Introduction to Decision Support based on Cake Cutting problem

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February 28, 2017

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What is Decision Support?

The goal of Decision Support

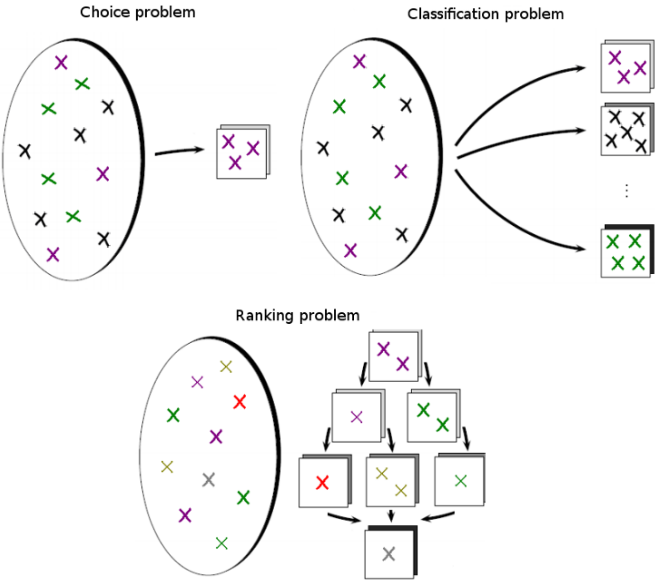
The main aim of decision support is proposing algorithms that simplify a process of making decisions i.e. choosing a new car, camera, etc. In other words, we look for a method which solves a specific decision problem and let us achieve a goal.

Decision problem

A situation where there is a necessity to choose one of at least two possible variants of actions. A decision maker has to answer one of the following questions:

- How to choose the best variant? (Choice problem)
- How to classify variants into decision classes? (Classification problem)
- How to order variants from the best to the worst? (Ordering problem)

What is Decision Support?



Specific areas of Decision Support

- What is a number of decision makers?
- What is a number of criteria?
- Is a consequence of action deterministic or uncertain?

	Theory of Social Choice	Multi-Criteria Decision Ma
Number of DM	many	one
Number of criteria	one	many
Risk and Uncertainty	No	No

What do we need to construct decision support algorithm?

Preference information

The information that is given by decision maker in order support solving a problem.

Preference model

Preference model allows to aggregate evaluations on each criterion of specific variant. It is built by preference information given by decision maker. We usually distinguish three types of preference model:

- function,
- relational system,
- set of decision rules.

Criterion

Criterion is a real-valued function reflecting a worth of variants from a particular point of view. Family of criteria should be consistent.

Cake cutting

Cake-cutting is a metaphor for a wide range of real-world problems that involve dividing some continuous object, whether its cake or, say, a tract of land, among people who value its features differently. The ideal method, which solves the problem, should:

- work for any number of players,
- make a division proportional,
- make a division envy-free,
- make a division equitable.

Historical background

The model

Let us denote a set of players by $N = 1, \ldots, n$ and our devisible good - the cake - by the interval $[0, 1]$. Moreover we assume that each player is endowed with a valuation function V_i (information preference), which maps a given subinterval $I \subseteq [0, 1]$ to it by player i, $V_i(I)$.
We are certainly interested in allocations $A = (A_1, \ldots, A_n)$, where each A_i is the piece of cake allocated to agent i. Now we can express our criteria in a more formal way:

- Proportionality: for all $i \epsilon N$, $V_i(A_i) \geq \frac{1}{n}$,
- Envy-freeness: for all $i, j \epsilon N$, $V_i(A_i) \geq V_i(A_j)$,
- Equitability: for all $i, j \epsilon N$, $V_i(A_i) = V_j(A_j)$

Proportionality for n = 2: Cut and Choose

1. Player 1 cuts the cake into two equally-valued pieces X_1 and X_2, such that $V_1(X_1) = V_1(X_2) = \frac{1}{2}$
2. Player 2 chooses its preferred piece and player 1 receives the remaining piece.

Proportionality for any n: Banach-Knaster

Proportionality for any n: Dubins-Spanier

1. In the first step each player $i \epsilon N$ makes a mark at the point x_i such that $V_i([0, x_i]) = \frac{1}{n}$. The player j that made leftmost mark exits with the piece $A_j = [0, x_j]$.
2. If there is only one player left, it receives uclaimed piece of cake else go to step 1.

Proportionality for any n: Even-Paz

1. In the first step each player from a subset $1, \ldots, k$ makes a mark at the point x_i such that $V_i([y, x_i]) = \frac{V_i([y,z])}{2}$.
2. Let $x_1^*, \ldots, x_k^*$ be the marks sorted from left to right. Call the algorithm recursively with players $i_1, \ldots, i_{k/2}$ and the piece $[y, x_{k/2}^*]$, and with players $i_{k/2+1}, \ldots, i_k$ and the piece $[x_{k/2+1}^*, z]$.
3. If there are only one player i and and interval I, assign $A_i = I$

Envy-freeness for n = 3: Selfridge-Conway

1. Agent 1 divides the cake into three eequally-valued pieces $X_1, X_2, X_3 : V_1(X_1) = V_1(X_2) = V_1(X_3) = \frac{1}{3}$
2. Agent 2 trims the most valuable piece according to V_2 to create a tie for most valuable.

How about an env-free algorithm for any number of players?

References

Scott Meyers (2002)
Effective C++: 50 Specific Ways to Improve Your Programs and Design

C++ reference
`http://en.cppreference.com/`

The End