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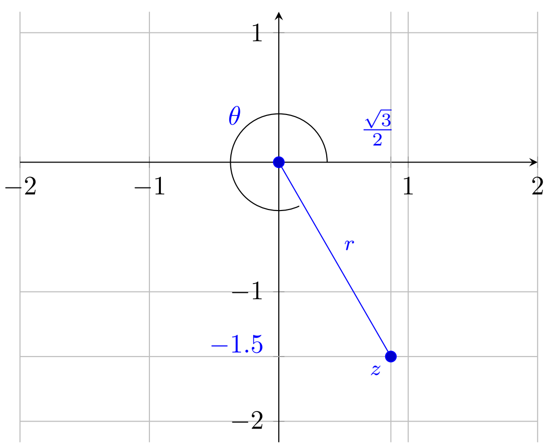
OBLIG 1 — Obligatorisk oppgave 1 av 2

> *"Skriv det komplekse tallet $z = \frac{6}{\sqrt{3}+3i}$ først på formen $a + ib$ også på polarformen $re^{i\theta}$."*

$$z = \frac{6}{\sqrt{3}+3i}$$
$$z = \frac{2}{\sqrt{\frac{1}{3}}+i}$$
$$z = \frac{2\times(\sqrt{\frac{1}{3}}-i)}{(\sqrt{\frac{1}{3}}+i)\times(\sqrt{\frac{1}{3}}-i)}$$
$$z = \frac{2\times(\sqrt{\frac{1}{3}}-i)}{\frac{1}{3}-i^2}$$
$$z = \frac{2\times(\sqrt{\frac{1}{3}}-i)}{\frac{4}{3}}$$
$$z = \frac{3\times(\sqrt{\frac{1}{3}}-i)}{2}$$
$$z = \frac{\sqrt{3}}{2} - \frac{3}{2}i$$

□

Using $z = \frac{\sqrt{3}}{2} - \frac{3}{2}i$:



With Pythagora's Theorem

$$r = \sqrt{(\frac{\sqrt{3}}{2})^2 + (\frac{3}{2})^2}$$
$$r = \sqrt{\frac{3}{4}+\frac{9}{4}}$$
$$r = \sqrt{3}$$

■

By the laws of trigonometry

$$\theta' := 2\pi - \theta$$
$$\sin(\theta') = \frac{-1.5}{\sqrt{3}}$$
$$\theta' = \arcsin(\frac{-1.5\sqrt{3}}{3})$$
$$\theta' = \arcsin(\frac{-\sqrt{3}}{2})$$
$$\theta' = -\frac{\pi}{3} \equiv \frac{5\pi}{3}$$
$$\therefore \theta = \frac{\pi}{3}$$

■

And since z is in the fourth quadrant, as $Re(z) > 0 \wedge Im(z) < 0$ we know to use θ':

$$r : \sqrt{3} \wedge \theta : \frac{5\pi}{3} \therefore z = \sqrt{3}e^{\frac{-i\pi}{3}} \equiv \sqrt{3}e^{\frac{5i\pi}{3}}$$

□

"*Finn de to løsningene til likningen $w^2 - w + 1 = \theta$, og bruk disse til å finne alle komplekse løsninger til likningen $z^4 - z^2 + 1 = \theta$.Gi en faktorisering av $z^4 - z^2 + 1$, først i komplekse førstegradspolynomer og så i reelle andregradspolynomer.*"

$\square$

"*Finn grensene $\lim_{n\to\infty} \frac{3n+2}{\sqrt{4n^2-1}}$ og $\lim_{n\to\infty}(\sqrt{n^2-5n} - n)$.*"

$$\lim_{n\to\infty} \frac{3n+2}{\sqrt{4n^2-1}} =$$

$$\lim_{n\to\infty} \frac{(3n+2)\frac{1}{n}}{\sqrt{4n^2-1}\frac{1}{n}} =$$

$$\lim_{n\to\infty} \frac{3+\frac{2}{n}}{\sqrt{\frac{4n^2-1}{n^2}}} =$$

$$\lim_{n\to\infty} \frac{3+\frac{2}{n}}{\sqrt{4-\frac{1}{n^2}}} =$$

$$\frac{3+\cancel{0}}{\sqrt{4-\cancel{0}}} =$$

$$\frac{3}{2}$$

$\square$

$$\lim_{n\to\infty}(\sqrt{n^2-5n} - n)$$

$$\lim_{n\to\infty} \frac{(\sqrt{n^2-5n} - n)(\sqrt{n^2-5n} + n)}{\sqrt{n^2-5n} + n}$$

$$\lim_{n\to\infty} \frac{n^2-5n-n^2}{\sqrt{n^2-5n} + n}$$

$$\lim_{n\to\infty} \frac{-5n}{\sqrt{n^2-5n} + n}$$

$$\lim_{n\to\infty} \frac{-5}{\sqrt{1-\frac{5}{n}} + 1}$$

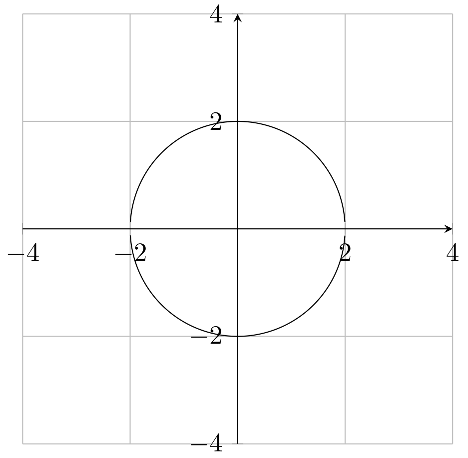
$$\frac{-5}{\sqrt{1-\cancel{0}} + 1}$$

$$\frac{-5}{2}$$

$\square$

"*Finn de komplekse tallene z som oppfyller likningen $2|z-1| = |z-4|$ og skisser løsningsmengden i det komplekse planet.(Hint: Sett inn $z = x+iy$ og finn en polynomlikning ix og y forløsningsmengden.)*"

$$\begin{aligned}
2|z-1| &= |z-4| \equiv \\
4|x+iy-1| &= |x+iy-4| \\
4((x-1)^2+(y)^2) &= (x-4)^2+(y)^2 \\
4(x-1)^2+4y^2 &= x^2-8x+16+y^2 \\
4(x^2-2x+1)+4y^2 &= x^2-8x+16+y^2 \\
4x^2-8x+4+4y^2 &= x^2-8x+16+y^2 \\
3x^2+3y^2 &= 12 \\
y^2 &= 4-x^2 \\
y &= \sqrt{4-x^2}
\end{aligned}$$



$$\therefore Im(z) = \pm\sqrt{4-Re(z)^2}, \exists z \forall \{x \in Re(z) | -2 < x < 2\}$$

$\square$

"*En følge $\{a_n\}$ er definert ved $a_1 = 3, a_n+1 = 3\sqrt{a_n}$ for $n \geq 1$. Vis at $a_n < 9$ og at $a_n+1 > a_n$ for alle n. Forklar hvorfor følgen konvergerer og finn $\lim_{n\to\infty} a_n$*"

$\square$.

Submitted by Rolf Vidar Hoksaas *on September 12, 2019.*