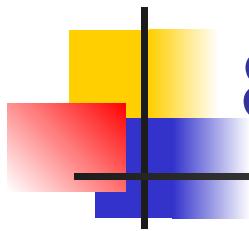


8-Puzzle

$$f(N) = g(N) + h(N)$$

with $h(N)$ = number of misplaced tiles

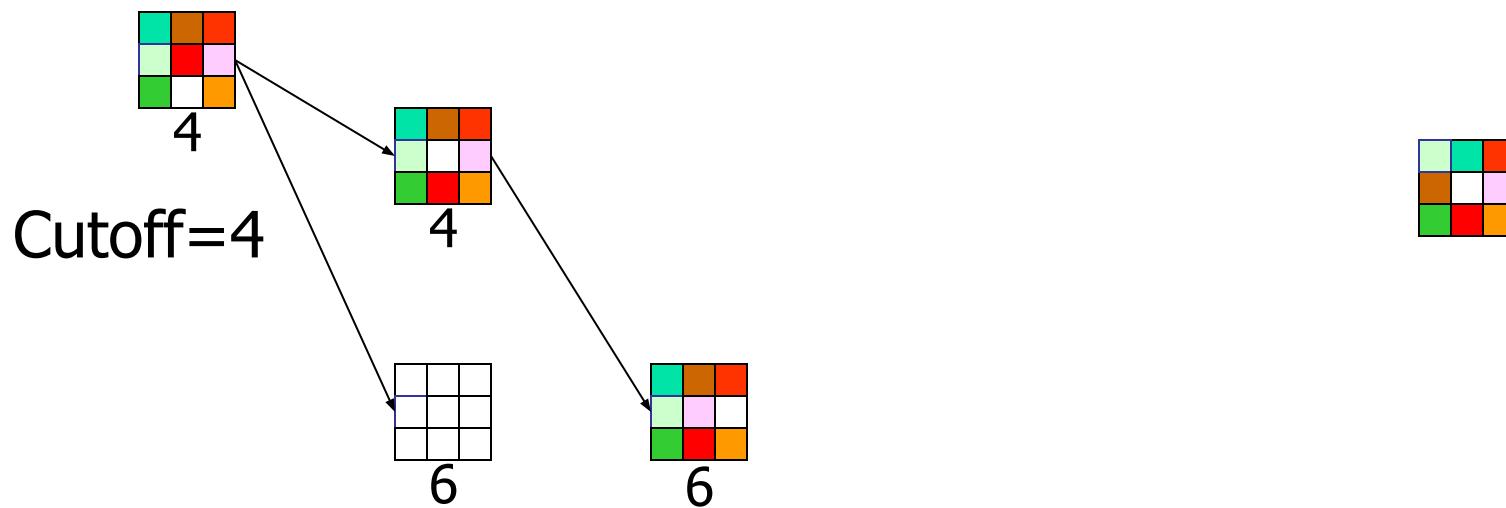


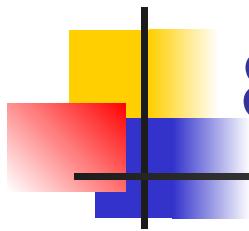


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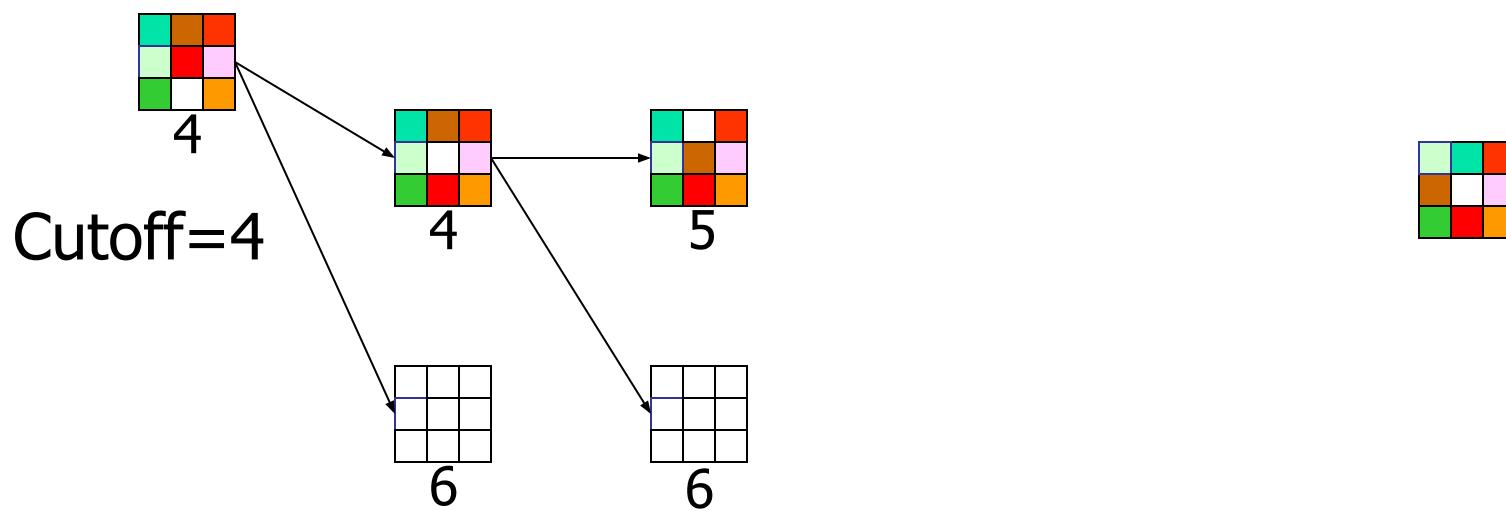


