# Final Exam

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**Question 1:** Given the differential equation $\frac{d^4y}{dx^4} + \frac{d^2y}{dx^2} = 0 \equiv y'''' + Ay''$ derive the equivalent system of first-order ordinary differential equations. Thsi is a fourth order differential equation. What order is the system of equations? Is the system linear or nonlinear? What does such a system of first-order ordinary differential equations represent?

**Question 2:** The maxwell-Bloch equations are a sophisticated model for a laser and describe the dynamics of the wlectric field $E$, the mean polarization of the atoms $P$, and the population inversion $D$:

$$\dot{E} = (P - E)$$
$$\dot{P} = \gamma_1(ED - P)$$
$$\dot{D} = \gamma_2(\lambda + 1 - D - EP)$$

where $\gamma_1$ and $\gamma_2$ are decay rates of the atomic polarization and population inversion, respectively, and $\lambda$ is a pumping energy parameter. The parameter $\lambda$ may be positive, negative, or zero; all other parameters are positive. In the simplest case, $P$ and $D$ relax rapidly to steady values, and hence may be eliminated as follows.

1. Assuming $\dot{D} \approx 0$ $\dot{P} \approx 0$, express $P$ and $D$ in terms of $E$, and thereby derive a first-order equation for the evolution of $E$.
2. Find all the fixed points of $E$.
3. Draw the bifurcation diagram of $E^*$ versus $\lambda$. Distinguish between stable and unstable branches.

**Question 3:** What is this an example of? What features are represented?

**Question 4:** For the Lorenz equations

$$\dot{x} = \sigma(y - x)$$
$$\dot{y} = rx - y - xz$$
$$\dot{z} = xy - bz$$

with $\sigma = 10, r = 28$, and $b = 2.66666$, and initial condition $x = 1.0 + \delta, y = 1.0$, and $z = 10$, determine how long it takes the absolute error between the "true $x$ solution" ($\delta = 0$) to grow from $\delta$ to 0.1. Calculate for $\delta$ values of 0.01, $10^{-4}, 10^{-6}, 10^{-8}$, and $10^{-10}$. What does this tell you abotu the predictability versus measurement error? Can you estimate the Liapunov exponent?

**Question 5:** Consider the iterated map given by

$$x_{n+1} = \begin{cases} rx_n & 0 \leq x_n \leq 0.5 \\ f(1 - x_n) & 0.5 \leq x_n \leq 1 \end{cases}$$

where $0 < r < 2$. What properties do you expect to see in the orbit diagram? Is there any condition that might cause different behavior? The Liapunov exponent is $\lambda = \ln r$. What does this tell you about the behavior?

**Question 6:** In your own words and using no more than one paragraph, describe the difference between complex and complicated systems. That is, in your own opinion what distinguishes the two?

**Question 7:** How are fractals and complexity related?

**Question 8:** Define what an adaptive agent-based model is and briefly describe its characteristics.

**Question 9:** In an engineering system consisting of various parts and mechanisms, what kinds of diversity are most applicable to determining complexity? How might that diversity be measured?

**Question 10:** What approaches are likely to [be] part of any attempt to harness complexity in an inherently complex system?