DOA Estimation Using Mixed Resolution Data

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***Abstract*—Direction of arrival (DOA) estimation is a crucial aspect of array processing and radar technology. In this context, the use of mixed-resolution data, which includes both analog and 1-bit quantized measurements, is being discussed. The impact of varying the number of low-resolution measurements used in different analytic algorithms is being investigated to explore ways to utilize mixed-resolution data in DOA estimation. Our goal is to uncover methods for effectively utilizing both low and high-resolution sensors, employing suitable algorithms, and adjusting the covariance matrix to enhance DOA estimation in mixed-resolution array case.**

I. Introduction

Array signal processing is an important branch of the signal processing field, widely used in radar signals, underwater sonar, wireless communication, radio astronomy, and other fields. Primarily, array signal processing focuses on handling received signals, boosting essential signals, reducing irrelevant interference and noise, and extracting vital parameters. Estimating DOA is a significant research area in this field. The traditional DOA estimation methods are mainly based on beamforming and 'null pattern' guidance techniques [1]. The DOA challenge has been extensively explored from various angles. One facet, that receives more and more mentions in the literature, involves examining the impact of sample quantization on DOA estimation. When dealing with systems having very large arrays, using 1-bit sampling can be a cost-effective option. This approach offers a practical estimation accuracy while also allowing for high-rate sampling. For this reason, lower-resolution Analog to digital convertor (ADC) have attracted ubiquitous attention in the field of signal processing and DOA estimation [2], [3]. As a pioneer work, the connection between the one-bit covariance matrix and the analog covariance matrix has been discussed in [4] in the realm of DOA estimation, where the relationship between the unquantized covariance matrix and the 1-bit quantized covariance matrix provided.

The connection between the one-bit covariance matrix and the analog covariance matrix has been discussed in [5] for strictly non-circular sources. In [6], the connections between the two matrices were examined as well, where for uncorrelated signals case in low Signal to noise ratio (SNR) region, a linear approximation of the 1-bit covariance matrix is offered. Thus, subspace-based methods can be straightforwardly applied without extra pre-processing. This leads to the so-called one-bit approach.

However, we would like to see whether using mixed-resolution data can narrow the disparity between the high-resolution to low-resolution approaches. We discuss the effects of mixed-resolution quantization of the input signals on the DOA's estimation accuracy. While there are methods that focus solely on quantized signals, less attention has been given to the mixed-resolution approach [7]. We compare the usage of analytic algorithms with and without transformation of the mixed-resolution covariance matrix and examine different analytic methods for mixed-resolution DOA estimation.

II. System Model

Assume that K narrowband far-field signals impinge on an M-element array from different directions $\{\theta_1, \theta_2, \ldots, \theta_K\}$. Under the assumption of infinite-resolution quantization, the output vector $\mathbf{x}(t) = [x_1(t), \ldots, x_M(t)]^T \in \mathbb{C}^M$ of the array at time t can be expressed as:

$$\mathbf{x}(t) = \mathbf{As(t)} + \mathbf{n(t)} \tag{1}$$

where $\mathbf{A} = [\mathbf{a}(\theta_\mathbf{1}), \ldots, \mathbf{a}(\theta_\mathbf{K})] \in \mathbb{C}^{\mathbf{M}\times\mathbf{K}}$ represents the steering matrix, and each column is steering vector

$$a(\boldsymbol{\theta}_k) = \left[1, \ldots, \mathrm{e}^{-j\pi sin(\theta_K)(M-1)}\right]^T \tag{2}$$

using the fact that element spacing is a half wavelength, $d = \frac{\lambda}{2}$.

$s(t) \sim \mathcal{CN}(0, \sigma_s^2\mathbf{I}_k)$ and $\mathbf{n(t)} \sim \mathcal{CN}(\mathbf{0}, \sigma_\mathbf{n}^\mathbf{2}\mathbf{I_M})$ are, respectively, the signal vector and noise vector, and $(\cdot)^T$ denotes the transpose. Note that the signal and noise are assumed to be uncorrelated, and both of them are modeled as independent, zero-mean, circular, complex, Gaussian random processes.

When one-bit ADC are employed for quantization, the array output should be modified as

$$\mathbf{x(t)} = \mathcal{Q}(\mathbf{y(t)}) = \mathcal{Q}(\mathbf{As(t)} + \mathbf{n(t)}) \tag{3}$$

where $\mathcal{Q}(\cdot)$ represents a complex-valued element-wise quantization function composed of two sign functions as

$$\mathcal{Q}(z) = \frac{1}{\sqrt{2}}\left(\text{sign}(\text{Re}\{z\}) + j\text{sign}(\text{Im}\{z\})\right)$$

and $\text{Re}\{z\}$ and $\text{Im}\{z\}$ denote the real part and imaginary part of a complex-valued number z, respectively.

