61A Lecture 20

Monday, March 11

Announcements

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- Project 3 due Thursday 3/12 @ 11:59pm

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  - Project party on Tuesday 3/10 5pm–6:30pm in 2050 VLSB

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  - Fill out conflict form if you cannot attend due to a course conflict

Time

The Consumption of Time

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Implementations of the same functional abstraction can require different amounts of time

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(Demo)

Space

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Interactive Diagram

Fibonacci Space Consumption

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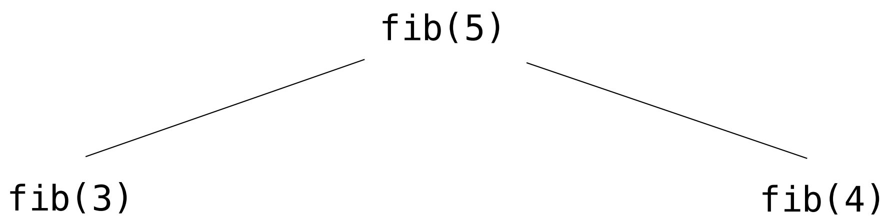
`fib(5)`

Fibonacci Space Consumption

```
        fib(5)
       /
fib(3)
```

Fibonacci Space Consumption



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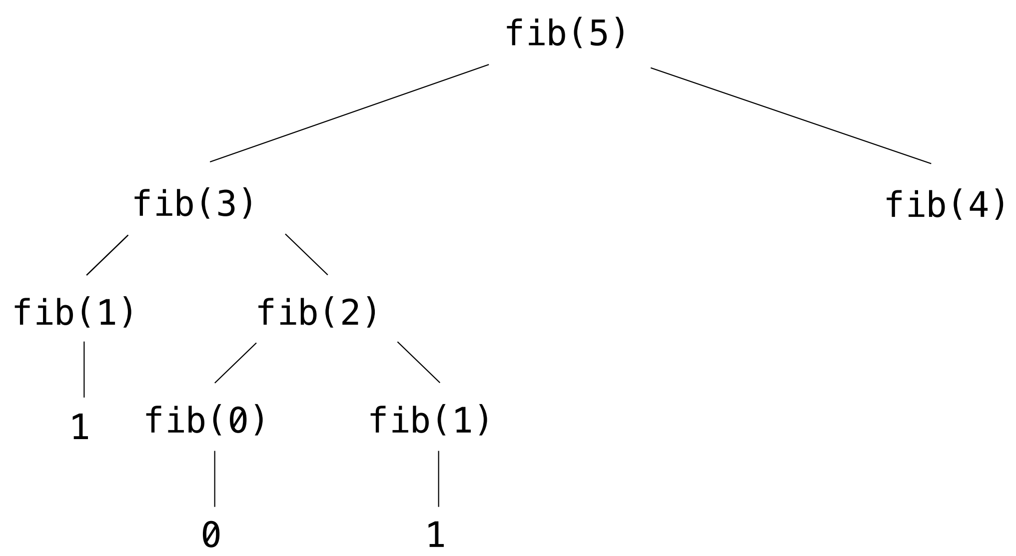

