Boolean Algebra

Mathematical Methods in the Physical Sciences

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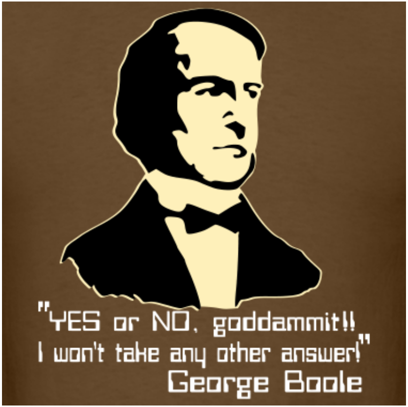
Introduction

In 1854 George Boole introduced a 2-state algebra designed to solve logic problems. Today this algebra is at the heart of network and computer science.

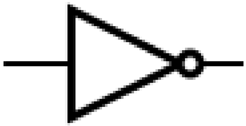


Receives input x and produces x' where

$$x' = \begin{cases} 1 & \text{if } x = 0 \\ 0 & \text{if } x = 1 \end{cases}$$

The output is the *compliment* of the input.

x	x'
1	0
0	1

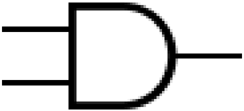


Receives input x_1 and x_2 and produces $(x_1 \wedge x_2)$ where

$$(x_1 \wedge x_2) = \begin{cases} 1 & \text{if } x_1 = x_2 = 1 \\ 0 & \text{otherwise} \end{cases}$$

There may be more than two inputs but the there is always one output.

x_1	x_2	$(x_1 \wedge x_2)$
0	0	0
0	1	0
1	0	0
1	1	1

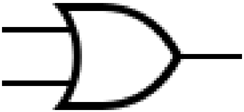


Receives input x_1 and x_2 and produces $(x_1 \vee x_2)$ where

$$(x_1 \vee x_2) = \begin{cases} 1 & \text{if } x_1 = 1 \text{ or } x_2 = 1 \\ 0 & \text{otherwise} \end{cases}$$

There may be more than two inputs but the there is always one output.

x_1	x_2	$(x_1 \wedge x_2)$
0	0	0
0	1	1
1	0	1
1	1	1

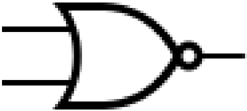


Receives input x_1 and x_2 and produces $(x_1 \vee x_2)'$ where

$$(x_1 \vee x_2)' = \begin{cases} 1 & \text{if } x_1 = x_2 = 0 \\ 0 & \text{otherwise} \end{cases}$$

There may be more than two inputs but the there is always one output.

x_1	x_2	$(x_1 \wedge x_2)'$
0	0	1
0	1	0
1	0	0
1	1	0

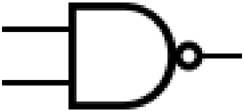


Receives input x_1 and x_2 and produces $(x_1 \wedge x_2)'$ where

$$(x_1 \wedge x_2)' = \begin{cases} 1 & \text{if } x_1 = 0 \text{ or } x_2 = 0 \\ 0 & \text{otherwise} \end{cases}$$

There may be more than two inputs but the there is always one output.

x_1	x_2	$(x_1 \wedge x_2)'$
0	0	1
0	1	1
1	0	1
1	1	0

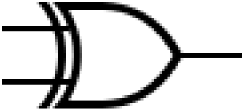


Receives input x_1 and x_2 and produces $(x_1 \oplus x_2)$ where

$$(x_1 \oplus x_2) = \begin{cases} 1 & \text{if only } x_1 = 0 \text{ or only } x_2 = 1 \\ 0 & \text{otherwise} \end{cases}$$

There may be more than two inputs but the there is always one output. The XNOR gate implements the logical expressions: $x_1 \cdot \overline{x_2} + \overline{x_1} \cdot x_2$ and $(x_1 + x_2) \cdot \overline{x_1 \cdot x_2}$.

x_1	x_2	$(x_1 \oplus x_2)$
0	0	0
0	1	1
1	0	1
1	1	0

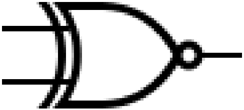


Receives input x_1 and x_2 and produces $(x_1 \oplus x_2)'$ where

$$(x_1 \oplus x_2)' = \begin{cases} 1 & \text{if } x_1 = x_2 \\ 0 & \text{otherwise} \end{cases}$$

There may be more than two inputs but the there is always one output. The XNOR gate implements the logical expression: $x_1 \cdot x_2 + \overline{x_1} \cdot \overline{x_2}$.

x_1	x_2	$(x_1 \oplus x_2)'$
0	0	1
0	1	0
1	0	0
1	1	1



Combinatorial Circuit

Boolean Expression

Equivalent Combinatorial Circuits

Dual of a Statement

Boolean Function

Various Normal Forms

Questions?