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OBLIG 1 — Obligatorisk oppgave 1 av 2

> “*Skriv det komplekse tallet* $z = \frac{6}{\sqrt{3}+3i}$ *først på formen* $a + ib$ *også på polarformen* $re^{i\theta}$. ”

$$z = \frac{6}{\sqrt{3}+3i}$$

$$z = \frac{2}{\sqrt{\frac{1}{3}}+i}$$

$$z = \frac{2 \times (\sqrt{\frac{1}{3}} - i)}{(\sqrt{\frac{1}{3}}+i) \times (\sqrt{\frac{1}{3}} - i)}$$

$$z = \frac{2 \times (\sqrt{\frac{1}{3}} - i)}{\frac{1}{3} - i^2}$$

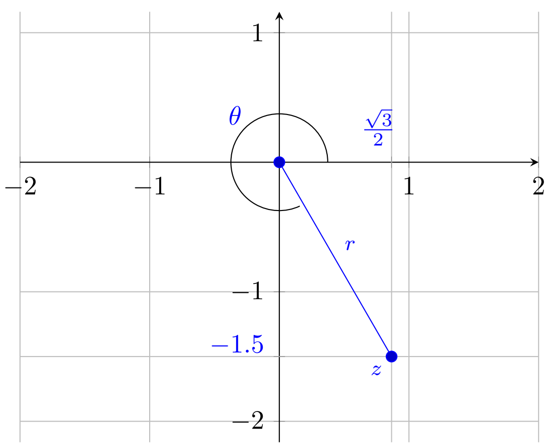
$$z = \frac{2 \times (\sqrt{\frac{1}{3}} - i)}{\frac{4}{3}}$$

$$z = \frac{3 \times (\sqrt{\frac{1}{3}} - i)}{2}$$

$$z = \frac{\sqrt{3}}{2} - \frac{3}{2}i$$

□

Using $z = \frac{\sqrt{3}}{2} - \frac{3}{2}i$:



With Pythagora's Theorem

$$r = \sqrt{(\frac{\sqrt{3}}{2})^2 + (\frac{3}{2})^2}$$

$$r = \sqrt{\frac{3}{4} + \frac{9}{4}}$$

$$r = \sqrt{3}$$

■

By the laws of trigonometry

$$\theta' := 2\pi - \theta$$

$$\sin(\theta') = \frac{-1.5}{\sqrt{3}}$$

$$\theta' = \arcsin(\frac{-1.5\sqrt{3}}{3})$$

$$\theta' = \arcsin(\frac{-\sqrt{3}}{2})$$

$$\theta' = -\frac{\pi}{3} \equiv \frac{5\pi}{3}$$

$$\therefore \theta = \frac{\pi}{3}$$

■

And since z *is in the fourth quadrant, as* $Re(z) > 0 \wedge Im(z) < 0$ *we know to use* θ'*:*

$$r : \sqrt{3} \wedge \theta : \frac{5\pi}{3} \therefore z = \sqrt{3}e^{\frac{-i\pi}{3}} \equiv \sqrt{3}e^{\frac{5i\pi}{3}}$$

□

"*Finn de to løsningene til likningen $w^2 - w + 1 = 0$, og bruk disse til å finne alle komplekse løsninger til likningen $z^4 - z^2 + 1 = 0$. Gi en faktorisering av $z^4 - z^2 + 1$, først i komplekse førstegradspolynomer og så i reelle andregradspolynomer.*"

Assuming $cis(x) \equiv cos(x) + i \cdot sin(x)$, and $i^2 = -1$.

$$\boldsymbol{a = 1, b = -1, c = 1} \tag{2-1}$$
$$\boldsymbol{w = \frac{1 \pm \sqrt{1-4}}{2} = \frac{1}{2} \pm \frac{\sqrt{3}}{2} i} \tag{2-2}$$
$$\tag{2-3}$$

With $\boldsymbol{t = z^2}$:

$$\boldsymbol{z^4 - z^2 + 1 = t^2 - t^2 + 1 = 0} \tag{2-4}$$
$$\boldsymbol{t = \frac{1}{2} \pm \frac{\sqrt{3}}{2} i} \tag{2-5}$$
$$\tag{2-6}$$

With $\boldsymbol{t_0 = \frac{1}{2} + \frac{\sqrt{3}}{2} i = cis(\frac{\pi}{3})}$:

$$\boldsymbol{\sqrt{cis(\frac{\pi}{3})}^4 - \sqrt{cis(\frac{\pi}{3})}^2 + 1 = 0} \tag{2-7}$$
$$\boldsymbol{cis^2(\frac{\pi}{3}) - cis(\frac{\pi}{3}) + 1 = 0} \tag{2-8}$$
$$\tag{2-9}$$

With De Moivres formula:

$$\boldsymbol{cis(\frac{2\pi}{3}) - cis(\frac{\pi}{3}) + 1 = 0} \tag{2-10}$$
$$\boldsymbol{cos(\frac{2\pi}{3}) + i \cdot sin(\frac{2\pi}{3}) - cos(\frac{\pi}{3}) - i \cdot sin(\frac{\pi}{3}) + 1 = 0} \tag{2-11}$$
$$\boldsymbol{-\frac{1}{2} + \frac{\sqrt{3}}{2} i - \frac{1}{2} - \frac{\sqrt{3}}{2} i + 1 = 0} \tag{2-12}$$
$$\boldsymbol{-\frac{1}{2} - \frac{1}{2} + 1 = 0} \tag{2-13}$$
$$\boldsymbol{-1 + 1 = 0} \tag{2-14}$$
$$\tag{2-15}$$

Therefore, $\boldsymbol{z = t_0^{\frac{1}{2}} = cis^{\frac{1}{2}}(\frac{\pi}{3}) = cis(\frac{\pi}{6})}$ is a root $\boldsymbol{z_0}$.
By the complex conjugate root theorem, $\boldsymbol{z = t_1^{\frac{1}{2}} = cis^{\frac{1}{2}}(-\frac{\pi}{3}) = cis(-\frac{\pi}{6})}$ is a root $\boldsymbol{z_0'}$.

□

“ *Finn grensene* $\lim_{n\to\infty} \frac{3n+2}{\sqrt{4n^2-1}}$ *og* $\lim_{n\to\infty}(\sqrt{n^2-5n}-n)$. ”

□

“ *Finn de komplekse tallenezsom oppfyller likningen* $2|z-1| = |z-4|$ *og skisser løsningsmengden i det komplekse planet.(Hint: Sett inn* $z = x+iy$ *og finn en polynomlikning ixogyforløsningsmengden.)* ”

□

“ *En følge* $\{a_n\}$ *er definert ved* $a_1 = 3, a_n+1 = 3\sqrt{a_n}$ *for* $n \geq 1$. *Vis at* $a_n < 9$ *og at* $a_n+1 > a_n$ ” *for alle* n. *Forklar hvorfor følgen konvergerer og finn* $\lim_{n\to\infty} a_n$

□.

Submitted by Rolf Vidar Hoksaas *on September 12, 2019.*