For mixed-resolution data, only the first Q elements of the observation vector, $\mathbf{x}$, will pass through the quantization operator.

Our final observation

$$\mathbf{x} = \left[\mathbf{x}_{\mathbf{q_1}}, \ldots, \mathbf{x}_{\mathbf{q_Q}}, \mathbf{x}_{\mathbf{a_1}}, \ldots, \mathbf{x}_{\mathbf{a_{(M-Q)}}}\right] \tag{4}$$

can be separated into quantized and analog elements, where $\mathbf{x}_q = Q(y)$.

We will utilize the covariance matrix based on $\mathbf{x}$ for our analytical algorithms, which will be discussed further on.

For T snapshots, the covariance matrix $\mathbf{R_{xx}}$ is define as:

$$\mathbf{R_{xx}} = \mathrm{E}\left[\mathbf{xx^H}\right] = \mathbf{A}(\boldsymbol{\theta})\mathbf{R_{ss}}\mathbf{A}(\boldsymbol{\theta})^\mathbf{H} + \sigma_\mathbf{n}^\mathbf{2}\mathbf{I_M} \tag{5}$$

where $R_{ss} = \mathbb{E}[s(t)s^H(t)]$ is the signal covariance matrix, σ_n^2 is noise power, and $\mathbf{I}_K$ is the identity matrix of size $M \times M$. Since we would like to estimate $\boldsymbol{\theta}$, we use the law of big numbers for reconstruct the covariance matrix, using T snapshots each time t:

$$\mathbf{R} \approx \frac{\mathbf{1}}{\mathbf{T}} \sum_{\mathbf{t=1}}^{\mathbf{T}} \mathbf{x(t)x(t)^H} \tag{6}$$

III. Mixed Resolution Approach

Inspired by [6], [5] our proposal involves utilizing the mixed resolution structure to alter the covariance matrix $\mathbf{R}_{xx}$. We will apply the arcsine law established by Julian J. Bussgang [8] to accomplish this. This law states that the covariance of variable $x_q = Q(y)$ is equivalent to:

$$\begin{aligned}\mathbf{R}_{x_q} =& \frac{2}{\pi}\Big[\arcsin\Big(\text{diag}\left[\mathbf{R}_y\right]^{-\frac{1}{2}} \mathcal{R}e\left[\mathbf{R}_y\right] \text{diag}\left[\mathbf{R}_y\right]^{-\frac{1}{2}}\Big) \\ &+ j \arcsin\Big(\text{diag}\left[\mathbf{R}_y\right]^{-\frac{1}{2}} \mathcal{I}m\left[\mathbf{R}_y\right] \text{diag}\left[\mathbf{R}_y\right]^{-\frac{1}{2}}\Big)\Big]\end{aligned} \tag{7}$$

Where diag$[A]$ represents a diagonal matrix with the same diagonal entries as matrix A. For the model described in II, we get diag$[\mathbf{R}_y] = \rho\mathbf{I}$, where $\rho = \sum_s \sigma_s^2 + \sigma_n^2 = K\sigma_s^2 + \sigma_n^2$.

When the argument of the $\arcsin$ function is small, we can approximate it to the identity function $f(x) = x$ using the Taylor series expansion as proposed in [6]. This approximation is suitable for cases where the SNR is low, and the correlation between y_i and y_j is small. Note that the diagonal elements of $\mathbf{R}_y$ are always get high value, and therefore we can say:

$$\hat{\mathbf{R}}_y = \frac{\rho\pi}{2}(\mathbf{R}_{x_q} - ((1 - \frac{2}{\pi})\mathbf{I})) \tag{8}$$

In addition, the cross-covariance between Gaussian variable x_a and quantized variable $x_q = Q(y)$ is [8] [9]:

$$\mathbf{R}_{x_a x_q} = \sqrt{\frac{2}{\pi}}\mathbf{R}_{x_a y}\text{diag}\left[\mathbf{R}_y\right]^{-\frac{1}{2}} \tag{9}$$

Where x_a and y are Gaussian zero-mean random variables. According to 9 we can write $\mathbf{R}x_a y$ as:

$$\hat{\mathbf{R}}_{x_a y} = \sqrt{\frac{\pi\rho}{2}}\mathbf{R}_{x_q y} \tag{10}$$

Consider the mentioned results (7),(9), we are getting a connection between covariance and cross-covariance of quantized (or partially quantized) measurements and their analog high-resolution measurements. Note that the covariance matrix $\mathbf{R}_x$ is a block matrix given by:

$$\mathbf{R}_x = \mathbf{C_x} = \begin{bmatrix} \mathbf{R}_{\mathbf{x}_q} & \mathbf{R}_{\mathbf{x}_q\mathbf{x}_a} \\ \mathbf{R}_{\mathbf{x}_a\mathbf{x}_q} & \mathbf{R}_{\mathbf{x}_a} \end{bmatrix} \tag{11}$$

