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ASSIGNMENT 3 — RC filter

> “*Find $H(j\omega)$ for a low-pass RC filter of the type illustrated in the figure.*”

Since the impedance Z_C of the condensator equals $\frac{1}{j\omega C}$, can we insert it into the expression:

$$
\begin{aligned}
V_{out} &= \frac{Z_C}{R + Z_C} \cdot V_{in} \\
V_{out} &= \frac{1}{{}^{R}/_{Z_C} + 1} \cdot V_{in} \\
V_{out} &= \frac{1}{R \cdot j\omega C + 1} \cdot V_{in}
\end{aligned}
$$

We therefore know the frequency response is $H(j\omega) = \frac{1}{1+j\omega RC}$. □

> “*Find the the expression describing the relationship between the amplitude of the input voltage V_{in} and the amplitude of the output voltage V_{out} of the RC filter as a function of frequency.*”

The relationship between the output value of the voltage and its input is given by

$$
\begin{aligned}
V_{out}(t) &= |H(j\omega)| \cdot V_{in}(\omega, t) \\
V_{out}(t) &= |H(j\omega)| \cdot cos(\omega t + \phi_c) \\
V_{out}(t) &= \frac{1}{\sqrt{1 + (\omega RC)^2}} \cdot cos(\omega t + \angle H(j\omega)) \\
V_{out}(t) &= \frac{1}{\sqrt{1 + (\omega RC)^2}} \cdot cos(\omega t + \arctan(\omega RC))
\end{aligned}
$$

and we know the boundaries of the cosinus is ± 1, therefore the amplitude is given by

$$
A_{out} = |H(j\omega)| = \frac{1}{\sqrt{1 + (\omega RC)^2}}
$$

□

“ *Assume $C = 22nF$, and that we want the cutoff frequency of the filter to be 200Hz. Find R. The cutoff frequency is defined as the frequency where the amplitude of the output signal is $1/\sqrt{2} \approx 0.7071$ times the amplitude of the input signal.* ”

Given

$$\boldsymbol{H(j\omega) = \frac{1}{\sqrt{1 + j\omega RC}}}$$

and the definition of cutoff frequency (where the cutoff frequency is $\boldsymbol{1/\sqrt{2}}$ the DC voltage, thereby the frequency being $\mathbf{0}$.)

$$\begin{aligned}
\boldsymbol{|H(j\omega_c)|} &= \boldsymbol{\frac{1}{\sqrt{1+(\omega_c RC)^2}} = \frac{1}{\sqrt{2}} H(j0)} \\
&\implies \boldsymbol{\frac{1}{\sqrt{1+(\omega_c RC)^2}} = \frac{1}{\sqrt{2}}} \\
&\implies \boldsymbol{\sqrt{1+(\omega_c RC)^2} = \sqrt{2}} \\
&\implies \boldsymbol{1+(\omega_c RC)^2 = 2} \\
&\implies \boldsymbol{(\omega_c RC)^2 = 1} \\
&\implies \boldsymbol{\omega_c = \frac{1}{RC}}
\end{aligned}$$

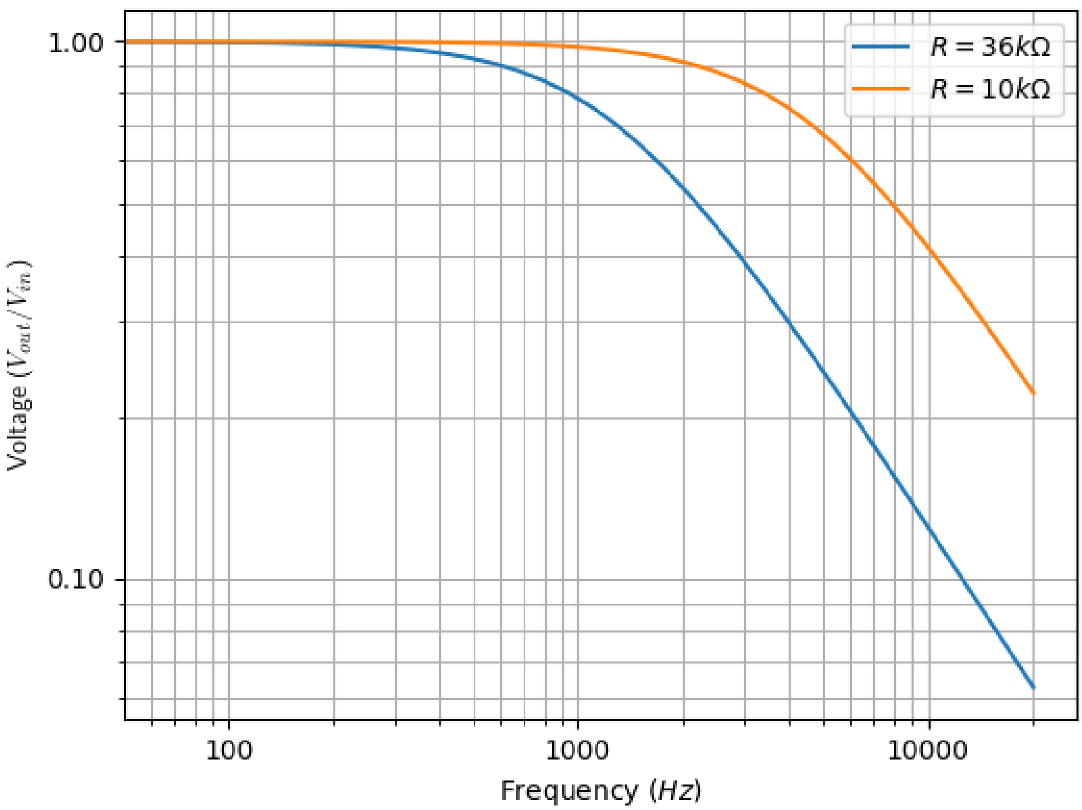
We can therefore say that the resistance for the desired cutoff frequency is $\boldsymbol{R = 1/w_c C}$, and since $\boldsymbol{w_c = \tau \cdot f_c}$, it is equivalent to saying

