Homework for Module 2

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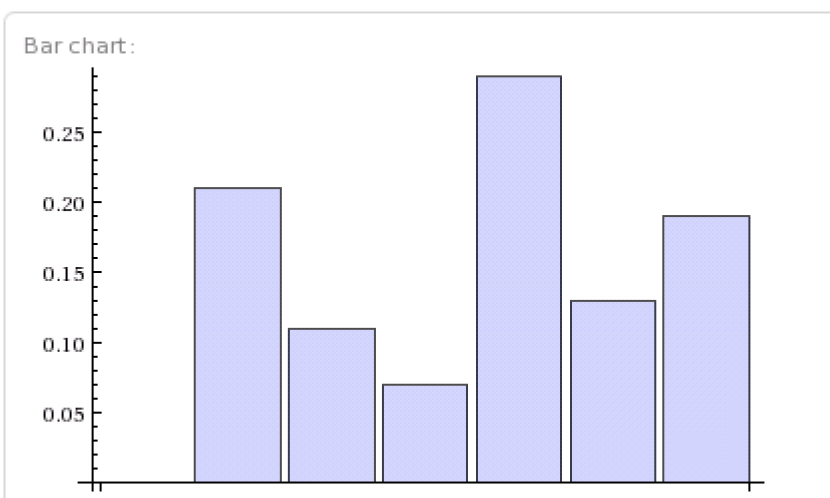
2.1.2

Table 1: Probability Mass Function

	$P(x)$
$-\infty$	0.00
-4	0.21
-1	0.11
0	0.07
2	0.29
3	0.13
7	0.19

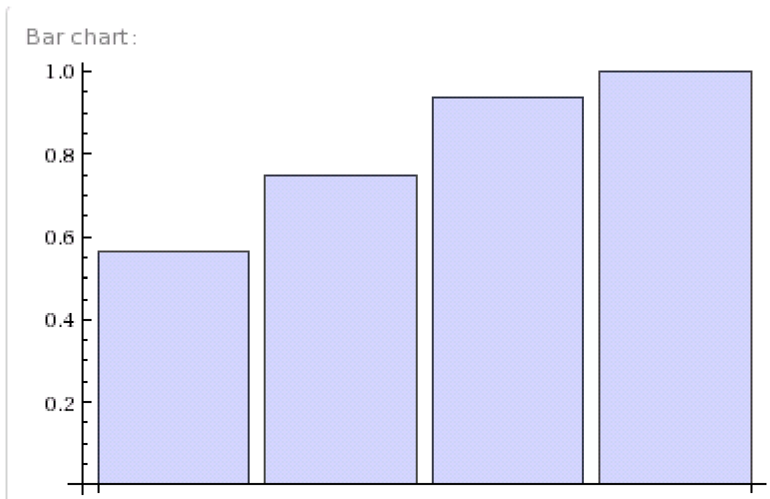


2.1.4

1. a) Probability mass function

$$P(0,0) = 0.75 \times 0.75 = 0.5625$$
$$P(0,1) = 0.75 \times 0.25 = 0.1875$$
$$P(1,0) = 0.25 \times 0.75 = 0.1875$$
$$P(1,1) = 0.25 \times 0.25 = 0.0625$$

   b)

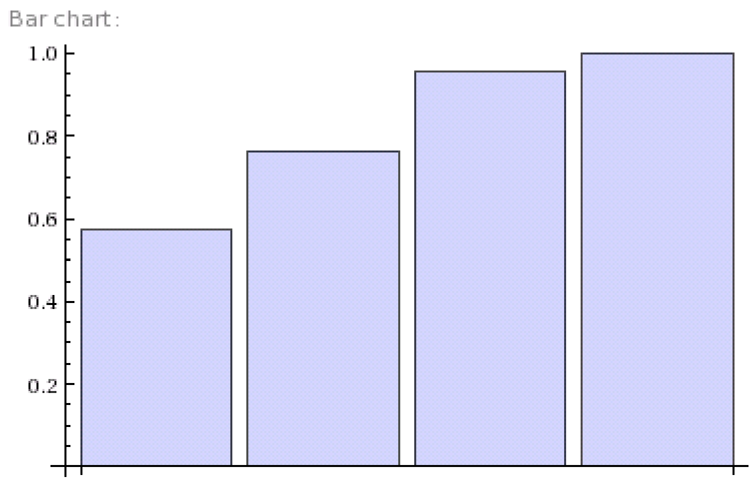


   c) 0 and 1 seem equally likely

2. a) Probability mass function

$$P(0,0) = \frac{39}{52} \times \frac{39}{51} \approx 0.5735$$
$$P(0,1) = \frac{39}{52} \times \frac{13}{51} \approx 0.1912$$
$$P(1,0) = \frac{13}{52} \times \frac{39}{51} \approx 0.1912$$
$$P(1,1) = \frac{13}{52} \times \frac{13}{51} \approx 0.637$$

   b)