And we define:

$$\hat{\mathbf{R}}_x = \begin{bmatrix} \hat{\mathbf{R}}_{\mathbf{x}_q} & \hat{\mathbf{R}}_{\mathbf{x}_q\mathbf{x}_a} \\ \hat{\mathbf{R}}_{\mathbf{x}_a\mathbf{x}_q} & \mathbf{R}_{\mathbf{x}_a} \end{bmatrix} \tag{12}$$

Where $\hat{\mathbf{R}}_{\mathbf{x}_q\mathbf{x}_a}$ and $\hat{\mathbf{R}}_{\mathbf{x}_q}$ are actually the reconstruction of $\mathbf{R}_{y\mathbf{x}_a}$ and $\mathbf{R}_y$ respectively, based on (9) and (7). We aim to investigate the impact of input $\hat{\mathbf{R}}_x$ instead of $\mathbf{R}$ to the mentioned algorithms. We will call it **Sin recon** in our simulations, where **Lin recon** refer to (8).

Algorithm 1 DOA estimation for mixed resolution data

Input: Received signal matrix $\mathbf{X}$
Output: Estimated angles of arrival $\boldsymbol{\theta} = [\theta_1, \ldots, \theta_D]$
Compute the sample covariance matrix
$\mathbf{R} = \frac{1}{T}\sum_{t=1}^{T}\mathbf{x}(t)\mathbf{x}^H(t)$
Apply Bussgang theorem on the sample covariance matrix $\mathbf{R}$ for getting approximation of $\mathbf{R}_{y,x}$: $\mathbf{R} \to \hat{\mathbf{R}}$
Apply Estimation of signal parameters via rotational invariance techniques (ESPRIT) algorithm using $\hat{\mathbf{R}}$
return $\boldsymbol{\theta}$

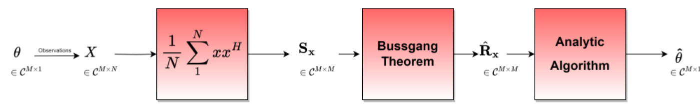


Fig. 1: Block diagram for mixed resolution DOA

IV. Simulations

In this part, simulations are carried out to verify the previous analysis, validate the effectiveness of the mixed-resolution data, and compare it with a single resolution system.

We have made simulations for the described Uniform linear array (ULA) array for angles in the range [-60°,60°], assuming 2 sources. A clear resolution was achieved by assuming a minimum separation of 0.5° degrees between the sources.

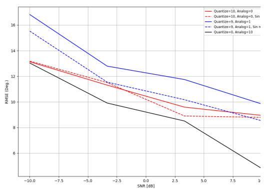
The Root Mean Square Error (RMSE) is define by:

$$\text{RMSE} = \sqrt{\frac{1}{RD}\sum_{i=1}^{D}\sum_{j=1}^{R}(\hat{\theta}_{i,j} - \theta_i)} \tag{13}$$

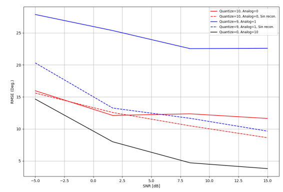
Where D is the number of sources, and R is the number of Monte Carlo iterations. The full analog system, which is expensive and energy-limited, constitutes as our bound.

As We see in figure 2, which has been done using 400 snapshots and 1000 Monte Carlo simulations, using naive $\mathbf{R}$ matrix for DOA algorithm is inconsistent in the performance regarding the number of high-resolution measurements. The results are not intuitive and can probably harm our system, which might be expensive and inefficient relative to a pure quantized array. Using the new covariance matrix, $\hat{\mathbf{R}}$, seems to improve the relation between different arrays and makes it more sensible, as we can see that using this method significantly improves the mixed resolution array's performance. However, the resource allocation problem is better addressed using the ESPRIT algorithm, where mixed resolution systems provide better performance than pure 1-bit systems, after reconstructing the covariance matrix (12). Due to figure 2, using $\hat{\mathbf{R}}$ has the advantage of eliminating non-monotonic behavior caused by 1-bit sensors, possibly due to dithering. This feature simplifies the process of system design, as we can ensure constant performance across all SNR levels.

