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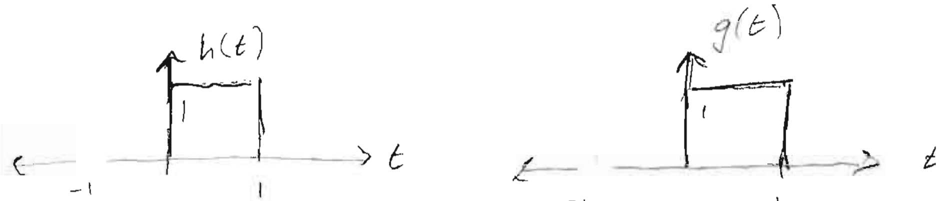
Main Points	Details
# 1	Given: 2 identical pulses of height 1.0. Show: their convolution is a triangle pulse of height 1.0.

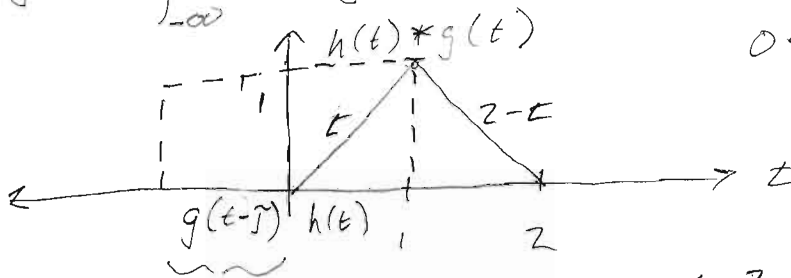
Given:



Apply:

$$h(t) * g(t) = \int_{-\infty}^{\infty} h(\tau)\, g(t-\tau)\, d\tau$$

$0 < t < 2$



- slide this from $t = 0 \rightarrow t = 2$
- Same as integration of the convolution from $t = 0 \rightarrow 2$
- This is the "interesting" region since their products are 0 where they do not overlap.

$$\int_0^1 1\, d\tau = t \qquad \int_1^2 1\, d\tau = 1 - (t-1) = 2 - t$$

$$= \begin{cases} t, & 0 < t < 1 \\ 0, & t < 0 \\ 0, & t > 2 \\ 2-t, & 1 < t < 2 \end{cases}$$

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Main Points **Details**

#2

Series coefficient $\begin{cases} \dfrac{(-i)^n + i^n}{2(n+1)!} & n \geq -1 \\ 0 & \text{otherwise} \end{cases}$

Using Dirichlet

Series expansion is getting me no where !!!

$$1 - \frac{x^2}{6} + \frac{x^4}{120} - \frac{x^6}{5040} + \ldots$$

Computer solution →

$$\int_0^\infty \frac{\sin x}{x}\,dx = \frac{\pi}{2}$$

* Using Laplace

this is the key

$$\mathcal{L}\sin kt = \frac{s}{s^2+k^2} = \frac{1}{s^2+1}$$

From tables

$$= \int_0^\infty \frac{1}{s^2+1}\,ds$$

$$= \arctan(s)\Big|_0^\infty = \frac{\pi}{2} - 0 = \frac{\pi}{2}$$

Summary

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#3. Given $x \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ $n = 9$
$y \{9, 8, 10, 12, 11, 13, 14, 16, 15\}$

$$\text{Correlation Coefficient } (r) = \frac{n\sum xy - (\sum x)(\sum y)}{\sqrt{n(\sum x^2) - (\sum x)^2}\sqrt{n(\sum y^2) - (\sum y)^2}}$$

x	1	2	3	4	5	6	7	8	9	$\sum x = 45$
y	9	8	10	12	11	13	14	16	15	$\sum y = 108$
$x \cdot y$	9	16	30	48	55	78	98	128	135	$\sum xy = 597$
x^2	1	4	9	16	25	36	49	64	81	$\sum x^2 = 285$
y^2	81	64	100	144	121	169	196	256	225	$\sum y^2 = 1356$

$$r = \frac{9 \cdot 597 - (45)(108)}{\sqrt{(9 \cdot 285) - (45)^2}\sqrt{(9 \cdot 1356) - (108)^2}}$$

