BEE1034 Equation Cheat Sheet

This provides you with all the basic equations you will need for the module. (Note that I am building these up as we go through the module, so while it contains everything you need so far, more equations will be added as we go along.) Note that while this file details the equations you will need, it does not provide a comprehensive list of the ideas for solving business problems that we discuss in the module.

(1) **Profit**
Profit is denoted by the Greek letter π or Π. The equation for profit is

$$\begin{aligned}\Pi &= \text{ Total Revenue - Total Cost} \\ &= P \times Q - TC \\ &= P \times Q - (FC + VC) \\ &= P \times Q - (FC + MC \times Q)\end{aligned}$$

where, P is price, Q is quantity sold, TC is total cost, FC is fixed cost, VC is variable cost, and MC is marginal cost.

(2) **Break-Even Quantity**
From the above expression for profit, we can solve for the break-even quantity. Breaking even is the same as making zero profit. So we simply rearrange the above expression to solve for the level of output at which profits are zero:

$$\Pi = P \times Q - (FC + MC \times Q) = 0.$$

Collecting terms in Q,

$$\begin{aligned}(P - MC)Q - FC &= 0 \\ (P - MC)Q &= FC \\ Q &= \frac{FC}{(P - MC)}\end{aligned}$$

This is the break-even quantity.

(3) **Average Cost and Average Revenue**
These are important for calculating profit per unit. First, average revenue, AR, is equal to price, P. To see this, recall that total revenue, $TR = P \times Q$, and since $AR = TR/Q$, we have $AR = P \times Q/Q = P$.

The reason it is important to look at AC and AR is because this tells you about profit per unit. To see this, divide the expression for total profit by Q as follows:

$$\begin{aligned}\frac{\Pi}{Q} &= AR - AC \\ & \quad P - (\frac{FC}{Q} + MC).\end{aligned}$$

The reason this is important is because for a business to be viable in the medium to long run it must be the case that $AR > AC$.

(4) **Profit Maximisation**

It is important to remember that to maximise profit we set $MR = MC$. This is what we mean when we say that FC and $AC = FC/Q$ are not relevant for profit maximisation. The easiest way to see this is to look carefully at Table 6.4 of FMSW. You do not need to know the mathematical derivation of why profit is maximised when $MR = MC$. Just focus on the intuition in the table.

(5) **Net Present Value**

Net present value is a useful way of calculating the economic profit of an investment, taking into account the cost of capital over time. The NPV formula takes the form:

$$X_t + \frac{X_{t+1}}{1+r} + \frac{X_{t+2}}{(1+r)^2} + \frac{X_{t+3}}{(1+r)^3} + ...$$

where a negative value denotes an investment and a positive value denotes cash flow that you receive on the investment subsequently.

For example, say your up-front investment is \$100 and from this you receive cash flow in years 1 and 2 of \$60 (and nothing after that), and your cost of capital is 14%. Then the NPV of this investment is

$$\begin{aligned} NPV &= -\$100 + \frac{\$60}{1.14} + \frac{\$60}{1.14^2} \\ &= -\$100 + \frac{\$60}{1.14} + \frac{\$60}{1.2996} \\ &= -\$100 + 52.63 + 46.17 \\ &= -\$1.20 \end{aligned}$$

Any NPV calculation follows this simple pattern.

(6) **Simultaneous Equations**

Often you will find yourself in a situation where your knowledge of the world is imperfect. Simultaneous equations are helpful here. If you are rusty, cut and paste this link into your browser: https://mathsmadeeasy.co.uk/gcse-maths-revision/simultaneous-equations-gcse-maths-revision/

In our case, for example, you might know that $FC + 4 \times MC = £60,000$, and $FC + 6 \times MC = £80000$. How do you find out FC and MC? Well, in each case you need to arrange the equations so you can eliminate the variable that you don't want to know. So if you want to know MC, eliminate FC. How? Subtract one equation from the other:

$$\begin{aligned} FC + 6 \times MC &= £80000 \\ FC + 4 \times MC &= £60,000 \\ 2 \times MC &= £20,000 \end{aligned}$$

where the last line is just the second line subtracted from the first. Then we can see that $MC = £10,000$.

What about finding FC? This is slightly trickier, because the coefficients on MC are not equal. So we have to multiply one equation by a constant in order to get them to be equal. In this case, if we multiply the second equation by 3/2, we get

$$\frac{3}{2} \times FC + 6 \times MC = \pounds 90,000.$$

Now, in exactly this same way as before, we can subtract one equation from the other to eliminate MC

$$\begin{aligned} \frac{3}{2} \times FC + 6 \times MC &= \pounds 90,000 \\ FC + 6 \times MC &= \pounds 80,000 \\ \frac{1}{2} FC &= \pounds 10,000 \end{aligned}$$

And so $FC = \pounds 20,000$.

