Problem: $(P \to Q) \vdash (P \to (A \to Q))$

1	$(P \to Q)$	Premise
2	P	Assumption
3	A	Assumption
4	Q	$\to$E 1,2
5	$(A \to Q)$	$\to$I 3-4
6	$(P \to (A \to Q))$	$\to$I 2-5

Problem: $\vdash \neg(P \land \neg P)$

1	$(P \land \neg P)$	Assumption
2	P	$\land$E 1
3	$\neg P$	$\land$E 1
4	$\bot$	$\neg$E 2,3
5	$\neg(P \land \neg P)$	$\neg$I 1-4

Problem: $(A \lor B) \vdash \neg(\neg A \land \neg B)$

1	$(A \lor B)$	Premise
2	$(\neg A \land \neg B)$	Assumption
3	A	Assumption
4	$\neg A$	$\land$E 2
5	$\bot$	$\neg$E 3,4
6	B	Assumption
7	$\neg B$	$\land$E 2
8	$\bot$	$\neg$E 6,7
9	$\bot$	$\lor$E 1,3-5,6-8
10	$\neg(\neg A \land \neg B)$	$\neg$I 2-9

Problem: $(A \lor \exists x Fx) \vdash \exists x(A \lor Fx)$

1	$(A \lor \exists x Fx)$	Premise
2	$\quad A$	Assumption
3	$\quad (A \lor Fa)$	$\lor$I 2
4	$\quad \exists x(A \lor Fx)$	$\exists$I 3
5	$\quad \exists x Fx$	Assumption
6	$\quad\quad Fa$	Assumption
7	$\quad\quad (A \lor Fa)$	$\lor$I 6
8	$\quad\quad \exists x(A \lor Fx)$	$\exists$I 7
9	$\quad \exists x(A \lor Fx)$	$\exists$E 5,6-8
10	$\exists x(A \lor Fx)$	$\lor$E 1,2-4,5-9

Problem: $\vdash \forall x \forall y((Fx \land \neg Fy) \to \neg x = y)$

1	a	Flag
2	$\quad b$	Flag
3	$\quad\quad (Fa \land \neg Fb)$	Assumption
4	$\quad\quad\quad a = b$	Assumption
5	$\quad\quad\quad Fa$	$\land$E 3
6	$\quad\quad\quad \neg Fb$	$\land$E 3
7	$\quad\quad\quad Fb$	=E 4,5
8	$\quad\quad\quad \bot$	$\neg$E 6,7
9	$\quad\quad \neg a = b$	$\neg$I 4-8
10	$\quad\quad ((Fa \land \neg Fb) \to \neg a = b)$	$\to$I 3-9
11	$\quad \forall y((Fa \land \neg Fy) \to \neg a = y)$	$\forall$I 2-10
12	$\forall x \forall y((Fx \land \neg Fy) \to \neg x = y)$	$\forall$I 1-11