Online Appendix

Union and Firm Labor Market Power

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This Appendix is organized as follows. Section A presents the model derivations for the baseline equilibrium. Section B shows the proofs of the propositions in the main text. Section C presents the details of our identification strategy and additional estimation results. Section D provides additional details about the counterfactual exercises. Section E provides details on our reduced form exercise and the theoretical link of the reduced-form with our model.

A Derivations

In this section we provide the derivations of the model that are not presented in the main text.

A.1 Establishment-occupation labor supply

To simplify the notation, we get rid of the occupation subscript o in this subsection. The indirect utility of a worker k that is employed in establishment i in sub-market m is:

$$u_{kim} = w_i z^1_{i|m} z^2_m,$$

where $z^1_{i|m}$ and z^2_m are independent utility shocks. They are both distributed Fréchet with shape and scale parameters ε_b and T_i for $z^1_{i|m}$, and η and 1 for z^2_m.

Workers first see shocks z^2_m for all local labor markets. After choosing their labor market, workers then observe the establishment specific shocks. Therefore, there is a two stage decision: first, the worker chooses the local labor market that maximizes her expected utility, and subsequently she chooses the establishment that maximizes her utility conditional on the chosen sub-market.

The goal is to compute the unconditional probability of a worker going to establishment i in sub-market m. This probability is equal to:

$$\Pi_i = P\left(w_i z^1_{i|m} \geq \max_{i' \neq i} w_{i'} z^1_{i'|m}\right) P\left(\mathbb{E}_m(\max_i w_i z^1_{i|m}) z^2_m \geq \max_{m' \neq m} \mathbb{E}_{m'}(\max_i w_i z^1_{i|m'}) z^2_{m'}\right)$$

We first solve for the left term. Let's define the following distribution function:

$$G_i(v) = P\left(w_i z^1_{i|m} < v\right) = P\left(z^1_{i|m} < v/w_i\right) = e^{-T_i w_i^{\varepsilon_b} v^{-\varepsilon_b}}.$$

To ease notation, define *conditional* utility $v_i = w_i z^1_{i|m}$ for all i, i'. We need to solve for $P\left(v_i \geq \max_{i' \neq i} v_{i'}\right)$. Fix $v_i = v$. Then we have:

$$P\left(v \geq \max_{i' \neq i} v_{i'}\right) = \bigcap_{i' \neq i} P\left(v_{i'} < v\right) = \prod_{i' \neq i} G_{i'}(v) = e^{-\Phi_m^{-i} v^{-\varepsilon_b}} = G_m^{-i}(v),$$

where $\Phi_m^{-i} \equiv \sum_{i' \neq i} T_{i'} w_{i'}^{\varepsilon_b}$. Similarly, the probability of having at most conditional utility v is:

$$G_m(v) = P\left(v \geq \max_{i'} v_{i'}\right) = e^{-\Phi_m v^{-\varepsilon_b}},$$

where $\Phi_m \equiv \sum_{i'} T_{i'} w_{i'}^{\varepsilon_b}$. Integrating $G_m^{-i}(v)$ over all possible values of v, we get:

$$\begin{aligned} P\left(v_i \geq \max_{i' \neq i} v_{i'}\right) &= \int_0^\infty e^{-\Phi_m^{-i} v^{-\varepsilon_b}} dG_i(v) = \int_0^\infty \varepsilon_b T_i w_i^{\varepsilon_b} v^{-\varepsilon_b - 1} e^{-\Phi_m v^{-\varepsilon_b}} dv \\ &= \frac{T_i w_i^{\varepsilon_b}}{\Phi_m} \int_0^\infty \varepsilon_b \Phi_m v^{-\varepsilon_b - 1} e^{-\Phi_m v^{-\varepsilon_b}} dv = \frac{T_i w_i^{\varepsilon_b}}{\Phi_m} \int_0^\infty dG_m(v) = \frac{T_i w_i^{\varepsilon_b}}{\Phi_m}. \end{aligned}$$

Now we need to find $P\left(\mathbb{E}_m(\max_i w_i z_{i|m}^1) z_m^2 \geq \max_{m' \neq m} \mathbb{E}_{m'}(\max_i w_i z_{i|m'}^1) z_{m'}^2\right)$. So, the expected utility of working in sub-market m is:

$$\mathbb{E}_m(\max_i w_i z_{i|m}^1) = \int_0^\infty v_i dG_m(v) = \int_0^\infty \varepsilon_b \Phi_m v^{-\varepsilon_b} e^{-\Phi_m v^{-\varepsilon_b}} dv.$$

We define this new variable:

$$x = \Phi_m v^{-\varepsilon_b} \qquad dx = -\varepsilon_b \Phi_m v^{-(\varepsilon_b + 1)} dv.$$

Now we can change variable in the previous integral and obtain:

$$\int_0^\infty x^{-1/\varepsilon_b} \Phi_m^{1/\varepsilon_b} e^{-x} dx = \Gamma\left(\frac{\varepsilon_b - 1}{\varepsilon_b}\right) \Phi_m^{1/\varepsilon_b},$$

where $\Gamma(\cdot)$ is the Gamma function. Defining $\Gamma_b \equiv \Gamma\left(\frac{\varepsilon_b - 1}{\varepsilon_b}\right)$, we can rewrite:

$$P\left(\mathbb{E}_m(\max_i w_i z_{i|m}^1) z_m^2 \geq \max_{m' \neq m} \mathbb{E}_{m'}(\max_i w_i z_{i|m'}^1) z_{m'}^2\right) = P\left(\Phi_m^{1/\varepsilon_b} \Gamma_b z_m^2 \geq \max_{m' \neq m} \Phi_{m'}^{1/\varepsilon_{b'}} \Gamma_{b'} z_{m'}^2\right).$$

Following similar arguments as above, this probability is equal to:

$$P\left(\Phi_m^{1/\varepsilon_b} \Gamma_b z_m^2 \geq \max_{m' \neq m} \Phi_{m'}^{1/\varepsilon_b} \Gamma_{b'} z_{m'}^2\right) = \frac{\Phi_m^{\eta/\varepsilon_b} \Gamma_b^\eta}{\Phi},$$

where $\Phi \equiv \sum_{b' \in \mathcal{B}} \sum_{m' \in \mathcal{M}_{b'}} \Phi_{m'}^{\eta/\varepsilon_{b'}} \Gamma_{b'}^\eta$. Finally, combining the two probabilities we obtain:

$$\Pi_i = \frac{T_i w_i^{\varepsilon_b}}{\Phi_m} \times \frac{\Phi_m^{\eta/\varepsilon_b} \Gamma_b^\eta}{\Phi}.$$

By integrating Π_i over the whole measure of workers L, we can obtain the labor supply for each establishment: $L_i = \Pi_i \times L$.

Workers' welfare. An obvious way to measure workers welfare would be to compute the average utility for workers. However this is not possible as the estimated shape parameter η is smaller than 1.[1] This implies that the mean for the Fréchet distributed utilities is not defined. Instead, we

[1] See Section 5 in the main text.

compute the median utility agents expect to receive in each local labor market. This is equal to:

$$\text{Median}\left[\max_m \mathbb{E}_m(\max_i w_i z^1_{i|m}) z^2_m\right] \propto \Phi^{1/\eta}.$$

A.2 Establishment decision

In the absence of bargaining, the profit maximization problem of establishment i is:

$$\max_{w_{io}, K_{io}} P_b \widetilde{A}_{io} K_{io}^{\alpha_b} L_{io}^{\beta_b} - w_{io} L_{io}(w_{io}) - R_b K_{io}, \tag{A1}$$

where $L_{io}(w_{io})$ is the labor supply (12) where they take Φ and L as given but internalize their effect on Φ_m. P_b and R_b are respectively the sector price and rental rate of capital.[2]. The first order conditions of this problem are:

$$\begin{aligned} w_{io} &= \beta_b \frac{e_{io}}{e_{io}+1} P_b \widetilde{A}_{io} K_{io}^{\alpha_b} L_{io}^{\beta_b - 1}, \\ R_b &= \alpha_b P_b \widetilde{A}_{io} K_{io}^{\alpha_b - 1} L_{io}^{\beta_b}. \end{aligned} \tag{A2}$$

$e_{io} = \varepsilon_b\,(1 - s_{io|m}) + \eta\, s_{io|m}$ is the perceived elasticity of supply for establishment i in occupation o.

We can use the first order conditions of capital to substitute it into the establishment's production function and obtain an expression that depends only in labor:

$$y_{io} = \left(\frac{\alpha_b}{R_b}\right)^{\frac{\alpha_b}{1-\alpha_b}} \widetilde{A}_{io}^{\frac{1}{1-\alpha_b}} L_{io}^{\frac{\beta_b}{1-\alpha_b}} P_b^{\frac{\alpha_b}{1-\alpha_b}}. \tag{A3}$$

In order to gain tractability in the solution of the model, we restrict the output elasticity with respect to capital, such that $1 - \frac{\beta_b}{1-\alpha_b} = \delta$, where $\delta \in [0, 1]$ is a constant across sectors. This specification would nest a constant returns to scale technology when $\delta = 0$. As long as $0 < \delta < 1$ the establishment faces decreasing returns to scale within occupations. Define a transformed productivity $A_{io} \equiv \widetilde{A}_{io}^{\frac{1}{1-\alpha_b}} \left(\frac{\alpha_b}{R_b}\right)^{\frac{\alpha_b}{1-\alpha_b}}$. The establishment-occupation production is: $y_{io} = P_b^{\frac{\alpha_b}{1-\alpha_b}} A_{io} L_{io}^{1-\delta}$.

Maximization (A1) is therefore equivalent to:

$$\max_{w_{io}} (1-\alpha_b) P_b^{\frac{1}{1-\alpha_b}} A_{io} L_{io}^{1-\delta} - w_{io} L_{io}(w_{io}), \tag{A4}$$

A.3 Markdown function

We get the markdown function from the establishment's optimality condition with respect to wages abstracting from wage bargaining. For the full derivation of the wage with bargaining, see the main Appendix. Establishments post a wage and choose capital quantity in order to maximize profits subject to their individual labor supply taking only into account the effect on their local labor market. As explained in the main text, this can happen because of a myopic behavior from the

[2]The construction details of the rental rate of capital are in the Supplemental Material

establishments or if there is a continuum of local labor markets. The establishment problem is:

$$\max_{w_{io}, K_{io}} P_b \sum_{o=1}^{O} \widetilde{A}_{io} K_{io}^{\alpha_b} L_{io}^{\beta_b} - \sum_{o=1}^{O} w_{io} L_{io}(w_{io}) - R_b \sum_{o=1}^{O} K_{io},$$

The first order condition with respect to the wage is:

$$P_b \frac{\partial F}{\partial L_{io}} \frac{\partial L_{io}}{\partial w_{io}} = L_{io}(w_{io}) + w_{io} \frac{\partial L_{io}}{\partial w_{io}},$$

where the derivative of the labor supply L_{io} with respect to the establishment-occupation wage w_{io} is:

$$\begin{aligned}
\frac{\partial L_{io}}{\partial w_{io}} &= \frac{L\Gamma_b^{\eta}}{\Phi} \left(\left[\frac{\varepsilon_b T_{io} w_{io}^{\varepsilon_b - 1} \Phi_m - T_{io} w_{io}^{\varepsilon_b} \varepsilon_b T_{io} w_{io}^{\varepsilon_b - 1}}{\Phi_m^2} \right] \Phi_m^{\eta/\varepsilon_b} + \eta \frac{T_{io} w_{io}^{\varepsilon_b}}{\Phi_m} \Phi_m^{\eta/\varepsilon_b - 1} T_{io} w_{io}^{\varepsilon_b - 1} \right) \\
&= \frac{\varepsilon_b T_{io} w_{io}^{\varepsilon_b - 1}}{\Phi_m} \frac{\Phi_m^{\eta/\varepsilon_b} \Gamma_b^{\eta}}{\Phi} L - \frac{\varepsilon_b T_{io} w_{io}^{\varepsilon_b - 1} \Phi_m^{\eta/\varepsilon_b} \Gamma_b^{\eta}}{\Phi_m \Phi} L \frac{T_{io} w_{io}^{\varepsilon_b}}{\Phi_m} + \eta \frac{T_{io} w_{io}^{\varepsilon_b}}{\Phi_m} \frac{T_{io} w_{io}^{\varepsilon_b - 1}}{\Phi_m} \frac{\Phi_m^{\eta/\varepsilon_b} \Gamma_b^{\eta}}{\Phi} L \\
&= \varepsilon_b \frac{L_{io}}{w_{io}} - \varepsilon_b \frac{L_{io}}{w_{io}} \frac{L_{io}}{L_m} + \eta \frac{L_{io}}{w_{io}} \frac{L_{io}}{L_m} = \frac{L_{io}}{w_{io}} \left(\varepsilon_b (1 - s_{io|m}) + \eta s_{io|m} \right).
\end{aligned}$$

Substituting this last derivative into the first order condition we get:

$$\begin{aligned}
& L_{io} + L_{io} \left(\varepsilon_b (1 - s_{io|m}) + \eta s_{io|m} \right) = P_b \frac{\partial F}{\partial L_{io}} \frac{L_{io}}{w_{io}} \left(\varepsilon_b (1 - s_{io|m}) + \eta s_{io|m} \right) \\
\Rightarrow \quad & w_{io} = \frac{\varepsilon_b (1 - s_{io|m}) + \eta s_{io|m}}{\varepsilon_b (1 - s_{io|m}) + \eta s_{io|m} + 1} P_b \frac{\partial F}{\partial L_{io}} = \mu(s_{io|m}) P_b \frac{\partial F}{\partial L_{io}}.
\end{aligned}$$

A.4 Hat algebra

This section shows that it is possible to compute the counterfactuals in general equilibrium by using revenue productivities (TFPRs), which are a function of prices determined in general equilibrium, and not just the underlying physical productivities. A priori, the issue is that counterfactually changing the labor wedge changes equilibrium prices and therefore the 'fundamental' TFPRs.

