Quick Reference 8

Truth Functional Operators (Ch. 3)

φ	ψ	$\neg\varphi$	$\varphi \wedge \psi$	$\varphi \vee \psi$	$\varphi \to \psi$	$\varphi \leftrightarrow \psi$
T	T	F	T	T	T	T
T	F	F	F	T	F	F
F	T	T	F	T	T	F
F	F	T	F	F	T	T

Deduction Rules for TFL (Ch. 4)

Conjunction Introduction

$$
\begin{array}{r|ll}
m & \varphi & \\
n & \psi & \\
 & \varphi \wedge \psi & \wedge\mathrm{I}\ m, n
\end{array}
$$

Conjunction Elimination

$$
\begin{array}{r|ll}
m & \varphi \wedge \psi & \\
 & \varphi & \wedge\mathrm{E}\ m
\end{array}
$$

$$
\begin{array}{r|ll}
m & \varphi \wedge \psi & \\
 & \psi & \wedge\mathrm{E}\ m
\end{array}
$$

Conditional Introduction

$$
\begin{array}{r|ll}
i & \quad \underline{\varphi} & \text{Assumption} \\
j & \quad \psi & \\
 & \varphi \to \psi & \to\mathrm{I}\ i\text{–}j
\end{array}
$$

Conditional Elimination

$$
\begin{array}{r|ll}
m & \varphi \to \psi & \\
n & \varphi & \\
 & \psi & \to\mathrm{E}\ m, n
\end{array}
$$

Biconditional Introduction

$$
\begin{array}{r|ll}
i & \quad \underline{\varphi} & \text{Assumption} \\
j & \quad \psi & \\
k & \quad \underline{\psi} & \text{Assumption} \\
l & \quad \varphi & \\
 & \varphi \leftrightarrow \psi & \leftrightarrow\mathrm{I}\ i\text{–}j, k\text{–}l
\end{array}
$$

Biconditional Elimination

$$
\begin{array}{r|ll}
m & \varphi \leftrightarrow \psi & \\
n & \varphi & \\
 & \psi & \leftrightarrow\mathrm{E}\ m, n
\end{array}
$$

m	$\varphi \leftrightarrow \psi$	
n	ψ	
	φ	$\leftrightarrow$E m, n

Negation Introduction

m	$\quad\vert\ \varphi$	Assumption
n	$\quad\vert\ \bot$	
	$\neg\varphi$	$\neg$I m–n

Absurdity Introduction

m	φ	
n	$\neg\varphi$	
	$\bot$	$\bot$I m, n

Indirect Proof

m	$\quad\vert\ \neg\varphi$	Assumption
n	$\quad\vert\ \bot$	
	φ	IP m–n

Disjunction Introduction

m	φ	
	$\varphi \vee \psi$	$\vee$I m

m	φ	
	$\psi \vee \varphi$	$\vee$I m

Disjunction Elimination

m	$\varphi \vee \psi$	
i	$\quad\vert\ \varphi$	Assumption
j	$\quad\vert\ \chi$	
k	$\quad\vert\ \psi$	Assumption
l	$\quad\vert\ \chi$	
	χ	$\vee$E m, i–j, k–l

Derived Rules for TFL (§4.11)

Sequent	**Derived Rule**
$\varphi \to \psi, \neg\psi \vdash \neg\varphi$	MT
$\varphi \vee \psi, \neg\psi \vdash \varphi$	DS
$\varphi \vee \psi, \neg\varphi \vdash \psi$	DS
$\varphi \vdash \psi \to \varphi$	PMI
$\neg\varphi \vdash \varphi \to \psi$	PMI
$\varphi \to \psi \dashv\vdash \neg\varphi \vee \psi$	Imp
$\neg(\varphi \to \psi) \dashv\vdash \varphi \wedge \neg\psi$	NegImp
$\neg(\varphi \wedge \psi) \dashv\vdash \neg\varphi \vee \neg\psi$	DeM
$\neg(\varphi \vee \psi) \dashv\vdash \neg\varphi \wedge \neg\psi$	DeM
$\varphi \dashv\vdash \neg\neg\varphi$	DN
$(\varphi \# \psi) \dashv\vdash (\neg\neg\varphi \# \neg\neg\psi) \dashv\vdash (\neg\neg\varphi \# \psi) \dashv\vdash (\varphi \# \neg\neg\psi)$	SDN
$\neg(\varphi \# \psi) \dashv\vdash \neg(\neg\neg\varphi \# \neg\neg\psi) \dashv\vdash \neg(\neg\neg\varphi \# \psi) \dashv\vdash \neg(\varphi \# \neg\neg\psi)$	SDN
$\varphi @ \psi \vdash \psi @ \varphi$	Com
$\bot \vdash \varphi$	EX
$\vdash \varphi \vee \neg\varphi$	LEM

Deduction Rules for FOL (Ch. 6)

Universal Elimination

$$
\begin{array}{r|ll}
m & \forall v\,\varphi(\ldots v\ldots) & \\
 & \varphi(\ldots c\ldots) & \forall\text{E } m
\end{array}
$$

Universal Introduction

$$
\begin{array}{r|l|ll}
m & & c & \text{Flag} \\ \cline{3-3}
n & & \varphi(\ldots c\ldots) & \\
 & \multicolumn{2}{l}{\forall v\,\varphi(\ldots v\ldots)} & \forall\text{I } m\text{–}n
\end{array}
$$

The Flag-ed name c may not occur outside the subproof.

Existential Introduction

$$
\begin{array}{r|ll}
m & \varphi(\ldots c\ldots) & \\
 & \exists v\,\varphi(\ldots v\ldots) & \exists\text{I } m
\end{array}
$$

Existential Elimination

$$
\begin{array}{r|l|ll}
m & \multicolumn{2}{l}{\exists v\,\varphi(\ldots v\ldots)} & \\
i & & \varphi(\ldots c\ldots) & \text{Assumption (flag } c\text{)} \\ \cline{3-3}
j & & \psi & \\
 & \multicolumn{2}{l}{\psi} & \exists\text{E } m,\ i\text{–}j
\end{array}
$$

The Flag-ed name c may not occur outside the subproof.

Identity Elimination

$$
\begin{array}{r|ll}
m & a = b & \\
n & \varphi(\ldots a\ldots a\ldots) & \\
 & \varphi(\ldots b\ldots a\ldots) & =\text{E } m, n
\end{array}
$$

$$
\begin{array}{r|ll}
m & a = b & \\
n & \varphi(\ldots b\ldots b\ldots) & \\
 & \varphi(\ldots a\ldots b\ldots) & =\text{E } m, n
\end{array}
$$

Identity Introduction

$$
\begin{array}{|ll}
c = c & =\text{I}
\end{array}
$$