Student Information

Full Name : Mustafa Barış Emektar
Id Number : 2304533

Answer 1

(a) The statement is a **tautology**.

Table 1: Truth table for Question 1.a

p	q	$p \to q$	$\neg q$	$(p \land \neg q)$	$(p \to q) \oplus (p \land \neg q)$
T	T	T	F	F	T
T	F	F	T	T	T
F	T	T	F	F	T
F	F	T	T	F	T

(b)

$$
\begin{aligned}
p \to ((q \lor \neg p) \to r) &\equiv p \to (\neg(q \lor \neg p) \lor r) && \text{Lemma Implication, Table 7} \\
&\equiv p \to ((\neg q \land p) \lor r) && \text{De Morgan's Law, Table 6} \\
&\equiv \neg p \lor ((\neg q \land p) \lor r) && \text{Lemma Implication, Table 7} \\
&\equiv (\neg p \lor (\neg q \land p)) \lor (\neg p \lor r) && \text{Distributive Law, Table 6} \\
&\equiv ((\neg p \lor \neg q) \land (\neg p \lor p)) \lor (\neg p \lor r) && \text{Distributive Law, Table 6} \\
&\equiv ((\neg p \lor \neg q) \land T) \lor (\neg p \lor r) && \text{Negation Law, Table 6} \\
&\equiv ((\neg p \lor \neg q)) \lor (\neg p \lor r) && \text{Identity Law, Table 6} \\
&\equiv ((\neg p \lor \neg p)) \lor (\neg q \lor r) && \text{Associative Law, Table 6} \\
&\equiv (\neg p) \lor (\neg q \lor r) && \text{Idempotent Law, Table 6} \\
&\equiv (\neg p \lor \neg q) \lor r && \text{Associative Law, Table 6} \\
&\equiv \neg(\neg p \lor \neg q) \to r && \text{Lemma Implication, Table 7} \\
&\equiv (p \land q) \to r && \text{De Morgan's Law, Table 6}
\end{aligned}
$$

(c)
- **F**
- **F**
- **F**
- **T**
- **T**

Answer 2

(a) There is at least one team (x) that plays in league L in which Can plays.
$\exists x(P(Can, x) \land T(x, L))$

(b) If a team (x) plays in league S, then there is at least one player y plays in team x whose nationality is Turkish
$\forall x \exists y(T(x, S) \rightarrow (P(y, x) \land N(y, Turkish)))$

(c) If a team x is in league S, then there is **exactly** one rival team (y) plays in league S.
$\forall x \exists y \exists z(T(x, S) \rightarrow (T(y, S) \land R(x, y) \land ((y \neq z) \rightarrow \neg(T(z, S) \land R(x, z))))$

or

$\forall x \exists y \forall z(T(x, S) \rightarrow (T(y, S) \land R(x, y) \land (T(z, S) \land R(x, z) \rightarrow (y = z)))$

(d) If a team (x) has at least one English player (y), then team M has never won against that team (x).
$\forall x \exists y((N(y, English) \land P(y, x)) \rightarrow \neg W(M, x))$

(e) There are **exactly** two players (x and y) play on team G whose nationalities are Turkish.
$\exists x \exists y \forall z((x \neq y) \land P(x, G) \land P(y, G) \land N(x, Turkish) \land N(y, Turkish) \land (((x \neq z) \lor (y \neq z)) \rightarrow \neg(P(y, G) \land N(x, Turkish))))$

(f) There is at least one team (x) that play in **more than one** league (y and z).
$\exists x \exists y \exists z(T(x, y) \land T(x, z) \land (y \neq z))$

Answer 3

$$p \rightarrow q, (r \land s) \rightarrow p, (r \land \neg q) \vdash \neg s$$

1.	$r \land \neg q$	Premise
2.	r	$\land$e, 1
3.	$\neg q$	$\land$e, 1
4.	$p \rightarrow q$	Premise
5.	s	Assume
6.	$r \land s$	$\land$i, 2, 5
7.	$r \land s \rightarrow p$	Premise
8.	p	$\rightarrow$e, 6, 7
9.	$p \rightarrow q$	Premise
10.	q	$\rightarrow$e, 8, 9
11.	$\perp$	$\neg$e, 3, 10
12.	$\neg s$	$\neg$ i, 5 - 11

Answer 4

(a) **Premise 1**: $\exists x(S(x) \to P(x))$
The statement says that there exists at least one student for whom the act of studying (S(x)) implies passing (P(x)), which means some students need to study in order to pass the exam.

Premise 2: $\forall x P(x)$
The statement says that for every student x, the predicate P(x) holds true, meaning every student passed the exam

Claim: : $\exists x S(x)$
There is at least one student that studied for the exam

$$\exists x(S(x) \to P(x)), \forall x P(x) \vdash \exists x S(x)$$

(b) The proof of the claim by natural deduction is:

1.	$\exists x(S(x) \to P(x))$	Premise
2.	$\forall x P(x)$	Premise
3.	$S(c) \to P(c)$	∃e, 1
4.	$P(c)$	∀e, 2
5.	$S(c)$	→e, 3
6.	$\exists x S(x)$	∀i, 5