CTAP Open Assessment

Exam no: Y0076159

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Contents

1 Breaking Stream Ciphers

i. Implementation and Challenge

The stream cipher implementation, found in appendix B (written in Python), can be ran using the Python 2.7.6 compiler with the command `python stream.py`. This will produce the challenge 25 bit output of 3 Lfsrs with initial states 97, 975 and 6420:

1 0 1 0 1 0 1 1 1 0 1 0 1 1 0 0 0 0 0 0 1 0 0 1 1

Code description

The class `Lfsr` implements an LFSR with a tap sequence, size and initial state. It contains 2 methods: `calc_tap`, which calculates the tap value for the current register value, and `shift` which calculates the tap value, shifts the register and outputs the register's value. The 3 LFSR's are combined using `combine_lfsr_outputs`, which uses a lookup table to implement the boolean operation.

ii. Cryptanalysis

Analysis

Linear func.	Walsh transform	Attack No	Attack description
R_3	0		
R_2	0		
$R_2 \oplus R_3$	0		
R_1	-4	1	No dependencies, has a single outlier agreement (see appendix A
$R_1 \oplus R_3$	-4	3	Requires R_1, attack with $R_1 \oplus R_3 \oplus K$
$R_1 \oplus R_2$	4	2	Requires R_1, attack with $R_1 \oplus R_2 \oplus K$
$R_1 \oplus R_2 \oplus R_3$	-4		

Using the Walsh transform displayed in appendix D, we can implement a divide-and-conquer strategy as first detailed by Meier and Staffelbach [19]. LFSR1 correlates quite strongly with the output (Walsh value of -4, 2 agreements and 6 disagreements resulting in a probability of 2/8 that it will agree with the function's output, with a bias of 0.25); this is reflected by the outlying value in the graph of sub-keys and their agreements with outputs in appendix A. As such, it can be brute-forced to obtain it's key value by iterating over the space of sub-keys, and finding the one with the highest agreement (sub-key value 27, with an agreement of 0.253).

The next attack targets $R_1 \oplus R_2$, as it has a Walsh value of 4 (6 agreements and 2 disagreements resulting in a probability of 6/8, and a bias of -0.25). It is not feasible to brute-force this LFSR due to the lack of any outlying sub-key agreement. The search space is reduced from $O(n^7 * n^{11}) = O(n^{18})$ to $O(n^7 + n^{11}) = O(n^{11})$) as we now know the key for LFSR1, allowing us to rapidly brute force the sub key for LFSR2 (value 1357, with agreement 0.247).

Finally LFSR3 can be attacked using $R_1 \oplus R_3$, with a Walsh value of -4 (same agreements, disagreements, probability of correlation and bias as previous). As LFSR1's key is known, the search space is also reduced for this attack from $O(n^7 * n^{13}) = O(n^{20})$ to $O(n^7 + n^{13}) = O(n^{13})$). This resulted in the sub-key for LFSR3 (value 7531, with agreement 0.245). This LFSR was also not brute-forcible due to a lack of outlying key agreements, as visible in the box-and-whisker plots in appendix A, where the biggest outlying agreement in LSFR3 is significantly closer to the median than the outlying agreement for LSFR1; this suggests a lower likelihood of the outlying key being correct.

Using this strategy, we have successfully attacked all 3 shift registers in a severely reduced search space $(O(n^{7+11+13}) > (O(n^7 + n^{11} + n^{13}) = O(n^{13}))$ (a reduction of 18 bits), resulting in the set of sub-keys: 27, 1357, and 7531.

Attack Implementation

The Python program which implements this attack loads a key-stream from `stream.txt`, and begins by brute forcing the first register. This is achieved by iterating over all possible keys, recording their agreement with the given data, and returning the key with the highest normalized agreement. The next 2 LFSR's are then broken by iterating over each respective set of possible keys in combination with the output of LFSR1 using the previously known key. This results in the set of initial states: `lfsr1: 27, lfsr2: 1357, lfsr3: 7531.`

iii. Improvement

The current cipher's key weakness is the correlation of a single shift register (LFSR1) with the output: this allows the key to be brute forced in $O(n^7)$, followed by the other two keys in On^{11} and On^{13}. As such, the current function is not correlation immune to any significant order, hence why it was significantly vulnerable to a divide and conquer attack. As demonstrated by Siegenthaler [25], improving this correlation immunity would greatly increase the complexity of a divide and conquer attack, as demonstrated below.