Once again, every simultaneous equation follows this same basic pattern.

(7) **Elasticity**

In economics, elasticity measures the responsiveness of one thing to another. Here we illustrate with price elasticity of demand, e:

$$e = (\%\text{ change in quantity demanded}) \div (\%\text{ change in price})$$

There are many ways to calculate this, some of which involve calculus. Notice that this will generally yield a negative number, since lowering price normally brings about an increase in quantity.

We will focus on a simple practical way that you can use in business, called the arc price elasticity:

$$\frac{Q_1 - Q_2}{Q_1 + Q_2} \div \frac{P_1 - P_2}{P_1 + P_2}.$$

Your initial price and quantity are P_1 and Q_1. Then by experimentation (say you have a sale), you drop the price to P_2 and your quantity demanded increases to Q_2.

Once we know e, we have a nice simple equation for MR. This is $MR = P(1 - 1/|e|)$, where $|e|$ is just the positive value of e. So if $e = -2$ then $|e| = 2$. We will not be going through the derivation of $MR = P(1 - 1/|e|)$ since it involves calculus. The thing to focus on is that this expression shows $MR < P = AR$, with the amount by which $MR < P$ depending on $|e|$.

(8) **Risk, Risk Aversion, and Insurance**

In economics, risk is captured using the idea of a lottery. An example of a lottery is as follows. Say you have a 50% chance of winning £10 and a 50% chance of winning £0. The expected value of this lottery is then calculated as $0.5 \times \pounds 10 + 0.5 \times \pounds 0 = \pounds 5$.

If an individual is risk neutral, then they are indifferent between the above lottery and receiving $\pounds 5$ with certainty. If they are risk averse then they would be prepared to give up the above lottery that gives an expected value of $\pounds 5$ (but with a risk of receiving £0) in exchange for receiving a lesser amount than £5 with certainty. The more risk averse they are, the less the amount they would be prepared to accept with certainty in exchange for a lottery with an expected value of £5. A moderately risk averse person would be prepared to accept £4 while a more risk averse person would be prepared to accept £3.

Insurance provides a way for a risk averse individual to exchange the risk of losing something they value in exchange for a lesser amount with certainty. For example, say an individual owns a bike valued at £100, and faces a 20% chance that the bike is stollen. This has the same structure as the lottery above. That is, the expected value of owning the bike is $0.8 \times £100 + 0.2 \times £0 = £80$.

If the individual is risk neutral then they would need to be given £80 with certainty in order to be prepared to give up the above lottery of owning the bike. But if they are risk averse, they would be prepared to accept an amount less than £80. Say that a given individual's risk aversion is such that they are prepared to give up the lottery involved in owning a bike in exchange for £70 with certainty. (A more risk averse person would be prepared to accept £60 with certainty ...) Now say an insurance company offers an insurance contract for £25 that will pay out the amount of £100 in the event that the bike gets stollen. So now, with probability 80% the bike is not stollen, in which case the individual is left with the bicycle worth £100 minus the £25 insurance premium, which equals a value of £75. With probability 20% the bike is stollen, in which case the individual is left with the insurance payout of £100 minus the £25 insurance premium, which equals a value of £75. Since they get the full value of the bike if it is stollen, we say they are 'fully insured'. (If the insurance contract only paid out a share of the value of the bike if it were stollen, we would say that they were partially insured.) So either way the individual is left with a value of £75. We can calculate this the same way as a lottery, by writing $0.8 \times (£100 - £25) + 0.2(£100 - £25) = £75$. In other words, they get £75 with certainty. Since their degree of risk aversion is such that they are prepared to give up the lottery involved in owning a bike in exchange for £70 with certainty, they would certainly be prepared to buy this insurance contract that yields a value of £75 with certainty.

Finally, consider the insurance company. We have seen that they get the £25 insurance premium if the bike is not stollen. And they get £25 but pay out £100 if the bike gets stollen. We can work out the expected value of this as $0.8 \times £25 + 0.2 \times (£25 - 100) = £5$. So on average, the insurance company expects to make a profit of £5 on each insurance contract like this that they sell. But notice on some contracts they will make £25 while on others they will lose £75. If they sell lots of contracts like this, the profit will average out at £5 across them all. That is the sense in which we say that the expected profit is £5.