Direction Of Arrival Estimation Using Graph Signal Processing

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***Abstract*—Direction of arrival (DOA) estimation plays a pivotal role in array processing and radar technology, with ongoing discussions centered around the utilization of mixed-resolution data incorporating both analog and 1-bit quantized measurements. Investigating the impact of varying the number of low-resolution measurements on different analytic algorithms aims to uncover methods for effectively leveraging both low and high-resolution sensors, enhancing DOA estimation in mixed-resolution array cases. Concurrently, DOA estimation, supported by graph signal processing, introduces innovative methodologies for discerning arrival directions across various applications. This work delves into the utilization of graph-based approaches to address this challenge, juxtaposing them against established methods such as the MUSIC algorithm. Specifically, a multi-objective DOA methodology is explored, leveraging a fully connected graph to emulate sensor connectivity. Simulation outcomes validate the efficacy of graph-based solutions, closely aligning with conventional approaches and highlighting their potential in resolving this pertinent issue.**

I. Introduction

In contemporary signal processing, the ability to accurately estimate the direction of arrival (DOA) of signals is paramount across various domains, including wireless communications, radar systems, sonar, and microphone arrays. DOA estimation serves as a fundamental building block for tasks such as beamforming, target tracking, and spatial localization. Traditionally, DOA estimation methods have relied heavily on classical signal processing techniques such as beamforming algorithms and subspaces methods, like Multiple signal classification (MUSIC), root-acmusic, and Estimation of signal parameters via rotational invariance techniques (ESPRIT).

Parallel to solving problems with classical methods, paradigms such as Graph Signal Processing (GSP) have been developed which have received increasing attention due to their ability to efficiently model and analyze data with complex structural dependencies.

GSP extends traditional signal processing concepts to data represented by graphs or networks, where nodes represent data samples and edges capture relationships or interactions between them. This framework provides a powerful toolkit for exploiting the underlying structure and topology of data, enabling more powerful and efficient signal processing techniques.

Using GSP for the DOA estimation problem offers another and innovative way to solve it. By translating sensor measurements into nodes within a graph and delineating their spatial connections using graph edges, GSP provides a systematic methodology for exploiting spatial dependencies, thereby refining DOA estimation.

In this work, we chose to explore the use of GSP techniques for DOA estimation and test their effectiveness in comparison to classical methods. We provide an overview of traditional DOA estimation methods and their limitations. Then, we present GSP-based algorithms suitable for the task of DOA estimation, discussing their theoretical foundations and practical applications. Finally, we compare the performance of these methods to traditional approaches using simulations and analyze the results.

Through this work, we aim to demonstrate the potential of graph signal processing as a valuable tool for advancing DOA estimation capabilities.

II. System Model

Assume that K narrowband far-field signals impinge on an M-element array from different directions $\{\theta_1, \theta_2, \ldots, \theta_K\}$. Under the assumption of infinite-resolution quantization, the output vector $\mathbf{x}(t) = [x_1(t), \ldots, x_M(t)]^T \in \mathbb{C}^M$ of the array at time t can be expressed as:

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(\mathbf{t}) + \mathbf{n}(\mathbf{t}) \tag{1}$$

where $\mathbf{A} = [\mathbf{a}(\theta_\mathbf{1}), \ldots, \mathbf{a}(\theta_\mathbf{K})] \in \mathbb{C}^{\mathbf{M}\times\mathbf{K}}$ represents the steering matrix, and each column is steering vector

$$a(\boldsymbol{\theta}_k) = \left[1, \ldots, \mathrm{e}^{-j\pi sin(\theta_K)(M-1)}\right]^T \tag{2}$$

using the fact that element spacing is a half wavelength, $d = \frac{\lambda}{2}$.

$s(t) \sim \mathcal{CN}(0, \sigma_s^2 \mathbf{I}_k)$ and $\mathbf{n}(\mathbf{t}) \sim \mathcal{CN}(\mathbf{0}, \sigma_\mathbf{n}^\mathbf{2}\mathbf{I_M})$ are, respectively, the signal vector and noise vector, and $(\cdot)^T$ denotes the transpose. Note that the signal and noise are assumed to be uncorrelated, and both of them are modeled as independent, zero-mean, circular, complex, Gaussian random processes.

