

September 13, 2019

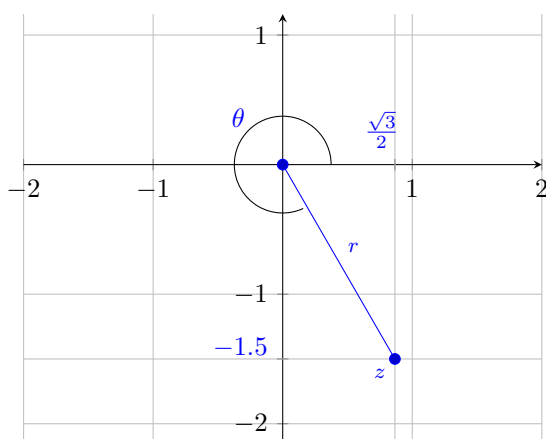
# OBLIG 1 — Obligatorisk oppgave 1 av 2

“Skriv det komplekse tallet  $z = \frac{6}{\sqrt{3}+3i}$  først på formen  $a + ib$  også på polarformen  $re^{i\theta}$ .”

$$\begin{aligned} z &= \frac{6}{\sqrt{3} + 3i} \\ z &= \frac{2}{\sqrt{\frac{1}{3}} + i} \\ z &= \frac{2 \times (\sqrt{\frac{1}{3}} - i)}{(\sqrt{\frac{1}{3}} + i) \times (\sqrt{\frac{1}{3}} - i)} \\ z &= \frac{2 \times (\sqrt{\frac{1}{3}} - i)}{\frac{1}{3} - i^2} \\ z &= \frac{2 \times (\sqrt{\frac{1}{3}} - i)}{\frac{4}{3}} \\ z &= \frac{3 \times (\sqrt{\frac{1}{3}} - i)}{2} \\ z &= \frac{\sqrt{3}}{2} - \frac{3}{2}i \end{aligned}$$

□

Using  $z = \frac{\sqrt{3}}{2} - \frac{3}{2}i$ :



With Pythagora's Theorem

$$\begin{aligned} r &= \sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{3}{2}\right)^2} \\ r &= \sqrt{\frac{3}{4} + \frac{9}{4}} \\ r &= \sqrt{3} \end{aligned}$$

■

By the laws of trigonometry

$$\begin{aligned} \theta' &:= 2\pi - \theta \\ \sin(\theta') &= \frac{-1.5}{\sqrt{3}} \\ \theta' &= \arcsin\left(\frac{-1.5\sqrt{3}}{3}\right) \\ \theta' &= \arcsin\left(\frac{-\sqrt{3}}{2}\right) \\ \theta' &= -\frac{\pi}{3} \equiv \frac{5\pi}{3} \\ \therefore \theta &= \frac{\pi}{3} \end{aligned}$$

■

And since  $z$  is in the fourth quadrant, as  $\text{Re}(z) > 0 \wedge \text{Im}(z) < 0$  we know to use  $\theta'$ :

$$r : \sqrt{3} \wedge \theta : \frac{5\pi}{3} \therefore z = \sqrt{3}e^{\frac{-i\pi}{3}} \equiv \sqrt{3}e^{\frac{5i\pi}{3}}$$

□

“  
 Finn de to løsningene til likningen  $w^2 - w + 1 = 0$ , og bruk disse til å finne alle komplekse løsninger til likningen  $z^4 - z^2 + 1 = 0$ . Gi en faktorisering av  $z^4 - z^2 + 1$ , først i komplekse førstegradspolynomer og så i reelle andregradspolynomer.  
 ”

Assuming  $\text{cis}(x) \equiv \cos(x) + i \cdot \sin(x)$ , and  $i^2 = -1$ .

$$a = 1, b = -1, c = 1 \quad (2-1)$$

$$w = \frac{1 \pm \sqrt{1-4}}{2} = \frac{1}{2} \pm \frac{\sqrt{3}}{2}i \quad (2-2)$$

$$(2-3)$$

With  $t = z^2$ :

$$z^4 - z^2 + 1 = t^2 - t + 1 = 0 \quad (2-4)$$

$$t = \frac{1}{2} \pm \frac{\sqrt{3}}{2}i \quad (2-5)$$

$$(2-6)$$

With  $t_0 = \frac{1}{2} + \frac{\sqrt{3}}{2}i = \text{cis}(\frac{\pi}{3})$ :

$$\sqrt{\text{cis}(\frac{\pi}{3})^4} - \sqrt{\text{cis}(\frac{\pi}{3})^2} + 1 = 0 \quad (2-7)$$

$$\text{cis}^2(\frac{\pi}{3}) - \text{cis}(\frac{\pi}{3}) + 1 = 0 \quad (2-8)$$

$$(2-9)$$

With De Moivre's formula:

$$\text{cis}(\frac{2\pi}{3}) - \text{cis}(\frac{\pi}{3}) + 1 = 0 \quad (2-10)$$

$$\cos(\frac{2\pi}{3}) + i \cdot \sin(\frac{2\pi}{3}) - \cos(\frac{\pi}{3}) - i \cdot \sin(\frac{\pi}{3}) + 1 = 0 \quad (2-11)$$

$$-\frac{1}{2} + \frac{\sqrt{3}}{2}i - \frac{1}{2} - \frac{\sqrt{3}}{2}i + 1 = 0 \quad (2-12)$$

$$-\frac{1}{2} - \frac{1}{2} + 1 = 0 \quad (2-13)$$

$$-1 + 1 = 0 \quad (2-14)$$

$$(2-15)$$

Therefore,  $z = t_0^{\frac{1}{2}} = \text{cis}^{\frac{1}{2}}(\frac{\pi}{3}) = \text{cis}(\frac{\pi}{6})$  is a root  $z_0$ .