The literature on misallocation has used the TFPRs, together with a modeling assumption on the sector price, to compute the normalized within sector productivity distribution. This has prevented performing general equilibrium counterfactuals that also take into account productivity differences across industries.[3] We show that we can: (i) carry out counterfactuals in general equilibrium by writing the model in relative terms from a baseline scenario; and (ii) compute the movement of production factors across industries.

Our approach is to write counterfactual sector prices relative to the baseline and to fix the transformed revenue productivities Z_{io}.[4] Using the definition for $Z_{io} = P_b^{\frac{1}{1-\alpha_b}} A_{io}$ and equation (25),

[3]For example, Hsieh and Klenow (2009) conduct a counterfactual where they remove distortions at the firm level and compute the productivity gains at the *sector* level. The productivity gains are a result of factors of production reallocating to more productive firms *within* each sector. This allows them to compute a *partial* equilibrium effect on total factor productivity, i.e. keeping the production factors constant *across* industries. A general equilibrium effect on total factor productivity takes into account, not only the reallocation of inputs within, but also across industries. They cannot do this as they can identify only relative productivity differences within each sector while normalizing average differences across industries. For more details, see equation (19) and the discussion below in their paper.

[4]Solving the counterfactuals in levels as stated in Section 4 would require to back out the productivities. It would be possible to do so by making some additional normalizations per sector. For example, one could assume that the minimum physical productivity (or Total Factor Productivity, TFP) is constant across industries and get rid of sector relative prices by normalizing the minimum TFP per sector.

wages are equal to:

$$w_{io} = \beta_b \lambda(\mu_{io}, \varphi_b) Z_{io} L_{io}^{-\delta}.$$

We denote with a prime the variables in the counterfactual (e.g. P_b') and with a hat the relative variables (e.g. $\widehat{P}_b = \frac{P_b'}{P_b}$). Writing the model as deviations from a baseline scenario has been dubbed 'exact-hat-algebra' by Costinot and Rodríguez-Clare (2014). We can then rewrite the revenue productivity in a counterfactual in hat terms as:

$$Z_{io}' = {P'}_b^{\frac{1}{1-\alpha_b}} A_{io} = \widehat{P}_b^{\frac{1}{1-\alpha_b}} Z_{io}.$$

The counterfactual revenue productivity is a function of the relative price $\widehat{P}_b$ and the observed revenue productivity Z_{io}. Let λ_{io}' be the counterfactual wedge, the counterfactual real wages are:

$$w_{io}' = \beta_b \lambda_{io}' Z_{io}' {L'}_{io}^{-\delta} = \beta_b \lambda_{io}' Z_{io} \widehat{P}_b^{\frac{1}{1-\alpha_b}} {L'}_{io}^{-\delta}, \tag{A5}$$

where in the last step we used the definition of the transformed TFPRs. In the counterfactuals Z_{io} is taken as a fixed fundamental and we have to solve for sector prices relative to the baseline $\widehat{P}_b$.

The system (B1) in the counterfactual writes as:

$$w_{io}' = \omega_{io} \left(\widehat{P}_b^{\frac{1}{1-\alpha_b}} \right)^{\frac{1}{1+\varepsilon_b \delta}} {\Phi'}_m^{\frac{\delta(\varepsilon_b - \eta)}{\varepsilon_b(1+\varepsilon_b \delta)}} \left(\frac{\Phi'}{L} \right)^{\frac{\delta}{1+\varepsilon_b \delta}}, \tag{A6}$$

where the establishment-occupation component in the counterfactual is: $\omega_{io} \equiv \left(\beta_b \lambda_{io}' \frac{Z_{io}}{(T_{io} \Gamma_b^{\eta})^{\delta}} \right)^{\frac{1}{1+\varepsilon_b \delta}}$. Finally, the counterfactual establishment-occupation components ω_{io} are enough to compute the employment shares $s_{io|m}'$ and $s_{m|b}'$, as shown in Propositions 1 and 3.

To see why the employment shares and wages in the baseline are sufficient statistics for the fundamentals, we can rewrite equation (16) in Proposition 1 with revenue productivities instead of physical productivities for the counterfactual:

$$s_{io|m}' = \frac{\left(T_{io}^{\frac{1}{\varepsilon_b}} \lambda_{io}' Z_{io} \right)^{\frac{\varepsilon_b}{1+\varepsilon_b \delta}}}{\sum_{j \in \mathcal{I}_m} \left(T_{jo}^{\frac{1}{\varepsilon_b}} \lambda_{jo}' Z_{jo} \right)^{\frac{\varepsilon_b}{1+\varepsilon_b \delta}}}.$$

Substituting the identified values for the revenue productivities $Z_{io} = \frac{w_{io} L_{io}^{\delta}}{\beta_b \lambda_{io}}$ (see equation 25 in the main text), and amenities $\frac{s_{io|m}}{w_{io}^{\varepsilon_b}} \left(\frac{L_m}{\Gamma_b} \right)^{\varepsilon_b / \eta}$ (see section C.4 of this Online Appendix for the derivation) into the expression above and simplifying, we get:

$$s_{io|m}' = \frac{s_{io|m} \left(\lambda_{io}' / \lambda_{io} \right)^{\frac{\varepsilon_b}{1+\varepsilon_b \delta}}}{\sum_{j \in \mathcal{I}_m} s_{jo|m} \left(\lambda_{jo}' / \lambda_{jo} \right)^{\frac{\varepsilon_b}{1+\varepsilon_b \delta}}}.$$

Therefore, it is equivalent to compute the counterfactual employment shares within a local labor market using the observed employment shares and wedges in the baseline, or the identified amenities and revenue productivities. We can then use the revenue productivities, which are themselves a function of observed wages, employment levels and wedges to aggregate the counterfactual economy at the sector level. Following the same steps as in the baseline, the sector level system of equations in the counterfactual is analogous to (20) but with relative variables. Solving for relative sector prices we can compute the sector employment L'_b. Propositions 2 and 4 apply also in the 'hat' economy. Therefore, the solution for the counterfactuals exists and is unique.

Summing the counterfactual wage w'_{io} from (A6) to $\Phi'_m = \sum_{i \in \mathcal{I}_m} T_{io} w'^{\varepsilon_b}_{io}$ and factoring out the industry or economy wide constants we find the following relation,

$$\Phi'_m = \widetilde{\Phi'}_m^{\frac{1+\varepsilon_b \delta}{1+\eta\delta}} \widehat{P}_b^{\frac{\varepsilon_b}{(1-\alpha_b)(1+\eta\delta)}} \left(\frac{\Phi'}{L'}\right)^{\frac{\varepsilon_b \delta}{1+\eta\delta}}, \quad \widetilde{\Phi}'_m \equiv \sum_{io \in \mathcal{I}_m} T_{io} \omega_{io}^{\varepsilon_b}$$

Using the definition of $\Phi'_b \equiv \sum_{m \in \mathcal{M}_b} \Phi'^{\eta/\varepsilon_b}_m$ and $\Phi' \equiv \sum_{b \in \mathcal{B}} \Phi'_b \Gamma_b^\eta$, we have that:

$$\Phi'_b = \widetilde{\Phi}'_b \widehat{P}_b^{\frac{\eta}{(1-\alpha_b)(1+\eta\delta)}} \left(\frac{\Phi'}{L'}\right)^{\frac{\eta\delta}{1+\eta\delta}}, \quad \widetilde{\Phi}'_b \equiv \sum_{m \in \mathcal{M}_b} \widetilde{\Phi'}_m^{\frac{(1+\varepsilon_b\delta)\eta}{(1+\eta\delta)\varepsilon_b}},$$

$$\Phi' = \widetilde{\Phi'}^{1+\eta\delta} L'^{-\eta\delta}, \quad \widetilde{\Phi}' \equiv \sum_{b \in \mathcal{B}} \widetilde{\Phi}'_b \widehat{P}_b^{\frac{\eta}{(1-\alpha_b)(1+\eta\delta)}} \Gamma_b^\eta.$$

Sector employment in the counterfactual is equal to:

$$L'_b = \frac{\widehat{P}_b^{\frac{\eta}{(1-\alpha_b)(1+\eta\delta)}} \widetilde{\Phi}'_b(\mathbf{s}'_b)\Gamma_b^\eta}{\sum_{b' \in \mathcal{B}} \widehat{P}_{b'}^{\frac{\eta}{(1-\alpha_{b'})(1+\eta\delta)}} \widetilde{\Phi}'_{b'}(\mathbf{s}'_{b'})\Gamma_{b'}^\eta} L',$$

where counterfactual sector employment is a function of relative prices $\{\widehat{P}_b\}_{b \in \mathcal{B}}$ and counterfactual employment shares $\{\mathbf{s}'_b\}_{b \in \mathcal{B}}$. Establishment-occupation output in the counterfactual is:

$$y'_{io} = P_b'^{\frac{\alpha_b}{1-\alpha_b}} A_{io} L'^{\,1-\delta}_{io} = P_b^{\frac{\alpha_b}{1-\alpha_b}} A_{io} \widehat{P}_b^{\frac{\alpha_b}{1-\alpha_b}} L'^{\,1-\delta}_{io} = \frac{\widehat{P}_b^{\frac{\alpha_b}{1-\alpha_b}}}{P_b} Z_{io} L'^{\,1-\delta}_{io}.$$

The analogue expression for the baseline is: $y_{io} = \frac{1}{P_b} Z_{io} L_{io}^{1-\delta}$. To aggregate this expression note that the revenue productivities are multiplied by a sector-level constant and therefore cancel out,

$$\Psi'_b \equiv \sum_{io \in \mathcal{I}_b} \frac{Z_{io}}{Z_b} s'^{\,1-\delta}_{io|m} s'^{\,1-\delta}_{m|b} = \sum_{io \in \mathcal{I}_b} \frac{A_{io}}{A_b} s'^{\,1-\delta}_{io|m} s'^{\,1-\delta}_{m|b} \equiv \Omega'_b,$$

where Ω'_b is a measure of sector productivity in the counterfactual relative to the productivity under the efficient allocation A_b. Aggregating up to sector level, the counterfactual output Y'_b is,

$$Y'_b = \frac{\widehat{P}_b^{\frac{\alpha_b}{1-\alpha_b}}}{P_b} Z_b \Omega'_b L'^{\,1-\delta}_b, \quad \Omega'_b \equiv \sum_{io \in \mathcal{I}_b} \frac{A_{io}}{A_b} s'^{\,1-\delta}_{io|m} s'^{\,1-\delta}_{m|b}, \quad Z_b \equiv \sum_{io \in \mathcal{I}_b} Z_{io} \tilde{s}_{io|m}^{1-\delta} \tilde{s}_{m|b}^{1-\delta}.$$