$$= \frac{5373 - 4860}{\sqrt{540}\sqrt{540}}$$

$$= \frac{513}{540}$$

$$= \frac{19}{20}$$

$$= 0.95$$

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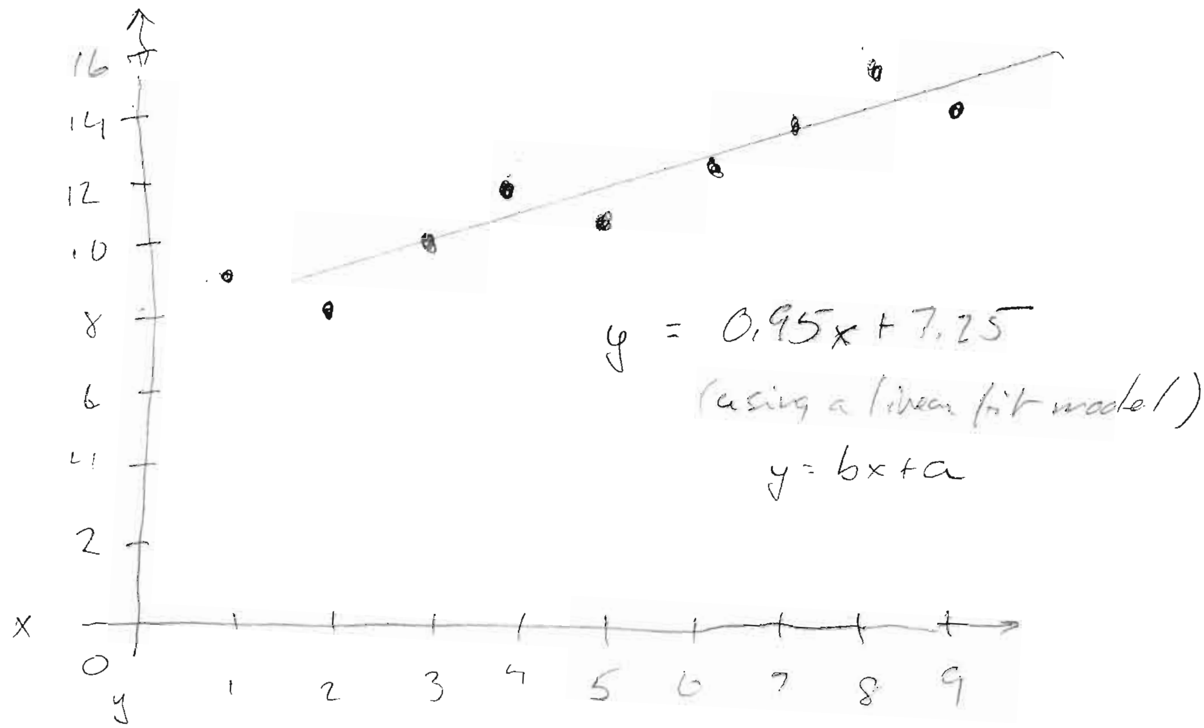
#3 continued



b was found previously ($b = r$)

$$a = \frac{(\sum y)(\sum x^2) - (\sum x)(\sum xy)}{n\sum x^2 - (\sum x)^2}$$

$$= \frac{(108 \cdot 285) - (45 \cdot 597)}{(9 \cdot 285) - (45)^2}$$

$$= \frac{30,780 - 26,865}{2565 - 2025}$$

$$= \frac{3915}{540}$$

$$= \frac{29}{4}$$

$$= 7.25$$

Solve $y = 0.95x + 7.25$ for $x = 6.2$

$y = 0.95 \cdot 6.2 + 7.25$

$y = 5.89 + 7.25$

$y = 13.14$

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#4

Equations from Hayter

$N = 5$

Given

x	0	1	2	3	4	$\sum x = 10$
y	1	0	3	10	21	$\sum y = 35$

Find a, b, c | $y = cx^2 + bx + a$

x^2	0	1	4	9	16	$\sum x^2 = 30$
$x \cdot x^2$	0	1	8	27	64	$\sum x^3 = 100$
$x^2 \cdot x^2$	0	1	16	81	256	$\sum x^4 = 354$
$x \cdot y$	0	0	6	30	84	$\sum xy = 120$
$x^2 y$	0	0	12	90	336	$\sum x^2 y = 438$