We would like to mention that when one-bit Analog to digital convertor (ADC) are employed for quantization, the array output should be modified as

$$\mathbf{x}(\mathbf{t}) = \mathcal{Q}(\mathbf{y}(\mathbf{t})) = \mathcal{Q}(\mathbf{A}\mathbf{s}(\mathbf{t}) + \mathbf{n}(\mathbf{t})) \tag{3}$$

where $\mathcal{Q}(\cdot)$ represents a complex-valued element-wise quantization function composed of two sign functions as

$$\mathcal{Q}(z) = \frac{1}{\sqrt{2}} \left(\mathrm{sign}(\mathrm{Re}\{z\}) + j\mathrm{sign}(\mathrm{Im}\{z\})\right)$$

and $\mathrm{Re}\{z\}$ and $\mathrm{Im}\{z\}$ denote the real part and imaginary part of a complex-valued number z, respectively.

Using mixed-resolution data, only the first Q elements of the observation vector, $\mathbf{x}$, will pass through the quantization operator.

Our final observation

$$\mathbf{x} = \left[\mathbf{x_{q_1}}, \ldots, \mathbf{x_{q_Q}}, \mathbf{x_{a_1}}, \ldots, \mathbf{x_{a_{(M-Q)}}}\right] \tag{4}$$

can be separated into quantized and analog elements, where $\mathbf{x}_q = Q(y)$. For a single resolution case, we will apply $Q = 0$, and the whole observations will be analog.

Since we would like to estimate $\boldsymbol{\theta}$, we use the law of big numbers for reconstruct the autocorrelation matrix $\mathbf{R}_x$, using T snapshots each time t:

$$\mathbf{R} \approx \mathbf{\frac{1}{T}\sum_{t=1}^{T} x(t)x(t)^{H}} \tag{5}$$

III. Subspace Methods

The MUSIC algorithm is a technique in array signal processing for estimating the Direction of Arrival (DOA) of multiple sources. Given an array of M sensors, the received signal vector at time instant t can be represented as $\mathbf{x}(t) = [x_1(t), \ldots, x_M(t)]^T$. The key idea behind MUSIC is to compute the eigenvalues and eigenvectors of the array covariance matrix $\mathbf{R} = \mathbb{E}[\mathbf{x}(t)\mathbf{x}^H(t)]$. By projecting the received signal onto the orthogonal complement of the signal subspace, MUSIC forms the spatial spectrum $P(\theta)$:

$$P(\theta) = \frac{1}{\mathbf{a}^H(\theta)\mathbf{Q}_n\mathbf{Q}_n^H\mathbf{a}(\theta)}$$

where θ represents the DOA angle, $\mathbf{a}(\theta)$ is the steering vector as define in (2), and $\mathbf{Q}_n$ is the noise covariance matrix. Peaks in the spatial spectrum correspond to potential DOA estimates.

$$\hat{\theta} = \arg\max_{\theta}\ P(\theta) \tag{6}$$

Since the analytic form of $\mathbf{R_{xx}}$ is not available, accurate estimation of the DOA relies on the reconstruction of the noise covariance matrix $\mathbf{Q}_n$. This matrix captures the statistical properties of the noise affecting the array sensors. To reconstruct $\mathbf{Q}_n$, the eigenvalue decomposition of the array covariance matrix $\mathbf{R}$ from (5) is performed. The smallest eigenvalues correspond to the noise subspace by ordering the eigenvalues in decreasing order. The eigenvectors corresponding to these small eigenvalues span the noise subspace, which is orthogonal to the signal subspace. The noise covariance matrix $\mathbf{Q}_n$ can then be approximated as:

$$\mathbf{Q}_n \approx \sum_{i=1}^{M-K} \lambda_i \mathbf{v}_i \mathbf{v}_i^H$$

where λ_i are the smallest eigenvalues, $\mathbf{v}_i$ are the corresponding eigenvectors, and K represents the number of signal sources. By effectively eliminating the signal components, the MUSIC algorithm exploits this reconstructed noise covariance matrix to enhance the accuracy of DOA estimation.

The Root-MUSIC method is another prominent approach in array signal processing for DOA estimation. Like the MUSIC algorithm, Root-MUSIC operates on the eigen-decomposition of the array covariance matrix to discern the angles of arrival of signals. However, Root-MUSIC focuses on finding the roots of the polynomial derived from the eigenvalues of the covariance matrix. It also exploits the fact that the eigenvalues of the covariance matrix can be related to the spatial frequencies corresponding to the DOAs. By computing the roots of a polynomial function $J(z)$, where:

$$\begin{aligned} J(z) &= \mathbf{a}^H(z)\mathbf{Q}_n\mathbf{Q}_n^H\mathbf{a}(z) \\ z &= \mathrm{e}^{j\pi sin(\theta_K)} \end{aligned} \tag{7}$$

Root-MUSIC directly yields the angles of arrival without the need for spatial spectrum estimation:

$$\theta_k = \arcsin(\frac{z_k}{\pi}) \tag{8}$$

where $z_k|_{k=1}^{D}$ are the D closest J(z) roots to the unit circle.