The baseline sector output is: $Y_b = \frac{1}{P_b} Z_b \Omega_b L_b^{1-\delta}$ with Ω_b analogue to the one defined for the counterfactual but with baseline employment shares, meaning $\Omega_b \equiv \sum_{io \in \mathcal{I}_b} \frac{A_{io}}{A_b} s_{io|m}^{1-\delta} s_{m|b}^{1-\delta}$. Taking the ratio, counterfactual sector output relative to the baseline is:

$$\widehat{Y}_b = \widehat{P}_b^{\frac{\alpha_b}{1-\alpha_b}} \widehat{\Omega}_b \widehat{L}_b^{1-\delta}, \tag{A7}$$

where $\widehat{\Omega}_b = \frac{\Omega'_b}{\Omega_b}$. Using L'_b and equation (3) we get a similar expression to (B8)

$$\widehat{P}_b^{\frac{1+\eta}{(1-\alpha_b)(1+\eta\delta)}} \widehat{\Omega}_b \left(\frac{\widetilde{\Phi}'_b \Gamma_b^\eta}{L_b}\right)^{1-\delta} = \prod_{b'\in\mathcal{B}} \left(\widehat{P}_{b'}^{\frac{\alpha_{b'}(1+\eta\delta)+\eta(1-\delta)}{(1-\alpha_{b'})(1+\eta\delta)}}\right)^{\theta_{b'}} \prod_{b'\in\mathcal{B}} \widehat{\Omega}_{b'}^{\theta_{b'}} \prod_{b'\in\mathcal{B}} \left(\frac{\widetilde{\Phi}'_{b'} \Gamma_{b'}^\eta}{L_{b'}}\right)^{(1-\delta)\theta_{b'}}. \tag{A8}$$

By taking the ratio, the elasticities θ_b and the economy wide constants cancel out on both sides. Rewriting, we get a similar expression to equation (B10) in Proposition 4 but with hat variables:

$$\widehat{P}_b = \widehat{X}_b \widehat{X}^{\frac{(1+\eta\delta)(1-\alpha_b)}{1+\eta}}, \quad \widehat{X}_b = \left(\frac{L_b^{1-\delta}}{\widehat{\Omega}_b \left(\widetilde{\Phi'}_b \Gamma_b^\eta\right)^{1-\delta}}\right)^{\frac{(1+\eta\delta)(1-\alpha_b)}{1+\eta}}, \quad \widehat{X} = \left(\prod_{b'\in\mathcal{B}} \widehat{X}_{b'}^{-\theta_{b'}}\right)^{\frac{1+\eta}{\sum_{b'\in\mathcal{B}}\theta_{b'}(1-\alpha_{b'})(1+\eta\delta)}}.$$

Fixed labor.
In the case where employment is fixed at the industry level b, the counterfactual wage (A6) becomes:

$$w'_{io} = \left(\beta_b \lambda_{io} \frac{Z_{io}}{T_{io}^\delta}\right)^{\frac{1}{1+\varepsilon_b\delta}} \widehat{P}_b^{\frac{1}{(1-\alpha_b)(1+\varepsilon_b\delta)}} \Phi'_m{}^{(1-\eta/\varepsilon_b)\frac{\delta}{1+\varepsilon_b\delta}} \left(\frac{\Phi'_b}{L'_b}\right)^{\frac{\delta}{1+\varepsilon_b\delta}}.$$

Fixing lower levels than b would only change the last element. Keeping total employment at the local labor market fixed, the last term would become: $\left(\frac{\Phi'_m}{L'_m}\right)^{\frac{\delta}{1+\varepsilon_b\delta}}$. The constant Γ_b does not appear in this case as workers can't move across industries and the functional Γ_b is the same for all the local labor markets within an industry. Also, fixing lower levels than b clearly implies that L'_b is known and equal to the baseline labor in the industry L_b.

The counterfactuals where employment at b or lower level employment is fixed will give rise to a condition similar to (A8). Given that L'_b is known, we have that:

$$\widehat{P}_b^{\frac{1}{1-\alpha_b}} \widehat{\Omega}_b = \prod_{b'\in\mathcal{B}} \left(\widehat{P}_{b'}^{\frac{\alpha_{b'}}{1-\alpha_{b'}}} \widehat{\Omega}_{b'}\right)^{\theta_{b'}}.$$

Propositions 2 and 4 therefore also apply in the relative counterfactuals with fixed labor at the sector level b (or at a lower level).

B Proofs

Proof of Proposition 1. Substituting the labor supply (12) into (15) and using the restriction $\frac{\beta_b}{1-\alpha_b} = 1-\delta \in [0,1]$, we obtain:

$$w_{io} = \left(\lambda(\mu_{io}, \varphi_b)\beta_b \frac{A_{io}}{(T_{io}\Gamma_b^\eta)^\delta}\right)^{\frac{1}{1+\varepsilon_b\delta}} \Phi_m^{\frac{\delta(1-\eta/\varepsilon_b)}{1+\varepsilon_b\delta}} P_b^{\frac{1}{(1-\alpha_b)(1+\varepsilon_b\delta)}} \left(\frac{\Phi}{L}\right)^{\frac{\delta}{1+\varepsilon_b\delta}}. \tag{B1}$$

Equation (11) implies that in equilibrium the employment share of the establishment-occupation is:

$$s_{io|m} = \frac{T_{io} w_{io}^{\varepsilon_b}}{\sum_{j\in\mathcal{I}_m} T_{jo} w_{jo}^{\varepsilon_b}} = \frac{T_{io}^{\frac{1}{1+\varepsilon_b\delta}} \lambda_{io}^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}} A_{io}^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}}}{\sum_{j\in\mathcal{I}_m} T_{jo}^{\frac{1}{1+\varepsilon_b\delta}} \lambda_{io}^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}} A_{jo}^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}}},$$

where we used equation (B1) in the second step and simplified terms. The solutions of the labor wedge $\lambda_{io}(\mu_{io}, \varphi_b)$ and the markdown come respectively from equations (15) and (14). $\square$

Proof of Proposition 2.

Existence. We follow closely the proof by Kucheryavyy (2012). Define the right hand side of (B1):

$$f_{io}(\mathbf{w}) = [\lambda(\mu(s(\mathbf{w})))]^{\frac{1}{1+\varepsilon_b\delta}} c_{io},$$

where $\mathbf{w}$ denotes the vector formed by $\{w_{io}\}$, we simplified the notation of the wedge $\lambda(\mu_{io}, \varphi_b)$ from the main text by getting rid of the second argument. $c_{io} = \left(\beta_b \frac{A_{io}}{(T_{io}\Gamma_b^{\eta})^{\delta}}\right)^{\frac{1}{1+\varepsilon_b\delta}} \Phi_m^{(1-\eta/\varepsilon_b)\frac{\delta}{1+\varepsilon_b\delta}}$ $P_b^{\frac{1}{(1-\alpha_b)(1+\varepsilon_b\delta)}} \left(\frac{\Phi}{L}\right)^{\frac{\delta}{1+\varepsilon_b\delta}}$ is an establishment-occupation specific parameter. This means we consider Φ_m and Φ as constants and not as functions of w_{io}.

Under the assumption $0 < \eta < \varepsilon_b$, the function $\mu(s) = \frac{\varepsilon_b(1-s)+\eta s}{\varepsilon_b(1-s)+\eta s+1}$ is decreasing in s, the employment share out of the local labor market. Therefore, we can conclude that the wedge $\lambda(\mu(s)) = (1-\varphi_b)\mu(s) + \varphi_b\frac{1}{1-\delta}$ is also decreasing in s. The employment share has bounds $0 \leq s \leq 1$, which implies $(1-\varphi_b)\frac{\eta}{\eta+1} + \varphi_b\frac{1}{1-\delta} \leq \lambda(\mu(s)) \leq (1-\varphi_b)\frac{\varepsilon_b}{\varepsilon_b+1} + \varphi_b\frac{1}{1-\delta}$. Also, $1+\varepsilon_b\delta > 0$. Therefore, it follows that $f_{io}(\mathbf{w})$ is bounded:

$$\left((1-\varphi_b)\frac{\eta}{\eta+1} + \varphi_b\frac{1}{1-\delta}\right)^{\frac{1}{1+\varepsilon_b\delta}} c_{io} \leq f_i(\mathbf{w}) \leq \left((1-\varphi_b)\frac{\varepsilon_b}{\varepsilon_b+1} + \varphi_b\frac{1}{1-\delta}\right)^{\frac{1}{1+\varepsilon_b\delta}} c_{io}.$$

If the number of participants in sub-market m is $N_m > 0$, we can define the compact set S where $f_{io}(\mathbf{w})$ maps into itself as:

$$S = \left[\left((1-\varphi_b)\frac{\eta}{\eta+1} + \varphi_b\frac{1}{1-\delta}\right)^{\frac{1}{1+\varepsilon_b\delta}} c_1, \left((1-\varphi_b)\frac{\varepsilon_b}{\varepsilon_b+1} + \varphi_b\frac{1}{1-\delta}\right)^{\frac{1}{1+\varepsilon_b\delta}} c_1\right] \times \ldots$$
$$\times \left[\left((1-\varphi_b)\frac{\eta}{\eta+1} + \varphi_b\frac{1}{1-\delta}\right)^{\frac{1}{1+\varepsilon_b\delta}} c_{N_m}, \left((1-\varphi_b)\frac{\varepsilon_b}{\varepsilon_b+1} + \varphi_b\frac{1}{1-\delta}\right)^{\frac{1}{1+\varepsilon_b\delta}} c_{N_m}\right].$$

The function $f_{io}(\mathbf{w})$ is continuous in wages on S. We can therefore apply Brouwer's fixed point theorem and claim that at least one solution exists. $\square$

Uniqueness.

In the Supplemental Material we present a Theorem and a Corollary from Allen, Arkolakis, and Li (2016) that we use to establish uniqueness in our proofs.

Define the function $g : \mathbb{R}^n_{++} \to \mathbb{R}^n$ for some $n \in \{1, ..., N\}$ as:

$$g_{io}(\mathbf{w}) = f_{io}(\mathbf{w}) - w_{io}, \quad \forall i \in \{1, .., N_m\}.$$

We want to prove that the solution satisfying $g(\mathbf{w}) = 0$ is unique. In order to do so, we first need to show that $g(\mathbf{w})$ satisfies the gross substitution property ($\frac{\partial g_{io}}{\partial w_{jo}} > 0$ for any $j \neq i$).

Taking the partial derivative of g_{io} with respect to w_{jo} for any $j \neq i$:

$$\frac{\partial g_{io}}{\partial w_{jo}} = \frac{\partial f_{io}(\mathbf{w})}{\partial \lambda(\mu(s(\mathbf{w}))} \times \frac{\partial \lambda(\mu(s_{io|m}))}{\partial \mu(s_{io|m})} \times \frac{\partial \mu(s_{io|m})}{\partial s_{io|m}} \times \frac{\partial s_{io|m}}{\partial w_{jo}},$$

where $\frac{\partial f_{io}(\mathbf{w})}{\partial \lambda(\mu(s(\mathbf{w}))} = \frac{1}{1+\varepsilon_b \delta} \frac{f_{io}(\mathbf{w})}{\lambda(\mu(s(\mathbf{w}))} > 0$. We have that $\frac{\partial \lambda(\mu(s_{io|m}))}{\partial \mu(s_{io|m})} > 0$. Furthermore, we previously established that $\frac{\partial \mu(s_{io|m})}{\partial s_{io|m}} < 0$ under the assumption that $0 < \eta < \varepsilon_b$. The share of an establishment i with occupation o in sub-market m is: $s_{io|m} = \frac{T_{io} w_{io}^{\varepsilon_b}}{\sum_{j \in \mathcal{I}_m} T_{jo} w_{jo}^{\varepsilon_b}}$. Clearly, $\frac{\partial s_{io|m}}{\partial w_{jo}} < 0$ for any $i \neq j$. Therefore $\frac{\partial g_{io}}{\partial w_{jo}} > 0$ for any $i \neq j$ and g satisfies the gross-substitution property required by Theorem 1 in the Supplemental Material.

