Power Series

Mathematical Methods in the Physical Sciences

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Power series are series where the n^{th} term is a constant times x^n or a constant times $(x - a)^n$ where a is also constant.

Definition

$$\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$$

$$\sum_{n=0}^{\infty} a_n (x-a)^n = a_0 + a_1(x-a) + a_2(x-a)^2 + a_3(x-a)^3 + \cdots$$

Examples

The following are two of the examples that can also be found in Boas: *Quantitative Methods of Systems Engineering* on page 20.

Example #1

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + \frac{(-1)^{x+1}x^n}{n} + \cdots$$

Example #2

$$1 + \frac{(x+2)}{\sqrt{2}} + \frac{(x+2)^2}{\sqrt{3}} + \cdots + \frac{(x+2)^n}{\sqrt{(n+1)}} + \cdots$$

Interval of Convergence

Convergence of the power series depends on the value of x. We can use the ratio test to find x such that the series converges.

Theorems About Power Series

Expanding Functions in Power Series

Techniques for Obtaining Power Series Expansions

We outlay and demonstrate several methods of obtaining power series expansions.

- Multiplying a series by polynomial or by another series
- Division of two series or of a series by a polynomial
- Binomial series
- Substitution of a polynomial or series or the variable in another series
- Combination of methods
- Taylor series using the basic Maclaurin series
- Using a computer

The Maclaurin series provides us with an alternative method to the formulas for obtaining a Taylor series.

Maclaurin Series

$$\ln x = \ln\,[1 + (x - 1)]$$

Then we replace x with $(x - 1)$

Substitution

$$\begin{aligned} \ln x &= \ln\,[1 + (x - 1)] \\ &= (x - 1) - \frac{1}{2}(x - 1)^2 + \frac{1}{3}(x - 1)^3 - \frac{1}{4}(x - 1)^4 \cdots \end{aligned}$$

Accuracy of Series Approximations

Some Uses of Series

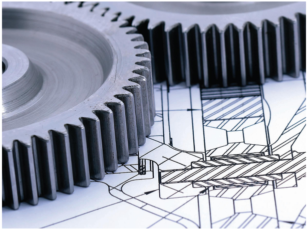
Example of columns 1

Contents of the first column

Contents split
into two lines

Example of columns 2

Contents of first column
split into two lines



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