Groups & Vector Spaces

Mathematical Methods in the Physical Sciences

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Groups

Definition of Groups

A group is a set of elements, G, together with a set operation, \cdot , that satisfies the following conditions:

Group Conditions

Closure: $\forall a, b \in G, a \cdot b \in G$

Association: $\forall a, b, c \in G, (a \cdot b) \cdot c = a \cdot (b \cdot c)$

Identity: \exists exactly 1 element, $i \in G \mid \forall \ a \in G, i \cdot a = a \cdot i = a$

Inversion: $\forall a \in G \exists b \mid a \cdot b = b \cdot a = i$, where *i* is the identity

element.

Operation Table

Group Symmetry

Conjugate Elements, Class, Character

Irreducible Representations

Infinite Groups

Vector Spaces

Definition of Vector Spaces

A vector space over field F is a set V together with two binary operations satisfying following conditions:

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Group Conditions

Closure: \forall a, b \in V, a \cdot b \in V

Association:

Addition: \forall a, b, c \in V, (a+b) + c = a + (b+c)
Multiplication: \forall a, b, c \in V, (a \cdot b) \cdot c = a \cdot (b \cdot c)

Identity:

Addition: \exists exactly 1 element, 0 \in V \mid \forall a \in V, 0 = a = a = 0 = a
Multiplication: \exists exactly 1 element, i \in V \mid \forall a \in V, i \cdot a = a \cdot i = a

Inversion:
Inversion:
Inversion:
Inversion:
Inversion:
Inversion:
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Inner Product, Norm, Orthogonality

Schwart's Inequality

Orthonormal Basis

Infinite Dimensional Spaces

Questions?