The remaining condition to prove to use Corollary 1 in the Supplemental Material is simply that $f_{io}(\mathbf{w})$ is homogeneous of a degree smaller than 1.[5] Clearly, $f_{io}(\mathbf{w})$ is homogeneous of degree 0 as a consequence that the markdown function itself $\mu(s_{io|m})$ is homogeneous of degree 0. Therefore, the function g satisfies the conditions of Corollary 1, and we can conclude that there exists at most one solution satisfying $g(\mathbf{w}) = 0$. □

Proof of Proposition 3. Aggregating establishment-occupation output (6)and using the restriction $\frac{\beta_b}{1-\alpha_b} = 1 - \delta \in [0, 1]$, the local labor market output is:

$$Y_m = \sum_{i \in \mathcal{I}_m} y_{io} = P_b^{\frac{\alpha_b}{1-\alpha_b}} \sum_{i \in \mathcal{I}_m} A_{io} L_{io}^{1-\delta} = P_b^{\frac{\alpha_b}{1-\alpha_b}} \sum_{i \in \mathcal{I}_m} A_{io} s_{io|m}^{1-\delta} L_m^{1-\delta} = P_b^{\frac{\alpha_b}{1-\alpha_b}} \Omega_m A_m L_m^{1-\delta},$$

where the local labor market productivity and misallocation are measured as:

$$\Omega_m \equiv \sum_{i \in \mathcal{I}_m} \frac{A_{io}}{A_m} s_{io|m}^{1-\delta}, \quad A_m \equiv \sum_{i \in \mathcal{I}_m} A_{io} \tilde{s}_{io|m}^{1-\delta}, \quad \tilde{s}_{io|m} = \frac{\left(T_{io}^{1/\varepsilon_b} A_{io}\right)^{\frac{\varepsilon_b}{1+\varepsilon_b \delta}}}{\sum_{j \in \mathcal{I}_m} \left(T_{jo}^{1/\varepsilon_b} A_{jo}\right)^{\frac{\varepsilon_b}{1+\varepsilon_b \delta}}}.$$

The definition of $\tilde{s}_{io|m}$ comes from Proposition 1 with constant labor wedges.

Further aggregating to sector level according to (4):

$$Y_b = \sum_{m \in \mathcal{M}_b} Y_m = P_b^{\frac{\alpha_b}{1-\alpha_b}} \sum_{m \in \mathcal{M}_b} \Omega_m A_m L_m^{1-\delta} = P_b^{\frac{\alpha_b}{1-\alpha_b}} \Omega_b A_b L_b^{1-\delta}. \tag{B2}$$

[5] The degree of homogeneity of $h_{io}(\mathbf{w}) = w_{io}$ is 1.

The sector level measures of productivity and misallocation are:

$$
\begin{aligned}
\Omega_b &\equiv \sum_{m\in\mathcal{M}_b} \Omega_m \frac{A_m}{A_b} s_{m|b}^{1-\delta} = \sum_{m\in\mathcal{M}_b}\sum_{io\in\mathcal{I}_m} \frac{A_{io}}{A_b} s_{io|m}^{1-\delta} s_{m|b}^{1-\delta},\\
A_b &\equiv \sum_{m\in\mathcal{M}_b} A_m \tilde{s}_{m|b}^{1-\delta} = \sum_{m\in\mathcal{M}_b}\sum_{io\in\mathcal{I}_m} A_{io} \tilde{s}_{io|m}^{1-\delta} \tilde{s}_{m|b}^{1-\delta},\\
\tilde{s}_{m|b} &= \frac{\left[\sum_{j\in\mathcal{I}_m}\left(T_{jo}^{1/\varepsilon_b} A_{jo}\right)^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}}\right]^{\frac{\eta(1+\varepsilon_b\delta)}{\varepsilon_b(1+\eta)}}}{\sum_{m'\in\mathcal{M}_b}\left[\sum_{j'\in\mathcal{I}_{m'}}\left(T_{j'o}^{1/\varepsilon_b} A_{j'o}\right)^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}}\right]^{\frac{\eta(1+\varepsilon_b\delta)}{\varepsilon_b(1+\eta)}}}.
\end{aligned}
$$

A_b is an employment weighted industry productivity with the employment shares that would arise with constant labor wedges. Similary, Ω_b is an employment weighted sum of productivities where employment shares incorporate the labor wedge normalized by A_b. The covariance between productivities and employment shares is key in order to determine sector productivity. As long as market power distorts the employment distribution making more productive firms to constrain their size, the covariance between productivity and employment is lower than in the case with constant wedges.

Turning to wages, from (15), the establishment wage bill is:

$$
w_{io}L_{io} = \beta_b P_b^{\frac{1}{1-\alpha_b}} \lambda_{io} A_{io} L_{io}^{1-\delta} = \beta_b \lambda_{io} P_b y_{io},
$$

where we used the production function (6). The local labor market wage bill is,

$$
\begin{aligned}
\sum_{i\in\mathcal{I}_m} w_{io}L_{io} &= \beta_b \sum_{i\in\mathcal{I}_m} \lambda_{io} P_b y_{io} = \beta_b \sum_{i\in\mathcal{I}_m} \lambda_{io} \frac{P_b y_{io}}{P_b Y_m} P_b Y_m = \beta_b \lambda_m P_b Y_m,\\
\lambda_m &\equiv \sum_{i\in\mathcal{I}_m} \lambda_{io} \frac{A_{io}}{\Omega_m A_m} s_{io|m}^{1-\delta},
\end{aligned}
$$

where λ_m is a value added weighted sum of establishment labor wedges. Aggregating to the sector,

$$
\begin{aligned}
\sum_{m\in\mathcal{M}_b}\sum_{i\in\mathcal{I}_m} w_{io}L_{io} &= \beta_b \sum_{m\in\mathcal{M}_b} \lambda_m \frac{P_b Y_m}{P_b Y_b} P_b Y_b = \beta_b \lambda_b P_b Y_b,\\
\lambda_b &\equiv \sum_{m\in\mathcal{M}_b} \lambda_m \frac{A_m \Omega_m}{\Omega_b A_b} s_{m|b}^{1-\delta} = \sum_{m\in\mathcal{M}_b}\sum_{i\in\mathcal{I}_m} \lambda_{io} \frac{A_{io}}{\Omega_b A_b} s_{io|m}^{1-\delta} s_{m|b}^{1-\delta}.
\end{aligned}
$$

Aggregate labor share. From the above, the sector labor share is,

$$
LS_b = \beta_b \lambda_b, \quad LS = \frac{\sum_{b\in\mathcal{B}}\sum_{m\in\mathcal{M}_b}\sum_{i\in\mathcal{I}_m} w_{io}L_{io}}{\sum_{b\in\mathcal{B}}\sum_{m\in\mathcal{M}_b}\sum_{i\in\mathcal{I}_m} P_b Y_{io}}. \tag{B3}
$$

Realizing that industry b expenditure share is equal to θ_b, the aggregate labor share is $LS = \sum_{b\in\mathcal{B}} \beta_b \lambda_b \theta_b$. For given parameters, knowing the industry wedges $\{\lambda_b\}_{b=1}^B$ is enough to compute the aggregate labor share. □

Proof of Proposition 4.

Equation (B1) can be separated into two terms. First, a local labor market m constant. Second, an establishment-occupation specific component which is enough to characterize the local equilibrium as shown in Proposition 1. We denote this second term as:

$$\widetilde{w}_{io} = \left(\beta_b \lambda(\mu_{io}, \varphi_b) \frac{A_{io}}{\left(T_{io}\Gamma_b^{\eta}\right)^{\delta}} \right)^{\frac{1}{1+\varepsilon_b \delta}}, \tag{B4}$$

where $\widetilde{w}_{io}$ is a function of the employment shares of all the establishment-occupations in the local labor market equilibrium. The real wage w_{io} is,

$$w_{io} = \widetilde{w}_{io} \Phi_m^{(1-\eta/\varepsilon_b)\frac{\delta}{1+\varepsilon_b \delta}} P_b^{\frac{1}{(1-\alpha_b)(1+\varepsilon_b \delta)}} \left(\frac{\Phi}{L}\right)^{\frac{\delta}{1+\varepsilon_b \delta}}.$$

We can use the definition of $\Phi_m \equiv \sum_{io \in \mathcal{I}_m} T_{io} w_{io}^{\varepsilon_b}$ to find:

$$\Phi_m = \widetilde{\Phi}_m^{\frac{1+\varepsilon_b \delta}{1+\eta\delta}} P_b^{\frac{\varepsilon_b}{(1-\alpha_b)(1+\eta\delta)}} \left(\frac{\Phi}{L}\right)^{\frac{\varepsilon_b \delta}{1+\eta\delta}}, \quad \widetilde{\Phi}_m \equiv \sum_{io \in \mathcal{I}_m} T_{io} \widetilde{w}_{io}^{\varepsilon_b}, \tag{B5}$$

where, as we mentioned before, $\widetilde{w}_{io}$ is a function of the employment share of io and $\widetilde{\Phi}_m$ is a function of the local labor market equilibrium $\{s_{io|m}\}_{io \in \mathcal{I}_m}$ that can be solved separated from aggregates as shown in Proposition 1.

Plugging the expression of Φ_m into the wage,

$$w_{io} = \widetilde{w}_{io} \widetilde{\Phi}_m^{\frac{(\varepsilon_b - \eta)\delta}{\varepsilon_b(1+\eta\delta)}} P_b^{\frac{1}{(1-\alpha_b)(1+\eta\delta)}} \left(\frac{\Phi}{L}\right)^{\frac{\delta}{1+\eta\delta}}. \tag{B6}$$

The establishment-occupation labor supply L_{io} can be written as $L_{io} = s_{io|m} s_{m|b} L_b$. Given the solution of normalized wages per sub-market $\widetilde{w}_{io}$, we can actually compute the employment share out of the local labor market $s_{io|m}$:

$$s_{io|m} = \frac{T_{io} w_{io}^{\varepsilon_b}}{\Phi_m} = \frac{T_{io} \widetilde{w}_{io}^{\varepsilon_b}}{\widetilde{\Phi}_m}, \quad \widetilde{\Phi}_m \equiv \sum_{i \in \mathcal{I}_m} T_{io} \widetilde{w}_{io}^{\varepsilon_b}.$$

We can also compute the employment share of the local labor market out of the industry $s_{m|b}$. Using the definition of $\Phi_b \equiv \sum_{m \in \mathcal{M}_b} \Phi_m^{\eta/\varepsilon_b}$ and (B5),

$$s_{m|b} = \frac{\Phi_m^{\eta/\varepsilon_b}}{\Phi_b} = \frac{\widetilde{\Phi}_m^{\frac{\eta(1+\varepsilon_b \delta)}{\varepsilon_b(1+\eta\delta)}}}{\widetilde{\Phi}_b}, \quad \widetilde{\Phi}_b \equiv \sum_{m \in \mathcal{M}_b} \widetilde{\Phi}_m^{\frac{\eta(1+\varepsilon_b \delta)}{\varepsilon_b(1+\eta\delta)}}.$$

where $\mathcal{M}_b$ is the set of all local labor markets that belong to industry b. This just formalizes the notion that, as long as we know the relative wages within an industry, we can compute the measure of workers that go to each establishment, conditioning on industry employment.

Using (B5), sector labor supply can be written as function of aggregators of 'tilde' variables that are functions of the local employment shares $\widetilde{\Phi}_b(\mathbf{s}_b)$, where $\mathbf{s}_b \equiv \{s_{io|m}\}_{io \in \mathcal{I}_b}$, and prices:

$$L_b = \frac{\Phi_b \Gamma_b^{\eta}}{\sum_{b' \in \mathcal{B}} \Phi_{b'} \Gamma_{b'}^{\eta}} L = \frac{P_b^{\frac{\eta}{(1-\alpha_b)(1+\eta\delta)}} \widetilde{\Phi}_b(\mathbf{s}_b) \Gamma_b^{\eta}}{\widetilde{\Phi}} L, \quad \widetilde{\Phi} \equiv \sum_{b' \in \mathcal{B}} P_{b'}^{\frac{\eta}{(1-\alpha_{b'})(1+\eta\delta)}} \widetilde{\Phi}_{b'}(\mathbf{s}_{b'}) \Gamma_{b'}^{\eta}. \tag{B7}$$

This is where the simplifying assumption on the labor demand elasticity $\delta \equiv 1 - \frac{\beta_b}{1-\alpha_b}$ being constant across industries buys us tractability. We can factor out the economy wide constant from (B5) and leave everything in terms of normalized wages and transformed prices.