This method is particularly advantageous in scenarios with a small number of sources. It offers an alternative and complementary approach to MUSIC, providing flexibility for various array processing applications and environmental conditions.

The ESPRIT method is another signal processing technique employed for DOA estimation, particularly in array signal processing scenarios. Another subspace method is the ESPRIT, which relies on the concept of subspace techniques. The fundamental idea behind ESPRIT is to leverage the array antenna's shift-invariant characteristics for estimating the direction-of-arrival (DOA) of received signals. It operates by jointly diagonalizing two submatrices obtained from the covariance matrix of the received signal, thereby separating the estimation of angles from the estimation of signal frequencies. In the context of DOA estimation, ESPRIT effectively decouples the angle estimation problem from the need to estimate the number of sources. This feature contributes to its robustness in situations involving closely spaced sources. ESPRIT provides a computationally efficient and high-resolution alternative for DOA estimation, making it particularly well-suited for practical applications in array signal processing.

IV. Graph Signal Processing

GSP provides a powerful framework for analyzing signals defined on irregular or structured data domains, such as graphs. In GSP, signals are represented as nodes on a graph, and the edges of the graph capture the relationships or interactions between these nodes. By exploiting the graph topology, GSP enables the development of robust signal processing algorithms for irregular sampling patterns, noise, and other challenges encountered in real-world data.

GSP extends classical signal processing concepts to graph-structured data, enabling the application of familiar tools such as Fourier transforms, filtering, and spectral analysis to analyze signals residing on graphs. This framework allows for exploring the fundamental properties of graph signals, including smoothness, bandlimitedness, and spectral representation.

The application of GSP is illustrated through examples such as urban mobility patterns and social network analysis. By modeling data as piecewise-smooth graph signals, GSP captures large-scale variations between entities while preserving small-scale variations within them. Such representations facilitate understanding complex phenomena, such as mobility patterns in urban environments and behavioral trends in social networks.

Utilizing GSP for DOA estimation involves translating sensor measurements into nodes within a graph and delineating their spatial connections using graph edges. This allows for the exploitation of spatial dependencies in the data, leading, as we believe and will check, to improve DOA estimation results.

V. GRAPH-BASED DOA ESTIMATION

A. Related Works

Graph-based approaches for solving DOA problems utilize the spatial configuration of sensor arrays to construct a graph representation where each sensor node is connected to its neighbors based on physical proximity. The signals these sensors receive are then treated as graph signals, with the edges capturing the spatial correlations between them.

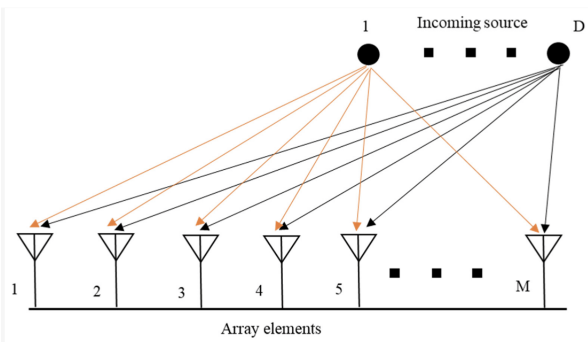


Fig. 1: DOA estimation

Once the sensors are represented as a graph, we can compute the adjacency matrix $\mathbf{A}$ where edge connections between any two elements in the radar array are established, resulting in a fully connected A with zero elements on the diagonal. The spatially phased shift of the radar array forms the structure of $\mathbf{A}$. Matrix $\mathbf{A}$ possesses properties similar to the covariance matrix, enabling a Graph Fourier Transform (GFT) analysis. GFT of the adjacency matrix yields eigenvalue decomposition, providing insights into signal analysis. The eigenvectors correspond to non-zero positions indicative of aligned targets. The orthogonality between steering vectors and noise subspaces is measured to determine accurate DOA.

To understand a bit more about the relationship between DOA and GSP and to see how we can solve DOA problem with it, we search relevant studies.