$$S_{\cancel{xx}_1} = 30 - \frac{10^2}{5} = 10$$

$$S_{x_1 x_2} = 100 - \frac{10 \cdot 30}{5} = 40$$

$$S_{x_2 x_2} = 354 - \frac{30^2}{5} = 174$$

$$S_{y_1} = 120 - \frac{35 \cdot 10}{5} = 50$$

$$S_{y_2} = 438 - \frac{35 \cdot 30}{5} = 228$$

$$\bar{x}_1 = 10/5 = 2$$

$$\bar{x}_2 = 30/5 = 6$$

$$\bar{y} = 35/5 = 7$$

Summary

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#4 continued

$$b = \frac{(50 \cdot 174) - (228 \cdot 40)}{(174 \cdot 10) - 40^2} = -3$$

$$c = \frac{(228 \cdot 10) - (50 \cdot 40)}{(174 \cdot 10) - 40^2} = 2$$

$$a = 7 - (-3 \cdot 2) - (2 \cdot 6) = 1$$

$$y = cx^2 + bx + a$$

$$\Rightarrow y = 2x^2 - 3x + 1$$

$$b = \frac{S_{y_1} \cdot S_{x_2 x_2} - S_{y_2} \cdot S_{x_1 x_2}}{S_{x_2 x_2} \cdot S_{x_1 x_1} - S_{x_1 x_2}^2}$$

$$c = \frac{S_{y_2} \cdot S_{x_1 x_1} - S_{y_1} \cdot S_{x_1 x_2}}{S_{x_2 x_2} \cdot S_{x_1 x_1} - S_{x_1 x_2}^2}$$

$$a = \bar{y} - b\bar{x}_1 - c\bar{x}_2$$

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Main Points

Boas 6.10.12

* Find electric field, E
  - between cylinders
  - inner cylinder
  - outer cylinder

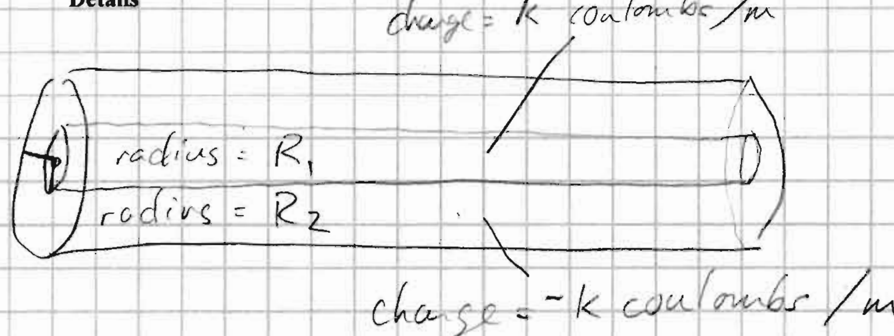
* find potential ϕ | $E = -\nabla\phi$

$$\oint \vec{E}\cdot dA = \frac{Q_e}{\epsilon_0}$$

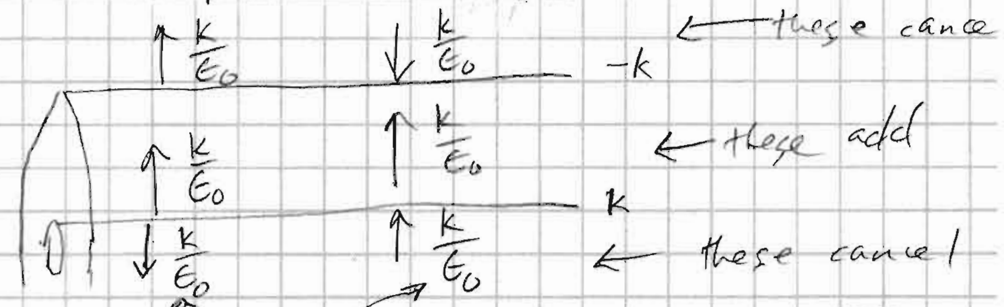
$$\vec{E}\cdot 2\pi R L = \frac{Q_e}{\epsilon_0}$$

$$\vec{E} = \frac{Q_e}{2\pi R\epsilon_0}$$

Details



Close up of a section of the above diagram



These are actually $k/2\pi R\epsilon_0$ for all terms.