In order to find equilibrium allocations, we need to solve for the transformed prices $\mathbf{P} = \{P_b\}_{b=1}^{\mathcal{B}}$. Using the intermediate input demand from the final good producer (3) and the above expression for industry labor supply L_b we get:

$$P_b^{\frac{1+\eta}{(1-\alpha_b)(1+\eta\delta)}} A_b \Omega_b \left(\widetilde{\Phi}_b \Gamma_b^{\eta}\right)^{1-\delta} = \theta_b \prod_{b' \in \mathcal{B}} \left(A_{b'} \Omega_{b'} \left(\widetilde{\Phi}_{b'} \Gamma_{b'}^{\eta}\right)^{1-\delta}\right)^{\theta_{b'}} \prod_{b' \in \mathcal{B}} \left(P_{b'}^{\frac{\alpha_{b'}(1+\eta\delta)+\eta(1-\delta)}{(1-\alpha_{b'})(1+\eta\delta)}}\right)^{\theta_{b'}}. \tag{B8}$$

Define $f_b \equiv \frac{1}{1-\alpha_b} \log(P_b)$ and $\mathbf{f}$ as a $B \times 1$ vector whose element b' is $f_{b'}$. Then, taking logs and rearranging the previous expressions for all $b \in \mathcal{B}$ we obtain:

$$\mathbf{f} = \mathbf{C} + \mathbf{D}\mathbf{f}, \tag{B9}$$

where $\mathbf{C}$ is a $B \times 1$ vector whose b element is

$$(\mathbf{C})_b = \frac{1+\eta\delta}{1+\eta} \left[\log\left(\frac{\theta_b}{A_b \Omega_b}\right) - (1-\delta)\log\left(\widetilde{\Phi}_b \Gamma_b^{\eta}\right) + \sum_{b' \in \mathcal{B}} \theta_{b'} \left(\log(A_{b'} \Omega_{b'}) + (1-\delta)\log(\widetilde{\Phi}_{b'} \Gamma_{b'}^{\eta})\right)\right],$$

and $\mathbf{D}$ is a $B \times B$ matrix whose b row b' column element is:

$$(\mathbf{D})_{bb'} = \frac{(\alpha_{b'}(1+\eta\delta) + \eta(1-\delta))\,\theta_{b'}}{1+\eta}.$$

A solution to the system (B9) exists and is unique if the matrix $\mathbf{I} - \mathbf{D}$ is invertible. This matrix has an eigenvalue of zero, and therefore is not invertible, if and only if $\mathbf{D}$ has an eigenvalue equal to one.[6] The matrix $\mathbf{D}$ has an eigenvalue equal to one if and only if the sum of the elements of the rows in matrix $\mathbf{D}$ are equal to 1. To see this, let $\mathbf{v}$ be the eigenvector associated with the unit eigenvalue of $\mathbf{D}$, i.e. $\mathbf{D}\mathbf{v} = \mathbf{v}$. If $\mathbf{v} = \mathbf{1}$, then, by the Perron-Frobenius theorem, it is the only eigenvector (up-to-scale) associated with the unit eigenvalue. Furthermore, if $\mathbf{v} = \mathbf{1}$, then $\sum_{b'} (D)_{bb'} = 1$ for all $b \in \mathcal{B}$. Conversely, if $\sum_{b'} (D)_{bb'} = 1$ for all $b \in \mathcal{B}$, then $\mathbf{v} = \mathbf{1}$ is a solution for the eigensystem $\mathbf{D}\mathbf{v} = \mathbf{v}$. But, by the Perron-Frobenius theorem, $\mathbf{v} = \mathbf{1}$ is the unique (up-to-scale) eigenvector associated with the unit eigenvalue. Therefore, the matrix $\mathbf{I} - \mathbf{D}$ is not invertible if and only if the sum of the elements of the rows in matrix $\mathbf{D}$ are equal to 1.

[6]Proof: If 1 is an eigenvalue of $\mathbf{D}$, then $\mathbf{D}\mathbf{v} = \mathbf{v}$ for a nonzero vector $\mathbf{v}$. Then $(\mathbf{I} - \mathbf{D})\mathbf{v} = 0$, so 0 is an eigenvalue of $\mathbf{I} - \mathbf{D}$ with the associated eigenvector $\mathbf{v}$. Conversely, if 0 is an eigenvalue of $\mathbf{I} - \mathbf{D}$, then $\mathbf{D}\mathbf{v} = \mathbf{v}$ and 1 is an eigenvalue of $\mathbf{D}$.

This sum is equal to 1 if and only if $\sum_b \alpha_b \theta_b = 1$ as:

$$\begin{aligned}\sum_{b'} (\mathbf{D})_{bb'} = 1 &\Leftrightarrow \sum_{b'} \left(\alpha_{b'}(1+\eta\delta) + \eta(1-\delta)\right)\theta_{b'} = 1+\eta \\ &\Leftrightarrow \sum_{b'} \alpha_{b'}\theta_{b'}(1+\eta\delta) = 1+\eta-\eta(1-\delta) \\ &\Leftrightarrow \sum_{b'} \alpha_{b'}\theta_{b'} = \frac{1+\eta-\eta(1-\delta)}{1+\eta\delta} \Leftrightarrow \sum_{b} \alpha_{b'}\theta'_{b} = 1.\end{aligned}$$

Therefore we can conclude that whenever $\sum_b \alpha_b \theta_b \neq 1$, $\mathbf{f}$ has a unique solution. Also, if $\alpha_b \neq 1$ for all $b \in \mathcal{B}$, then the vector of prices $[P_b]_{b\in\mathcal{B}}$ has a unique solution as well.

Solving for P_b in (B8) we get:

$$P_b = X_b X^{\frac{(1+\eta\delta)(1-\alpha_b)}{1+\eta}}, \quad X_b = \left(\frac{\theta_b}{A_b\Omega_b(\widetilde{\Phi}_b\Gamma_b^{\eta})^{(1-\delta)}}\right)^{\frac{(1+\eta\delta)(1-\alpha_b)}{1+\eta}}, \quad X = \left(\prod_{b'\in\mathcal{B}}\left(\frac{\theta_{b'}}{X_{b'}}\right)^{\theta_{b'}}\right)^{\frac{1+\eta}{(1+\eta\delta)\sum_{b'\in\mathcal{B}}\theta_{b'}(1-\alpha_{b'})}}, \tag{B10}$$

for all $b \in \mathcal{B}$ where we used the aggregate price index $1 = \prod_{b\in\mathcal{B}}\left(\frac{P_b}{\theta_b}\right)^{\theta_b}$ to find the economy wide constant X. The above is the closed-form solution of prices in Proposition 4.

The sector price P_b depends positively on the final good elasticity θ_b, reflecting that a higher demand for goods of sector b will increase its price. It also negatively depends on the product of productivity and misallocation $A_b\Omega_b$ and the labor supply shifter for sector b, Γ_b. An increase in any of both terms translates into more supply of sector b goods, either by being more productive or by increasing the labor employed in sector b. This in turn would reduce its price. □

C Identification and estimation

C.1 Identification of common parameters η and δ

In order to identify the across markets labor supply elasticity η and the labor demand elasticity δ we exploit the fact that in local labor markets where there is only one establishment, the wedge $\lambda(\mu, \phi_b)$ is constant within industries b. We denominate this type of establishments as *full monopsonists.* Additionally, the effect of wages on the labor supply of full monopsonists is only affected by the parameter η as the within market labor supply elasticity ε_b is irrelevant in local labor markets with only one establishment. Taking the logarithm for the labor supply that full monopsonists face (12), we get:

$$\ln(L_{io,s=1}) = \eta\ln(w_{io}) + \ln(\tilde{T}_{io}) + \ln(\Gamma_b^{\eta}L/\Phi),$$

where $\tilde{T}_{io} = T_{io}^{\eta/\varepsilon_b}$. As mentioned in the main text, full monopsonists apply a constant markdown equal to $\mu(s=1) = \frac{\eta}{\eta+1}$ that in turn will imply a constant wedge $\lambda(\mu, \varphi_b)$ within industry b. Their labor demand (15) in logs is:

$$\ln(w_{io,s=1}) = \ln(\beta_b) + \ln\left((1-\varphi_b)\frac{\eta}{\eta+1} + \varphi_b\frac{1}{1-\delta}\right) + \ln(A_{io}) - \delta\ln(L_{io}) + \frac{1}{1-\alpha_b}\ln(P_b).$$

In order to get rid of industry and economy wide constants, we demean $\ln(L_{io,s=1})$ and $\ln(w_{io,s=1})$ by removing the industry b averages per year. Denoting with $\overline{\ln(X)}$ the demeaned variables, we rewrite the labor supply and demand equations as:

$$\begin{aligned} \overline{\ln(L_{io})} &= \eta\,\overline{\ln(w_{io})} + \overline{\ln(\tilde{T}_{io})}, \\ \overline{\ln(w_{io})} &= -\delta\,\overline{\ln(L_{io})} + \overline{\ln(A_{io})}. \end{aligned} \tag{C1}$$

The above system is a traditional demand and supply setting and as it is well known, is under-identified. It is the classic textbook example of simultaneity bias. The reason for this under-identification is the following: while the variance-covariance matrix of $\left(\overline{\ln(L_{io})}, \overline{\ln(w_{io})}\right)$ gives us three moments from the data, the system above has five unknowns, which are the elasticities, η and δ, plus the three components of the variance-covariance matrix of the structural errors $\overline{\ln(\tilde{T}_{io})}$ and $\overline{\ln(A_{io})}$. Therefore, in absence of valid instruments that would exogenously vary either the supply or demand equations in (C1) we can not identify the elasticities through exclusion restrictions.

In order to identify the elasticities using the labor supply and demand equations in (C1), we impose restrictions on the variance-covariance matrix of the structural errors while exploiting the differences in the variance-covariance matrix of the employment and wages across occupations. This way of achieving identification is known in the literature as *identification through heteroskedasticity* (see Rigobon (2003)). We classify our four occupations into two broader categories $S \in \{1, 2\}$ which we denote as blue collar and white collar. Our identification assumption is that the covariance between the transformed productivity $\overline{\ln(A_{io})}$ and amenities $\overline{\ln(\tilde{T}_{io})}$, that we denote σ_{TA}, is constant within each category S. The fact that the elasticities are the same across occupational groups within the categories, in addition to the assumption of common covariance of the structural errors within broad categories, are the reason we can achieve identification. While the four occupational categories give us $3 \times 4 = 12$ moments, the unknowns to be identified are also 12: 2, δ and η, plus 2, the broad category covariances, plus 8, the variances of the transformed productivities and amenities for each of the four occupational categories.[7]

We can rewrite the system (C1) in the following way:

$$\begin{aligned} \overline{\ln(\tilde{T}_{io})} &= \overline{\ln(L_{io})} - \eta\,\overline{\ln(w_{io})}, \\ \overline{\ln(A_{io})} &= \delta\,\overline{\ln(L_{io})} + \overline{\ln(w_{io})}. \end{aligned} \tag{C2}$$

Denote the covariance matrix of the structural errors for occupation o in category S (meaning the left hand side of system (C2)) by E_{oS}. Denote the covariance matrix between employment and wages of the full monosponists by Ω_{oS}. The covariance of system (C2) writes as:

$$E_{oS} = D\Omega_{oS}D^T, \quad D = \begin{pmatrix} 1 & -\eta \\ \delta & 1 \end{pmatrix}, \quad \Omega_{oS} = \begin{pmatrix} \sigma^2_{L,oS} & \sigma_{LW,oS} \\ \sigma_{LW,oS} & \sigma^2_{W,oS} \end{pmatrix},$$

[7] Of course we could have a more stringent identification assumption that would leave us with an overidentified system, for example, that all covariances are equal to zero. As an additional exercise we also estimated the parameters following a different identification strategy: we assume that the covariances of the structural errors were the same among all the occupational groups. This gives us a system with one overidentification restriction. The point estimates using this assumption and the one we mentioned above are pretty similar.