[1] explores an enhanced DOA estimation method utilizing GSP, particularly focusing on scenarios with multiple targets and angles. By leveraging GSP theory, the researchers propose a fully connected adjacency matrix based on the spatial phase shift of radar arrays. The use of the GFT method achieves multi-target orientation DOA estimation. Comparative analysis reveals that GSP approaches the performance of the MUSIC algorithm, especially in low Signal to noise ratio (SNR) ratio conditions.

[2], The methodology involves representing the spatial shift of sensor arrays using a graph framework, where the signal vector corresponds to an eigenvector of the adjacency matrix. Unlike conventional approaches, this method obviates the need for arranging data in a cyclic buffer. By employing the concept of graph product, the study establishes a relationship between Uniform Linear Array (ULA) snapshots and time series representing signals received by sensors. The proposed space-time graph structure enables the use of the Graph Fourier Transform to devise an objective function for DoA estimation. We can learn the effectiveness of the method for modulated signals, even in challenging environments characterized by multipath and interference.

These studies highlight the potential of GSP in advancing DOA assessment techniques, offering innovative solutions, and addressing the limitations of traditional approaches. Based on the observed results, we have decided to attempt to implement an algorithm for solving the DOA problem using GSP ourselves.

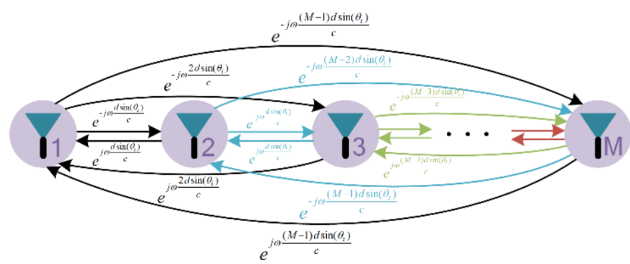


Fig. 2: Radar array signal model

B. Graph Based Algorithm

We focused on the full-connected graph, which is described in figure 2. For such a graph, the adjacency matrix will be:

$$A_s = \frac{1}{M-1}\begin{pmatrix} 0 & e^{-j\omega\tau_{1,2}} & e^{-j\omega\tau_{1,3}} & \cdots & e^{-j\omega\tau_{1,M}} \\ e^{-j\omega\tau_{2,1}} & 0 & e^{-j\omega\tau_{2,3}} & \cdots & e^{-j\omega\tau_{2,M}} \\ e^{-j\omega\tau_{3,1}} & e^{-j\omega\tau_{3,2}} & 0 & \cdots & e^{-j\omega\tau_{3,M}} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ e^{-j\omega\tau_{M,1}} & \cdots & \cdots & e^{-j\omega\tau_{M,M-1}} & 0 \end{pmatrix}$$
$$= \alpha_{\theta_i}\alpha_{\theta_i}^H - \frac{1}{M-1}\mathrm{I}$$

Fig. 3: Adjacency matrix

For this case, we will get one largest value, λ_I, which is referred to as the eigenvector α_{θ_i}, and eigenvalue with a multiplicity of $M-1$, which belongs to the noise subspace. The projection of the input signal $x(t)$, which is created by averaging of the received snapshots, is actually the GFT of the received input. The normalized GFT coefficients in such case should be approximately $\delta[K-M]$, as it is the projection of $x(t)$ on $\mathbf{A}_s(\theta)$.

For the case when $x(t)$, which is imposed on $\mathbf{A}_s(\theta_2)$, is obtained by other angles, the spectral energy will disperse uniformly:

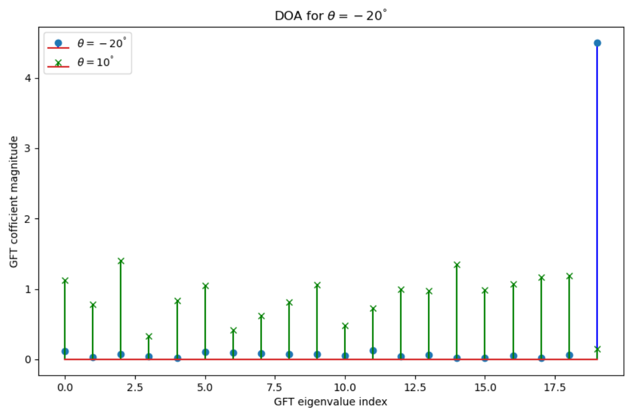


Fig. 4: GFT representation of signal vector impinging standard Uniform linear array (ULA) at $-20°$. Disperse uniform when GFT comes from an incorrect DOA, and a single peak when the GFT corresponds to the correct DOA.

