Homework 4 Solutions **hw04.zip (hw04.zip)**

Solution Files

You can find the solutions in hw04.py (hw04.py).

Required questions

Trees

Q1: Replace Leaf

Define `replace_leaf`, which takes a tree `t`, a value `old`, and a value `new`. `replace_leaf` returns a new tree that's the same as `t` except that every leaf value equal to `old` has been replaced with `new`.

```
def replace_leaf(t, old, new):
    """Returns a new tree where every leaf value equal to old has
    been replaced with new.

    >>> yggdrasil = tree('odin',
    ...                  [tree('balder',
    ...                        [tree('thor'),
    ...                         tree('loki')]),
    ...                   tree('frigg',
    ...                        [tree('thor')]),
    ...                   tree('thor',
    ...                        [tree('sif'),
    ...                         tree('thor')]),
    ...                   tree('thor')])
    >>> laerad = copy_tree(yggdrasil) # copy yggdrasil for testing purposes
    >>> print_tree(replace_leaf(yggdrasil, 'thor', 'freya'))
    odin
      balder
        freya
        loki
      frigg
        freya
      thor
        sif
        freya
      freya
    >>> laerad == yggdrasil # Make sure original tree is unmodified
    True
    """
    if is_leaf(t) and label(t) == old:
        return tree(new)
    else:
        bs = [replace_leaf(b, old, new) for b in branches(t)]
        return tree(label(t), bs)
```

Use Ok to test your code:

```
python3 ok -q replace_leaf
```

Q2: Pruning Leaves

Define a function `prune_leaves` that given a tree `t` and a tuple of values `vals`, produces a version of `t` with all its leaves that are in `vals` removed. Do not attempt to try to remove non-leaf nodes and do not remove leaves that do not match any of the items in `vals`. Return `None` if pruning the tree results in there being no nodes left in the tree.

```
def prune_leaves(t, vals):
    """Return a modified copy of t with all leaves that have a label
    that appears in vals removed.  Return None if the entire tree is
    pruned away.

    >>> t = tree(2)
    >>> print(prune_leaves(t, (1, 2)))
    None
    >>> numbers = tree(1, [tree(2), tree(3, [tree(4), tree(5)]), tree(6, [tree(7)])])
    >>> print_tree(numbers)
    1
      2
      3
        4
        5
      6
        7
    >>> print_tree(prune_leaves(numbers, (3, 4, 6, 7)))
    1
      2
      3
        5
      6
    """
    if is_leaf(t) and (label(t) in vals):
      return None
    new_branches = []
    for b in branches(t):
      new_branch = prune_leaves(b, vals)
      if new_branch:
        new_branches += [new_branch]
    return tree(label(t), new_branches)
```

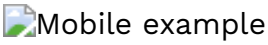
Use Ok to test your code:

```
python3 ok -q prune_leaves
```

Mobiles

Acknowledgements. This mobile example is based on a classic problem from Structure and Interpretation of Computer Programs, Section 2.2.2 (https://mitpress.mit.edu/sites/default/files/sicp/full-text/book/book-Z-H-15.html#%_sec_2.2.2).

> **Hint:** for more information on this problem (with more pictures!) please refer to this document (assets/mobiles.pdf)



A mobile (http://upload.wikimedia.org/wikipedia/commons/7/7e/Modern_mobile-art_mobiles_mobius.jpg) is a type of hanging sculpture. A binary mobile consists of two sides. Each side is a rod of a certain length, from which hangs either a weight or another mobile.

We will represent a binary mobile using the data abstractions below.

- A `mobile` has a left `side` and a right `side`.
- A `side` has a positive length and something hanging at the end, either a `mobile` or `weight`.
- A `weight` has a positive size.

Q3: Weights

Implement the `weight` data abstraction by completing the `weight` constructor and the `size` selector so that a weight is represented using a two-element list where the first element is the string `'weight'`. The `total_weight` example is provided to demonstrate use of the mobile, side, and weight abstractions.

```
def mobile(left, right):
    """Construct a mobile from a left side and a right side."""
    assert is_side(left), "left must be a side"
    assert is_side(right), "right must be a side"
    return ['mobile', left, right]

def is_mobile(m):
    """Return whether m is a mobile."""
    return type(m) == list and len(m) == 3 and m[0] == 'mobile'

def left(m):
    """Select the left side of a mobile."""
    assert is_mobile(m), "must call left on a mobile"
    return m[1]

def right(m):
    """Select the right side of a mobile."""
    assert is_mobile(m), "must call right on a mobile"
    return m[2]
```

```
def side(length, mobile_or_weight):
    """Construct a side: a length of rod with a mobile or weight at the end."""
    assert is_mobile(mobile_or_weight) or is_weight(mobile_or_weight)
    return ['side', length, mobile_or_weight]

def is_side(s):
    """Return whether s is a side."""
    return type(s) == list and len(s) == 3 and s[0] == 'side'

def length(s):
    """Select the length of a side."""
    assert is_side(s), "must call length on a side"
    return s[1]

def end(s):
    """Select the mobile or weight hanging at the end of a side."""
    assert is_side(s), "must call end on a side"
    return s[2]
```

```
def weight(size):
    """Construct a weight of some size."""
    assert size > 0
    return ['weight', size]

def size(w):
    """Select the size of a weight."""
    assert is_weight(w), 'must call size on a weight'
    return w[1]

def is_weight(w):
    """Whether w is a weight."""
    return type(w) == list and len(w) == 2 and w[0] == 'weight'
```

Use Ok to test your code:

```
python3 ok -q total_weight
```

Q4: Balanced

> **Hint:** for more information on this problem (with more pictures!) please refer to this document (assets/mobiles.pdf#page=3)

Implement the `balanced` function, which returns whether `m` is a balanced mobile. A mobile is balanced if two conditions are met:

1. The torque applied by its left side is equal to that applied by its right side. Torque of the left side is the length of the left rod multiplied by the total weight hanging from that rod. Likewise for the right.
2. Each of the mobiles hanging at the end of its sides is balanced.

