Homework 1

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Problem 1

Problem 2

The problem describes a step-wise function as follows:

$$f(t) = T, t \geq T \tag{1}$$
$$f(t) = t, 0 > t > T \tag{2}$$
$$f(t) = 0, t \leq 0 \tag{3}$$

In order to obtain the Laplace transform, the functions must be made continuous. There are at least two ways to achieve this. The first is to integrate.

$$\int_0^T t e^{-st} \, \mathrm{d}t + \int_T^\infty T e^{-st} \, \mathrm{d}t \tag{4}$$

The other is to apply the unit step function (let's use this method).

$$f(t) = t(u(t) - u(t - T)) + Tu(t - T) \tag{5}$$
$$f(t) = tu(t) - tu(t - T) + Tu(t - T) \tag{6}$$
$$F(s) = \frac{1}{s^2} + \frac{\mathrm{d}}{\mathrm{d}s}\left[e^{-Ts}\frac{1}{s} \right] + T\left(e^{-Ts}\frac{1}{s} \right) \tag{7}$$
$$F(s) = \frac{1}{s^2} - \frac{1 - e^{-Ts}(Ts + 1)}{s^2} + \frac{T\left(e^{-Ts}\right)}{s} \tag{8}$$
$$F(s) = \frac{1 - e^{-Ts}(Ts + 1)}{s^2} + \frac{T\left(e^{-Ts}\right)}{s} \tag{9}$$

Problem 3

The partial-fraction expansion is obtained in MATLAB as follows:

```
>> num = [1 5 6 9 30];
>> den = [1 6 21 46 30];
>> [r,p,k] = residue(num,den)
```

```
r =

  -1.0812 + 1.7051i
  -1.0812 - 1.7051i
  -0.1154 + 0.0000i
   1.2778 + 0.0000i


p =

  -1.0000 + 3.0000i
  -1.0000 - 3.0000i
  -3.0000 + 0.0000i
  -1.0000 + 0.0000i


k =

     1
```

The corresponding formatted equation for the solution is

$$F(s) = 1 + \frac{-1.0812 + 1.7051i}{s + 1 - 3i} + \frac{-1.0812 - 1.7051i}{s + 1 + 3i} + \frac{-0.1154}{s + 3} + \frac{1.2778}{s + 1} \tag{10}$$

The inverse Laplace transform is obtained in MATLAB as follows:

```
>> syms s
>> F = (s^4+5*s^3+6*s^2+9*s+30)/(s^4+6*s^3+21*s^2+46*s+30);
>> ilaplace(F)


ans =

(23*exp(-t))/18 - (3*exp(-3*t))/26 + dirac(t) - (253*exp(-t)*(cos(3*t)
+ (399*sin(3*t))/253))/117
```

The corresponding formatted equation for the solution is

$$\frac{23e^{-t}}{18} - \frac{3e^{-3t}}{26} + \delta(t) - \frac{253e^{-t}\left(\dfrac{\cos(3t) + 399\sin(3t)}{253}\right)}{117} \tag{11}$$

Please also see the corresponding MATLAB file for additional work.

Problem 4

Please also see the corresponding MATLAB file for additional work.

Problem 5

Please also see the corresponding MATLAB file for additional work.

Problem 6

Please also see the corresponding MATLAB file for additional work.