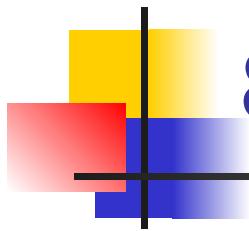


8-Puzzle

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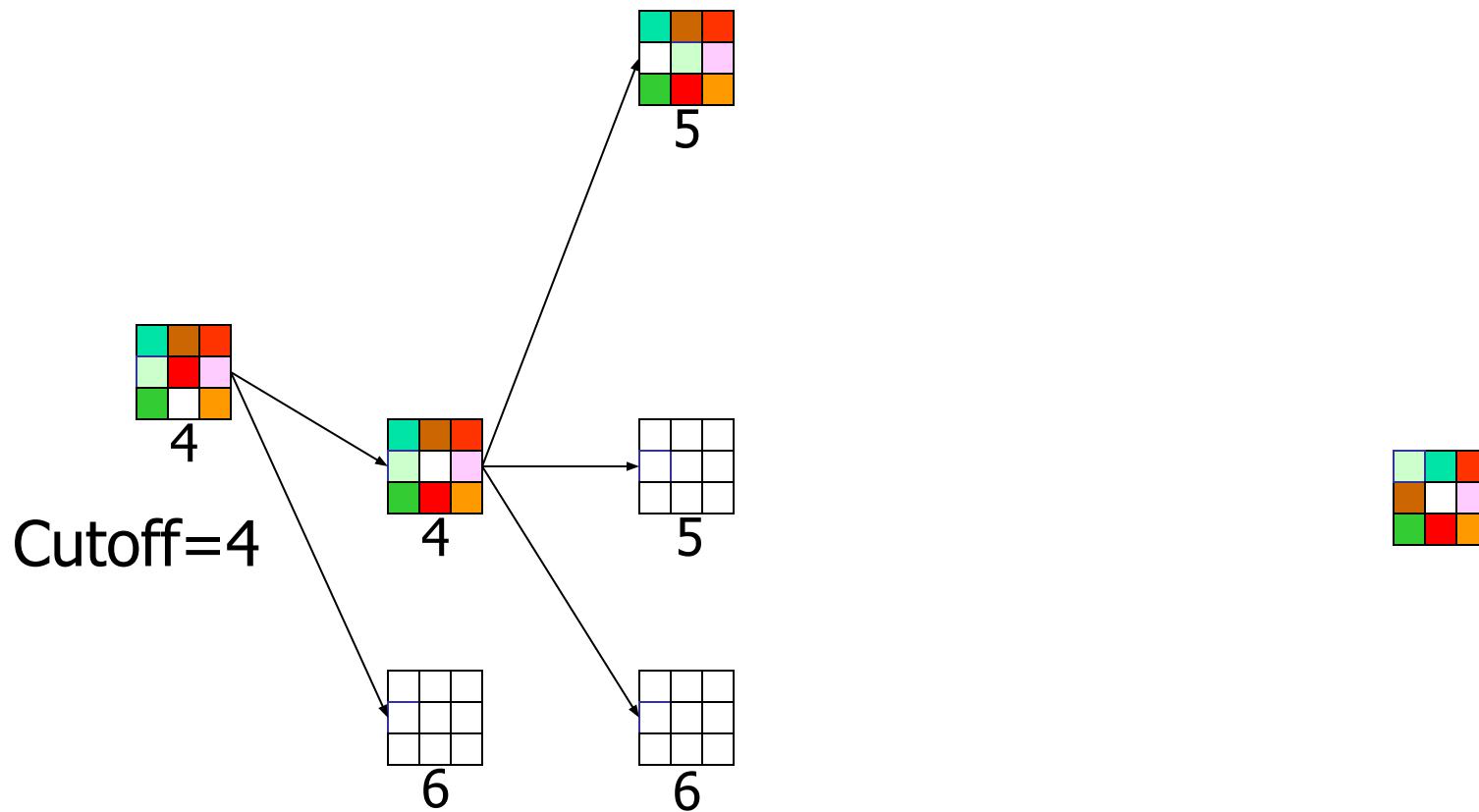


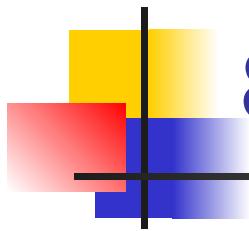


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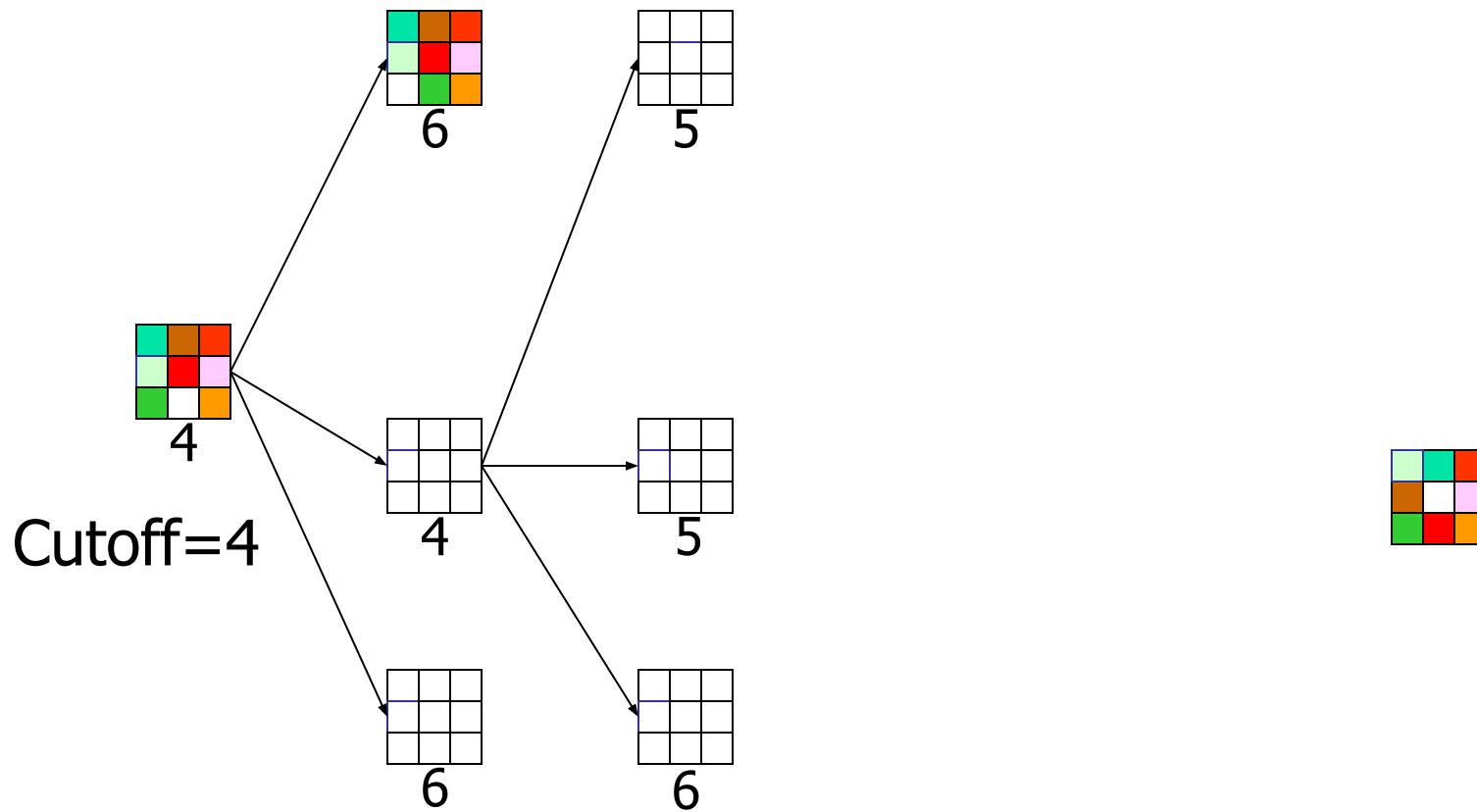