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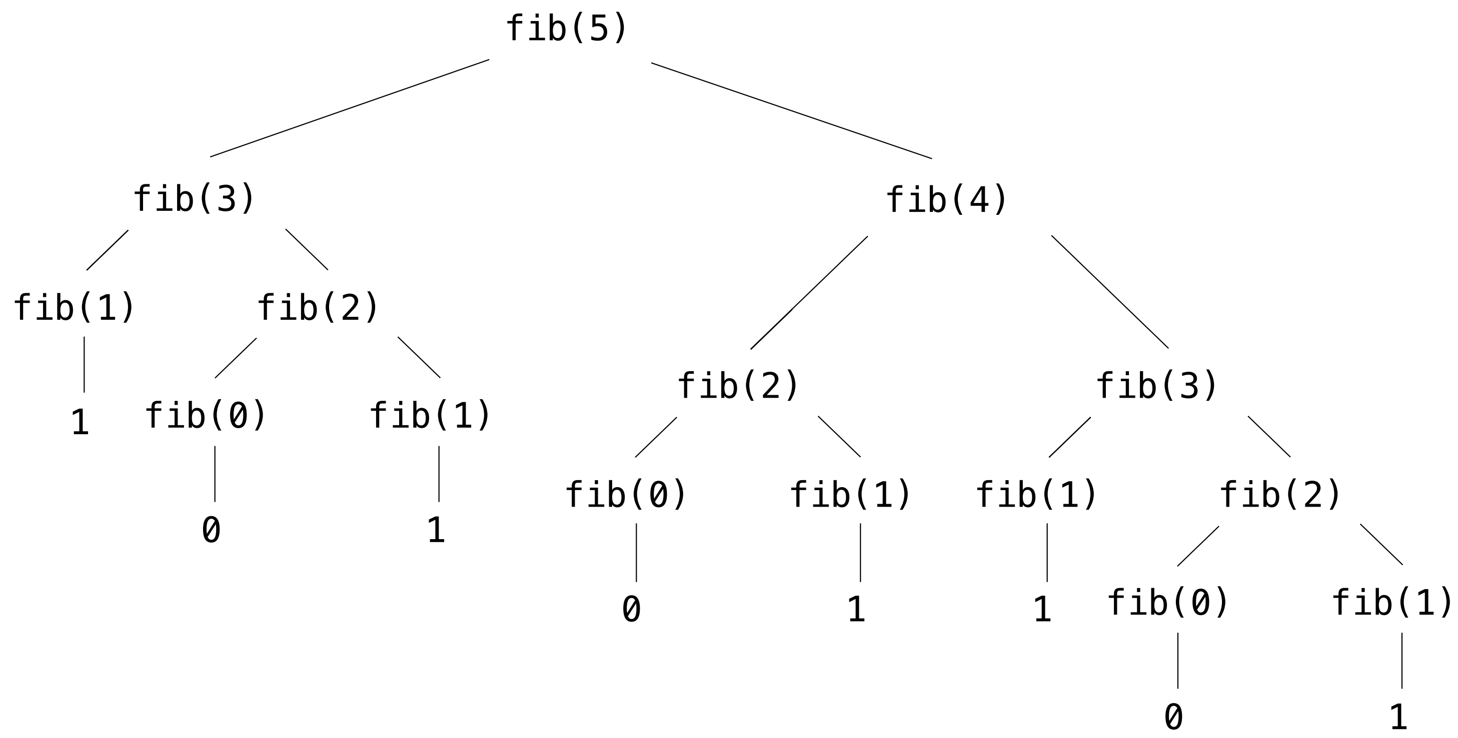

