Vector Analysis

Mathematical Methods in the Physical Sciences

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Introduction

We will extend our discussion of vectors from chapters 3, 4, & 5 with the following broad overview of topics

- Vector products
- Differentiation
- Integration

The end of the chapter contains discussions of vector theorems, which we will save for another lecture.

We can apply the dot and cross products introduced in Chapter 3, Section 4, to calculate

Work: $Fd\ \cos\theta = \vec{F} \cdot \vec{d}$

Torque: $rF\ \sin\theta = \vec{r} \times \vec{F}$

$\angle$ Velocity: $\omega r\ \sin\theta = |\vec{\omega} \times \vec{r}|$

Angular velocity is solved by considering that the linear velocity $\vec{v}$ of some point P is equal to $\vec{\omega} \times \vec{r}$, and that the magnitude of $\vec{v} = |\vec{\omega} \times \vec{r}|$, which is also equal to $\omega r\ \sin\theta$.

Triple Products

Triple Scalar Product

$$\vec{A} \cdot \left(\vec{B} \times \vec{C}\right)$$

- Can be thought of as the volume of a parallelepiped.
- The order of the factors is cyclical (3.3, p. 279).

Triple Vector Product

$$\vec{A} \times \left(\vec{B} \times \vec{C}\right) = \left(\vec{A} \cdot \vec{C}\right) \vec{B} - \left(\vec{A} \cdot \vec{B}\right) \vec{C}$$

Triple Products (continued)

6.3.2

Differentiation of Vectors

Definition

$$\frac{d\vec{A}}{dt} = \vec{i}\frac{dA_x}{dt} + \vec{j}\frac{dA_y}{dt} + \vec{k}\frac{dA_z}{dt}$$

Differentiation of Vectors (continued)

6.4.3

Fields

Gradient

Expressions Involving ∇

Questions?