By changing the combination function's output to 1 0 0 1 0 1 1 0 (two bit flips on L001 and L101, found by manual experimentation), we can ensure the function is correlation immune to order 2, as the Walsh-Hadamard values for R_1, R_2, R_3, $R_1 \oplus R_2$, $R_1 \oplus R_3$ and $R_1 \oplus R_3$, are now 0, and therefore have no exploitable correlation exist other than all 3 registers combined; this is visible in the table below.

Input	f	w
000	1	0
001	0	0
010	0	0
011	1	0
100	0	0
101	1	0
110	1	0
111	0	-8

This would restrict the attack vector to a combination of LFSR1, LFSR2 and LFSR3 simultaneously, with a Walsh-Hadamard value of -8 for $R_1 \oplus R_2 \oplus R_3$; this is acceptable as exploiting this would require iterating over the complete key-set $O(n^{7+11+13}) = O(n^{31})$, causing the time complexity to be significantly higher than the previous $O(n^{13})$).

It would also be possible to add additional LSFR's to the cipher and adapting the boolean function to require all 4 LSFRs to be cracked simultaneously. In the same way that increasing the length of the registers, this would increase the complexity to $O(n^{7+11+13+x})$ where `x` is the length of the additional register, or the added number of bits to an LFSR.

These methods would provide resilience to a divide-and-conquer attack by increasing the order of correlation immunity, causing the key-space to remain as the product of the key-space sizes of each register rather than their sum. However, other forms of attacks on stream ciphers can make them vulnerable if the cipher is poorly designed. Keeping in mind that the previously suggested combination function may not be robust these cases, we will now explore various other attack methodologies on stream ciphers, and methods to improve the robustness of the cipher to these attacks.

The simplest of these is a reused key attack, where the same key is used to encrypt two different plain-texts. Using the commutative property of XOR, the two cipher texts can be XOR'd to cancel out the use of the random stream, and reveal $A \oplus B$ (where A and B are plain-texts). If one of these message is known, retrieving the other's content is therefore trivial. This method was famously used to break the Lorenz cipher during WW2 [24], but is easily fixable by using a pre-agreed method to generate one-time keys from a master key and initialisation vector.

In addition, a bit-flipping attack is feasible which allows a man-in-the-middle attack or replay attack to vary the contents of a cipher-text without knowing it's key. This is feasible if the attacker knows part of the content of the cipher-text, as well as the location of the known content in the cipher-text. By XORing that section of the cipher text with his content, and XORing the result with the message which is being tampered. This results in a message which corresponds to what the sender would have sent using the information the attacker wished to insert into the message. This could be prevented by including a message authentication codes (MAC) such as the hash of the plain-text in the communication, which could then be used to verify whether the message was tampered without equivalent tampering of the MAC. This is detailed by Isaacs [12], with a case study of vulnerabilities in the Wired Equivalent Privacy (WEP) algorithm for wireless networks.

Courtois and Meier [9] details a newer form of attack on synchronous LFSR-based stream ciphers (”where each state is generated from the previous state independently of the plain-text”) using a set of over-defined algebraic equations to approximate the output function, which allow him to greatly reduce the key-space; the case study used to detail his methodology involves breaking a single 128 bit LFSR cipher in 2^{49} clock cycles. This vulnerability occurs due to a stream cipher's function using only a subset of state bits; this was novel for it's era as it showed a vulnerability in spite of satisfying all previously known security criteria. Courtois and Meier adds to this criterion by stating that ”there should be no non-trivial multivariate relations of low degree that relate key bits and one or many outputs bits of the cipher”.

Measuring the robustness of a boolean function to algebraic attack was defined by Armknecht et al. [4], whilst Carlet et al. [7] presented a methodology to generate boolean functions with maximum possible algebraic immunity, although the produced solutions had insufficient non-linearity, signifying a potential weakness to correlation attacks.

An additional attack was demonstrated by Ronjom and Helleseth [23], who formulates a methodology based on using a minimal amount of key-stream bits. As such, to maximise a resistance to such an attack, increasing the cyclic period of the LFSRs would be necessary. This could be achieved by changing the tap function to one with a longer period, or extending the bit length of the register. A new class of boolean functions which are resistant to fast correlation attacks, Helleseth-Ronjom, Berlekamp-Massey and fast algebraic attacks was also presented by Carlet and Feng [8].