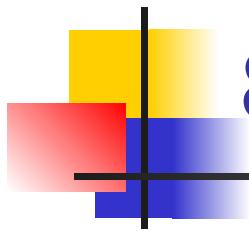


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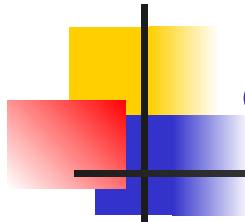


8-Puzzle

$$f(N) = g(N) + h(N)$$

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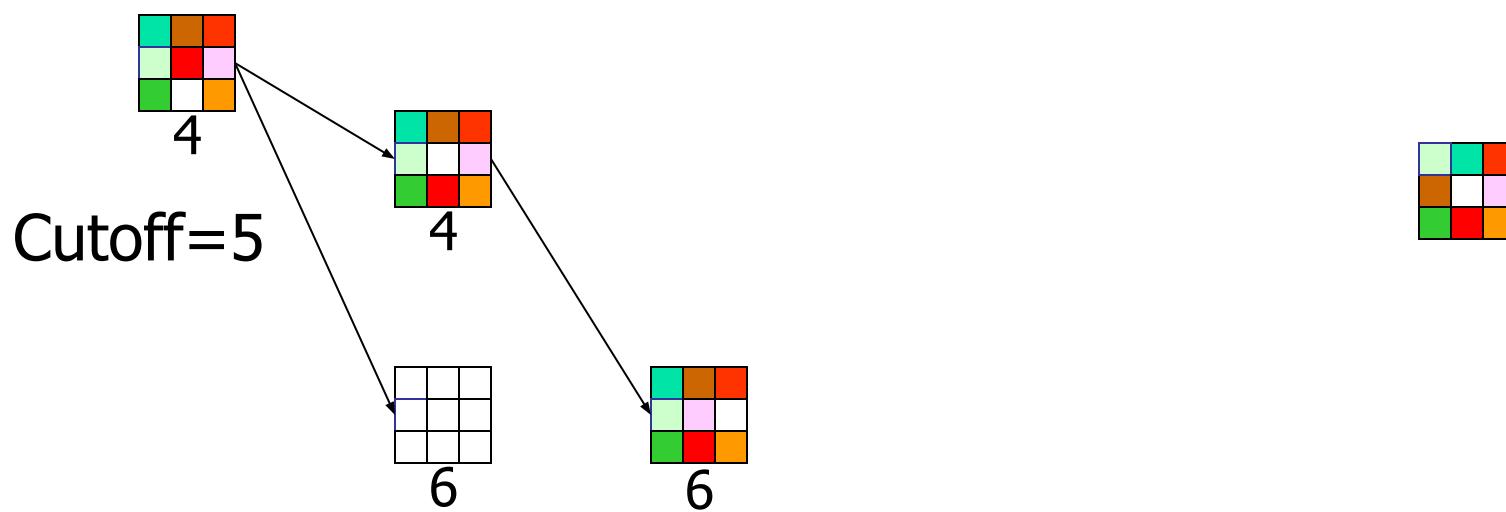


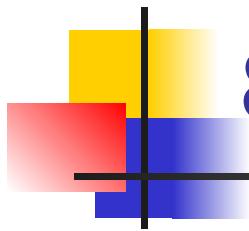


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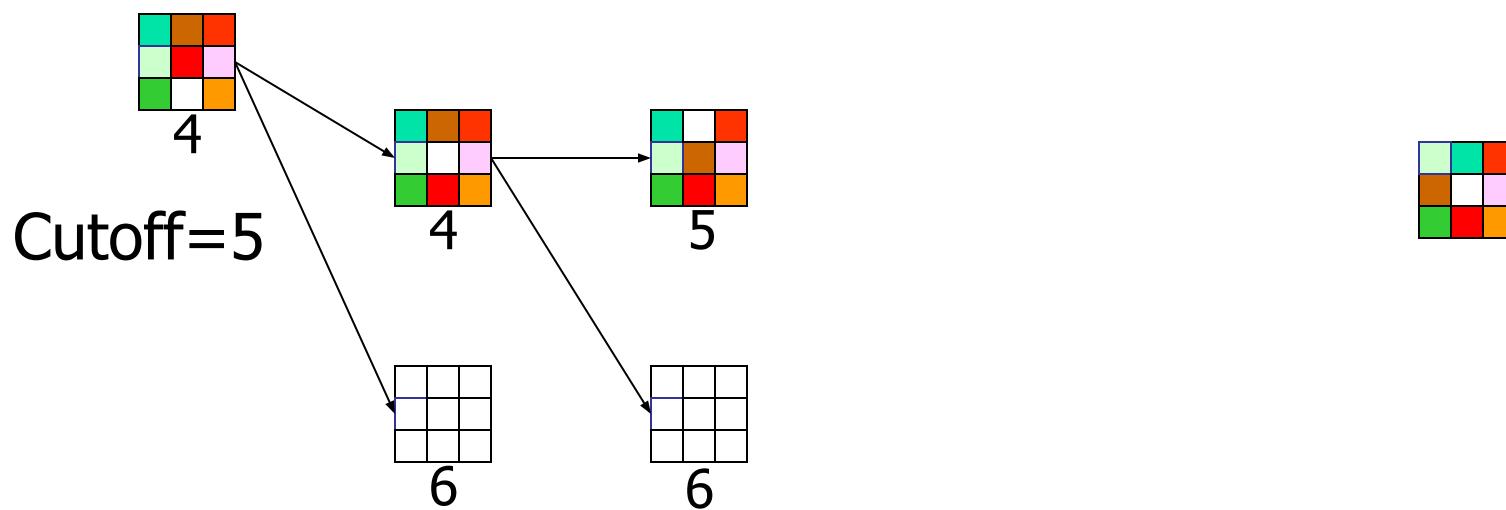