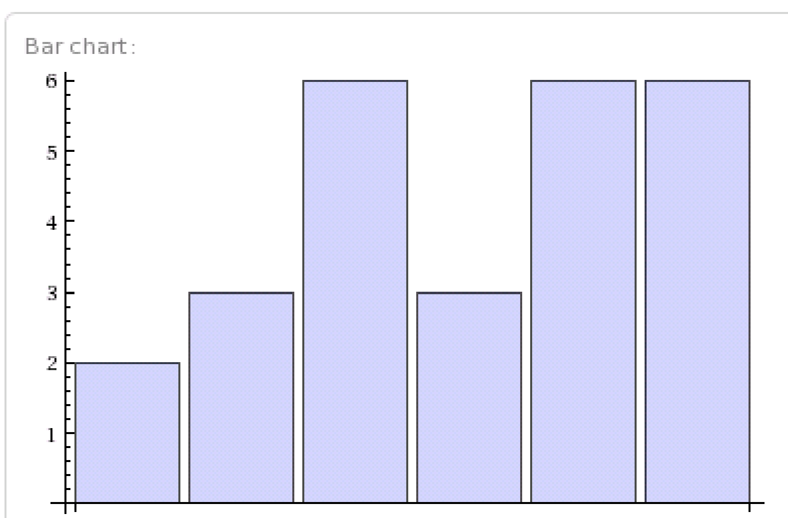


c) 0 and 1 seem equally likely

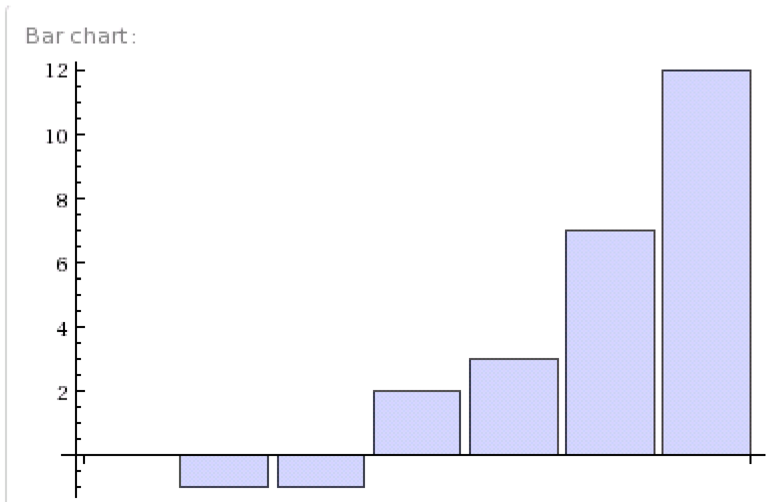
2.1.8 Possible combinations in the form WIN − COST = NET (all are equally likely):

$$(1,2) = 3 - 4 = -1$$
$$(1,3) = 4 - 4 = 0$$
$$(1,6) = 7 - 4 = 3$$
$$(2,3) = 5 - 4 = 1$$
$$(2,6) = 8 - 4 = 4$$
$$(3,6) = 9 - 4 = 5$$

Probability mass function:

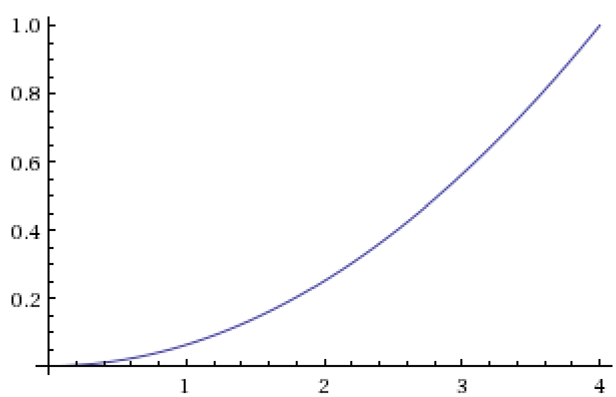


Cumulative distribution function:



