Homework 1

Steve Mazza

July 12, 2013

Because I feel like I have a lot of ground to make up, I have attempted all of the problems. I would very much appreciate any feedback you can provide in the interest of aiding my understanding.

Problem 1

There may be a more clever way to do this, but my approach is to find all values for s such that $e^{-as} = 1$. Solving for s we get $s = \frac{2i\pi n}{a}$. There are an infinite number of solutions to this periodic function.

Problem 2

The problem describes a step-wise function as follows:

$$f(t) = T, t \geq T \tag{1}$$
$$f(t) = t, 0 > t > T \tag{2}$$
$$f(t) = 0, t \leq 0 \tag{3}$$

In order to obtain the Laplace transform, the functions must be made continuous. There are at least two ways to achieve this. The first is to integrate.

$$\int_0^T te^{-st}\,\mathrm{d}t + \int_T^\infty Te^{-st}\,\mathrm{d}t \tag{4}$$

The other is to apply the unit step function (let's use this method).

$$f(t) = t(u(t) - u(t-T)) + Tu(t-T) \tag{5}$$
$$f(t) = tu(t) - tu(t-T) + Tu(t-T) \tag{6}$$
$$F(s) = \frac{1}{s^2} + \frac{\mathrm{d}}{\mathrm{d}s}\left[e^{-Ts}\frac{1}{s}\right] + T\left(e^{-Ts}\frac{1}{s}\right) \tag{7}$$
$$F(s) = \frac{1}{s^2} - \frac{1 - e^{-Ts}\left(Ts+1\right)}{s^2} + \frac{T\left(e^{-Ts}\right)}{s} \tag{8}$$
$$F(s) = \frac{1 - e^{-Ts}\left(Ts+1\right)}{s^2} + \frac{T\left(e^{-Ts}\right)}{s} \tag{9}$$

Problem 3

The partial-fraction expansion is obtained in MATLAB as follows:

```
>> num = [1 5 6 9 30];
>> den = [1 6 21 46 30];
>> [r,p,k] = residue(num,den)

r =

  -1.0812 + 1.7051i
  -1.0812 - 1.7051i
  -0.1154 + 0.0000i
   1.2778 + 0.0000i


p =

  -1.0000 + 3.0000i
  -1.0000 - 3.0000i
  -3.0000 + 0.0000i
  -1.0000 + 0.0000i


k =

     1
```

The corresponding formatted equation for the solution is

$$F(s) = 1 + \frac{-1.0812 + 1.7051i}{s + 1 - 3i} + \frac{-1.0812 - 1.7051i}{s + 1 + 3i} + \frac{-0.1154}{s+3} + \frac{1.2778}{s+1} \tag{10}$$

The inverse Laplace transform is obtained in MATLAB as follows:

```
>> syms s
>> F = (s^4+5*s^3+6*s^2+9*s+30)/(s^4+6*s^3+21*s^2+46*s+30);
>> ilaplace(F)

ans =

(23*exp(-t))/18 - (3*exp(-3*t))/26 + dirac(t) - (253*exp(-t)*(cos(3*t)
+ (399*sin(3*t))/253))/117
```

The corresponding formatted equation for the solution is

$$f(t) = \frac{23e^{-t}}{18} - \frac{3e^{-3t}}{26} + \delta(t) - \frac{253e^{-t}\left(\cos(3t) + \dfrac{399\sin(3t)}{253}\right)}{117} \tag{11}$$

Please also see the corresponding MATLAB file for additional work.

Problem 4

```
>> z = [-1; -2];
>> p = [0; -4; -6; 2+3i; 2-3i];
>> k = 5;
>> [num,den] = zp2tf(z,p,k);
>> printsys(num,den,'s')

num/den =

          5 s^2 + 15 s + 10
   -----------------------------------
   s^5 + 6 s^4 - 3 s^3 + 34 s^2 + 312 s
```

The corresponding formatted equation for the solution is

$$F(s) = \frac{5s^2 + 15s + 10}{s^5 + 6s^4 - 3s^3 + 34s^2 + 312s} \tag{12}$$

Please also see the corresponding MATLAB file for additional work.

Problem 5

We are asked to first derive the inverse Laplace transform of the function $F(s) = \dfrac{5}{s^2\left(s^2+\omega^2\right)}$ by hand.

$$F(s) = \frac{5}{s^2\left(s^2+\omega^2\right)} \tag{13}$$

$$F(s) = \frac{5}{\omega^3}\left(\frac{\omega^3}{s^2\left(s^2+\omega^2\right)}\right) \tag{14}$$

$$f(t) = \left(\frac{5}{\omega^3}\right)\omega t - \sin(\omega t) \tag{15}$$

$$f(t) = \frac{5\omega t - 5\sin(\omega t)}{\omega^3} \tag{16}$$

Next we find the solution using MATLAB.

```
>> syms w
>> F = 5/(s^2*(s^2+w^2));
>> ilaplace(F)

ans =

(5*t)/w^2 - (5*sin(t*w))/w^3
```

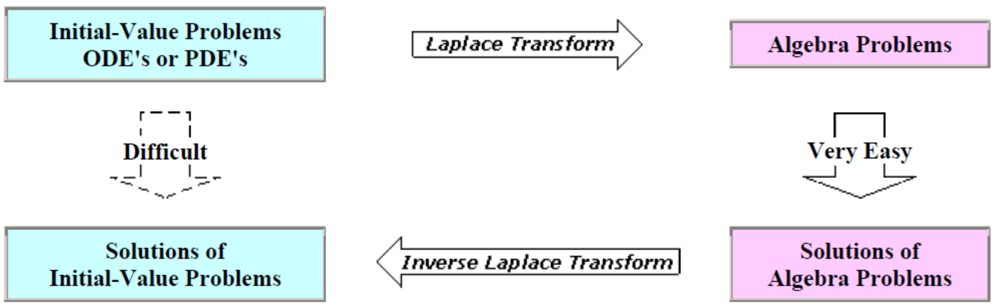
The corresponding formatted equation for the solution is

$$f(t) = \frac{5t}{\omega^2} - \frac{5\sin(t\omega)}{\omega^3} \tag{17}$$

Please also see the corresponding MATLAB file for additional work.

Problem 6

We apply the technique described in the introduction handout.



$$
\begin{aligned}
s^2F(s) - s(-1) - 2 + 3(sF(s) - (-1)) + 2F(s) &= 0 & (18)\\
F(s)(s^2 + 3s + s) &= -(s+1) & (19)\\
F(s) &= \frac{-(s+1)}{s^2+3s+2} & (20)\\
F(s) &= -\frac{1}{s+2} & (21)\\
f(t) &= -e^{-2t} & (22)
\end{aligned}
$$

I also provide a solution computed directly through MATLAB:

```
>> dsolve('D2x+3*Dx+2*x=0','x(0)=-1, Dx(0)=2')


ans =

-exp(-2*t)
```

Please also see the corresponding MATLAB file for additional work.