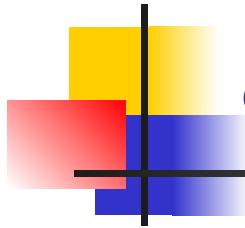


8-Puzzle

$$f(N) = g(N) + h(N)$$

with $h(N)$ = number of misplaced tiles

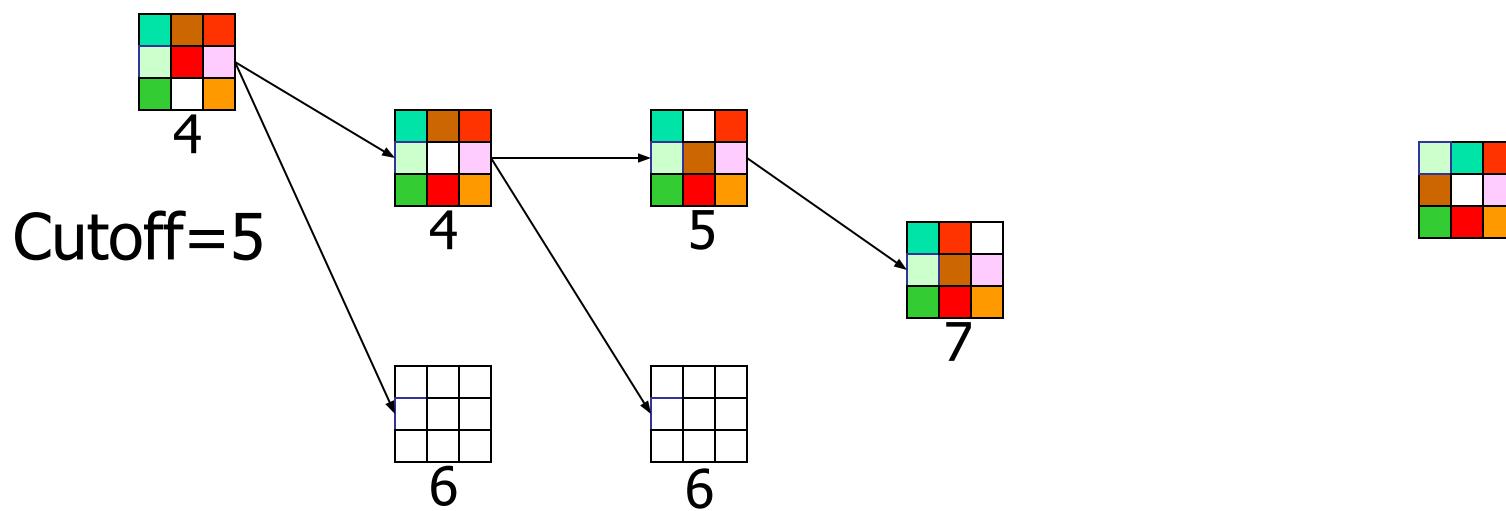


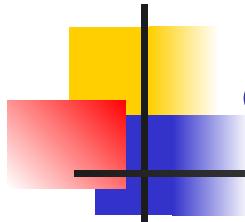


8-Puzzle

$$f(N) = g(N) + h(N)$$

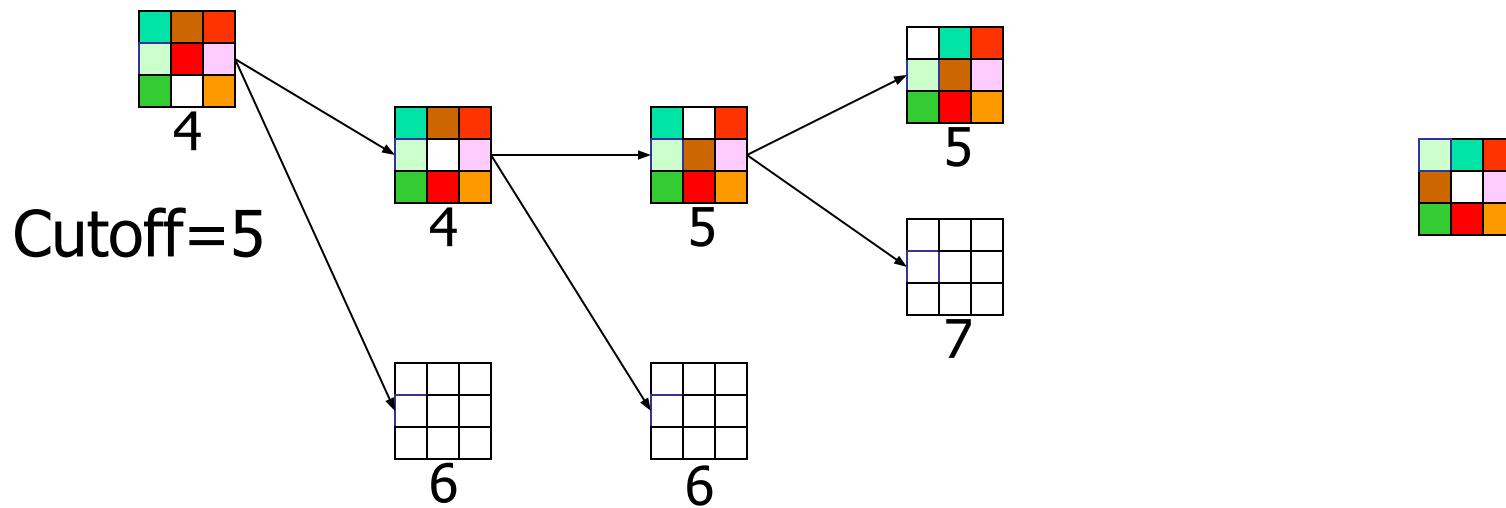
with $h(N)$ = number of misplaced tiles





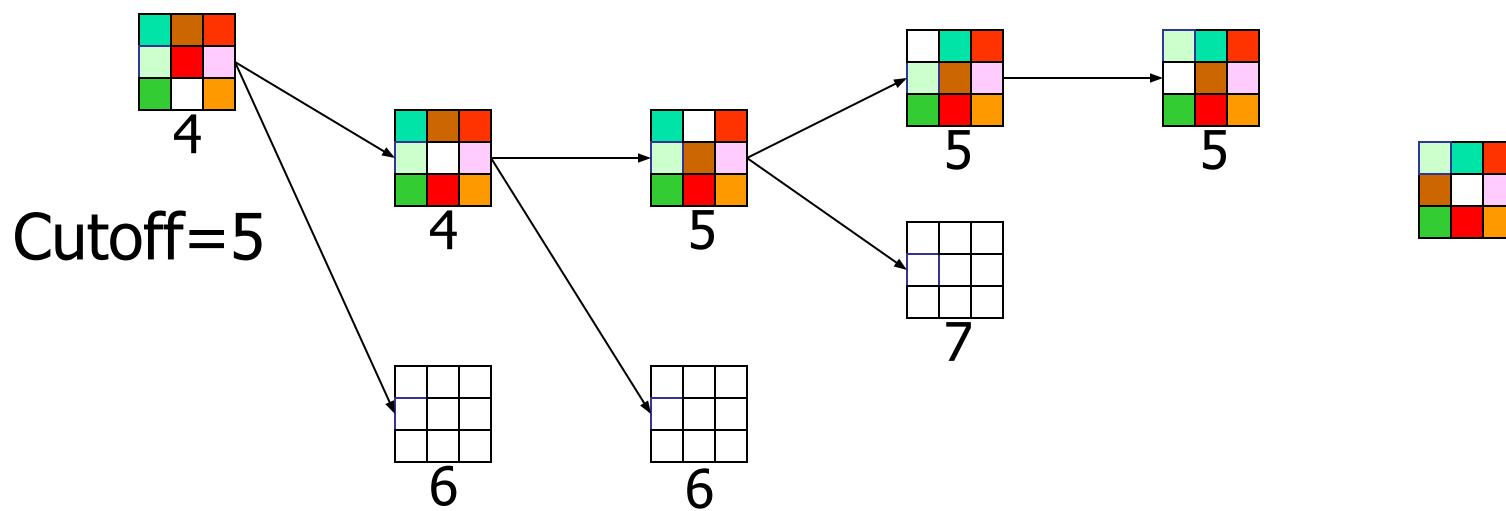
8-Puzzle

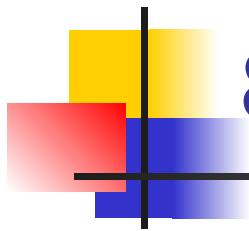
$f(N) = g(N) + h(N)$
with $h(N)$ = number of misplaced tiles