where D^T denotes the transpose of matrix D. Defining an auxiliary parameter $\widetilde{\delta} = -\delta$ and using our identifying assumption that $\sigma_{AT,oS} = \sigma_{AT,o'S} = \sigma_{AT,S}$ for occupations that belong to the same category S, the system writes as:

$$\begin{pmatrix} \sigma^2_{T,oS} & \sigma_{TA,S} \\ \sigma_{TA,S} & \sigma^2_{A,oS} \end{pmatrix} = \begin{pmatrix} 1 & -\eta \\ -\widetilde{\delta} & 1 \end{pmatrix} \begin{pmatrix} \sigma^2_{L,oS} & \sigma_{LW,oS} \\ \sigma_{LW,oS} & \sigma^2_{W,oS} \end{pmatrix} \begin{pmatrix} 1 & -\widetilde{\delta} \\ -\eta & 1 \end{pmatrix}$$

This system only allows us to identify η and δ. Denote by $\Omega_S \equiv \Omega_{oS} - \Omega_{o'S}$ the difference between the variance covariance matrix within category S, $\Delta_S \equiv E_{oS} - E_{o'S}$, and $\Omega_{S,[i,j]} = \omega_{ij,S}$ the element on ith row and jth column of Ω_S. The system of differences is:

$$\Delta_S = D\Omega_S D^T, \quad \forall S \in \{1,2\}$$

With the identification assumption of equal covariance within category, we have that:

$$\Delta_{S,[1,2]} = 0 = -\eta\omega_{22,S} + (1+\eta\widetilde{\delta})\omega_{12,S} - \widetilde{\delta}\omega_{11,S}.$$

Solving for η,

$$\eta = \frac{\omega_{12,S} - \widetilde{\delta}\omega_{11,S}}{\omega_{22,S} - \widetilde{\delta}\omega_{12,S}}, \quad \forall S \in \{1,2\}$$

Equalizing the above across both occupation categories we get a quadratic equation in $\widetilde{\delta}$ that solves:

$$\widetilde{\delta}^2[\omega_{11,1}\omega_{12,2} - \omega_{11,2}\omega_{12,1}] - \widetilde{\delta}[\omega_{11,1}\omega_{22,2} - \omega_{11,2}\omega_{22,1}] + \omega_{12,1}\omega_{22,2} - \omega_{12,2}\omega_{22,1} = 0. \tag{C3}$$

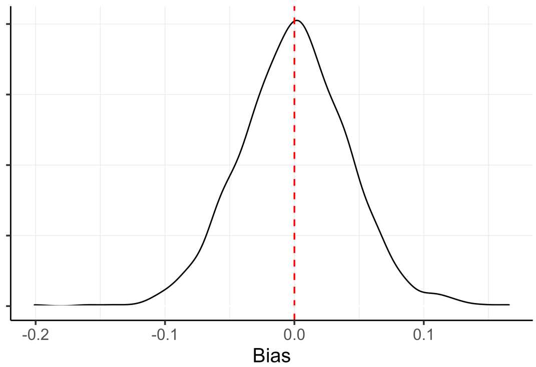
This is the same system as the simple case with zero covariance between the fundamental shocks in Rigobon (2003). Different to him, Ω_S is not directly the estimated variance-covariance matrix of each of the 4 occupations but rather the matrix of covariance differences within category or state S. As mentioned by Rigobon (2003) there are two solutions to the previous equation. One can show that if $\widetilde{\delta}^*$ and η^* are a solution then the other solution is equal to $\widetilde{\delta} = 1/\eta^*$ and $\eta = 1/\widetilde{\delta}^*$. This means that the solutions are actually the two possible ways the original structural system (C1) can be written. We have that by assumption η is positive while $\widetilde{\delta}$ is negative. Therefore as long as the two possible solutions for $\widetilde{\delta}$ have different signs, we just need to pick the negative one.

C.2 Validation of the identification of ε_b

In this section, we validate our identification strategy of the within-labor market labor supply elasticities via simulations. We perform 1000 simulations of an economy populated with 200 local labor markets. For each simulation we have 14 years as in the application. The number of competitors in the local labor market follows an exponential distribution with mean 4 and standard deviation of 1, and the logarithm of productivities and amenities are normally distributed with means of 1 for both and standard deviations of 0.8 and 0.1 respectively. Population is assumed to be symmetrically distributed across local labor markets. We simulate productivities, amenities and number of competitors in local labor markets of the *Food* sector. We solve for each local labor market independently of aggregates and therefore characterize $w_{io} = \left(T_{io}^{\frac{1}{\varepsilon_b}} \lambda_{io} A_{io}\right)^{\frac{1}{1+\varepsilon_b \delta}}$ for each establishment.

We estimate equation (23) in the simulated equilibrium by regressing the logarithm of establishment employment on the logarithm of wages. We control for the strategic interactions within the local labor market by introducing local labor market fixed effects and therefore only use within-local labor market variation to identify the local elasticity of substitution. Figure C1 presents the bias of the IV estimates when we instrument for contemporaneous log wages by a proxy of establishment revenue productivity: $\widehat{A}_{iot} = \frac{P_{bt} Y_{jt}}{L_{iot}^{1-\delta}}$. The figure shows that even in the presence of amenities, which are labor supply shifters that correlate with wages, our identification strategy recovers the local elasticities of substitution as the density is centered around 0.

Figure C1: Bias of estimated ε_b



Note: The figure presents the estimation results from simulating local labor markets of sector 15 *Food*. It shows the density of the difference between the estimated local elasticity of substitution and the true parameter when simulating the model.

C.3 Identification of φ_b

In order to identify the sector specific workers bargaining power, we need to construct the model counterparts of the industry labor share at every period t:

$$LS_{bt}^{M}(\varphi_b) = \frac{\beta_b \sum_{io \in \mathcal{I}_b} w_{iot} L_{iot}}{\sum_{io \in \mathcal{I}_b} w_{iot} L_{iot} / \lambda(\mu_{io}, \varphi_b)},$$

$\mathcal{I}_b$ being the set of all establishment-occupations that belong to sector b. We target the average across time industry labor share. That is, we pick ϕ_b such that:

$$\mathbb{E}_t \left[LS_{bt}^{M}(\varphi_b) - LS_{bt}^{D} \right] = 0, \tag{C4}$$

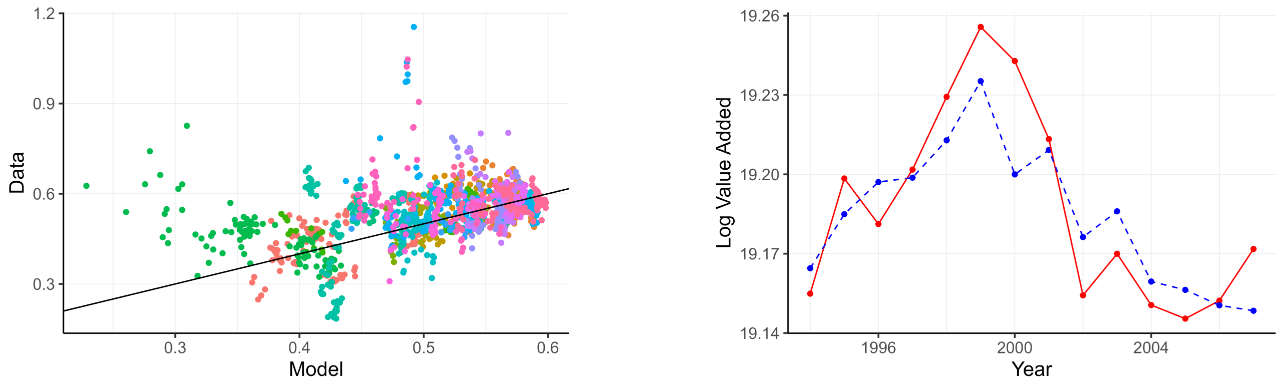
where LS_{bt}^{D} is the labor share of sector b at time t that we observe in the data. Given that the wedge $\lambda(\mu_{io}, \varphi_b)$ is increasing in φ_b, then $LS_{bt}^{M}(\varphi_b)$ is increasing in φ_b as well. Therefore, if a solution exists for (C4) with $\varphi_b \in [0, 1]$ this has to be unique.[8]

C.4 Amenities

In order to perform counterfactuals we still need to compute other policy invariant parameters, or fundamentals, from the data. In particular we need to recover establishment-occupation amenities

[8]It can be the case that the solution does not exist. For example, if given values of β_b, ε_b and η, even with $\varphi_b = 1$ the labor share generated by the model is too small to the one in the data.

Figure C2: Model Fit Non Targeted Moments



(a) Sub-industry Labor Share

(b) Aggregate Value Added. Model in dashed blue, data in red.

and TFPRs, while ensuring that in equilibrium the wages and labor allocations are the same as in the data.

Using the establishments labor supply (12), we can back out amenities, up to a constant:

$$T_{io} = \frac{s_{io|m}}{(w_{io})^{\varepsilon_b}} \Phi_m.$$

The sub-market level Φ_m is a function of the amenities of all plants in m. We proceed by normalizing one particular local labor market. Note that the allocation of resources is independent from this normalization. We denote the local labor market that we normalize as 1. The relative employment share of market m with respect to the normalized one is: $\frac{L_m}{L_1} = \frac{\Phi_m^{\eta/\varepsilon_b}}{\Phi_1^{\eta/\varepsilon_{b'}}} \frac{\Gamma_b}{\Gamma_1}$. The local labor market aggregate is then: $\Phi_m = \left(\frac{L_m}{L_1} \frac{\Gamma_1}{\Gamma_b} \Phi_1^{\frac{\eta}{\varepsilon_{b'}}} \right)^{\frac{\varepsilon_b}{\eta}}$. Substituting into the above we have that:

$$T_{io} \propto \frac{s_{io|m}}{(w_{io})^{\varepsilon_b}} \left(\frac{L_m}{\Gamma_b} \right)^{\varepsilon_b/\eta}.$$

C.5 Non targeted moments

In panel (a) of Figure C2 we have 3-digit industry labor shares per year. On the horizontal axis, we have the model generated moments, while on the vertical axis, we have the corresponding observed moment in the data. If the fit was perfect, each dot would be on the 45 degree line. Each color represents a 2-digit industry. We see that most of the dots are aligned around the 45 degree line.

Panel (b) shows the model matches the evolution of aggregate value added. This in fact might not be surprising as there is a very strong relationship between establishment's production and wage bill in the model and in the data. Since the model matches the establishment's wages and labor allocations exactly, it also has a good fit of the value added.

C.6 Additional estimation results

Table C1 presents the estimated output elasticities with respect to labor, within industry elasticities, the workers' bargaining power for every 2-digit industry, the elasticities of the final good production function and the rental rates of capital for 2007.

Table C1: Sector Estimates

Sector Code	Sector Name	$\widehat{\beta}_b$	$\widehat{\varepsilon}_b$	$\widehat{\varphi}_b$	θ_b	R_b
15	Food	0.74	1.69	0.22	0.13	0.11
17	Textile	0.74	1.49	0.51	0.02	0.14
18	Clothing	0.84	1.41	0.31	0.01	0.14
19	Leather	0.85	2.09	0.26	0.01	0.14
20	Wood	0.77	1.51	0.42	0.02	0.13
21	Paper	0.61	3.06	0.55	0.02	0.13
22	Printing	0.84	1.52	0.18	0.06	0.13
24	Chemical	0.67	3.25	0.06	0.14	0.16
25	Plastic	0.73	2.51	0.35	0.06	0.15
26	Other Minerals	0.65	1.62	0.43	0.05	0.15
27	Metallurgy	0.61	3.77	0.59	0.03	0.14
28	Metals	0.81	1.22	0.38	0.10	0.14
29	Machines and Equipments	0.79	2.18	0.32	0.09	0.17
30	Office Machinery	0.81	3.33	0.20	0.00	0.17
31	Electrical Equipment	0.65	3.02	0.67	0.04	0.23
32	Telecommunications	0.62	3.54	0.73	0.04	0.23
33	Optical Equipment	0.75	1.91	0.45	0.04	0.23
34	Transport	0.57	4.05	0.69	0.04	0.19
35	Other Transport	0.72	3.49	0.44	0.06	0.19
36	Furniture	0.81	1.57	0.43	0.03	0.14

Notes: All the estimated parameters are 2-digit sector specific. $\widehat{\beta}_b$ are the estimated output elasticities with respect of labor, $\widehat{\varepsilon}_b$ are the within local labor market elasticities and $\widehat{\varphi}_b$ are union bargaining powers. θ_b are the intermediate good elasticities in the final good production function and R_b are the capital rental rates for 2007. We construct the rental rates following Barkai (2020).