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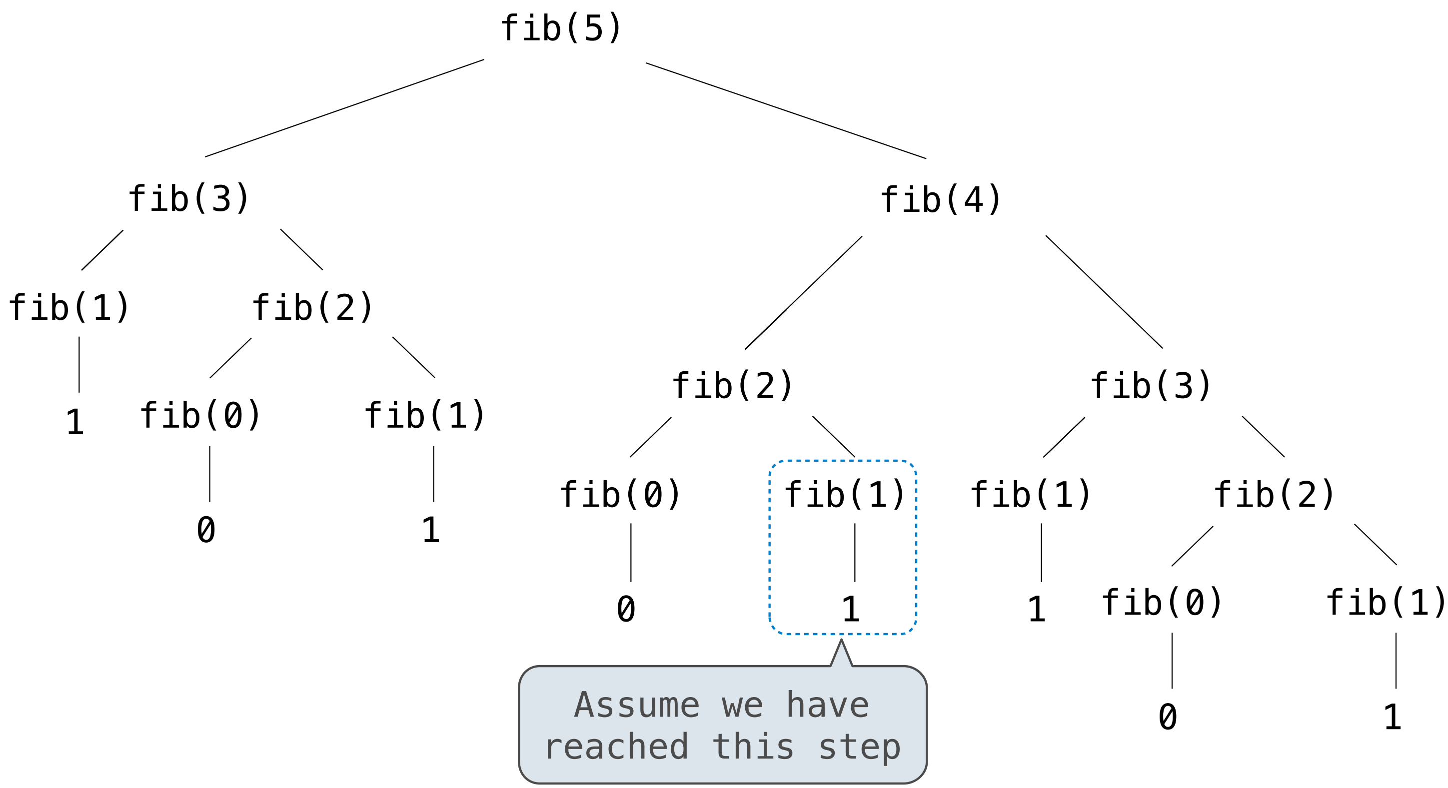

