Homework

Steve Mazza

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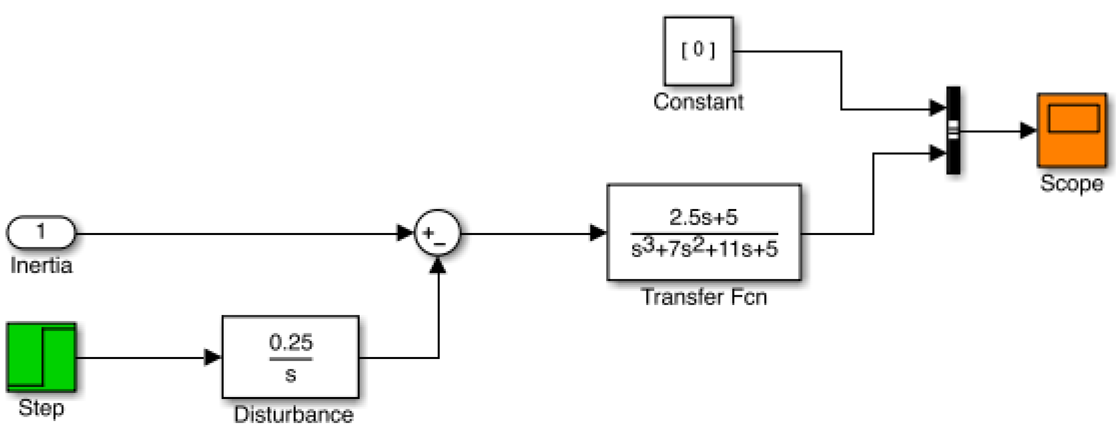
Homework 2

Problem 1

$$
\begin{aligned}
\frac{s\omega(s)}{0.25} &= \frac{2.5(s+2)}{(s+5)(s+1)^2} \\
\omega(s) &= \frac{5}{8}\left(\frac{s+2}{s(s+5)(s+1)^2}\right) \\
\omega(s) &= \frac{5}{8}\left(-\frac{7}{16(s+1)} + \frac{3}{80(s+5)} - \frac{1}{4(s+1)^2} + \frac{2}{5s}\right) \\
\omega(t) &= \frac{5}{8}\left(-\frac{1}{4}te^{-t} + \frac{3e^{-5t}}{80} - \frac{7e^{-t}}{16} + \frac{2}{5}\right)
\end{aligned}
$$

Problem 6



See the attached file, `homework2.slx`.

Homework 3

Problem 2

$$
\begin{aligned}
\frac{C(s)}{R(s)} &= \frac{1}{Ts+1} \\
C(s) &= \frac{1}{s}\left(\frac{1}{Ts+1}\right) \\
C(s) &= \frac{1}{s} - \frac{T}{Ts+1} \\
C(s) &= \frac{1}{s} - \frac{1}{s+(1/T)} \\
c(t) &= 1 - e^{-1/T} \\
0.98 &= 1 - e^{-1/T} \\
0.02 &= e^{-1/T} \\
ln(0.02) &= -\frac{1}{T} \\
T &= -\frac{1}{ln(0.02)} \\
T &\approx 0.2556 \text{ minutes} \\
&\approx 15.3373 \text{ seconds}
\end{aligned}
$$

Problem 3

Based on the information supplied, we determine that $J = 1$ and $B = 14$. Next we use the equation $\frac{K}{J} = \omega_n^2$ to determine that $K = \omega_n^2$. Finally we use the equation $\frac{B}{J} = 2\zeta\omega_n$ as follows:

$$
\begin{aligned}
B &= 2\zeta\omega_n \\
7 &= \zeta\omega_n \\
7 &= 0.7\omega_n \\
10 &= \omega_n \\
10^2 &= K \\
100 &= K
\end{aligned}
$$

Problem 4

We use the information given to determine the following three sets of inequalities in terms of ω_d and σ.

Given what we know about *percent overshoot*:

$$\begin{aligned} M_p &\leq e^{-(\sigma/\omega_d)\pi} \\ ln(0.05) &\leq -\left(\frac{\sigma}{\omega_d}\right)\pi \\ \frac{ln(0.05)}{\pi} &\leq -\frac{\sigma}{\omega_d} \end{aligned}$$

Given what we know about *settling time*:

$$\begin{aligned} t_s &< \frac{4}{\sigma} \\ 4 &> \frac{4}{\sigma} \\ \sigma &> 1 \end{aligned}$$

Given what we know about *peak time*:

$$\begin{aligned} 1 &> \frac{\pi}{\omega_d} \\ \omega_d &> \pi \end{aligned}$$

We now graph these inequalities to determine the permissible area for poles of $T(s)$.