8-Puzzle

$f(N) = g(N) + h(N)$
with $h(N)$ = number of misplaced tiles

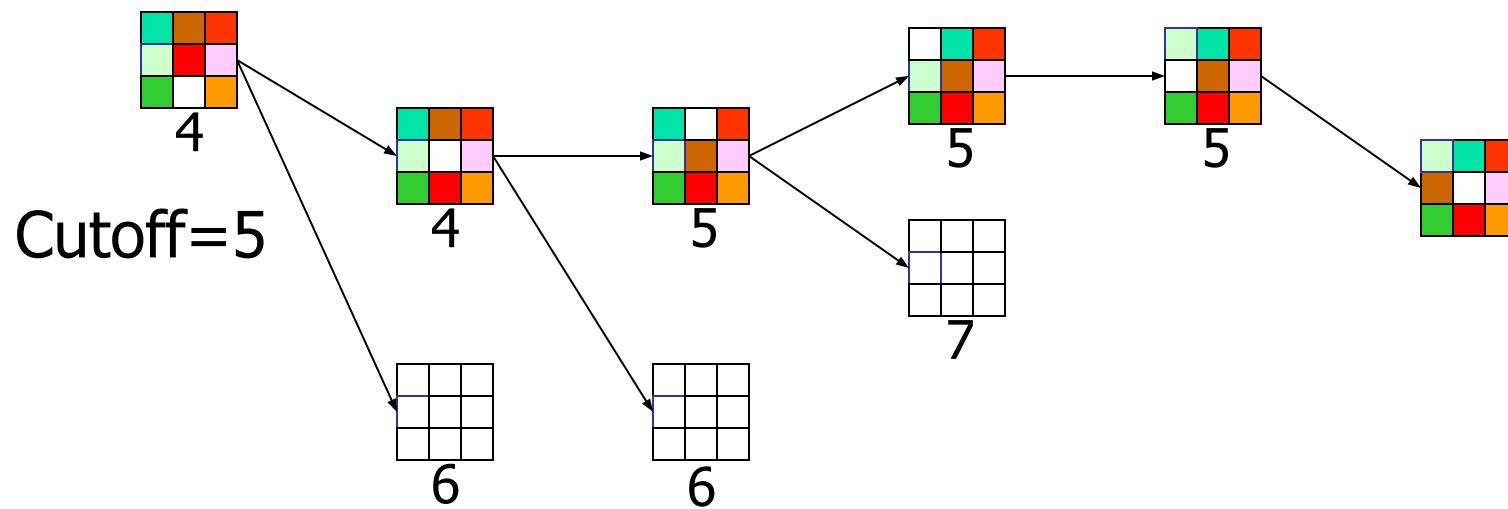




8-Puzzle

$$f(N) = g(N) + h(N)$$

with $h(N)$ = number of misplaced tiles

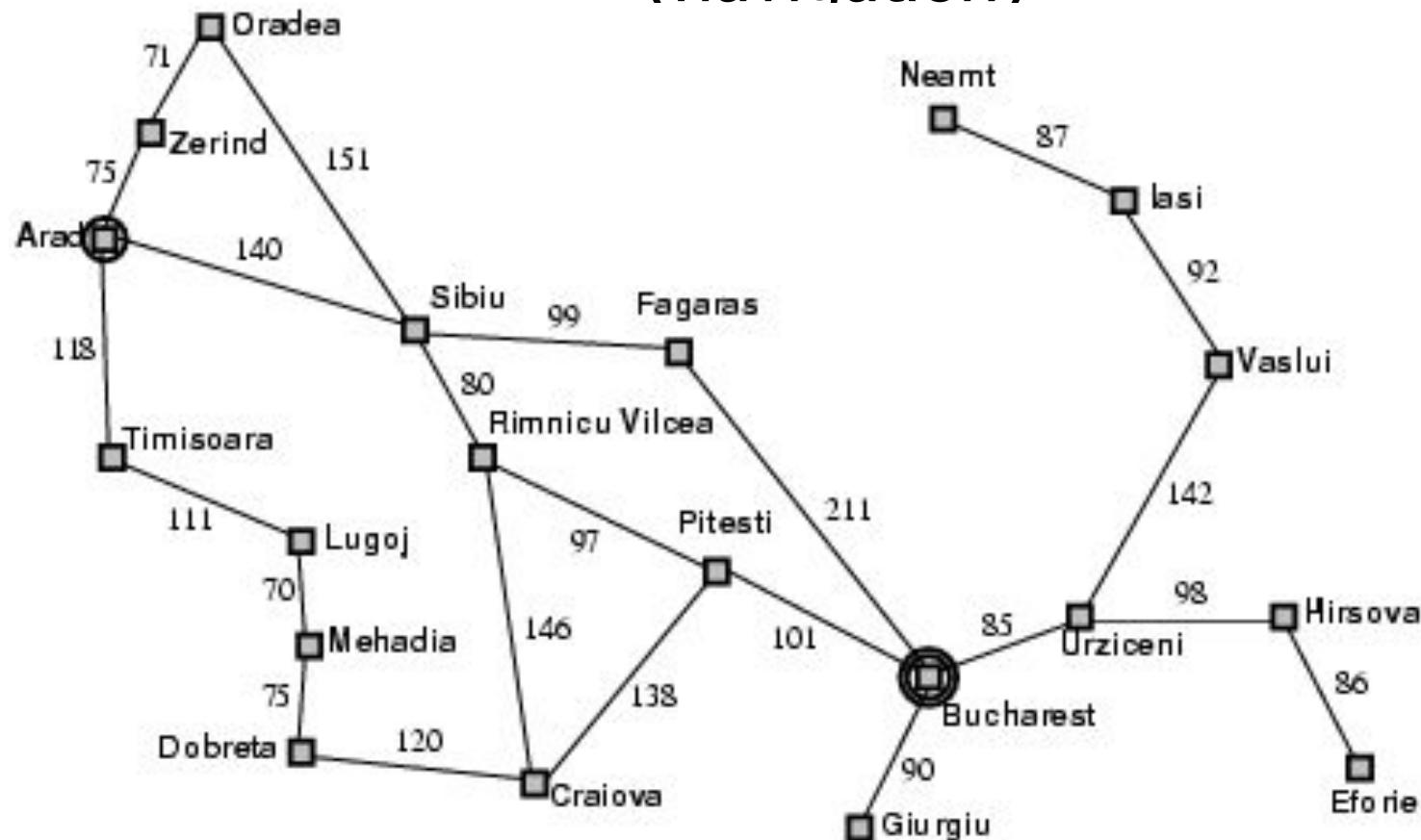


When to Use Search Techniques

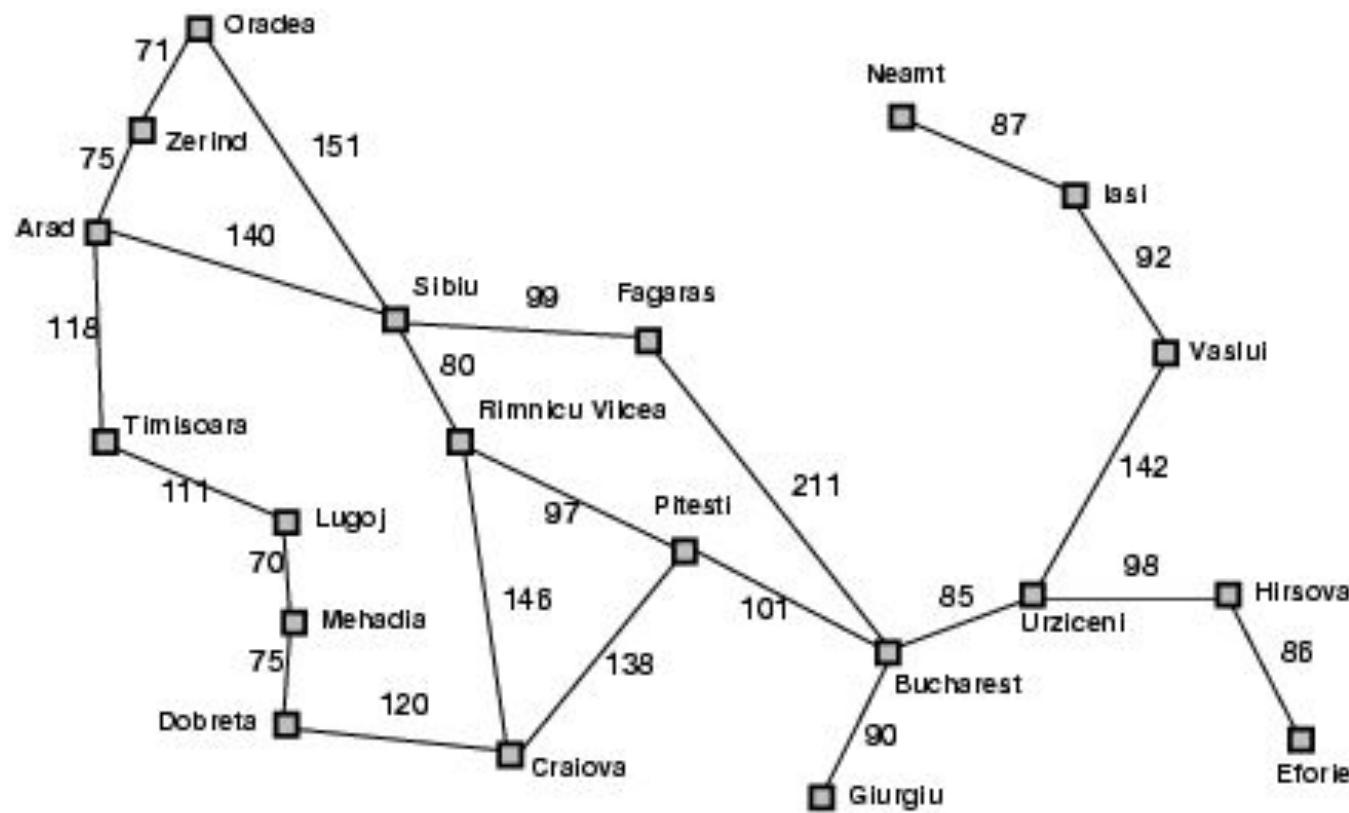
- The search space is small, and
 - There are no other available techniques, or
 - It is not worth the effort to develop a more efficient technique
- The search space is large, and
 - There is no other available techniques, and
 - There exist “**good**” heuristics

Popular AI Search Problems

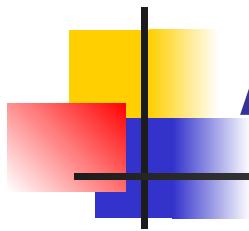
Classic AI search problems, Map searching
(navigation)



Romania with step costs in km

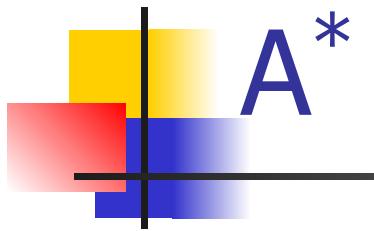