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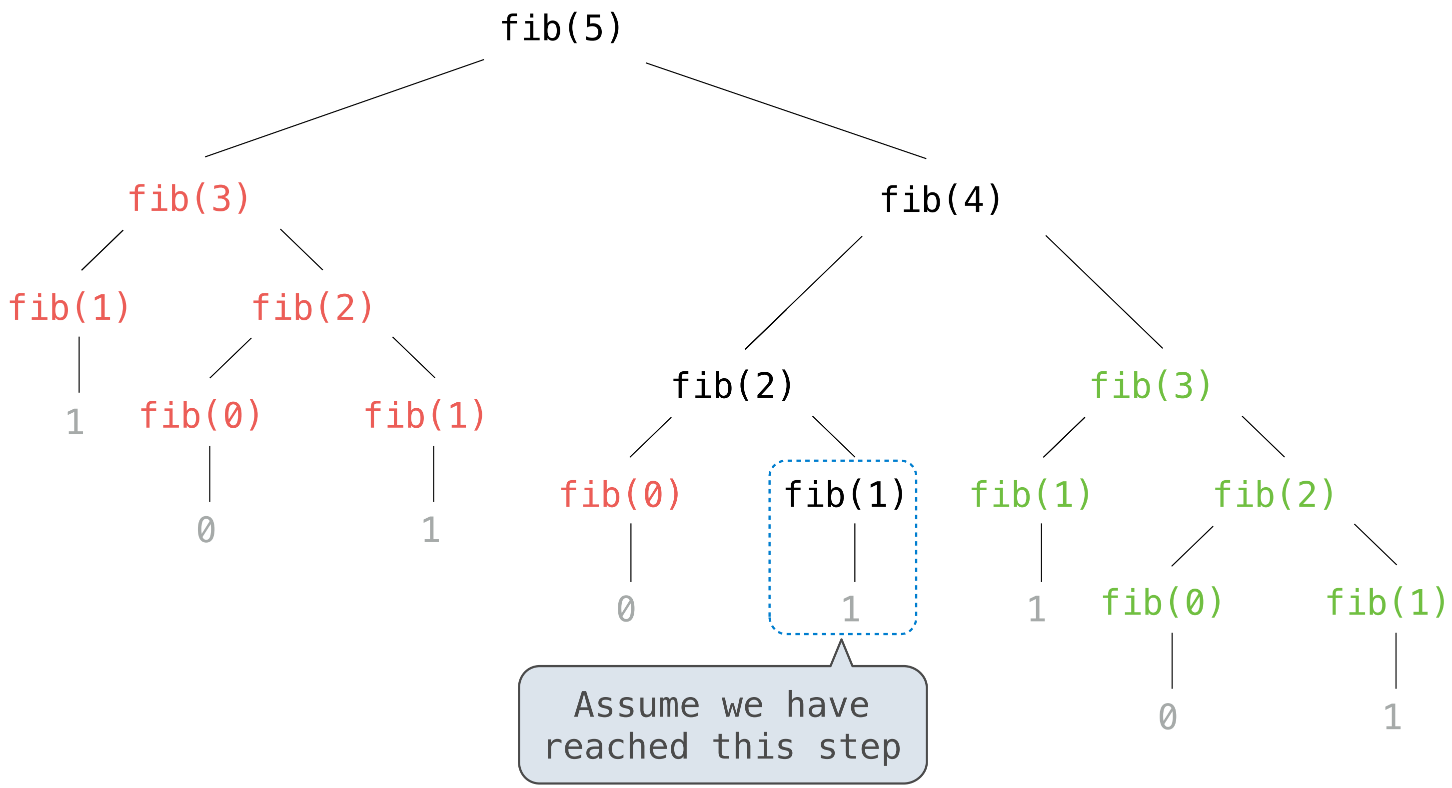