For 2 courses case, where the observations obtained from 2 DOA, $x(t)$ is linear combination of two courses and as so we won't get pure δ for any case. For this reason , our algorithm is probably efficient for the case where two sources are pretty close to each other. However, most of the signal energy will still concentrated in the highest index:

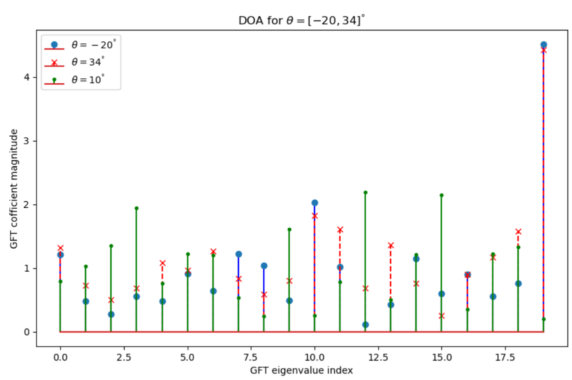


Fig. 5: GFT representation of signal vector impinging standard ULA at 2 directions. The difference between correct or incorrect direction reflect.

As we looking to find the DOA, we are looking for a function that depends on the direction and will get the maximum value in the correct direction. Typically, the pursuit of the peak value aims to ascertain the most accurate DOA outcome. Consequently, the objective of identifying the peak value is attained by eliminating the inverse of the largest eigenvalue [1]:

$$F_{GSP}(\theta) = \frac{1}{\sqrt{\sum_{i=0}^{M-2} |\mathbf{Q}_A^H x|^2}} \tag{9}$$

where $\mathbf{Q}_A$ is the A_s eigenvectors matrix. It may be conceivable to add up $M - D$ elements, but we didn't find this approach efficient, hence we opted to sum $M - 1$ elements.

The concept of multiple roots in the eigenvalue matrix typically constrains the function's result from being excessively large [2]:

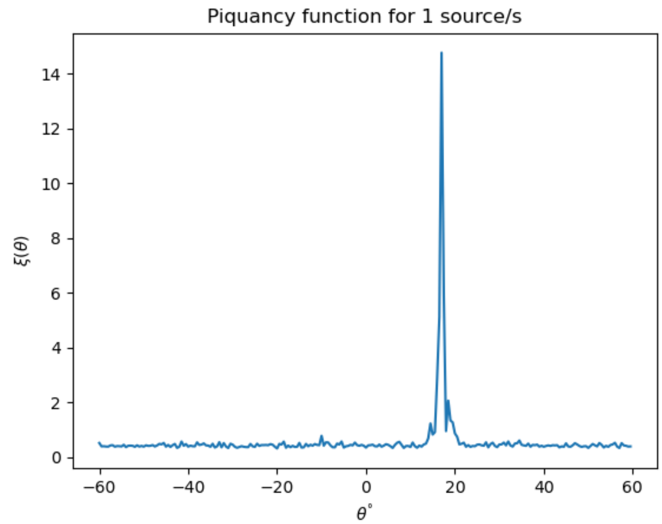


Fig. 6: Signal spectrum, $F_{GSP}(\theta)$ using 1 source, noiseless case

Algorithm 1: DOA using GSP

Input: Received signal matrix $\mathbf{X}$
Output: Estimated angles of arrival $\boldsymbol{\theta} = [\theta_1, \ldots, \theta_D]$
1: compute average $\mathbf{x}$
2: compute GFT[$\mathbf{x}$] using adjacency matrix of A_s
3: compute $F_{GSP}(\theta)$ as described in (9)
4: find the D highest peaks of $F_{GSP}(\theta)$

VI. Simulations And Results

In this part, simulations are carried out to verify the previous analysis and validate the effectiveness of using GSP to solve the DOA problem. We have made simulations for the described ULA array for angles in the range [-100°,100°], assuming 2 sources. A clear resolution was achieved by assuming a minimum separation of 20° degrees between the sources.

For our simulations, we used the model offered in [2], which includes a narrowband signal as the original signal input. Consider a ULA of 20 microphones, as illustrated in Fig. 7, where the processing block following the A/D converter is an analytical bandpass filter centered at ω_0, the central frequency of the incoming narrowband signal. This block is designed to eliminate the negative portion of the input signal spectrum, effectively introducing a delay of τ samples through multiplication by a complex exponential $\mathrm{e}^{-i\omega_0\tau}$. In the scenario of $s_m(t)$ representing a single cosine, the signal from the m-th microphone after passing through the bandpass filter can be represented as $s(k) = \mathrm{e}^{i\omega_0 k}$.