Hint: You may find it helpful to assume that weights themselves are balanced.

```
def balanced(m):
    """Return whether m is balanced.

    >>> t, u, v = examples()
    >>> balanced(t)
    True
    >>> balanced(v)
    True
    >>> w = mobile(side(3, t), side(2, u))
    >>> balanced(w)
    False
    >>> balanced(mobile(side(1, v), side(1, w)))
    False
    >>> balanced(mobile(side(1, w), side(1, v)))
    False
    """
    if is_weight(m):
        return True
    else:
        left_end, right_end = end(left(m)), end(right(m))
        torque_left = length(left(m)) * total_weight(left_end)
        torque_right = length(right(m)) * total_weight(right_end)
        return balanced(left_end) and balanced(right_end) and torque_left == torque_right
```

Use Ok to test your code:

```
python3 ok -q balanced
```

The balanced weights assumption is important, since we will be solving this recursively like many other tree problems (even though this is not explicitly a tree).

- **Base case:** if we are checking a weight, then we know that this is balanced. Why is this an appropriate base case? There are two possible approaches to this:
    1. Because we know that our data structures so far are trees, weights are the simplest possible tree since we have chosen to implement them as leaves.
    2. We also know that from an ADT standpoint, weights are the terminal item in a mobile. There can be no further mobile structures under this weight, so it makes sense to stop check here.
- **Otherwise:** note that it is important to do a recursive call to check if both sides are balanced. However, we also need to do the basic comparison of looking at the total

weight of both sides as well as their length. For example if both sides are a weight, trivially, they will both be balanced. However, the torque must be equal in order for the entire mobile to balanced (i.e. it's insufficient to just check if the sides are balanced).

Q5: Totals

Implement `totals_tree`, which takes a `mobile` (or `weight`) and returns a `tree` whose root is its total weight and whose branches are trees for the ends of the sides.

```
def totals_tree(m):
    """Return a tree representing the mobile with its total weight at the root.

    >>> t, u, v = examples()
    >>> print_tree(totals_tree(t))
    3
      2
      1
    >>> print_tree(totals_tree(u))
    6
      1
      5
        3
        2
    >>> print_tree(totals_tree(v))
    9
      3
        2
        1
      6
        1
        5
          3
          2
    """
    if is_weight(m):
        return tree(size(m))
    else:
        branches = [totals_tree(end(f(m))) for f in [left, right]]
        return tree(sum([label(b) for b in branches]), branches)
```

Use Ok to test your code:

```
python3 ok -q totals_tree
```

Just for fun Question

> *This question is out of scope for 61a. Do it if you want an extra challenge!*

Q6: Church numerals

The logician Alonzo Church invented a system of representing non-negative integers entirely using functions. The purpose was to show that functions are sufficient to describe all of number theory: if we have functions, we do not need to assume that numbers exist, but instead we can invent them.

Your goal in this problem is to rediscover this representation known as *Church numerals*. Here are the definitions of `zero`, as well as a function that returns one more than its argument:

```
def zero(f):
    return lambda x: x

def successor(n):
    return lambda f: lambda x: f(n(f)(x))
```

First, define functions `one` and `two` such that they have the same behavior as `successor(zero)` and `successsor(successor(zero))` respectively, but *do not call `successor` in your implementation*.

Next, implement a function `church_to_int` that converts a church numeral argument to a regular Python integer.

Finally, implement functions `add_church`, `mul_church`, and `pow_church` that perform addition, multiplication, and exponentiation on church numerals.

```
def one(f):
    """Church numeral 1: same as successor(zero)"""
    return lambda x: f(x)

def two(f):
    """Church numeral 2: same as successor(successor(zero))"""
    return lambda x: f(f(x))

three = successor(two)

def church_to_int(n):
    """Convert the Church numeral n to a Python integer.

    >>> church_to_int(zero)
    0
    >>> church_to_int(one)
    1
    >>> church_to_int(two)
    2
    >>> church_to_int(three)
    3
    """
    return n(lambda x: x + 1)(0)

def add_church(m, n):
    """Return the Church numeral for m + n, for Church numerals m and n.

    >>> church_to_int(add_church(two, three))
    5
    """
    return lambda f: lambda x: m(f)(n(f)(x))

def mul_church(m, n):
    """Return the Church numeral for m * n, for Church numerals m and n.

    >>> four = successor(three)
    >>> church_to_int(mul_church(two, three))
    6
    >>> church_to_int(mul_church(three, four))
    12
    """
    return lambda f: m(n(f))

def pow_church(m, n):
    """Return the Church numeral m ** n, for Church numerals m and n.
```

```
    >>> church_to_int(pow_church(two, three))
    8
    >>> church_to_int(pow_church(three, two))
    9
    """
    return n(m)
```

Use Ok to test your code:

```
python3 ok -q church_to_int
python3 ok -q add_church
python3 ok -q mul_church
python3 ok -q pow_church
```

Church numerals are a way to represent non-negative integers via repeated function application. The definitions of `zero`, `one`, and `two` show that each numeral is a function that takes a function and repeats it a number of times on some argument `x`.

The `church_to_int` function reveals how a Church numeral can be mapped to our normal notion of non-negative integers using the increment function.

Addition of Church numerals is function composition of the functions of `x`, while multiplication is composition of the functions of `f`.

CS 61A (/)

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