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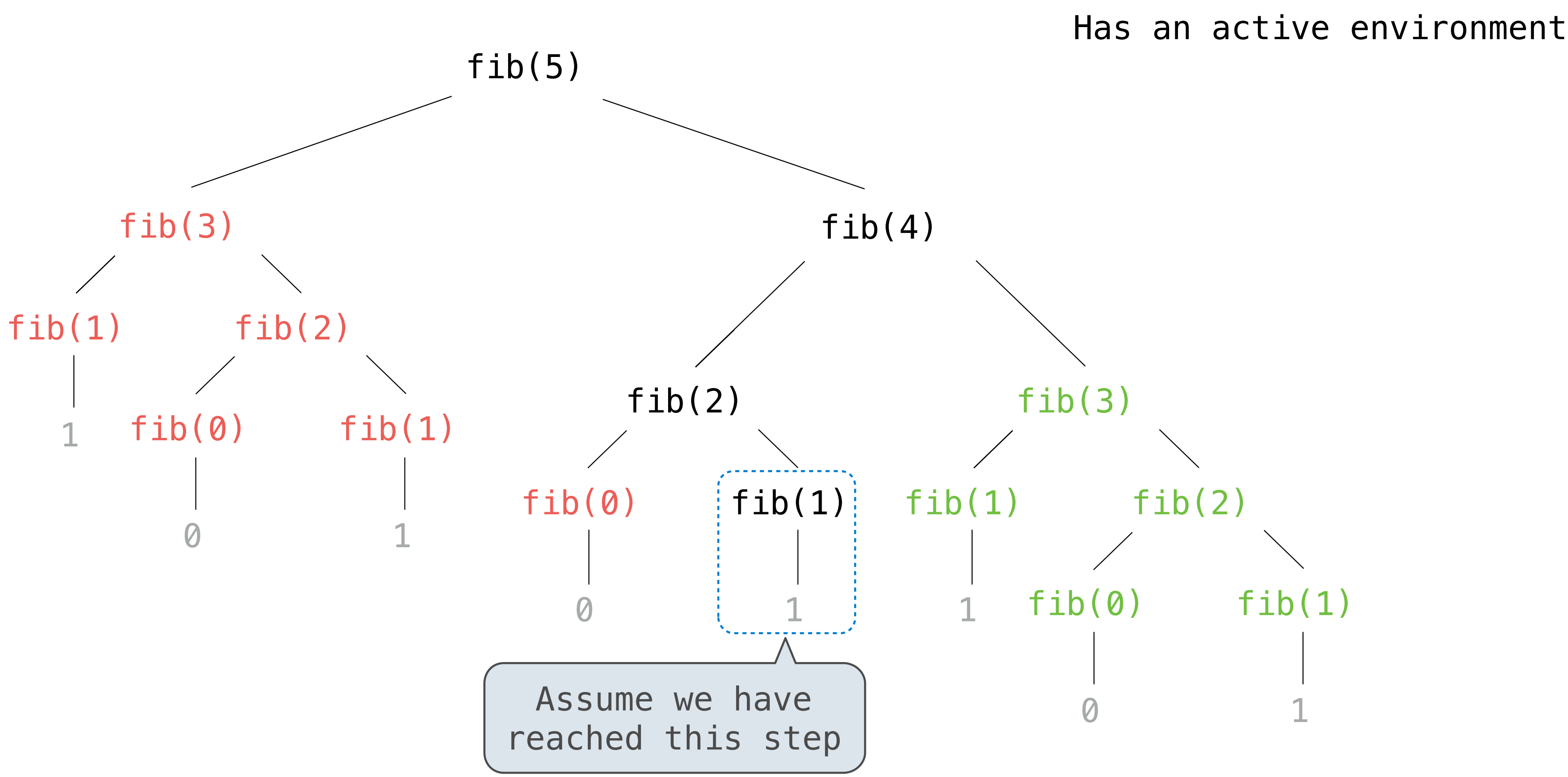

