Groups & Vector Spaces
Mathematical Methods in the Physical Sciences

Steve Mazza

Naval Postgraduate School
Monterey, CA

SE3030, Winter/2014
Quantitative Methods of Systems Engineering

Definition of Groups

A group is a set of elements, G, together with a set operation, $\cdot$, that satisfies the following conditions:

Group Conditions

Closure:	$\forall\, a, b \in G, a \cdot b \in G$
Association:	$\forall\, a, b, c \in G, (a \cdot b) \cdot c = a \cdot (b \cdot c)$
Identity:	$\exists$ exactly 1 element, $i \in G \mid \forall\, a \in G, i \cdot a = a \cdot i = a$
Inversion:	$\forall\, a \in G\ \exists\, b \mid a \cdot b = b \cdot a = i$, where i is the identity element.

Definition of Vector Spaces

A group is a set of elements, G, together with a set operation, $\cdot$, that satisfies the following conditions:

Group Conditions

Closure:	$\forall \, a, b \in G, a \cdot b \in G$
Association:	$\forall \, a, b, c \in G, (a \cdot b) \cdot c = a \cdot (b \cdot c)$
Identity:	$\exists$ exactly 1 element, $i \in G \mid \forall \, a \in G, i \cdot a = a \cdot i = a$
Inversion:	$\forall \, a \in G \, \exists \, b \mid a \cdot b = b \cdot a = i$, where i is the identity element.