By the complex conjugate root theorem,  $z = t_1^{\frac{1}{2}} = \text{cis}^{\frac{1}{2}}(-\frac{\pi}{3}) = \text{cis}(-\frac{\pi}{6})$  is a root  $z'_0$ .

□

“ Finn grensene  $\lim_{n \rightarrow \infty} \frac{3n+2}{\sqrt{4n^2-1}}$  og  $\lim_{n \rightarrow \infty} (\sqrt{n^2-5n} - n)$ . ”

$$\lim_{n \rightarrow \infty} \frac{3n+2}{\sqrt{4n^2-1}} =$$

$$\lim_{n \rightarrow \infty} \frac{(3n+2) \frac{1}{n}}{\sqrt{4n^2-1} \frac{1}{n}} =$$

$$\lim_{n \rightarrow \infty} \frac{3 + \frac{2}{n}}{\sqrt{\frac{4n^2-1}{n^2}}} =$$

$$\lim_{n \rightarrow \infty} \frac{3 + \frac{2}{n}}{\sqrt{4 - \frac{1}{n^2}}} =$$

$$\frac{3 + \emptyset}{\sqrt{4 - \emptyset}} =$$

$$\frac{3}{2}$$

$$\lim_{n \rightarrow \infty} (\sqrt{n^2-5n} - n)$$

$$\lim_{n \rightarrow \infty} \frac{(\sqrt{n^2-5n} - n)(\sqrt{n^2-5n} + n)}{\sqrt{n^2-5n} + n}$$

$$\lim_{n \rightarrow \infty} \frac{n^2 - 5n - n^2}{\sqrt{n^2-5n} + n}$$

$$\lim_{n \rightarrow \infty} \frac{-5n}{\sqrt{n^2-5n} + n}$$

$$\lim_{n \rightarrow \infty} \frac{-5}{\sqrt{1 - \frac{5}{n}} + 1}$$

$$\frac{-5}{\sqrt{1 - \emptyset} + 1}$$

$$\frac{-5}{2}$$

□

□

“ Finn de komplekse tallene  $z$  som oppfyller likningen  $2|z-1| = |z-4|$  og skisser løsningsmengden i det komplekse planet. (Hint: Sett inn  $z = x+iy$  og finn en polynomlikning i  $x$  og  $y$  for løsningsmengden.) ”

$$2|z-1| = |z-4| \equiv$$

$$4|x+iy-1| = |x+iy-4|$$

$$4((x-1)^2 + (y)^2) = (x-4)^2 + (y)^2$$

$$4(x-1)^2 + 4y^2 = x^2 - 8x + 16 + y^2$$

$$4(x^2 - 2x + 1) + 4y^2 = x^2 - 8x + 16 + y^2$$

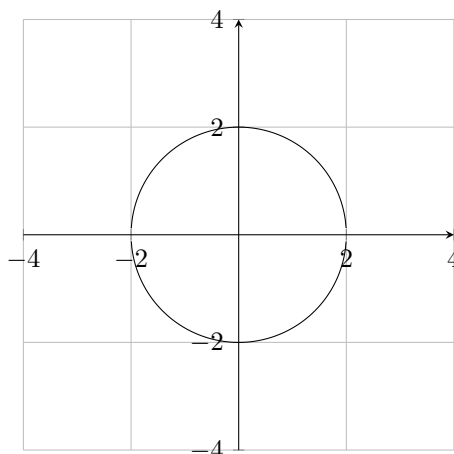
$$4x^2 - 8x + 4 + 4y^2 = x^2 - 8x + 16 + y^2$$

$$3x^2 + 3y^2 = 12$$

$$y^2 = 4 - x^2$$

$$y = \sqrt{4 - x^2}$$

$$\therefore \operatorname{Im}(z) = \pm \sqrt{4 - \operatorname{Re}(z)^2}, \exists z \forall \{x \in \operatorname{Re}(z) \mid -2 < x < 2\}$$



□

“ En følge  $\{a_n\}$  er definert ved  $a_1 = 3, a_n + 1 = 3\sqrt{a_n}$  for  $n \geq 1$ . Vis at  $a_n < 9$  og at  $a_n + 1 > a_n$  for alle  $n$ . Forklar hvorfor følgen konvergerer og finn  $\lim_{n \rightarrow \infty} a_n$  ”

□.

Submitted by Rolf Vidar Hoksaaas on September 13, 2019.