

Fourier Series Fundamentals

Mathematical Methods in the Physical Sciences

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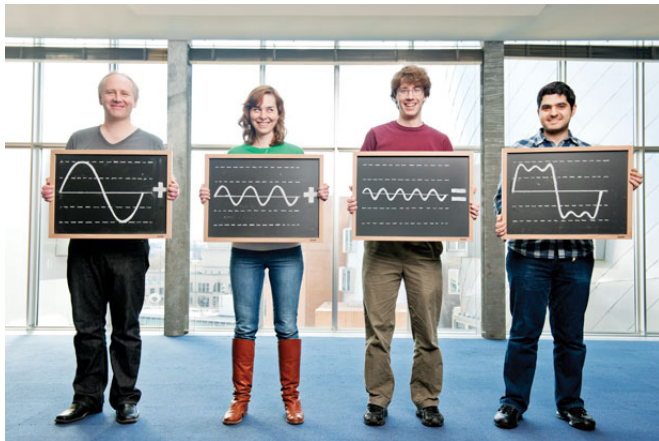
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Introduction

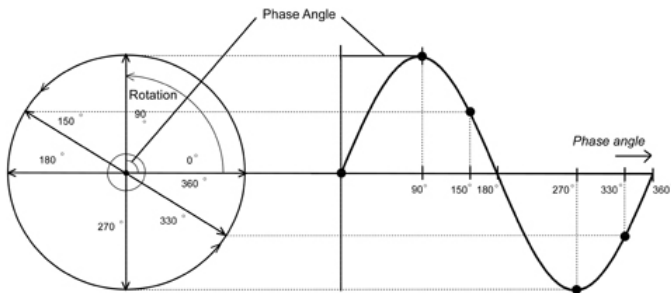
Fourier series are like power series but are only used to represent periodic functions.



Periodic Functions

Simple Harmonic Motion

An object executing simple harmonic motion if its displacement from equilibrium can be written as $A \sin \omega t$ or $A \cos \omega t$ or $A \sin (\omega t + \phi)$.



Periodic Functions (continued)

The x and y components are $(A \cos \omega t, A \sin \omega t)$. In the complex plane this could be rewritten as

$$\begin{aligned} z &= x + iy \\ &= A(\cos \omega t + i \sin \omega t) \\ &= Ae^{i\omega t} \end{aligned}$$

The *amplitude* is the maximum displacement from equilibrium and the *period* is the time of one complete oscillation.

Applications of Fourier Series

In application,

- Fourier series do not tend to converge as rapidly as power series.
- Fourier series can represent discontinuous functions.

Often applied to problems involving,

- Sound
- Light
- Radio waves

Applications of Fourier Series (continued)

Average Value of a Function

Definition

$$\text{average of } f(x) \text{ on } (a, b) = \frac{\int_a^b f(x) dx}{b - a}$$

When the average of a function over a period of time is 0 then the average of the square of the function is often of interest. The average value over 1 period of $\sin^2 nx$ and $\cos^2 nx$ are the same:

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} \sin^2 nx \, dx = \frac{1}{2\pi} \int_{-\pi}^{\pi} \cos^2 nx \, dx = \frac{\pi}{2\pi} = \frac{1}{2}$$

Fourier Coefficients

We begin by considering only periodic functions of period 2π in terms of $\sin nx$ and $\cos nx$.

Coefficient Form

$$f(x) = \frac{1}{2}a_0 + a_1 \cos x + a_2 \cos 2x + a_3 \cos 3x + \cdots \\ + b_1 \sin x + b_2 \sin 2x + b_3 \sin 3x + \cdots$$

The integrals on page 351 are used to find coefficients a_n and b_n .

Coefficient Formulae

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, dx$$
$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx$$

Fourier Coefficients Example

Dirichlet Conditions

Complex Form of Fourier Series

Other Intervals

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