Homework for Module 1

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1.1.10 The probability values of the three outcomes are as follows:

$$\begin{aligned} P(I) &= 0.6 \\ P(II) &= 0.3 \\ P(III) &= 0.1 \end{aligned}$$

1.2.6 The probability that the red die will have a score that is *strictly greater* than the blue die is $15/36$. The probability is < 0.05 because of the requirement that the value of the second die be *strictly greater* than value of the first. The compliment of this event is $21/36$.

1.3.2 The probabilities of the events are as follows:

$$\begin{aligned} P(B) &= 0.08 + 0.13 + 0.06 + 0.01 + 0.11 + 0.05 + 0.02 \\ &= 0.46 \\ P(B \cap C) &= 0.02 + 0.05 + 0.11 \\ &= 0.18 \\ P(A \cup C) &= 0.07 + 0.05 + 0.01 + 0.02 + 0.05 + 0.08 + 0.04 + 0.11 + 0.11 + 0.07 \\ &= 0.61 \\ P(A \cap B \cap C) &= 0.02 + 0.05 \\ &= 0.07 \end{aligned}$$

1.4.14 For the following $S =$ size, $T =$ taste, and $A =$ appearance.

a)

$$\begin{aligned} P(S \mid T) &= \frac{P(S \cap T)}{P(T)} \\ &= \frac{0.69}{0.78} \\ &= 0.88 \end{aligned}$$

b) From what was given in the text we can derive that

$$\begin{aligned} 0.84 &= 1 - P(S' \cap A') \\ 0.16 &= P(S' \cap A') \end{aligned}$$

And substituting back into the original equation we get

$$\begin{aligned} P(T \mid (S' \cap A')) &= P(0.78 \mid 0.16) \\ &= \frac{P(T \cap S' \cap A')}{P(S' \cap A')} \\ &= \frac{0.04}{0.16} \\ &= 0.25 \end{aligned}$$

1.5.7 For clarity we will refer to switch #1 as S_1, switch #2 as S_2, and switch #3 as S_3. Calculating the combined probability for S_1 and S_2 we get

$$S_{1+2} = 0.81$$

and we can infer the following

$$\begin{aligned} S'_{1+2} &= 0.19 \\ S'_3 &= 0.1 \end{aligned}$$

and so

$$\begin{aligned} P(S_{1+2} \cup S_3) &= P(S'_{1+2} \cap S'_3) \\ &= 1 - (0.19 \times 0.1) \\ &= 0.98 \end{aligned}$$

1.5.8 The general formula for determining the probability of a concurrent birthday (P_{CB}) given n people is

$$P_{CB} = 1 - \frac{365!}{365^n(365 - n)}$$

The following values were obtained using this formula:

$$\begin{aligned} n &= 10 \\ &= 0.116 \\ n &= 15 \\ &= 0.252 \\ n &= 20 \\ &= 0.411 \\ n &= 25 \\ &= 0.569 \\ n &= 30 \\ &= 0.706 \\ n &= 35 \\ &= 0.814 \end{aligned}$$

Twenty-three (23) people is the smallest value for which the probability is larger than a half.

Given that all dates are equally likely (i.e., ignoring February 29) I believe that birthdays are equally likely to be on any day of the year.

1.6.4 For the following exercise species 1 = S_1, species 2 = S_2, species 3 = S_3, and T is the probability any given species is tagged.

$$\begin{aligned}
P(S_1 \mid T) &= \frac{P(S_1)P(T_1 \mid S_1)}{P(T)} \\
&= \frac{0.45 \times 0.1}{0.187} \\
&= 0.24 \\
P(S_3 \mid T) &= \frac{P(S_2)P(T_2 \mid S_2)}{P(T)} \\
&= \frac{0.38 \times}{0.187} \\
&= 0.30 \\
P(S_2 \mid T) &= \frac{P(S_3)P(T_3 \mid S_3)}{P(T)} \\
&= \frac{0.17 \times 0.5}{0.187} \\
&= 0.45
\end{aligned}$$

1.7.4

$$(5 + 3) \times 7 \times 6 \times 8 = 2688$$

1.7.10

a)

$$\begin{aligned}
C_{52,5} &= \frac{52!}{5! \times 47!} \\
&= 2598960
\end{aligned}$$

b)

$$\begin{aligned}
C_{13,5} &= \frac{13!}{5! \times 8!} \\
&= 1287
\end{aligned}$$

1.7.12

$$\begin{aligned}
P_{6,6} &= \frac{6!}{(6-6)!} \\
&= 720
\end{aligned}$$

1.9.33 For the following exercise a lit warning light will be represented by W and a fault will be represented by F.

$$\begin{aligned} P(F \mid W) &= \frac{P(F)P(W \mid F)}{P(F)P(W \mid F) + P(F')P(W \mid F')} \\ &= \frac{0.004 \times 0.992}{(0.004 \times 0.992) + (0.996 \times 0.003)} \\ &= 0.57 \end{aligned}$$