Inside the inner cylinder, $Q_e = 0$, so $\vec{E} = 0$

Outside the outer cylinder, $Q_e = 0$, so $\vec{E} = 0$

Between the cylinders, $Q_e = \frac{2k}{2\pi R\epsilon_0}$

so $\vec{E} = \frac{k}{\pi R\epsilon_0}$

Summary

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Boas 8.2.19 (a)

Given: intensity $= I_0$ at distance $s = 0$ (surface)

Find intensity, I, at depth s, $\forall s > 0$.

$$\frac{dI}{ds} = -\mu I$$

Re-write separable eqn. Integrate both sides

$$\frac{dI}{I} = -\mu \, ds$$

$$\ln I = -\mu s + C$$

since $I = I_0$ at $s = 0$, $C = \ln I_0$

$$\ln I = -\mu s + \ln I_0$$

$$\boxed{I = I_0 e^{-\mu s}}$$

Let $\mu = 10^{-2}/ft.$

surface: $I = I_0 e^{10^{-2} \cdot 0} = I_0 \cdot e^0 = I_0 \cdot 1$

1-foot: $I = I_0 e^{10^{-2} \cdot -1} = I_0 \cdot \frac{1}{\sqrt[100]{e}} \approx I_0 \cdot 0.99$

50-feet: $I = I_0 e^{10^{-2} \cdot -50} = I_0 \cdot \frac{1}{\sqrt{e}} \approx I_0 \cdot 0.607$

500-feet: $I = I_0 e^{10^{-2} \cdot -500} = I_0 \cdot \frac{1}{e^5} \approx I_0 \cdot 0.007$

5,280-feet:

$I = I_0 e^{10^{-2} \cdot -5280}$

These numbers seem to make sense to me

Summary

$= I_0 \cdot \frac{1}{e^{264/5}} \approx 1.729 \times 10^{-23}$

Not very much light!

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Main Points	Details
Boas 8.2.19 continued	

(a)

$$e^{10^{-2}(-s)} = 1/2$$

$$e^{s/100} = 2 \quad \text{(recip)} \qquad e^{-\mu s} = 1/2$$

$$\frac{s}{100} = \ln 2 \quad (\ln) \qquad e^{s/\mu} = 2$$

$$s = 100 \ln 2 \qquad \frac{s}{\mu} = \ln 2$$

$$s = \ln 2 \cdot \mu$$

$s \approx 69ft$ ← seems right based on previous findings

Similarly,

(b)

$$e^{-\lambda t} = \frac{N_0}{2} = T$$

$$T = t \,/\, N = \frac{N_0}{2}$$

$$e^{-\lambda t} = \frac{1}{2}$$

$$e^{\lambda t} = 2$$

$$\lambda t = \ln 2$$

$$t = \frac{\ln 2}{\lambda} = T$$

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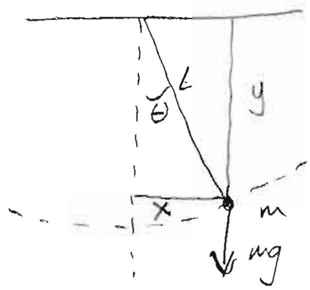
Main Points	Details
Boas 9.5.4	 Given: (1) $L = T - V$ (2) $T = \frac{1}{2} m v^2$ (3) $v = ds/dt$
	$x = l \sin\theta$ $y = l \cos\theta$
	$r = l(\sin\theta, \cos\theta)$
d/dt	$\dot{r} = v = l\dot{\theta}(\cos\theta, -\sin\theta)$
Apply (2)	$T = \frac{1}{2} m v^2 = \frac{1}{2} m l^2 \dot{\theta}^2$
Apply (3)	$V = -mgy = -mg\, l\cos\theta \Leftarrow F = -\nabla V = mg$
Apply (1)	$L = T - V = \frac{1}{2} m l^2 \dot{\theta}^2 + mgl\cos\theta$
from 5.8	$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} - \frac{\partial L}{\partial \theta} = 0$
subst.	$\frac{d}{dt}(ml^2\dot{\theta}) - \frac{\partial V}{\partial \theta} = 0$
subst.	$\frac{d}{dt}(ml^2\dot{\theta}) - (-mgl\sin\theta) = 0$
	$ml^2\ddot{\theta} + mgl\sin\theta = 0$ $m(l^2\ddot{\theta} + gl\sin\theta) = 0$ $l^2\ddot{\theta} = -gl\sin\theta$ $l\ddot{\theta} = -g\sin\theta$
	This result is verified by 11.5 (I discovered after the fact.)