We calibrate the elasticities of the final good production function $\{\theta_b\}_{b\in\mathcal{B}}$ for every year of the sample such that the industry expenditure shares are equal to the shares of industry value added in the data. Table C1 has the calibrated final good production function elasticities of the intermediate good $\{\theta_b\}_{b=1}^{\mathcal{B}}$ and the rental rate of capital $\{R_b\}_{b=1}^{\mathcal{B}}$ for the year 2007.

In the Supplemental Material we have additional estimation results that compare different estimates of the within local labor market labor supply elasticities with two-period lagged instruments. The results are similar to our baseline estimation with a one-period lagged instrument.

D Counterfactuals

We present additional results on the implications of labor market power on urban-rural differences.

D.1 The effect of labor market power on urban-rural differences

Figure 4 in the main text suggests an important labor reallocation from rural areas to cities in the counterfactual without unions. This section explores the impact of employer and union labor market power on the urban-rural wage gap. In the Supplemental Material we further explore the effect on employment changes over time.

Table D1 presents wage levels and the urban-rural wage gap.[9] Both urban and rural areas experience important wage gains in the counterfactual. Gains being bigger outside cities the wage

[9]We consider urban the 10 biggest commuting zones: Paris, Marseille, Lyon, Toulouse, Nantes, and the Paris surrounding areas of Boulogne-Billancourt, Creteil, Montreuil, Saint-Denis and Argenteuil. Rural is the rest of the commuting zones.

Table D1: Wage Gap

	Rural Wage	Urban Wage	Gap (%)
Baseline	33.319	45.210	36
Counterfactual. Oligopsony	24.592	36.861	50
Counterfactual. No wedges	49.486	60.675	23

Note: Wages in constant 2015 euros. We classify as *Urban* the 10 biggest commuting zones: Paris, Marseille, Lyon, Toulouse, Nantes, and the Paris surrounding areas of Boulogne-Billancourt, Creteil, Montreuil, Saint-Denis and Argenteuil. The rest are considered as *Rural*. Wages are employment weighted averages per urban/rural location in 2007.

gap is reduced from 36% to 23% in the counterfactual. This reveals that labor market distortions account for more than a third of the urban-rural wage gap.

E Empirical evidence

In this section, we provide the link between the reduced form relating labor market power to wages and our structural framework. We also present additional results, robustness checks and results on rent sharing elasticities.

E.1 Labor market power and wages

E.1.1 Instrument: Mass layoff shock

The mass layoff shock instrument we use intends to capture the effect of a negative idiosyncratic productivity shock on close competitors. To provide some intuition on how the instrument works, it will be helpful to focus on a local labor market with only 2 competitors. Using Proposition 1 and getting rid of the occupational subscript o and assuming constant amenities for simplicity, the employment share of establishment 1 is:

$$s_{1|m} = \frac{\left(\lambda(s_{1|m})A_1\right)^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}}}{\left(\lambda(s_{1|m})A_1\right)^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}} + \left(\lambda(1-s_{1|m})A_2\right)^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}}},$$

where the numerator is the aggregator Φ_m. This equation completely characterizes the equilibrium in the local labor market as the employment share of the other establishment in the market is equal to $1 - s_{1|m}$.

The above equation implicitly defines $s_{1|m}$ as a function of A_2 and $\lambda(g(s_{1|m}))$, where $g(s_{1|m}) = s_{1|m}$ or $g(s_{1|m}) = 1 - s_{1|m}$. We can represent the above system as: $F\left(s_{1|m}, A_2, \lambda(g(s_{1|m}))\right)$. Using the implicit function theorem we have that: $\frac{ds_{1|m}}{dA_2} = -\frac{\frac{\partial F(\cdot)}{\partial A_2}}{\frac{\partial F(\cdot)}{\partial s_{1|m}}}$. Developing the partial derivatives, we get:

$$\frac{\partial F(\cdot)}{\partial A_2} = -\Phi_m^2 \frac{\varepsilon_b}{1+\varepsilon_b\delta}\lambda(1-s_{1|m})^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}} A_2^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}-1} < 0,$$

and,

$$\frac{\partial F(\cdot)}{\partial s_{1|m}} = \Phi_m^{-2} \frac{\varepsilon_b}{1+\varepsilon_b\delta} \frac{\partial \lambda(s_{1|m})}{\partial s_{1|m}} \left(\lambda(s_{1|m})A_1\right)^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}} \lambda(s_{1|m})^{-1}\Phi_m - \left(\lambda(s_{1|m})A_1\right)^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}} \frac{\varepsilon_b}{1+\varepsilon_b\delta}\Phi_m^{-2}$$

$$\left[\left(\lambda(s_{1|m})A_1\right)^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}} \lambda(s_{1|m})^{-1}\frac{\partial \lambda(s_{1|m})}{\partial s_{1|m}} - \left(\lambda(1-s_{1|m})A_2\right)^{\frac{\varepsilon_b}{1+\varepsilon_b\delta}} \lambda(1-s_{1|m})^{-1}\frac{\partial \lambda(1-s_{1|m})}{\partial(1-s_{1|m})}\right] - 1$$

$$= \frac{\varepsilon_b}{1+\varepsilon_b\delta} s_{1|m} \left\{\frac{\partial \lambda(s_{1|m})}{\partial s_{1|m}}\lambda(s_{1|m})^{-1} - \left[s_{1|m}\frac{\partial \lambda(s_{1|m})}{\partial s_{1|m}}\lambda(s_{1|m})^{-1} - (1-s_{1|m})\lambda(1-s_{1|m})^{-1}\frac{\partial \lambda(1-s_{1|m})}{\partial(1-s_{1|m})}\right]\right\} - 1$$

$$= \frac{\varepsilon_b}{1+\varepsilon_b\delta} s_{1|m}(1-s_{1|m}) \left\{\frac{\partial \lambda(s_{1|m})}{\partial s_{1|m}}\lambda(s_{1|m})^{-1} + \lambda(1-s_{1|m})^{-1}\frac{\partial \lambda(1-s_{1|m})}{\partial(1-s_{1|m})}\right\} - 1 < 0,$$

where we used the expression of the employment share above and the fact that $\frac{\partial \lambda(s_{1|m})}{\partial s_{1|m}} < 0$ and $\frac{\partial \lambda(1-s_{1|m})}{\partial(1-s_{1|m})} < 0$. We therefore have that: $\frac{ds_{1|m}}{dA_2} < 0$. In turn, abstracting from market level constants, $\log(w_1) = \left(\lambda(s_{1|m})A_1\right)^{\frac{1}{1+\varepsilon_b\delta}}$. The effect of a change in A_2 on $\log(w_1)$ is:

$$\begin{aligned} \frac{d\log(w_1)}{dA_2} &= \frac{\partial \log(w_1)}{\partial A_2} + \frac{\partial \log(w_1)}{\partial s_{1|m}}\frac{ds_{1|m}}{dA_2} \\ &= \quad 0 \quad + \frac{\partial \log(w_1)}{\partial s_{1|m}}\frac{ds_{1|m}}{dA_2} \\ &= \underbrace{\frac{\partial \log(w_1)}{\partial \log(\lambda(s_{1|m}))}}_{>0}\underbrace{\frac{\partial \log(\lambda(s_{1|m}))}{\partial s_{1|m}}}_{<0}\underbrace{\frac{ds_{1|m}}{dA_2}}_{<0} > 0. \end{aligned}$$

Therefore, when a shock occurs to a competitor's productivity, the covariance between employment shares and log wages becomes negative. If we use an IV regression, we can identify the reduced-form effect. However, the reduced-form effect would be different from the structural estimate obtained when there is a change in the employment share—triggered by a change in a competitor's productivity—while holding everything else constant. This is because, as explained in section 5.2 of the main text, strategic interactions can trigger responses from other market participants, which changes the underlying environment. However, as explained by Berger, Herkenhoff, and Mongey (2022), the reduced-form estimate is still informative of the structural response. Reassuringly, our reduced-form estimate provides the same qualitative result as the structural one: a negative relation between employment share and log wages after a competitor's shock.

Definition of a mass layoff. The definition of a mass layoff is firm-occupation specific. Denote by ML the set of firm-occupations with a *national* mass layoff. That is, firm-occupations with all the establishments suffering a mass layoff. We instrument the employment share of the establishments of firm-occupations not suffering the national mass layoff $j \notin ML$ by the exogenous event of a firm present at the local labor market having a negative shock. We restrict the analysis to non-shocked multi-location firm-occupations with at least one establishment in a sub-market where a competitor has suffered a mass layoff and another establishment whose competitors do not belong to firms in ML.

Defining a cut-off value κ, we identify a firm-occupation $j \in ML$ if employment at t is less than κ% employment last year for all the establishment-occupations. That is, a firm j at occupation o has a mass layoff shock if $L_{io,t}/L_{io,t-1} < \kappa \, \forall i$ belonging to firm j. A local labor market is identified as shocked $D_{m,t} = 1$ if at least one establishment at the local market belongs to a firm in ML.

The first stage is:

$$s_{io|m,t} = \psi_{\mathbf{J}(i),o,t} + \delta_{\mathbf{N}(i),t} + \gamma \, D_{m,t} + \epsilon_{io,t}$$

where as before, $\psi_{\mathbf{J}(i),o,t}$ is a firm-occupation-year fixed effect and $\delta_{\mathbf{N}(i),t}$ is a commuting zone times year fixed effect. Using the fitted values we consider the following model for the second stage:

$$\log(w_{io,t}) = \psi_{\mathbf{J}(i),o,t} + \delta_{\mathbf{N}(i),t} + \alpha \, \widehat{s_{io|m,t}} + u_{io,t} \tag{E1}$$

E.1.2 Robustness checks

This section presents robustness checks of the reduced form evidence. First, we consider a different instrument for the employment shares and we change the main specification by taking commuting zone fixed effects. The results in the main text are with commuting zone-year fixed effects. Second, we present a robustness check to a different definition of local labor markets.

Instrument. Panel (a) of Figure E1 shows a robustness check where the new instrument is not binary anymore and takes into account the original employment share of the mass layoff establishments. Panel (b) of the same figure shows the results using the main text speciication but with commuting zone fixed effects. Results are qualitatively unchanged from the baseline in both cases.

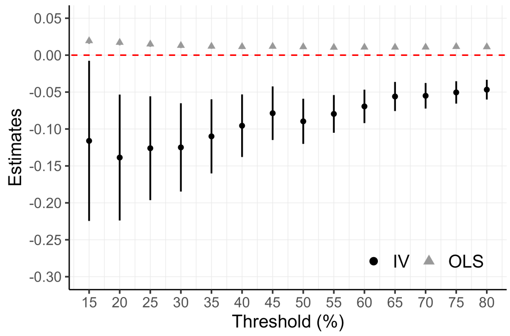
Controlling for labor demand. When there are decreasing returns to scale, establishments would have a demand with a negative slope. Thus, an increase in the employment level could lead to wage reductions if there is movement along the labor demand curve. To take into account the potential effects of changes along the labor demand curve after the mass-layoff shock, we control for the logarithm of establishment-occupation employment level as in the following model:

$$\log(w_{io,t}) = \beta s_{io|m,t} + \gamma \log(L_{io,t}) + \psi_{\mathbf{J}(i),o,t} + \delta_{\mathbf{N}(i),t} + \epsilon_{io,t} \, , \tag{E2}$$

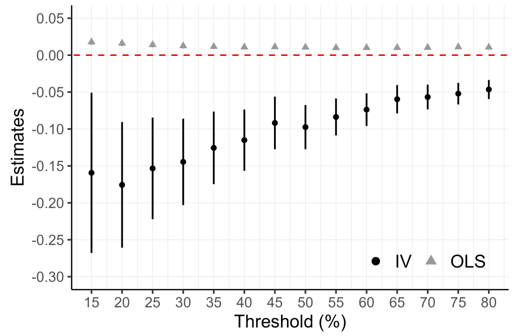
where $\log(w_{io,t})$ is the log average wage at plant i of firm j and occupation o at local labor market m in year t, $s_{io|m,t}$ is the employment share of the plant out of the market m, $\log(L_{io,t})$ is the logarithm of the establishment-occupation employment, $\psi_{\mathbf{J}(i),o,t}$ is a firm-occupation-year fixed effect, $\delta_{\mathbf{N}(i),t}$ is a commuting zone-year fixed effect and $\epsilon_{io,t}$ is an error term. Our parameter of interest is β.