| Straight-line distance to Bucharest | |
|-------------------------------------|-----|
| Arad | 366 |
| Bucharest | 0 |
| Craiova | 160 |
| Dobreta | 242 |
| Eforie | 161 |
| Fagaras | 176 |
| Giurgiu | 77 |
| Hirsova | 151 |
| Iasi | 226 |
| Lugoj | 244 |
| Mehadia | 241 |
| Neamt | 234 |
| Oradea | 380 |
| Pitesti | 10 |
| Rimnicu Vilcea | 193 |
| Sibiu | 253 |
| Timisoara | 329 |
| Urziceni | 80 |
| Vaslui | 199 |
| Zerind | 374 |

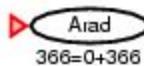


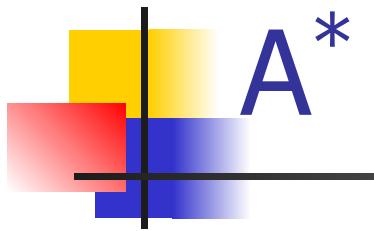
A* search

- Idea: avoid expanding paths that are already expensive
- Evaluation function $f(n) = g(n) + h(n)$
 - $g(n)$ = cost so far to reach n
 - $h(n)$ = estimated cost from n to goal
- $f(n)$ = estimated total cost of path through n to goal

