Homework I

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Problem 1.1.16

Some number of particles, a leaves $x = 0$ and heads toward $x = 1$. If r is the fraction of particles reflected back toward $x = 0$ at $x = 1$ (iteration $n = 1$) then ar particles are returned and $a(1 - r)$ particles escape. At iteration $n = 2$ (back at $x = 0$) we have $ar \times r$ particles or ar^2 particles reflected back and, consequently, $ar \times (1 - r)$ particles escaping. We write the sequence as

$$ar^0(1 - r), ar^1(1 - r), ar^2(1 - r), ar^3(1 - r), \ldots$$

and we can generalize this as

$$ar^{n-1}(1 - r) \tag{1}$$

We further notice that odd values of n occur at $x = 1$ and even values of n occur at $x = 0$, so we can write the sequence for each as

$$x = 0 : ar^1(1 - r), ar^3(1 - r), ar^5(1 - r), ar^7(1 - r), \ldots$$
$$x = 1 : ar^0(1 - r), ar^2(1 - r), ar^4(1 - r), ar^6(1 - r), \ldots$$

which we can generalize as

$$x = 0 : ar^{2n-1}(1 - r) \tag{2}$$
$$x = 1 : ar^{2n-2}(1 - r) \tag{3}$$

In general, the sums for these series is determined by the equation

$$S = \frac{a}{1 - r}$$

So to sum the series at $x = 0$ we substitute values $a = ar^{2n-1}$ $r = (1 - r)$ and get

$$\begin{aligned} S &= \frac{ar^{2n-1}}{1 - (1 - r)} \\ &= \frac{ar^{2n-1}}{1 - 1 + r} \\ &= \frac{ar^{2n-1}}{r} \\ &= ar^{2n-2} \end{aligned}$$

Summing the series at $x = 1$ we substitute values $a = ar^{2n-2}$ $r = (1 - r)$ and get

$$\begin{aligned}
S &= \frac{ar^{2n-2}}{1 - (1 - r)} \\
&= \frac{ar^{2n-2}}{1 - 1 + r} \\
&= \frac{ar^{2n-2}}{r} \\
&= ar^{2n-3}
\end{aligned}$$

Since the particles begin at $x = 0$ and head toward $x = 1$ first, the largest fraction of particles which can escape at $x = 0$ $(n = 2)$ is $\frac{1}{2}$.

Problem 1.6.27

Problem 1.10.2

Problem 2.4.12

Problem 2.5.5

Problem 2.5.41

Problem 2.5.59