

Homework

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Homework 2

Problem 1

$$\begin{aligned}\frac{s\omega(s)}{0.25} &= \frac{2.5(s+2)}{(s+5)(s+1)^2} \\ \omega(s) &= \frac{5}{8} \left(\frac{s+2}{s(s+5)(s+1)^2} \right) \\ \omega(s) &= \frac{5}{8} \left(-\frac{7}{16(s+1)} + \frac{3}{80(s+5)} - \frac{1}{4(s+1)^2} + \frac{2}{5s} \right) \\ \omega(t) &= \frac{5}{8} \left(-\frac{1}{4}te^{-t} + \frac{3e^{-5t}}{80} - \frac{7e^{-t}}{16} + \frac{2}{5} \right)\end{aligned}$$

Problem 6

See the attached file, `homework2.slx`.

Homework 3

Problem 2

$$\frac{C(s)}{R(s)} = \frac{1}{Ts + 1}$$

$$C(s) = \frac{1}{s} \left(\frac{1}{Ts + 1} \right)$$

$$C(s) = \frac{1}{s} - \frac{T}{Ts + 1}$$

$$C(s) = \frac{1}{s} - \frac{1}{s + (1/T)}$$

$$c(t) = 1 - e^{-1/T}$$

$$0.98 = 1 - e^{-1/T}$$

$$0.02 = e^{-1/T}$$

$$\ln(0.02) = -\frac{1}{T}$$

$$T = -\frac{1}{\ln(0.02)}$$

$$T \approx 0.2556 \text{ minutes}$$

$$\approx 15.3373 \text{ seconds}$$

Problem 3

Based on the information supplied, we determine that $J = 1$ and $B = 14$. Next we use the equation $\frac{K}{J} = \omega_n^2$ to determine that $K = \omega_n^2$. Finally we use the equation $\frac{B}{J} = 2\zeta\omega_n$ as follows:

$$B = 2\zeta\omega_n$$

$$7 = \zeta\omega_n$$

$$7 = 0.7\omega_n$$

$$10 = \omega_n$$

$$10^2 = K$$

$$100 = K$$

Problem 4

We use the information given to determine the following three sets of inequalities in terms of ω_d and σ .

Given what we know about *percent overshoot*:

$$M_p \leq e^{-(\sigma/\omega_d)\pi}$$

$$\ln(0.05) \leq -\left(\frac{\sigma}{\omega_d}\right)\pi$$

$$\frac{\ln(0.05)}{\pi} \leq -\frac{\sigma}{\omega_d}$$

Given what we know about *settling time*:

$$t_s < \frac{4}{\sigma}$$

$$4 > \frac{4}{\sigma}$$

$$\sigma > 1$$

Given what we know about *peak time*:

$$1 > \frac{\pi}{\omega_d}$$

$$\omega_d > \pi$$

We now graph these inequalities to determine the permissible area for poles of $T(s)$.