

Problem: $(P \rightarrow Q) \vdash (P \rightarrow (A \rightarrow Q))$

1		$(P \rightarrow Q)$	Premise
2		P	Assumption
3		A	Assumption
4		Q	\rightarrow E 1,2
5		$(A \rightarrow Q)$	\rightarrow I 3-4
6		$(P \rightarrow (A \rightarrow Q))$	\rightarrow I 2-5

Problem: $\vdash \neg(P \wedge \neg P)$

1		$(P \wedge \neg P)$	Assumption
2		P	\wedge E 1
3		$\neg P$	\wedge E 1
4		\perp	\perp I 2,3
5		$\neg(P \wedge \neg P)$	\neg I 1-4

Problem: $(A \vee B) \vdash \neg(\neg A \wedge \neg B)$

1		$(A \vee B)$	Premise
2		$(\neg A \wedge \neg B)$	Assumption
3		A	Assumption
4		$\neg A$	\wedge E 2
5		\perp	\perp I 3,4
6		B	Assumption
7		$\neg B$	\wedge E 2
8		\perp	\perp I 6,7
9		\perp	\vee E 1,3-5,6-8
10		$\neg(\neg A \wedge \neg B)$	\neg I 2-9

Problem: $(A \vee \exists x Fx) \vdash \exists x(A \vee Fx)$

1		$(A \vee \exists x Fx)$	Premise
2		A	Assumption
3		$(A \vee Fa)$	$\vee I$ 2
4		$\exists x(A \vee Fx)$	$\exists I$ 3
5		$\exists x Fx$	Assumption
6		Fa	Assumption
7		$(A \vee Fa)$	$\vee I$ 6
8		$\exists x(A \vee Fx)$	$\exists I$ 7
9		$\exists x(A \vee Fx)$	$\exists E$ 5,6-8
10		$\exists x(A \vee Fx)$	$\vee E$ 1,2-4,5-9

Problem: $\vdash \forall x \forall y ((Fx \wedge \neg Fy) \rightarrow \neg x = y)$

1		a	Flag
2		b	Flag
3		$(Fa \wedge \neg Fb)$	Assumption
4		$a = b$	Assumption
5		Fa	$\wedge E$ 3
6		$\neg Fb$	$\wedge E$ 3
7		Fb	$=E$ 4,5
8		\perp	$\perp I$ 6,7
9		$\neg a = b$	$\neg I$ 4-8
10		$((Fa \wedge \neg Fb) \rightarrow \neg a = b)$	$\rightarrow I$ 3-9
11		$\forall y ((Fa \wedge \neg Fy) \rightarrow \neg a = y)$	$\forall I$ 2-10
12		$\forall x \forall y ((Fx \wedge \neg Fy) \rightarrow \neg x = y)$	$\forall I$ 1-11