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Main Points	Details
Boas 11.9.4 (a)	Show $\int_{-\infty}^{\infty} e^{-y^2/2}\, dy = \sqrt{2\pi}$ by 9.5, 9.2a
	We break the problem into 2 halves where 9.2a covers $\int_{-\infty}^{0}$ and 9.5 covers $\int_{0}^{\infty}$.
Bottom half 9.2a	Let $t = y \Rightarrow dt = dy$, let $x = 0$. $\int_{-\infty}^{0} e^{-y^2/2}\, dy = \sqrt{\frac{\pi}{2}}$
Top half 9.5	Let $t = y/\sqrt{2} \Rightarrow dt = \frac{1}{\sqrt{2}} dy$ $\sqrt{2}\int_{0}^{\infty} e^{-y^2/2} \frac{1}{\sqrt{2}}\, dy = \sqrt{2} \cdot \frac{\sqrt{\pi}}{2} = \sqrt{\frac{\pi}{2}}$
Add the 2 halves	$\sqrt{\frac{\pi}{2}} + \sqrt{\frac{\pi}{2}} = \sqrt{2\pi}$

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Main Points	Details
Boas 12.16.2	Solve $y'' + 4x^2 y = 0$ by 16.1, 16.2

Assume this is of type (16.1) then

$1 - 2a = 0$ $\quad 2a = 1$ $\quad a = 1/2$

$(bc)^2 = 4$ $\quad b \cdot 2 = 2$ $\quad b = 1$

$a^2 - p^2 c^2 = 0$

$2(c-1) = 2$ $\quad 2c - 2 = 2$ $\quad 2c = 4$ $\quad c = 2$

$\frac{1}{4} - 4p^2 = 0$

$4p^2 = \frac{1}{4}$

$p^2 = \frac{1}{16}$

$p = \frac{1}{4}$

So...

$$y = x^{\frac{1}{2}} Z_{1/4}(x^2)$$

← This looks like the solution in the book for similar problems but can be generalized as

$$y = x^{1/2}\left[A J_{1/4}(x^2) + B N_{1/4}(x^2)\right]$$

where A + B are arbitrary constants.

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Main Points | **Details**

Boas 13.8.1

Show $V = \frac{-Gm}{r}$ satisfies Laplace by showing $\nabla^2 \frac{1}{r} = 0$ where $r^2 = x^2+y^2+z^2$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

Since $r^2 = x^2+y^2+z^2$, $\frac{1}{r} = \frac{1}{\sqrt{x^2+y^2+z^2}}$

Let $f = \frac{1}{r}$, $r = \sqrt{x^2+y^2+z^2}$

$$\nabla^2 \frac{1}{r} = \frac{\partial^2}{\partial x^2}\left(\frac{1}{\sqrt{x^2+y^2+z^2}}\right) + \frac{\partial^2}{\partial y^2}\left(\frac{1}{\sqrt{x^2+y^2+z^2}}\right) + \frac{\partial^2}{\partial z^2}\left(\frac{1}{\sqrt{x^2+y^2+z^2}}\right)$$

$$= \frac{2x^2-y^2-z^2}{(x^2+y^2+z^2)^{5/2}} + \frac{-x^2+2y^2-z^2}{(x^2+y^2+z^2)^{5/2}} + \frac{-x^2-y^2+2z^2}{(x^2+y^2+z^2)^{5/2}}$$

