Homework

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Homework 4

Problem 1

(a)

$$
\begin{aligned}
G(s) &= \frac{50}{(s+1)(s+5)(s+50)} \\
G(s) &= \frac{0.2}{\left(1+\frac{s}{1}\right)\left(1+\frac{s}{5}\right)\left(1+\frac{s}{50}\right)} \\
\text{decay constants} &= 1; 5; 10 \\
G_{simpl}(s) &= \frac{0.2}{s+1}
\end{aligned}
$$

(b)

$$
\begin{aligned}
G(s) &= \frac{100}{(s+1)(s^2+12s+20)} \\
G(s) &= \frac{5}{\left(1+\frac{s}{1}\right)\left(1+\frac{3}{5}s+\frac{s^2}{20}\right)} \\
\text{decay constants} &= 0.6; 1 \\
G_{simpl}(s) &= \frac{100}{(s^2+12s+20)}
\end{aligned}
$$

(c)

$$
\begin{aligned}
G(s) &= \frac{10}{(s+5)(s^2+2s+2)(s^2+4)} \\
G(s) &= \frac{0.25}{\left(1+\frac{s}{5}\right)\left(1+s+\frac{s^2}{4}\right)\left(1+\frac{S^2}{4}\right)} \\
\text{decay constants} &= 0; 1; 5 \\
G_{simpl}(s) &= \frac{2}{\left(s^2+2s+2\right)\left(s^2+4\right)}
\end{aligned}
$$

(d)

$$
\begin{aligned}
G(s) &= \frac{72(s+8)}{(s+4)(s+12)(s^2+8s+12)} \\
G(s) &= \frac{1 \times \left(1+\frac{s}{8}\right)}{\left(1+\frac{s}{4}\right)\left(1+\frac{s}{12}\right)\left(1+\frac{2s}{3}+\frac{s^2}{12}\right)} \\
\text{decay constants} &= 0.\overline{66}; 4; 12 \\
G_{simpl}(s) &= \frac{12}{s^2+8s+12}
\end{aligned}
$$

Problem 2

Please see the attached file.

Problem 4

Applying the negative feedback rule, $C(s) = \dfrac{G(s)}{1-G(s)H(s)}R(s)$ twice, we obtain a transfer function for the simplified block diagram,

$$
\frac{100}{s^2+100Ks+100}
$$

We then proceed to solve for ω_n,

$$
\begin{aligned}
\omega_n &= \sqrt{100} \\
\omega_n &= 10
\end{aligned}
$$

and ζ,

$$
\begin{aligned}
100K &= 2\zeta\omega_n \\
50K &= 10\zeta \\
\zeta &= 5K
\end{aligned}
$$

To achieve a zero overshoot and as rapid a response time as possible, we select $\zeta = 1$, or *critical damping*, from which we derive K,

$$\begin{aligned} \zeta &= 5K \\ 1 &= 5K \\ K &= 0.2 \end{aligned}$$

Assuming a Type 2 system, we determine our steady-state error from the formula $E_{ss} = \dfrac{1}{K} = 5$.

Homework 5

Problem 1

(a)

$$\begin{array}{cc} 1 & 6 \\ 4 & 6 \\ 9/2 & \\ 6 & \end{array}$$

$s^3 + 4s^2 + 6s + 6$ is stable. There are no roots in the right-hand plane.

(b)

$$\begin{array}{cc} 1 & 2+K \\ 3K & 5 \\ K+2-5/3K & \\ 5 & \end{array}$$

$s^3 + 3Ks^2 + (2+K)s + 5$ is stable for approximate values of $K, -2.63 < K < 0$ and $K > 0.63$. The number of roots in the right-hand plane will be determined by the value of K.

(c)

$$\begin{array}{ccc} 1 & 2 & 8 \\ 1 & 10 & \\ -8 & 8 & \\ 11 & & \\ 8 & & \end{array}$$

$s^4 + s^3 + 2s^2 + 10s + 8$ is unstable. There are 2 roots in the right-hand plane.

(d)

$$\begin{array}{ccc} 1 & 3 & K \\ 1 & 2 & \\ 1 & K & \\ 2-K & & \\ K & & \end{array}$$

$S^4 + s^3 + 3s^2 + 2s + K$ is stable for $K, 0 < K < 2$. The number of roots in the right-hand plane will be determined by the value of K.

(e)

$$\begin{array}{ccc} 1 & 2 & 11 \\ 1 & 0 & 5 \\ 2 & -4 & \\ 2 & 5 & \\ -9 & & \\ 5 & & \end{array}$$

$s^5 + s^4 + 2s^3 + s + 5$ is unstable. There are 2 roots in the right-hand plane.

(f)

$$\begin{array}{ccc} 1 & 2 & 1 \\ 1 & 1 & K \\ 1 & 1-K & \\ K & K & \\ -K & & \\ K & & \end{array}$$

$s^5 + s^4 + 2s^3 + s^2 + s + K$ is unstable. There are 2 roots in the right-hand plane.

Problem 4

We simplify the block diagram to obtain

$$G(s) = \frac{\omega_n^2}{s(s + 2\zeta\omega_n) + \omega_n^2}$$

Then we continue as follows,

$$\begin{aligned} \frac{C(s)}{R(s)} &= \frac{\omega_n^2(1 + T_d s)}{s(s + 2\zeta\omega_n) + \omega_n^2} \\ \frac{E(s)}{R(s)} &= 1 - \frac{\omega_n^2(1 + T_d s)}{s(s + 2\zeta\omega_n) + \omega_n^2} \\ E(s) &= \frac{1}{s^2}\left[\frac{s^2 + 2\zeta\omega_n s - T_d s\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}\right] \\ \lim_{s\to 0} sE(s) &= \frac{2\zeta\omega_n - T_d\omega_n^2}{\omega_n^2} \\ &= \frac{2\zeta - T_d s\omega_n}{\omega_n} \end{aligned}$$

And so it turns out that the *proper* value for T_d is, in fact, $\dfrac{2\zeta}{\omega_n}$.