Since our signal is deterministic, we found the subspace methods described in III irrelevant for this model.

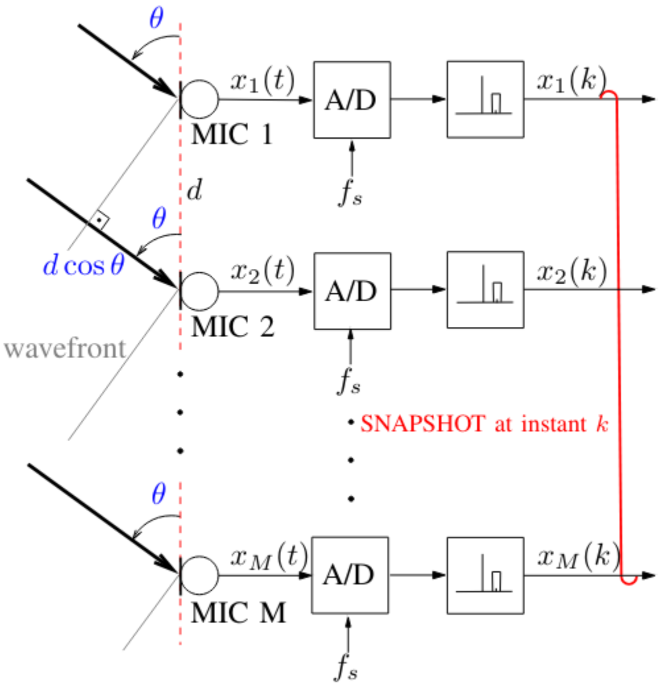
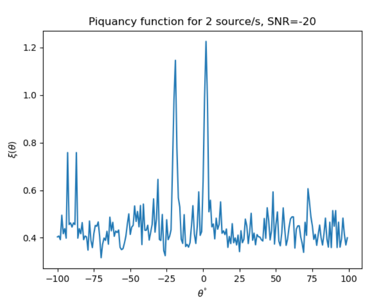
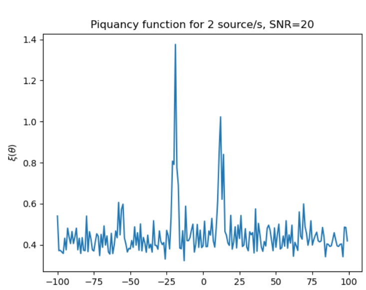


Fig. 7: ULA with M sensors

As depicted in Figure 5, in a system with two sources, the energy of $\hat{x} = \mathbf{Q}_A^H x$ is concentrated in the last index. Consequently, the spectrum is not as smooth as in the single-source scenario.



(a) MUSIC



(b) ROOT MUSIC

Fig. 8: Signal spectrum, $F_{GSP}(\theta)$ using 2 sources

Even with a spectrum that's not conventionally smooth, it's evident that we can still discern the peaks. This observation aids in understanding why the algorithm isn't highly sensitive to SNR changes, as can be shown in figure 9.

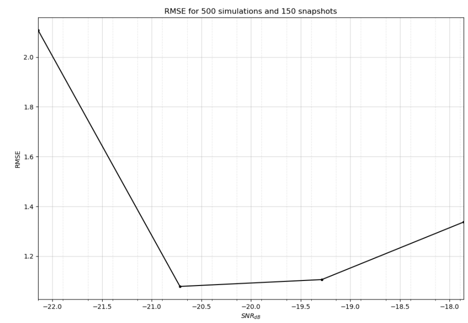


Fig. 9: Root Mean Square Error (RMSE) of GSP algorithm, as function of SNR

VII. Conclusions

In this project, we investigated the appropriate way to solve DOA estimation using GSP. We found it valuable for deterministic signal and pretty consistent under SNR modification. Future fields of research can be done in order to leverage the graph properties to improve the algorithm and expand the usage of GSP for more robust cases in the context of DOA estimation in comparison to another known algorithm. The usage in low-resolution or mixed-resolution data can be examined as well.

References

[1] ,, "Multi-target direction-of-arrival estimation method based on graph signal processing," *CIE International Conference on Radar (Radar)*, 2021.

[2] A. S. Moreira, L. L. Ramos,L. R. de Campos, A. Apolinario Jr. , G. Serrenho, "A graph signal processing approach to direction of arrival estimation," *EUSIPCO*, 2019.