Note: $2x^2-y^2-z^2-x^2+2y^2-z^2-x^2-y^2+2z^2 = 0$

$$\frac{0}{(x^2+y^2+z^2)^{5/2}} = 0$$

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Boas 14.5.3	Given $\oint_C f(z)\,dz = 2\pi i \cdot \sum_{n=1}^{k} \mathrm{Res}(f(z))_n$
	Obtain $f(a) = \frac{1}{2\pi i} \oint_C \frac{f(z)}{z-a}\,dz$, a inside C
*Let $f(z) = \frac{f(z)}{z-z_0}$	$\oint_C \frac{f(z)}{z-z_0}\,dz = 2\pi i \cdot \sum \mathrm{Res}$.
Note: Def, p. 680	Res of $f(z)$ at $z = z_0$ is b_1

* The substitution $f(z) = \frac{f(z)}{z-z_0}$ provides us with a solution to a specific term (by the definition above) which is that $f(z) = b$ at $z = z_0$. To find this we take the $\lim_{z \to z_0}$ around the singularity.

And so the Residue Theorem clearly generalizes Cauchy's Integral Formula since the Residue Theorem looks at the whole function $f(z)$ over C and the sum of all residues, not just the $\frac{1}{z-z_0}$ term.

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Boas 15.9.3

Apply Poisson distribution (9.8).

$$\mu = \frac{6000}{50 \cdot 60} = 2$$

Calculate $P_n = \frac{\mu^n}{n!} e^{-\mu}$, $\mu = 2$, $n = \{1, 2, 3, 4, 5\}$

P_1 $\quad \frac{2^1}{1!} e^{-2} = \frac{2}{e^2} \approx 0.270$

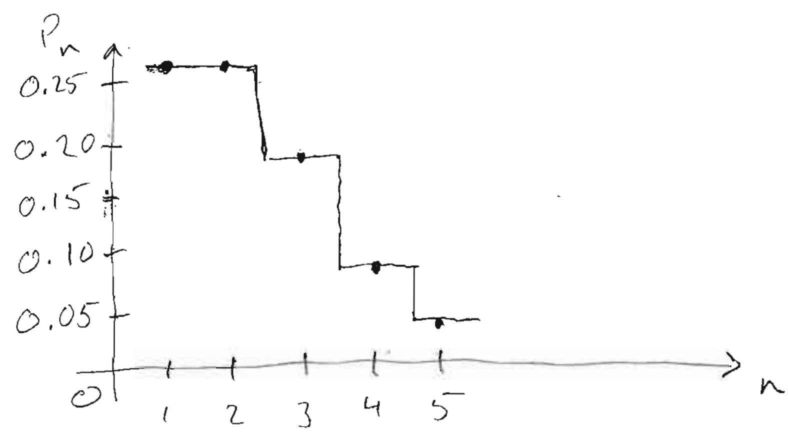
P_2 $\quad \frac{2^2}{2!} e^{-2} = \frac{2}{e^2} \approx 0.270$

P_3 $\quad \frac{2^3}{3!} e^{-2} = \frac{4}{3e^2} \approx 0.180$

P_4 $\quad \frac{2^4}{4!} e^{-2} = \frac{2}{3e^2} \approx 0.090$

P_5 $\quad \frac{2^5}{5!} e^{-2} = \frac{4}{15e^2} \approx 0.036$

Plot results:



Summary

Advice to Myself After the Fact

This class is going to move quickly so make sure you are prepared for the lessons. Do ALL of the reading in advance. The lesson time is best used to clarify parts of the lesson that you didn't understand from the reading. Attempt all of the homework problems in advance. At least have a working understanding of what the homework problems are asking. This will allow you to use class time to the best advantage. Don't avoid problems that you don't immediately see a solution for. You will often learn more from failure than from easy success. Don't be afraid to be wrong. There may be a lot of really smart people in class, but playing it safe is no way to learn. Open your mouth and make mistakes. Participate. Remember to be supportive when others do the same. The textbook (Boas) is very dense so be prepared to supplement the text with other resources. Ask for help when something isn't clear. Like all other classes, you will get out of this what you put into it. Be prepared to work hard and to also have fun.