Finally, Massey [17] provided an adaptation of the Berklekamp algorithm which allows the estimation of the length of an LFSR given Massey [17] given an upper-bounded number of bits of the output sequence. Given that this could reveal a hidden structure of the cipher, strengthening against this attack may be crucial. As previously stated, Carlet and Feng provides a boolean function generation which gives a good resistance to this attack in addition to others, whilst the more computationally expensive options of increasing the key-space through increased key-lengths and a larger number of LFSRs is also an alternative.

As such, suggested modifications to the cipher to improve it's robustness to common attacks would include lengthening the bit-width of each LFSR, adding LFSRs and utilising tap functions with maximal periods to increase the size of the key-space and resist Berlekamp-Massey attacks. The boolean combining function should be generated using Carlet and Fengś methodology [8], (or alternately Tang et al. [26]) although given the pace at which this area of research continues to surpass itself it would not be surprising to find a better function generation methodology in the near future.

2 Differential Cryptanalysis

i. Implementation and Challenge

The block cipher found in appendix E (written in Python), can be ran using the Python 2.7.6 compiler with the command `python block.py`. This will produce the challenge output of 45858 (1011001100100010 in binary).

ii. Code description

The block cipher is implemented using the `do_4_rounds` method, which controls the execution of substitutions, permutations or combinations of intermediary results with sub-keys dependent on the round. These sub-methods are implemented in `do_substitution(), permute(), and combine_key()` respectively.

iii. Cryptanalysis

Analysis

Following Hey's tutorial, a differential cryptanalysis attack was undertaken on the S-box. The XOR table of differences was generated using `calculate_xor_profiles()` in appendix H. This produced the difference distribution table found in appendix F. This indicated a number of high probability difference pairs:

$\triangle X$	$\triangle Y$	Difference
1	3	8
5	8	8
8	14	8
12	5	6
13	6	6
4	11	6
9	13	6

Using as many of these high-probability difference pairs as possible a set of high probability paths (which go through a minimal amount of S-boxes) can be used to find pairs of plain-text differences and cipher-text differences. These are illustrated in appendix G, which were picked through trial-and-error to find paths with maximal probability which only impacted a single set of 4 bits, so as to avoid iterating over more than 2^4 bits per attack; as such, 4 consecutive $O(2^4)$ attacks occurred to recover the 16 bit sub key. These paths could also have been obtained using Matsui's algorithm [18].
Using these paths, we were able to find a degree of certainty with which we can brute force the keys for 4-bit clusters to find the sub-key which maximally agrees with the provided plain-text/cipher-text pairs. This can be seen in the table below.

Sub-key bits	$\triangle P$	$\triangle U$	Path probability	Sub-key	Agreement with plain-text/cipher-text pairs
1..4	12	12288	0.0234	0xD	0.2083
5..8	13	768	0.0059	0xD	0.1181
9..11	2	48	0.0078	0xD	0.1627
12..16	17	1	0.0039	0x5	0.0855

As such, we were able to fully extract sub-key K_5 with a value of `0xDDD5`.

Implementation

As detailed above, the difference distribution table found in appendix F was generated using `calculate_xor_profiles()`, which simply iterated over every pair of possible 4 bit differences, and evaluated them against the configuration of the S-box. The list of differential pairs is then printed to console in descending probability of occurrence.

Using the manually-found paths through the substitution/permutation network, the sub-key brute-forcing was implemented in the `crack_section_subkey()` method which can be found in appendix H. It takes in $\triangle P$ and $\triangle U$ values, along with a mask and shift to indicate which group of 4 bits is attacked by the iteration. The data is then loaded from block.txt, and each possible 4-bit sub-key is iterated over each plain-text/cipher-text combination. Another plain-text/cipher-text combination is found by xor-ing the current plain-text with the requested difference, resulting in a second cipher-text. Both cipher-texts are then xor'd with the hypothetical sub-key (shifted to match the correct 4 bit cluster), and masked. They are then passed through the S-box in reverse, before being compared to the expected difference.