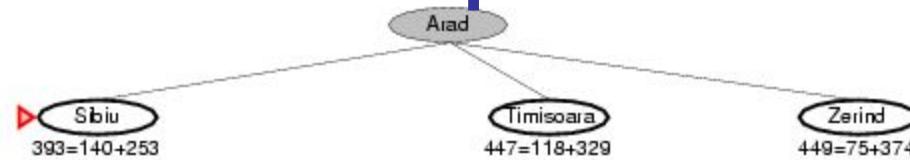


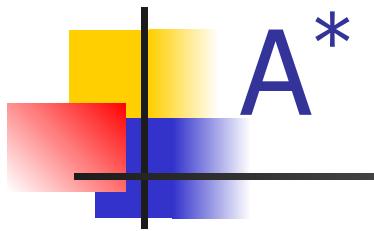
A* search example



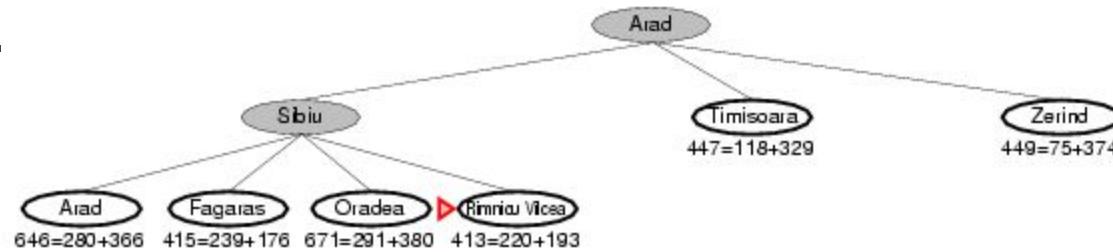


A* search example

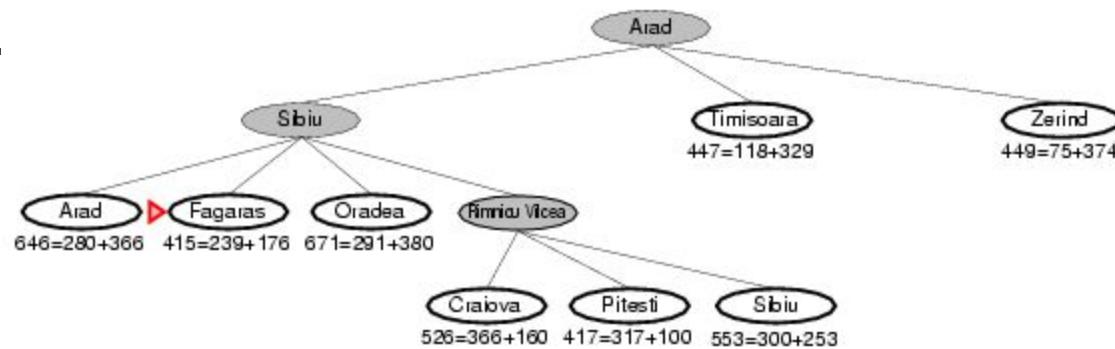




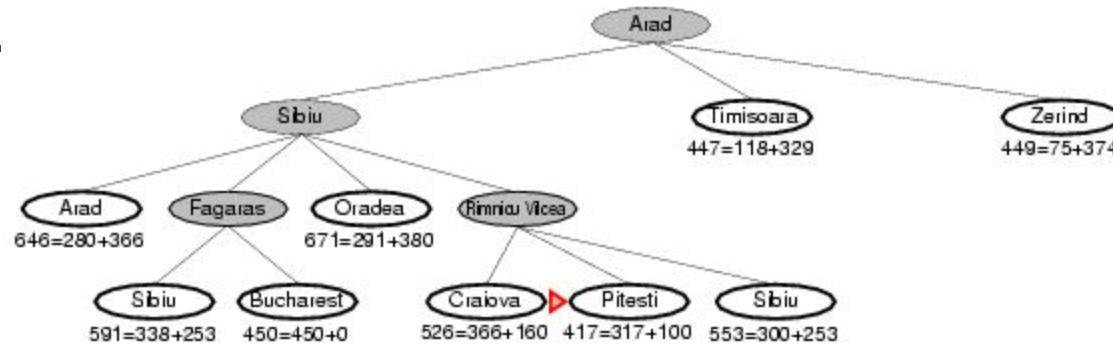
A* search example



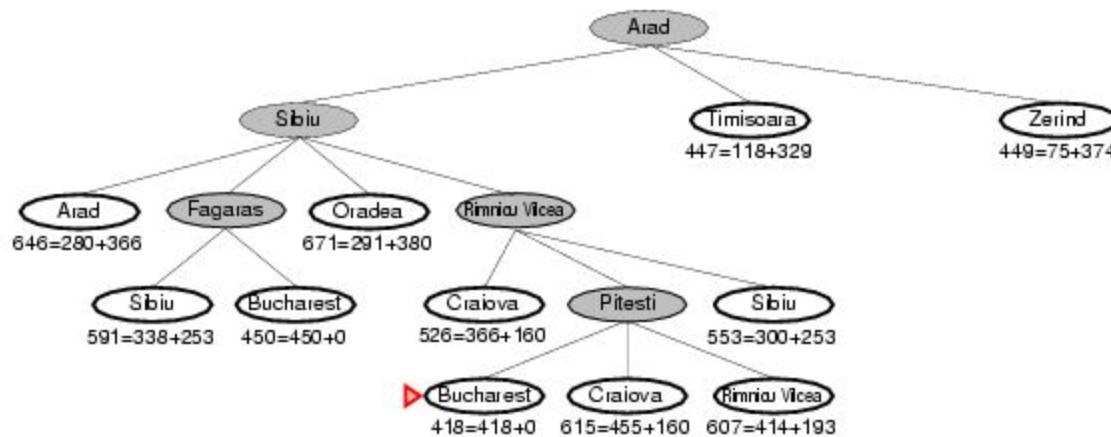
A* search example

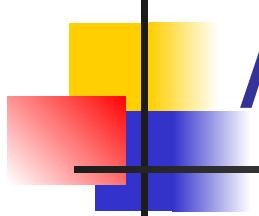


A* search example



A* search example



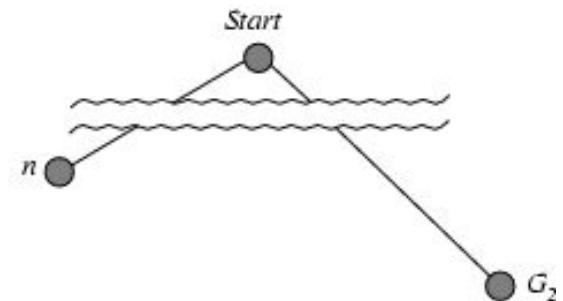


Admissible heuristics

- A heuristic $h(n)$ is **admissible** if for every node n , $h(n) \leq h^*(n)$, where $h^*(n)$ is the **true** cost to reach the goal state from n .
- An admissible heuristic **never overestimates** the cost to reach the goal, i.e., it is **optimistic**
- Example: $h_{SLD}(n)$ (never overestimates the actual road distance)
- **Theorem:** If $h(n)$ is admissible, A* using TREE-SEARCH **is optimal**

Optimality of A* (proof)

- Suppose some suboptimal goal G_2 has been generated and is in the fringe. Let n be an unexpanded node in the fringe such that n is on a shortest path to an optimal goal G .

