
Algorithm 1 Gaussian elimination algorithm with partial pivoting

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1: procedure GAUSSPIVOT( $N, A, b, x$ )      ▷ Solves linear system  $Ax = b$ 
2:   for  $k = 1$  to  $N$  step 1 do          ▷ Forward reduction
3:     PIVOTSWAP( $N, A, b, k$ )             ▷ Call PIVOTSWAP
4:     if  $A_{k,k} = 0$  then
5:       STOP                             ▷ Matrix  $A$  is not invertible!
6:     end if
7:     for  $i = k + 1$  to  $N$  step 1 do
8:        $b_i \leftarrow b_i - A_{i,k}b_k/A_{k,k}$ 
9:       for  $j = k + 1$  to  $N$  step 1 do
10:         $A_{i,j} \leftarrow A_{i,j} - A_{i,k}A_{k,j}/A_{k,k}$ 
11:      end for
12:       $A_{i,k} \leftarrow 0$ 
13:    end for
14:  end for
15:  for  $i = N$  to 1 step  $-1$  do          ▷ Backward substitution
16:     $s \leftarrow 0$                         ▷ Initialize the sum
17:    for  $j = i + 1$  to  $N$  step 1 do
18:       $s \leftarrow s + A_{i,j}x_j$ 
19:    end for
20:     $x_i \leftarrow (b_i - s)/A_{i,i}$ 
21:  end for
22:  return  $x$                              ▷ Return the solution
23: end procedure
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1: procedure PIVOTSWAP( $N, A, b, k$ )           ▷ Swaps the pivot if needed
2:    $A_{\max} \leftarrow A_{k,k}$ 
3:    $p \leftarrow k$                                ▷ Pivot row index
4:   for  $i = k + 1$  to  $N$  step 1 do           ▷ Find the maximum pivot
5:     if  $|A_{i,k}| > |A_{\max}|$  then
6:        $A_{\max} \leftarrow A_{i,k}$ 
7:        $p \leftarrow i$ 
8:     end if
9:   end for
10:  if  $p \neq k$  then                               ▷ Swap rows
11:     $D \leftarrow b_p$                                ▷ Dummy variable  $D$ 
12:     $b_p \leftarrow b_k$ 
13:     $b_k \leftarrow D$ 
14:    for  $j = 1$  to  $N$  step 1 do
15:       $D \leftarrow A_{p,j}$ 
16:       $A_{p,j} \leftarrow A_{k,j}$ 
17:       $A_{k,j} \leftarrow D$ 
18:    end for
19:  end if
20:  return  $A, b$                                      ▷ Return the modified matrices
21: end procedure

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