2.2.4

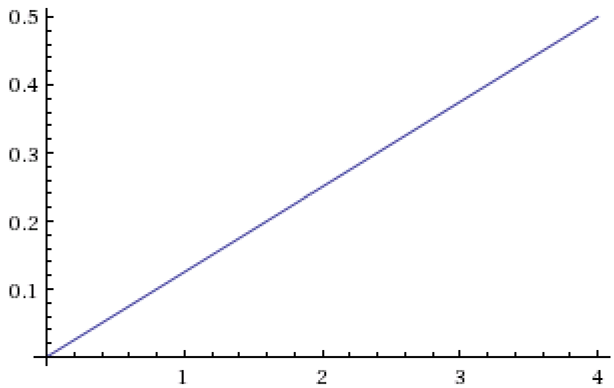
a) Cumulative distribution function:



b) $P(X \geq 2) = 7/6$

c) $P(1 \leq X \leq 3) = 13/24$

d) Probability density function:



2.3.8 I would expect net winnings on each ticket to be negative. People play the Lottery for many reasons. The psychology of random reward is very powerful. Furthermore, the incremental investment (risk) is very small compared to the reward (should the participant win). Also, most people cannot do math.

2.3.11

a)

$$\begin{aligned}\int_0^4 \frac{x^2}{16}dx &= \frac{x^3}{48}\,|_0^4 \\ &= \frac{64-0}{48} \\ &= \frac{4}{3}\end{aligned}$$

b)

$$\begin{aligned}\frac{x^2}{16} \rightarrow F(x) &= 0.5 \\ x &= 2 \\ &= \frac{4}{16} \\ &= \frac{1}{4}\end{aligned}$$

2.4.4

$$\begin{aligned}Var(X) &= E(x^2) - (E(x))^2 \\ &= \frac{35}{6} - 3.7636 \\ &\approx 2.07\end{aligned}$$

$$
\begin{aligned}
E(x^2) &= \sum_i p_i x_i^2 \\
&= (\frac{1}{6} \times 0^2) + (\frac{5}{18} \times 1^2) + (\frac{2}{9} \times 2^2) + (\frac{1}{6} \times 3^2) + (\frac{1}{9} \times 4^2) + (\frac{1}{18} \times 5^2) \\
&= 0 + \frac{5}{18} + \frac{8}{9} + \frac{9}{6} + \frac{16}{9} + \frac{25}{18} \\
&= \frac{35}{6} \\
&= 5.8\overline{33}
\end{aligned}
$$

I would prefer a small variance.

2.5.6

a)

Table 2: Joint & Marginal Probability Mass Function

	0	1	2	
0	0.25	0	0.125	0.375
1	0	0.125	0	0.25
2	0.125	0	0	0.125
	0.375	0.125	0.125	

b) See a)

c) The variables X and Y are dependent because there is no replacement.

d) I am confused on this point and cannot calculate these values.

e)

$$
\begin{aligned}
\mathrm{Cov}(X, Y) &= E(XY) - E(X)E(Y) \\
&= 0.062 - 0.140625 \\
&= -0.078125
\end{aligned}
$$

f)

$$
\frac{-0078125}{\sqrt{0.0087890625}} = -0.8\overline{33}
$$

g) I don't know how to calculate this value.

2.5.9

a) I don't know how to calculate this value.

b) I don't know how to calculate this value.

c)

$$\begin{aligned}E(X) &= 2.94\\ \mathrm{Var}(X) &= 0.942 - (2.94)^2\\ &= -7.7016\end{aligned}$$

d)

$$\begin{aligned}E(4) &= 2.86\\ \mathrm{Var}(4) &= -7.3118\end{aligned}$$

e) Yes. Yes. I don't know.

f) 0.3

g)

$$\begin{aligned}\mathrm{Cov}(X,Y) &= E(XY) - E(X)E(Y)\\ &= 9.29 - 8.4084\\ &= 0.8816\end{aligned}$$

h)

$$\begin{aligned}\mathrm{Cor}(X,Y) &= \frac{0.8816}{\sqrt{-7.7016 \times -7.3118}}\\ &\approx 0.1175\end{aligned}$$

2.6.4 Suppose $P = A_1 + A_2 + B$

$$\begin{aligned}E(P) &= E(A_1) + E(A_2) + E(B)\\ &= 37 + 37 + 24\\ &= 98\end{aligned}$$

$$\begin{aligned}\mathrm{Var}(P) &= \mathrm{Var}(A_1) + \mathrm{Var}(A_2) + \mathrm{Var}(B)\\ &= 0.7^2 + 0.7^2 + 0.3^2\\ &= 1.07\end{aligned}$$

2.6.14

$$\begin{aligned}
E(Y) &= aE(X) + b \\
1000 &= 77a + b \\
\text{–and–} & \\
100 &= 81a^2 \\
\frac{100}{81} &= a^2 \\
\pm\sqrt{\frac{100}{81}} &= a \\
\pm\frac{10}{9} &= a \\
a &= 1.\overline{11}
\end{aligned}$$