There are two potentially endogenous variables, $s_{io|m,t}$ and $\log(L_{io,t})$, so we follow two approaches. First, we instrument $s_{io|m,t}$ with the presence of mass-layoff shocks in the local labor market and add the contemporaneous logarithm of employment as a control. Even if this last instrument would not satisfy the standard exclusion restriction, we can still get a consistent estimate of β with a different conditional mean independence assumption. To see this, let Z be the mass-layoff shock instrument, and W is the vector of controls, which includes the logarithm of

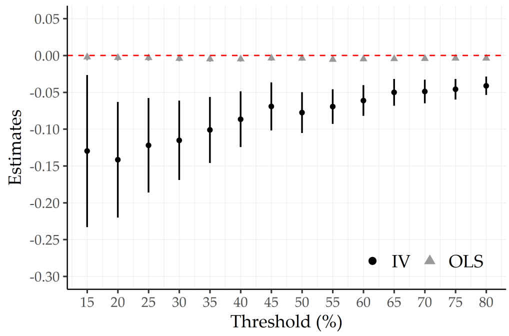
Figure E1: Robustness



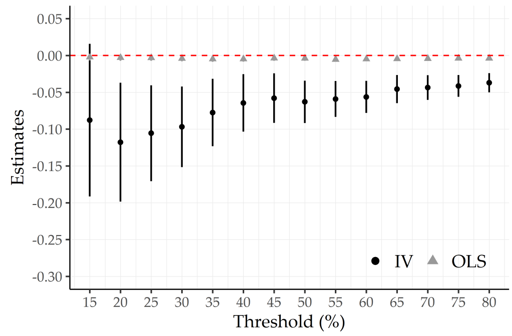
(a) Instrument: Intensive Share



(b) CZ fixed effects



(c) Control $\log(L_{io,t})$



(d) Control $\log(L_{io,t})$. Instrument $\log(L_{io,t-1})$

Notes: These figures present the point estimates and 95% confidence bands of the OLS and IV exercises on the y-axis. The x-axis presents different thresholds κ that define a mass layoff shock. In all cases we focus on non-affected competitors (not suffering a mass layoff shock). The instrument in Panel (a) is the presence of a mass layoff shock firm in the local labor market interacted with the employment share of the affected firm. Panel (b) presents the results with commuting zone fixed effects. For Panels (c) and (d) the specification is equation (E2). The figure in Panel (c) controls directly for $\log(L_{io})$, while Panel (d) instruments the logarithm of employment with its lagged value.

employment and the fixed effects. We have abstracted from subscripts to ease on notation. Then, if $\mathbb{E}(\epsilon|Z,W) = \mathbb{E}(\epsilon|W) = W\xi$ we can still obtain a consistent estimate of β using instrumental variables.[10] In the second approach, we use lagged values of the employment logarithm as an instrument instead of its contemporaneous value. In Panel (c) of Figure E1 we present the estimates for β estimating the model (E2) using the first approach. In Panel (d), we do the same but using the second approach.

Alternative instrument. We build an additional instrument for the employment share by lagged concentration measures. More specifically, we instrument the employment share $s_{io|m,t}$ by the lagged inverse of the number of competitors in the local labor market $1/N_{m,t-1}$. Lagged concentration measures exclude potential endogeneity of the market structure to current period shocks. The correlation between employment shares and lagged concentration measures is 0.77.

Table E1 shows the results. The first two columns recover estimates of the specification (1) with commuting zone (CZ) fixed effects and the last two columns with commuting zone-year fixed effects. Columns 1 and 3 present the Ordinary Least Squares (OLS) estimates. The model reflects both labor demand and supply therefore a direct estimation by OLS is problematic and expected

[10]Proof: Let the original regression be $y = \beta s + W\widetilde{\gamma} + \epsilon$. Then, assume that $\mathbb{E}(\epsilon|Z,W) = \mathbb{E}(\epsilon|W) = W\xi$. This implies that $y = \beta s + W\widetilde{\gamma} + \epsilon - \mathbb{E}(\epsilon|W) + \mathbb{E}(\epsilon|W) = \beta s + W(\widetilde{\gamma} + \xi) + \widetilde{\epsilon}$, where $\widetilde{\epsilon} = \epsilon - \mathbb{E}(\epsilon|W)$. Then $\mathbb{E}(\widetilde{\epsilon}|Z,W) = \mathbb{E}(\epsilon|Z,W) - \mathbb{E}(\epsilon|W) = \mathbb{E}(\epsilon|Z,W) - \mathbb{E}(\epsilon|Z,W) = 0$. Thus, an IV regression can obtain consistent estimates of β and $(\widetilde{\gamma} + \xi)$.

Table E1: Wage Regression. Multilocation firm-occupations

	Dependent variable: $log(w_{io,t})$			
	OLS	IV	OLS	IV
$s_{io\|m,t}$	0.010***	−0.030***	0.007***	−0.030***
	(0.001)	(0.002)	(0.001)	(0.002)
Firm-occ-year FE	Y	Y	Y	Y
CZ FE	Y	Y	N	N
CZ-year FE	N	N	Y	Y
Observations	792,656	733,576	792,656	733,576
R^2	0.833	0.861	0.853	0.862

Notes: The instruments in this table are lagged concentration measures $1/N_{m,t-1}$. Columns 1 and 2 present estimates with commuting zone (CZ) fixed effects for the ordinary least squares (OLS) and instrumental variable (IV) exercises. Columns 3 and 4 present the analogous with commuting zone-year fixed effects. The dependent variable $log(w_{io,t})$ is the logarithm of establishment-occupation wage at time t. $s_{io|m,t}$ is the establishment-occupation employment share at time t. *p<0.1; **p<0.05; ***p<0.01

to be biased towards zero. We indeed find that both OLS estimates are very close to zero and positive. Columns 2 and 4 present the results once we instrument for the employment share. Both specifications (with CZ and CZ-year fixed effects) give the same point estimates. These estimates imply that an increase of one percentage point (p.p. henceforth) of the local labor market share is associated with a decrease of 0.03% of the plant wage. This implies that the same establishment passing from the first to the third quartile of the employment share distribution reduces wages by 0.68%. This elasticity translates into a reduction of roughly 190 euros of the median yearly establishment-occupation wage.

E.2 Labor market concentration and the labor share

We follow similarly to the literature by establishing the relationship between aggregate concentration measures and the labor share. A standard measure of concentration is the Herfindahl-Hirschman Index (HHI). From our definition of local labor market m, the HHI of market m at time t, HHI_{mt}, is the sum of the squared employment shares of the plants present in m at a given year. The labor share at the 3-digit industry level, LS_{ht}, is the ratio of the wage bill over value added at time t. Due to data restrictions of observing value added only at the firm level, we cannot compute labor shares at the local labor market level. We therefore build a sub-industry concentration index $\overline{HHI}_{ht}$ by taking the employment weighted mean of HHI_{mt} across different local labor markets.[11]

We run the following linear regression:

$$\log(LS_{h,t}) = \delta_{b,t} + \beta \log(\overline{HHI}_{h,t}) + \varepsilon_{h,t}. \tag{E3}$$

Table E2 presents the results which indicate that more concentrated sub-industries have a lower labor share. Sector fixed effects capture differences in the usage of capital. The focus of the paper being the cross sectional allocation of resources we also take sector-year fixed effects to use

[11]The HHI index at market m and year t is: $HHI_{mt} = \sum_{i\in\mathcal{I}_{m,t}} s^2_{io|m,t}$ where shares at the market are accounted as shares of full time equivalent employees and $\mathcal{I}_{m,t}$ is the set of all firms in the sub-market m at year t. The sub-industry concentration index $\overline{HHI}_{ht}$ is:

$$\overline{HHI}_{ht} = \frac{1}{|\mathcal{M}_{ht}|} \sum_{m\in\mathcal{M}_{ht}} HHI_{mt} \frac{L_{mt}}{L_{ht}},$$

where $|\mathcal{M}_{ht}|$ is the number of local labor markets that belong to h in t, L_{mt} is the local labor market employment and L_{ht} is the 3-digit industry employment.

Table E2: Concentration and Labor Share

	Dependent variable: $\log(LS_{h,t})$		
	(1)	(2)	(3)
$\log(\overline{HHI}_{h,t})$	-0.064^{***}	-0.054^{***}	-0.056^{***}
	(0.013)	(0.013)	(0.014)
Sector FE	N	Y	N
Sector-year FE	N	N	Y
R^2	0.017	0.290	0.343

Notes: The number of observations is 1,357. This table presents estimates of equation (E3). Column 1 presents the estimate without any fixed effect. Column 2 shows results with sector fixed effects and column 3 has sector-year fixed effects. The dependent variable is the logarithm of 3-digit industry h labor share $\log(LS_{h,t})$ at time t. $\log(\overline{HHI}_{h,t})$ is the logarithm of the employment weighted average of the local labor market Herfindahl Index. *p<0.1; **p<0.05; ***p<0.01

only cross sectional variation.[12] Column 3 shows that the negative relation between employment concentration and the labor share is robust to controlling for sector and sector-year fixed effects.

This regression gives a sense of the importance of the labor wedge heterogeneity to generate output and labor share losses. At face value, the estimate with sector fixed effects (column 2) implies a reduction of 1 percentage point of the labor share when passing from the first to the third quartile of concentration.[13] Estimates in column 3 with sector-year fixed effects are very similar. The low estimated effects imply that wages, and therefore labor shares, are not very responsive to differentiated levels of concentration. Nevertheless, one cannot interpret that they rule out employer labor market power because in a setting where all the firms acted as pure monopsonists facing an equal labor supply elasticity, wages (and the labor share) would be insensitive to concentration as all establishments would have the same markdown.

The small estimated coefficient is most likely a result of level effects as the regression does not take into account the effect of concentration on the average level of the labor share as this is absorbed by the fixed effects.

E.3 Unions

Tables E3 and E4 present respectively the rent sharing elasticities for industries and occupations. As it is clear from comparing the tables, there is more heterogeneity in the rent sharing elasticities across industries than across occupations. This is one reason why we choose the bargaining powers to vary across industries instead of across occupations.

[12] The inclusion of fixed effects absorbs changes in the HHI that stem from the entry of more establishments in the economy.

[13] Local labor market summary statistics including quartiles of $HHI(s_{io|m})$ are in Table VII2 in Appendix **??**.

Table E3: Rent Sharing: Industry

Code	Industry Name	Rent Sharing	Std Err ($\times 10^2$)	Code	Industry Name	Rent Sharing	Std Err ($\times 10^2$)
15	Food	0.40	0.09	22	Printing	0.34	0.11
17	Textile	0.22	0.23	24	Chemical	0.17	0.17
18	Clothing	0.31	0.18	25	Plastic	0.23	0.21
19	Leather	0.31	0.39	26	Other Minerals	0.25	0.18
20	Wood	0.32	0.24	27	Metallurgy	0.14	0.40
21	Paper	0.22	0.37	28	Metals	0.37	0.12
22	Printing	0.34	0.11	29	Machines and Equip.	0.30	0.14
24	Chemical	0.17	0.17	30	Office Machinery	0.33	0.56
25	Plastic	0.23	0.21	31	Electrical Equipment	0.25	0.23
26	Other Minerals	0.25	0.18	32	Telecommunications	0.23	0.27

Table E4: Rent Sharing: Occupation

Occupation	Rent Sharing	Std Err ($\times 10^2$)
Top management	0.38	0.08
Supervisor	0.27	0.06
Clerical	0.29	0.06
Blue collar	0.30	0.05

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