4CCS1CM2 Coursework I – Due Thursday, 23th February

1. Use the appropriate integration technique to solve the following integral. Mention which technique and rules you follow in each step, in order to solve the problem.

a) $\int \frac{4t^3-t^2+16t}{t^2+4}\,dt$

b) $\int_0^{\frac{\pi}{4}} \frac{1+\sin\theta}{\cos\theta}\,d\theta$

c) $\int x^3\sqrt{x^2+1}\,dx$

d) $\int e^{2x}\cos 3x \quad dx$

e) $\int_{\frac{5\pi}{6}}^{\pi} \frac{cos^4 x}{\sqrt{1-\sin x}}\,dx$

f) $\int \frac{t^2-t+2}{t^3-1}\,dt$

[total 30 marks, 5 each part]

2. Find the volume of the solid generated by revolving the region in the first quadrant bounded by the coordinate axes and the curve y = cos x, 0 ≤ x ≤ π/2, about,

a) the y-axis

b) the line x= π/2.

[15 marks, 5 each part, 5 for baseline formulation]

3. (a) Solve the following double integral, (b) Sketch the region of integration, and (c) write an equivalent double integral with the order of integration reversed.

$$\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} 3y\,dx\,dy$$

[15 marks, 5 for each part]

4. Which of the following sequences converge or diverge? Find the limit of converging sequences.

a) $a_n = \frac{1-2n}{1+2n}$

b) $a_n = (1+\frac{7}{n})^n$

c) $a_n = \frac{\ln n}{n^{\frac{1}{n}}}$

[15 marks, 5 each part]

5.

a) Find a formula for the n-th partial sum of the series $\sum_{n=1}^{\infty}(\ln\sqrt{n+1} - \ln\sqrt{n})$, and use it to determine whether the series converges or diverges.

b) Use the integral test to determine if series$\sum_{n=1}^{\infty}(\frac{n}{n^2+4})$ converge or diverge.

c) Use the Limit Comparison test to determine if the following series converges or diverges.

$$\sum_{n=1}^{\infty}\left(\frac{n-2}{n^3-n^2+3}\right)$$

[15 marks, 5 each part]

6. Solve the following differential equations:

a) $y^{(3)} - 5y'' - 22y' + 56y = 0$

b) $\mathrm{t}y'' + 4y' = t^2$

[10 marks, 5 each part]