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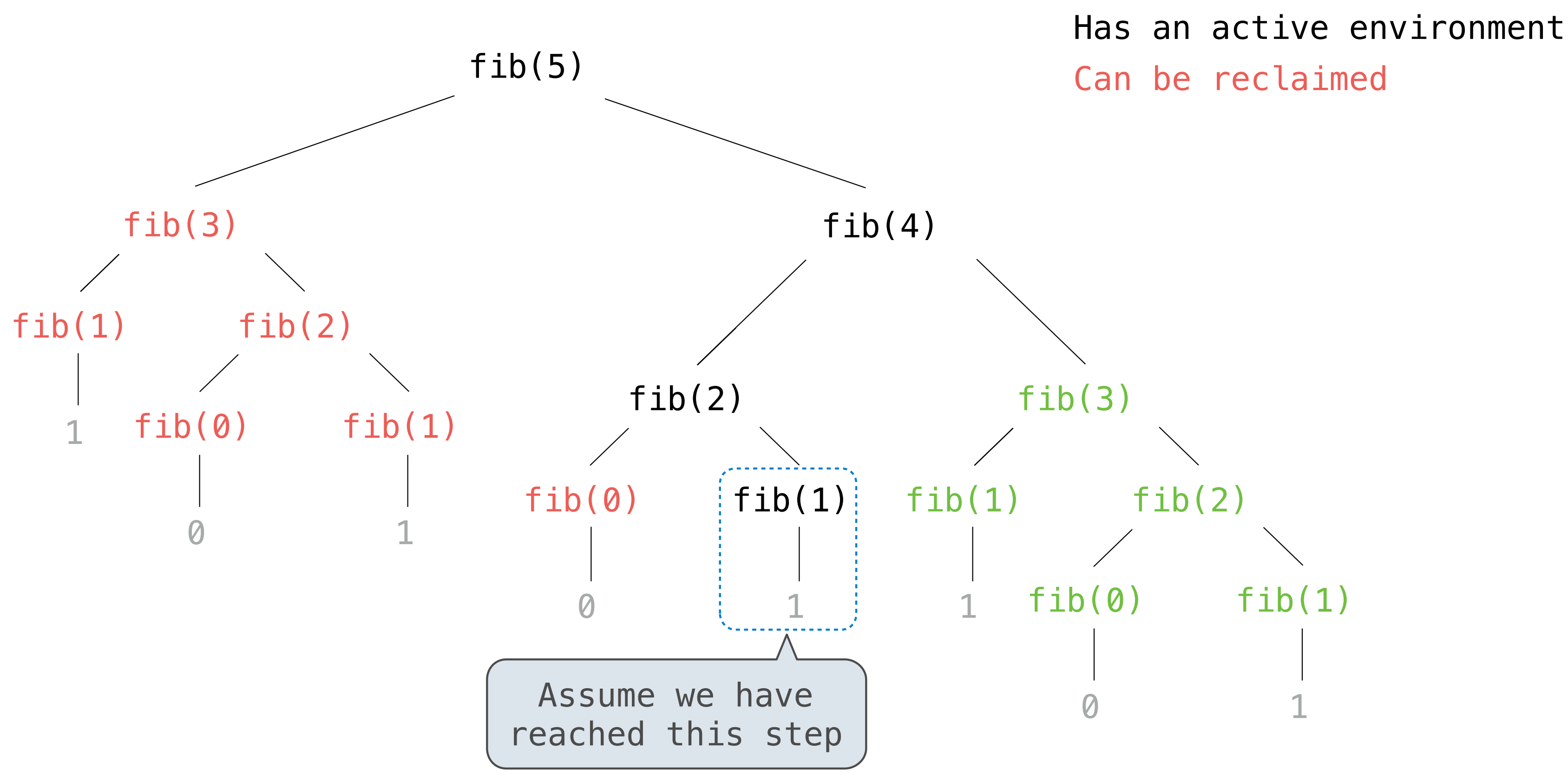

