

# Requirement 1

$$\frac{dS}{dt} = -a S(t) I(t)$$

$$\frac{dI}{dt} = -b S(t) I(t)$$

with initial conditions  $S(0) = S_0$  &  $I(0) = I_0$ , the formula for the S forces is:

$$\text{EQ (1)} \quad S(t) = S_0 \left( \frac{b S_0 - a I_0}{b S_0 - a I_0 e^{-(a I_0 - b S_0)t}} \right) \quad \text{for } b S_0 \neq a S_0$$

$$\text{or} \quad \text{EQ (2)} \quad S(t) = \frac{S_0}{1 + b S_0 t} \quad \text{if } b S_0 = a S_0$$

The formula for the I forces is:

$$\text{EQ (3)} \quad I(t) = I_0 \left( \frac{a I_0 - b S_0}{a I_0 - b S_0 e^{-(a I_0 - b S_0)t}} \right) \quad \text{for } b I_0 \neq a I_0$$

or

$$\text{EQ (4)} \quad I(t) = \frac{I_0}{1 + a I_0 t} \quad \text{if } a I_0 = b I_0$$

Since  $a$  is known to be 0 and 7000 Incas were killed, EQ (1) & EQ (3) are to be used

$$\text{So, } S(t) = S_0 \left( \frac{b S_0}{b S_0} \right) = S_0 = 168$$

$$I(t) = I_0 \left( \frac{-b S_0}{-b S_0 e^{-(-b S_0)t}} \right) = I_0 \left( \frac{1}{e^{b S_0 t}} \right)$$

$$\text{Rearrange } e^{b S_0 t} = \frac{I_0}{I(t)} \quad \rightarrow \quad b S_0 t = \ln \left( \frac{I_0}{I(t)} \right)$$

$$b = \frac{\ln(I_0 / I(t))}{S_0 t}$$

$$\frac{I(480)}{S_0 = 168} = 73000 \quad b = \frac{\ln \left( \frac{80000}{73000} \right)}{168(480)} = 1.136 \times 10^{-6} = b$$