The list of potential sub-keys is then ordered by highest agreement, and the most promising key is returned. This attack is repeated 4 times to attack each 4 bit section of the sub-key, as visible in `main()`.

iv. Improvements

The resilience of the cipher could be improved in two approaches. Firstly, the addition of additional rounds would significantly decrease the likelihood of finding high-probability paths through the substitution-permutation network. This could cause brute-forcing algorithms to produce hypothetical sub-key sections which are proportionally less agreeable to plain-text/cipher-text pairs than erroneous keys, and therefore less liable to have a superior agreement over other sub-key sections. This could however be overcome with the use of additional computational time and a larger set of plain-text/cipher-text pairs to improve the significance of the obtained statistics.

S-boxes present the other significant area for resilience; an S-box's strength can be seen as a combination of it's non-linearity (avoid the possibility of finding linear combinations of inputs which agree with linear combinations of subsets of outputs) and minimal autocorrelation (correlations of inputs which satisfy a difference with outputs). Nyberg [21] details a mapping for S-boxes which are 'differentially uniform', which entails that "for every non-zero input difference and any output difference the number of possible inputs has a uniform upper bound". As such, mappings obeying this property would cause further difficulty in finding biased paths through the substitution-permutation network.

The search for methods to develop maximally non-linear S-boxes have led to the adoption of bent functions (which were discovered prior to their application to cryptography) to the definition of the strict avalanche criterion (SAC) by Forré [10]. This defines a criteria for maximally non-linear functions, whereby a change in a maximally high number of input bits causes each output bit to change with a 50% probability. A method to construct such functions is details in Forré details a method of constructing SAC-fulfilling functions which flattens the difference distribution table. These methods are further explored by Adams and Tavares [3], who developed a quicker method for generating S-boxes fulfilling this criteria.

3 Timing Analysis

i. Example of a real-world timing analysis attack

Brumley and Boneh [6] details an attack on the SSL algorithm which allows an attacker to reveal a server's private keys, thereby allowing the attacker to decode any subsequent messages encrypted using the victim's public keys. This attack assumes that the victim's server resides on the same machine as the attacker's and that the machine has a sufficiently fine clock granularity. The attack focuses on the Montgomery reduction used in RSA decryption, which can perform an additional subtraction to perform a reduction modulo, causing a time difference for different inputs. As such, a correlation between the probability of an extra reduction during an exponentiation $g^d * (mod(q))$ is proportional to the difference between g and q.

This relationship is exploited as follows: let $N = pq$ with $q < p$. Approximations of q are constructed and refined as the attack continues, one bit at a time (most significant first), followed by Coppersmith's algorithm to retrieve the full factorisation. The decryption of the top few bits will show two timing peaks: openSSL will always display a peak for q first, followed by one for p. After decrypting these top few bits, the difference in time taken to decrypt a number with these same top bits (and one with the next least significant bit set to 1) is measured, and a large difference in timing values will suggest that the next bit is 0, whereas a small difference will suggest the next bit is 1.

This is used in an SSL handshake, where the SSL server performs a decryption from a CLIENT-KEY-EXCHANGE (CKE) message from a client. If improperly formatted, a master key cannot be computed by both parties, and instead the server sends an ALERT message to the client indicating a handshake failure. Sending guesses instead of CKE messages causes the server to decrypt the message and check it's formatting (which is invalid on purpose) and send an Alert. The client measures the time from sending the fake CKE to receiving the Alert message, and repeats this for each bit. As $N = pq$, once q is known, the server's private key is exposed.

Brumley and Boneh provides a viable attack with severe practical limitations: firstly, it requires the attacker to operate from a machine with sufficiently consistent response times from the server so that variations in network response times do not obscure variations in decryption time. Secondly, OpenSSL already includes a built-in blinding (see following section for details) function which Brumley and Boneh has found to cause this attack to fail; the only drawbacks of this defence include the need for a good source of random keys with which to combine the data, and a small performance degradation. Finally, the RSA algorithm could be implemented using bucketing or balancing operations to force all decryptions to take the same amount of time as the maximum decryption time, thereby removing any timing variations from the algorithm.

As such, it is a viable method of attack on poorly setup servers, but is severely restricted in it's topographical applicability due to the need for consistent response times from a server, and is blinded by default in newer OpenSSL versions. As such it demonstrates the need to consider timing attacks in cryptography, but would not present a viable approach to attack many systems. This conclusion is confirmed by Wong et al. [28] who finds that Brumley and Bonehś publication led to the widespread usage of blinding techniques in SSL implementations, thereby preventing this form of timing attack.

ii. Countering timing attacks

Removing data or key dependent branching

The hamming weight of a key can be estimated from the execution time of a data-dependent encryption algorithm due to key-dependent branch execution. An example of this issue occurs in RC5 [11] due to variations in the computational time of rotations, or as previously discussed in the RSA algorithm's use of modular exponentiation. This can be mitigated by ensuring the algorithm does not conditionally branch, causing it to take a longer but consistent time to execute the encryption.