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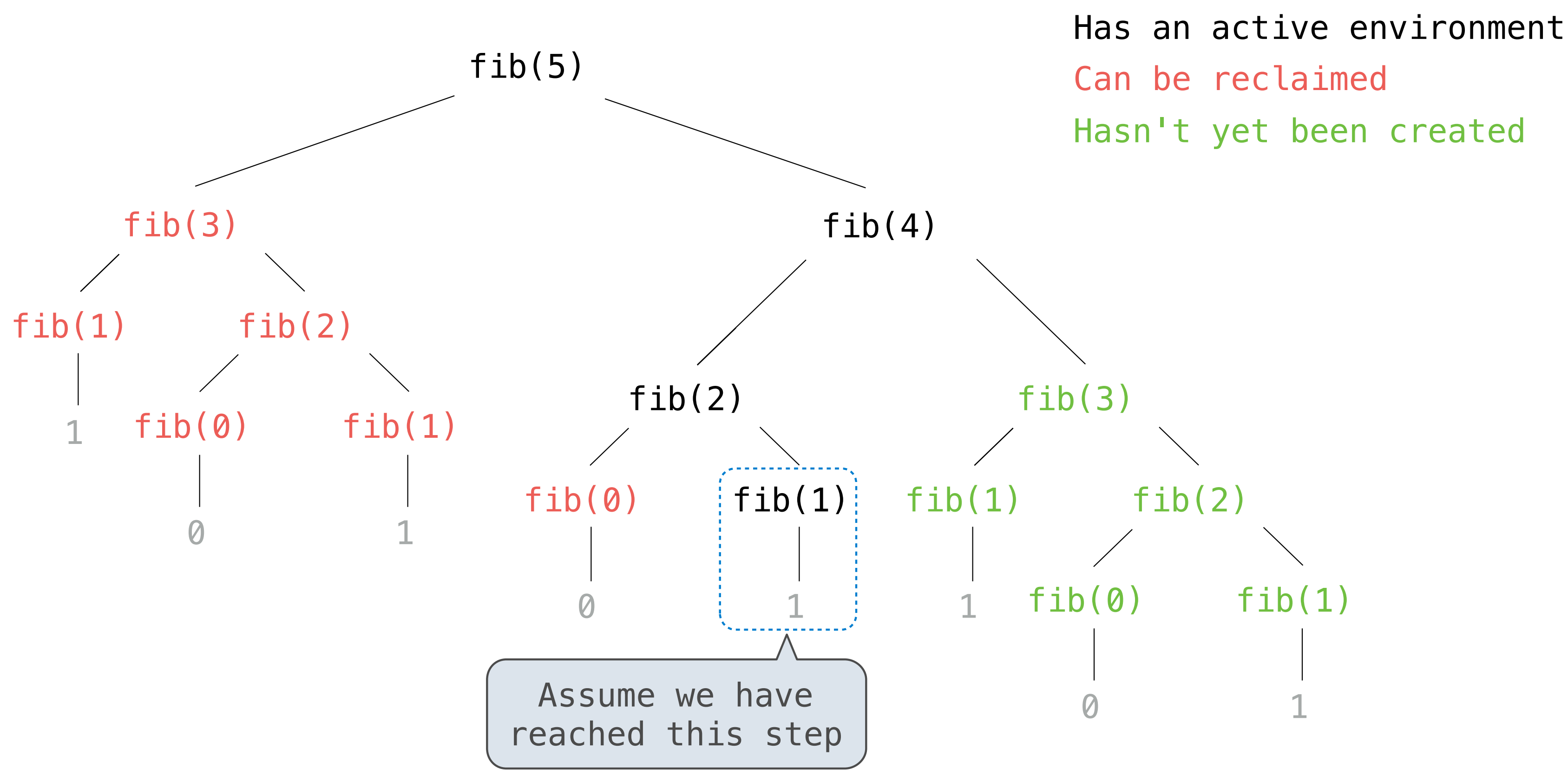

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- We choose $\mathbf{k_2} = 5\mathbf{p}$ and $\mathbf{m} = 25$

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Order of Growth of Counting Factors

Implementations of the same functional abstraction can require different amounts of time

Problem: How many factors does a positive integer n have?

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def factors(n):
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Assumption: integers occupy a fixed amount of space

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def exp(b, n):
    if n == 0:
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Goal: one more multiplication lets us double the problem size

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def square(x):
    return x*x

def exp_fast(b, n):
    if n == 0:
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    elif n % 2 == 0:
        return square(exp_fast(b, n//2))
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	Time	Space
`exp`	$\Theta(n)$	$\Theta(n)$
`exp_fast`	$\Theta(\log n)$	$\Theta(\log n)$

Comparing Orders of Growth

Properties of Orders of Growth

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Constants: Constant terms do not affect the order of growth of a process

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$$\Theta(n)$$

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Properties of Orders of Growth

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$$\Theta(n) \qquad \Theta(500 \cdot n) \qquad \Theta(\frac{1}{500} \cdot n)$$

Properties of Orders of Growth

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Lower-order terms: The fastest-growing part of the computation dominates the total

$$\Theta(n^2) \qquad \Theta(n^2 + n) \qquad \Theta(n^2 + 500 \cdot n + \log_2 n + 1000)$$

Comparing orders of growth (n is the problem size)

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$$\Theta(b^n)$$

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$\Theta(b^n)$ `Exponential growth.  Recursive fib takes`

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