Groups & Vector Spaces
Mathematical Methods in the Physical Sciences

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Definition of Groups

A group is a set of elements, G, together with a set operation, $\cdot$, that satisfies the following conditions:

Group Conditions

Closure:	$\forall\, a, b \in G, a \cdot b \in G$
Association:	$\forall\, a, b, c \in G, (a \cdot b) \cdot c = a \cdot (b \cdot c)$
Identity:	$\exists$ exactly 1 element, $i \in G \mid \forall\, a \in G, i \cdot a = a \cdot i = a$
Inversion:	$\forall\, a \in G \; \exists\, b \mid a \cdot b = b \cdot a = i$, where i is the identity element.

Operation Table

Definition of Vector Spaces

A vector space over field F is a set V together with two binary operations satisfying following conditions:

Group Conditions

Closure: $\forall\, a, b \in V, a \cdot b \in V$

Association:

         Addition: $\forall\, a, b, c \in V, (a + b) + c = a + (b + c)$

         Multiplication: $\forall\, a, b, c \in V, (a \cdot b) \cdot c = a \cdot (b \cdot c)$

Identity:

         Addition: $\exists$ exactly 1 element, $0 \in V \mid \forall\, a \in V, 0 = a = a = 0 = a$

         Multiplication: $\exists$ exactly 1 element, $i \in V \mid \forall\, a \in V, i \cdot a = a \cdot i = a$

Inversion:

Inversion:

Inversion:

Inversion:

Inversion:

Orthonormal Basis