King's College London

This paper is part of an examination of the College counting towards the award of a degree. Examinations are governed by the College Regulations under the authority of the Academic Board.

Degree Programmes	BSc, MSci, BEng, MEng
Module Code	5CCS2FC2
Module Title	Foundations of Computing II
Examination Period	January 2019 (Period 1)

Time Allowed	Two hours
Rubric	ANSWER ALL OF QUESTIONS 1–10, AND ANSWER TWO OF THREE FROM QUESTIONS 11–13. Questions 1–10 carry a total of FIFTY marks and each have ONE or MORE correct choices. In order to obtain full marks you must select all correct choices and only those. **Marks will be deducted for incorrect choices selected in those questions.** Questions 11–13 each carry TWENTY FIVE marks. The answers to questions **1–10** need to be clearly made by pen on the appropriate grid on the **answer sheet** provided at the back of the exam paper. The answers to questions **11–13** need to be written by pen on the separate **answer book** provided.
Calculators	Calculators may be used. The following models are permitted: Casio fx83 / Casio fx85.
Notes	Books, notes or other written material may not be brought into this examination

PLEASE DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

Please answer TWO out of THREE from Questions 11–13 in the separate answer book provided.

11. a. Show that the following language is undecidable

$$\mathsf{REJECT}_{TM} \; = \; \{\langle M, w\rangle : M \text{ encodes a TM that rejects } w\}.$$

from first principles (*i.e.* without using a reduction from another known undecidable problem).

[13 marks]

b. Show that REJECT_{TM} is recursively enumerable by constructing a sound and complete algorithm that recognises all words $\langle M, w\rangle \in \mathsf{REJECT}_{TM}$, where M encodes a TM that rejects w.

[7 marks]

c. Hence, or otherwise, show that the complement $\overline{\mathsf{REJECT}_{TM}}$ is *not* recursively enumerable.

[5 marks]

12. a. Consider the following recursion relation

$$\begin{aligned} T(0) &= 0 \\ T(1) &= 1 \\ T(n+1) &= T(n) + 2\,T(n-1) \end{aligned}$$

for all $n \geq 1$. Prove, by induction on the input n, that

$$T(n) = \frac{2^n - (-1)^n}{3}$$

for all $n \geq 0$. You should clearly state your induction hypothesis, base case(s) and induction step.

[10 marks]

b. Consider the following set of propositional clauses (numbered for reference):

$$(\neg P \vee R) \quad (1)$$

$$(\neg P \vee Q \vee T) \quad (2)$$

$$(Q \vee \neg R \vee \neg T) \quad (3)$$

$$(P \vee R \vee S) \quad (4)$$

$$(P \vee \neg R) \quad (5)$$

$$(P \vee R \vee \neg S) \quad (6)$$

$$(\neg P \vee \neg Q) \quad (7)$$

Use the *Davis-Putnam-Logemann-Loveland (DPLL)* algorithm to determine whether the above set of clauses is satisfiable. You should state which rule is being applied in each instance.

[10 marks]

QUESTION 12 CONTINUES ON NEXT PAGE

c. Prove that SAT is polynomially reducible to 3SAT by describing a polynomial-time algorithm for converting an instance of SAT into an instance of 3SAT. You should explain why your algorithm terminates.

[5 marks]

13. The problem CLIQUE takes as input a graph $G = (V, E)$ and integer $k > 0$ and returns `True` if there is a set $C \subseteq V$ of size k such that every pair of vertices in C are connected with an edge from E.

a. What does it mean to say that the Boolean Satisfiability problem SAT is *polynomially reducible* to CLIQUE?

[4 marks]

b. Given the following input formula

$$F \;=\; (\neg P \vee Q \vee \neg R) \wedge (P \vee Q) \wedge (\neg Q \vee R)$$

construct a graph G_F such that F is satisfiable if and only if G_F has a clique of size $k = 3$.

[9 marks]

c. Explain why the existence of a clique of size $k = 3$ guarantees that there is a satisfying assignment for F?

[6 marks]

d. Describe a non-deterministic algorithm for CLIQUE that runs in polynomial time, *i.e.*, show that CLIQUE belongs to the class NP.

[6 marks]