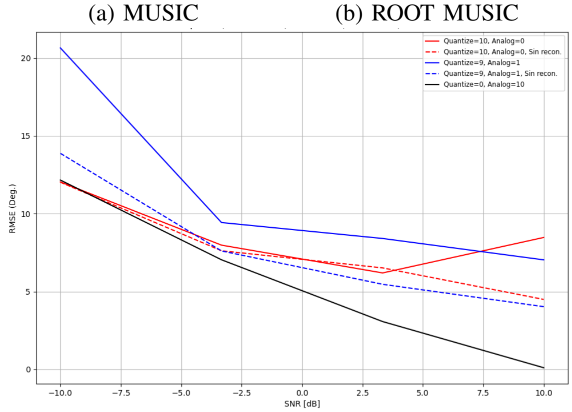
The second simulation focused on using linear reconstruction as discussed in equation (8). We conducted 1000 Monte



(a) MUSIC



(b) ROOT MUSIC



(c) ESPRIT

Fig. 2: RMSE as a function of the sensors's partition and the algorithm used

Carlo simulations and determined a minimum gap of 2° between the sources. As expected, when the SNR is low, both the Sin and Lin methods provide the same results. However, as the SNR increases, the linear approach does not yield good results as the Taylor approximation becomes invalid.

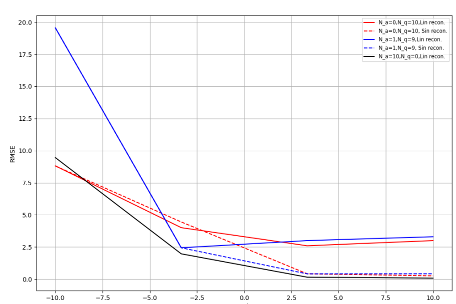


Fig. 3: RMSE as a function of the sensors's partition, using both Sin and Lin reconstruction for $\mathbf{R}$

We also conducted a study on the effects of adjusting the number of snapshots for the ESPRIT algorithm. Our findings indicate that utilizing a single high-resolution sensor in a low-resolution system without transforming the $\mathbf{R}$ is not cost-effective and does not offer optimal performance. However, using the estimated C matrix $\hat{\mathbf{R}}_{x_q}$ yields significantly better results.

Finally, we are analyzing the probability of successfully resolving the mixed resolution ESPRIT algorithm. We assume that the DOA are at 20° degrees and 20°+Δ degrees, where Δ represents the angular separation. a successful resolution is defined when the deviations of both DOA estimates are below $\frac{1}{2}\Delta$. The resulting probability of resolution is depicted in Fig. 5, varying with angular separation. We can see that the difference between a pure 1-bit system with or without employing the Bussgang theorem is minimal. However, the application of the theorem in mixed-resolution systems is crucial and substantially enhances detection performance. As previously observed, the simplistic use of a single high-resolution sensor can degrade performance despite its higher cost.

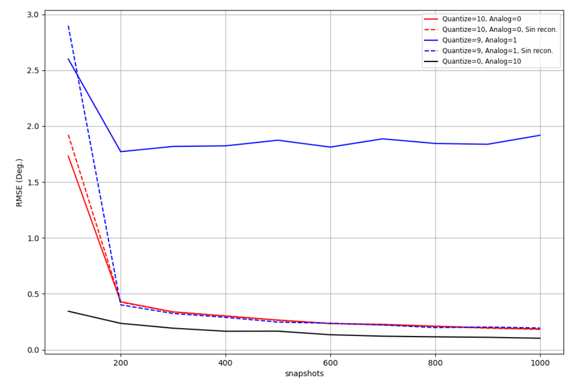


Fig. 4: Comprasion of the system performance for different partitions, as a function of snapshots

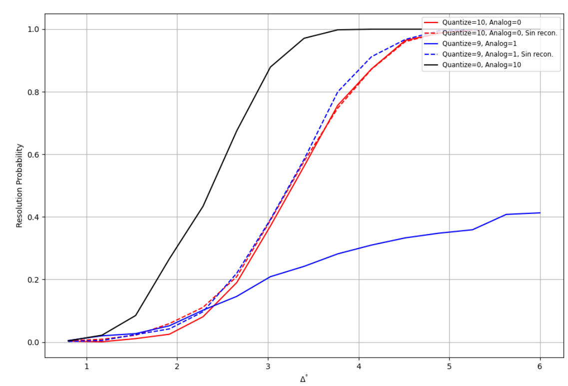


Fig. 5: Resolution probability as a function of angular separation

V. Conclusions

Our investigation focused on enhancing existing algorithms by leveraging mixed-resolution data. Through our experimentation, we discovered that a systematic adjustment of the covariance matrix using the Bussgang theorem consistently yielded superior results and must be done in order to use a non-analog resolution array properly. Our analysis has led us to conclude that the ESPRIT method is the most suitable algorithm for a mixed-resolution array. However, there is ample room for innovation by considering various system properties and exploring alternative transformations of the covariance matrix ($\mathbf{R}$). In addition, the exploration of model-based networks within the context of DOA estimation presents an intriguing area for research. Thus, our ongoing exploration of these facets promises to contribute significantly to the advancement of mixed-resolution DOA estimation techniques.

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