$$\begin{aligned}
\boldsymbol{R} &= \boldsymbol{\frac{1}{\tau \cdot f_c \cdot C}} \\
&= \boldsymbol{\frac{1}{\tau \cdot 200[Hz] \cdot 22[nF]}} \\
&= \boldsymbol{\frac{10^7}{\tau \cdot [Hz] \cdot 44[F]}} \\
&\approx \boldsymbol{36.2k\Omega}
\end{aligned}$$

□

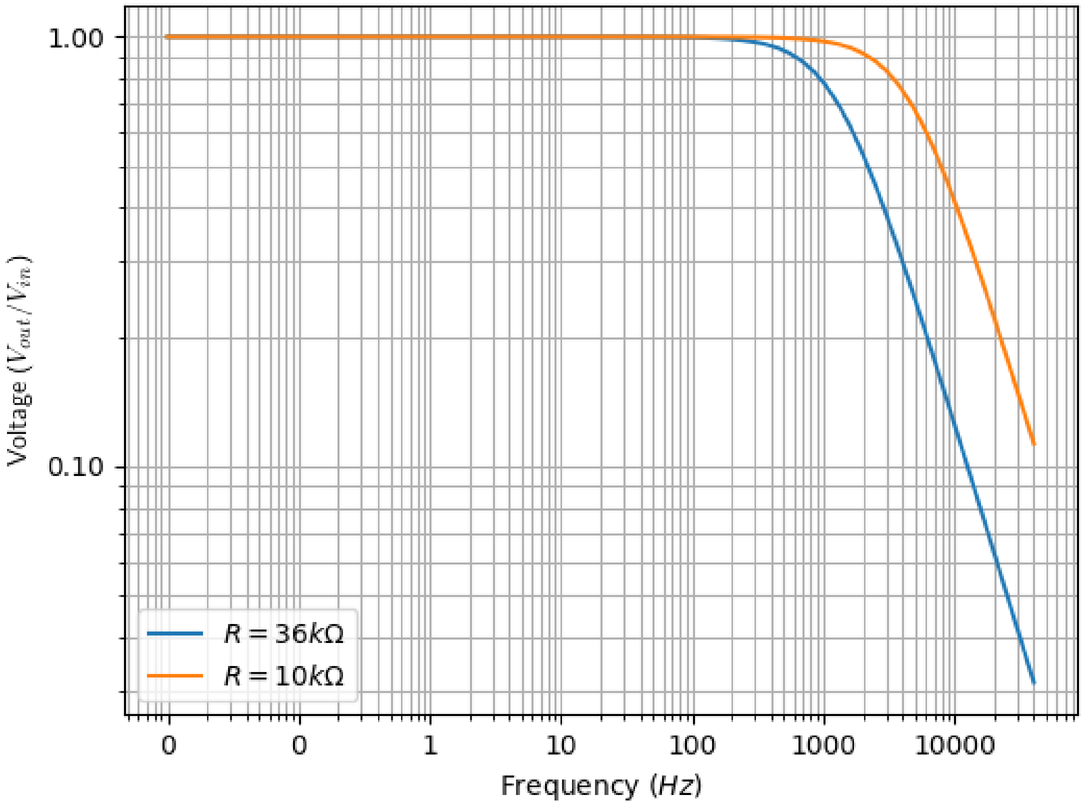
“ *For the value you have found for R, and a capacitor value of $C = 22nF$, use Matlab and plot the ratio of the amplitude of the input signal to the amplitude of the output signal ($|H(j\omega)|$), for frequencies in the range 0-20kHz.* ”

“ *Do the same for $R = 10k\Omega$ and combine the two curves together in a single plot.* ”



□

“ *Make an new plot of the curves found in question 4/5 using logarithmic axes by replacing ”plot()” with”loglog()” in Matlab. Follow with the command ”ylim([0.01 2])” and ”grid on”, to get a better view* ”



□

Submitted by Rolf Vidar Hoksaas on March 6, 2020.