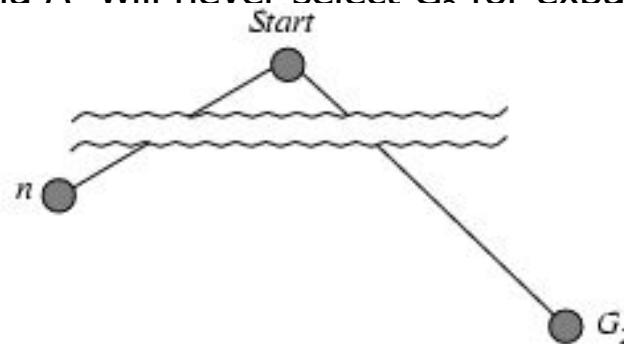


- $f(G_2) = g(G_2) + h(G_2)$
- $f(G_2) = g(G_2)$ [since $h(G_2) = 0$](1)
- Again, $f(G) = g(G) + h(G)$
- $f(G) = g(G)$ [since $h(G) = 0$](2)
- But, $g(G_2) > g(G)$ [since G_2 is suboptimal].....(3)
- Therefore, $f(G_2) > f(G)$ [from equation (1), (2) and (3)] (4)

Optimality of A* (proof)

- Suppose some suboptimal goal G_2 has been generated and is in the fringe. Let n be an unexpanded node in the fringe such that n is on a shortest path to an optimal goal G .
- Therefore, $f(G_2) > f(G)$ (4) [from equation (1), (2) and (3)]
- Again, $h(n) \leq h^*(n)$ (5) [since h is admissible; Here, $h^*(n)$ is the **true** cost to reach the goal state from n]
- $g(n) + h(n) \leq g(n) + h^*(n)$ (6) [Adding $g(n)$ in both sides of equation (5)]
- $f(n) \leq f(G)$ (7) [Because, $f(n) = g(n) + h(n)$; and $f(G) = g(n) + h^*(n)$; Here, $h^*(n)$ is the **true** cost to reach the goal state from n]
 $f(n) \leq f(G) < f(G_2)$ [From equation (4) and (7)]

Therefore, $f(G_2) > f(n)$. and A^* will never select G_2 for expansion before expanding n to reach at optimal goal G .

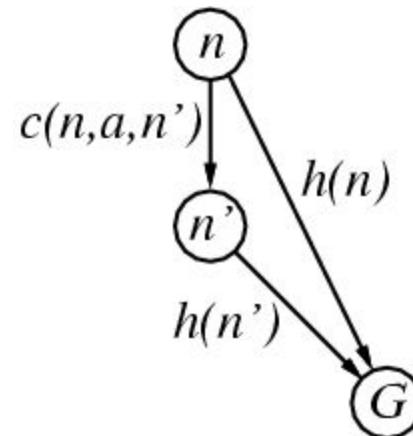


Consistent heuristics

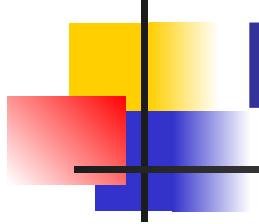
- A heuristic is **consistent** if for every node n , every successor n' of n generated by any action a , follows the triangular inequality.

$$h(n) \leq c(n,a,n') + h(n')$$

- If h is consistent, we have
$$\begin{aligned}f(n') &= g(n') + h(n') \\&= g(n) + c(n,a,n') + h(n') \\&\geq g(n) + h(n) \\&= f(n)\end{aligned}$$

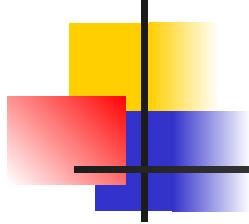


- i.e., $f(n)$ is non-decreasing along any path.
- Theorem:** If $h(n)$ is consistent, A* using GRAPH-SEARCH is optimal



Properties of A*

- Complete? Yes (unless there are infinitely many nodes with $f \leq f(G)$)
- Time? Depends on the quality of heuristic but still exponential.
- Space? Keeps all nodes in memory. A* has worst case $O(b^d)$ space complexity
- Optimal? Yes



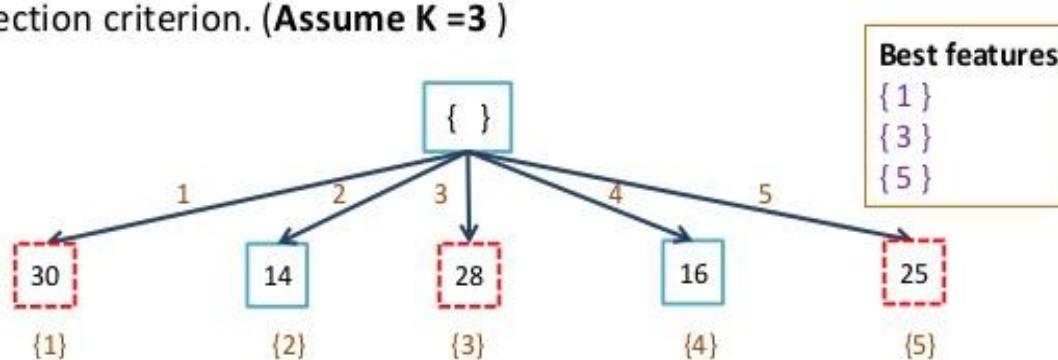
Local beam search

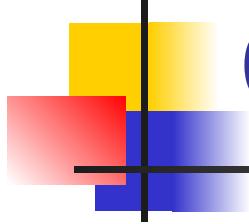
- Keep track of k states rather than just one
 - Start with k randomly generated states
 - At each iteration, all the successors of all k states are generated
 - If any one is a goal state, stop; else select the k best successors from the complete list and repeat.

Local Beam Search

- Begin with k random states
- Generate all successors of these states
- Keep the k best states
- Stochastic beam search: Probability of keeping a state is a *function* of its heuristic value

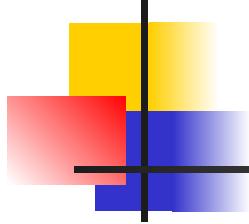
- Select the best K (beam width) features based on a pre-defined selection criterion. (**Assume $K = 3$**)





Conclusions

- Frustration with *uninformed* search led to the idea of using domain specific knowledge in a search so that one can intelligently explore only the relevant part of the search space that has a good chance of containing the goal state. These new techniques are called informed (heuristic) search strategies.
- Even though heuristics improve the performance of informed search algorithms, they are still time consuming especially for large size instances.



References

- University of Berkeley, USA
- <http://www.aima.cs.berkeley.edu>