Partial Differentiation

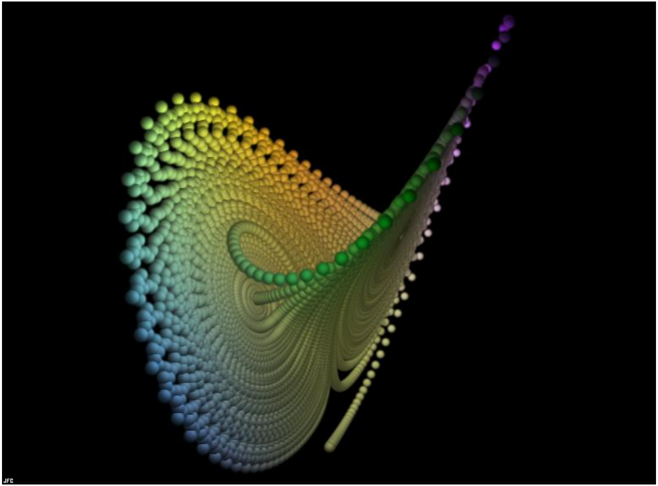
Mathematical Methods in the Physical Sciences

Steve Mazza

Naval Postgraduate School
Monterey, CA

SE3030, Winter/2014
Quantitative Methods of Systems Engineering

Lorenz Attractor



Introduction

Definition

Derivatives of a multi-variable function where all variables are held fixed during differentiation except the variable of interest.

To denote this we write $\dfrac{\partial}{\partial y}$, generically.

Let $f(x,y) = x^2y^3$, calculate $\dfrac{\partial f}{\partial x}(x,y)$

$$
\begin{aligned}
g(x) &= x^2u^3, y = u \\
\frac{dg}{dx}(x) &= 2b^3x \\
\frac{\partial f}{\partial x}(x,y) &= 2xy^3, u = y
\end{aligned}
$$

Power Series in Two Variables

Our standard power series expansions can be re-written as partial differential equations.

Definition

$$f(x, y) = \sum_{n=0}^{\infty} \frac{1}{n!} \left(h\frac{\partial}{\partial x} + k\frac{\partial}{\partial y} \right)^{n} f(a, b)$$

Total Differentials

Total Differentiation in 2 Variables

$$dz = \frac{\partial}{\partial x} dx + \frac{\partial}{\partial y} dy$$

Approximations Using Differentials

Chain Rule

In General

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dv}\frac{dv}{dx}$$

Find dy/dx if $y = \ln \sin 2x$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{\sin 2x} \cdot \frac{d}{dx}(\sin 2x) \\ &= \frac{1}{\sin 2x} \cdot \cos 2x \cdot \frac{d}{dx}(2x) \\ &= 2\cot 2x \end{aligned}$$

Implicit Differentiation

Chain Rule (Redux)

Applications

Questions?