Partial Differentiation

Mathematical Methods in the Physical Sciences

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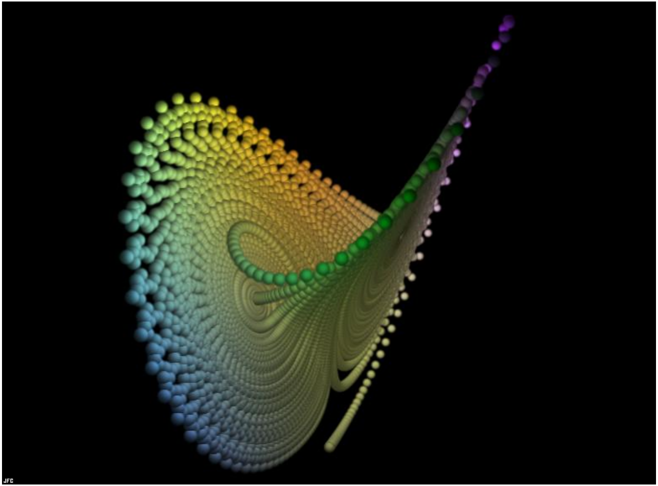
Lorenz Attractor



Introduction

Definition

Derivatives of a multi-variable function where all variables are held fixed during differentiation except the variable of interest.

To denote this we generically write $\frac{\partial}{\partial r}$, which means the partial derivative with respect to r. We more frequently see $\frac{\partial z}{\partial r}$, which means the partial derivative of z with respect to r. In equations of more than two variables we may see $\left(\frac{\partial z}{\partial r}\right)_x$, which denotes the partial derivative of z with respect to r, holding x constant.

Example

4.1.12

$$
\begin{aligned}
z &= x^2 + 2y^2, x = r\cos\theta, y = r\sin\theta \\
z &= x^2 + 2y^2 \\
z &= r^2\cos^2\theta + 2r^2\sin^2\theta \\
\left(\frac{\partial z}{\partial y}\right)_\theta &= r^2\cos^2\theta + 2r^2\sin^2\theta
\end{aligned}
$$

Power Series in Two Variables

Our standard power series expansions can be re-written as partial differential equations.

Definition

$$f(x, y) = \sum_{n=0}^{\infty} \frac{1}{n!} \left(h \frac{\partial}{\partial x} + k \frac{\partial}{\partial y} \right)^n f(a, b)$$

Total Differentials

Total Differentiation in 2 Variables

$$dz = \frac{\partial}{\partial x} dx + \frac{\partial}{\partial y} dy$$

Approximations Using Differentials

Chain Rule

In General

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dv}\frac{dv}{dx}$$

Find dy/dx if $y = \ln \sin 2x$

$$\begin{aligned}\frac{dy}{dx} &= \frac{1}{\sin 2x} \cdot \frac{d}{dx}(\sin 2x) \\ &= \frac{1}{\sin 2x} \cdot \cos 2x \cdot \frac{d}{dx}(2x) \\ &= 2\cot 2x\end{aligned}$$

We can differentiate with respect to a given term in some cases where we cannot solve for that term with respect to another.

Given $x + e^x = t$, find dx/dt

We realize that x is a function of t even though we cannot solve x for t directly.

$$\begin{aligned} x + e^x &= t \\ \frac{dx}{dt} + e^x \frac{dx}{dt} &= 1 \\ \frac{dx}{dt} &= \frac{1}{1 + e^x} \end{aligned}$$

This example can be found in Boas, p 202.

Chain Rule (Redux)

We can extend our earlier examples of the Chain Rule where $z = f(x, y)$ where x and y were functions of t by considering the case where x and y are two variables, s and t.

Applications

Questions?