Noise

Timing attacks can also be mitigating by randomizing the execution time of the algorithm Kocher [15] details a method of decreasing the accuracy of timing measurements by adding random delays to the processing time, causing the need for a larger number of cipher-texts to attack the algorithm. This produces a result similar to bucketing, where increasing the time spent waiting reduces performance but increases security, and vice-versa.

Blinding

Blinding is the obfuscation of data to harden RSA (or similar) algorithms to various side-channel attacks. This involves the encoding of data before and after the execution of attackable sections of code using a bijective function, resulting in unusable information if the attacker uses a standard timing attack as the algorithm's state will be significantly less predictable. Kocher [15] has concluded this to be insufficient to fully mitigate timing attacks due to maliciously-designed modular exponentiation causing timing spikes corresponding to exponent bits, revealing the hamming weight of the exponent.

Balancing

Balancing is a timing attack mitigation implemented in a program by executing every operation on the complement of the data as well as the data. This diminishes the correlation to single data bits [14], resulting in a lack of correlation between hamming weight and timing data. This can be applied to a large number of operations, such as fixed offset shifts, bitwise operations and arithmetic operations.

Bucketing

Bucketing is a method of trading off computational performance while limiting the benefit of timing attack measurements. This was first achieved by Köpf and Dürmuth [16], and consists of a discretization of possible execution times. Bucketing is implemented by partitioning the system's execution times into intervals (called buckets) of variable length, where computations wait until the end of the current bucket's time before returning results of a computation. The effectiveness of this method can be varied to either minimise computational penalty (by reducing bucket size to minimise waiting) or force the attacker to increase the sampling rate used to conduct the timing attack. Köpf and Dürmuth provide an algorithm to find the optimal bucket size and frequency, allowing the program to maximise both it's performance and security. One should note that bucketing would not improve resilience to power-monitoring attacks, as a long period of low power usage would indicate a wait.

Cache timing attacks

Bernstein [5] details an attack based on cache hits for data-dependent lookups, for example in the AES algorithm. On the assumption that the attacker can monitor the time taken by the victim to encrypt each character in the input, a split in the location of a stored array (between cache and slower RAM) will reveal which section of the array was requested. In the case of AES, this could indicate which part of the S-box was requested, and therefore which value was imputed to the S-box. This attack vector can be mitigated by removing S-box lookups and instead implementing them using constant time bit operations [5]; this results in a timing attack immune software which is unfortunately much slower than using lookups. Bernstein also notes that maintaining an S-box in cache is not reliably feasible due to lines of the S-box being kicked out of cache (or between cache levels) by computation other than AES.

4 Open cryptography

Should the public be allowed to use strong cryptographic algorithms?

Snowden's disclosure on global surveillance in 2013 has brought to light the public right to cryptography as a mean of individual privacy. Global surveillance conducted by intelligence agencies and governments is currently an over-reach of individual rights to privacy as defined by article 12 of the United Nations declaration of Human Rights [27]. As 'strong' cryptography would be a means of preventing governmental use of electronic surveillance cryptography has begun to gain ground amongst individuals. This has led government agencies to be unable to obtain evidence through legal means to incriminate criminals [13]. As such, the usage of cryptography to the general public enables the conduct of criminal activity, which may be detrimental to the security of society as a whole. It is therefore simple to see that there is a discourse between an individual's wish to protect himself from his government and peers, or give his government the power to improve the safety of his environment.

There are currently two significant contrasting paradigms of the implications of cryptography as a means of information security against government surveillance. James B. Coney [13] (Director of the FBI) illustrates the need to sacrifice individual privacy to the benefit of government-provided security. Examples of this exist in most western countries, with the passing of law allowing the government to monitor individuals [1]. As this can easily be thwarted, an individual's right to use cryptography has become a technological barrier to law enforcement, and therefore the public's right to use strong cryptographic algorithms are preventing a government's law enforcement from operating effectively.

In contrast, the cipherpunk movement [22] sees universal surveillance as a facilitator for totalitarianism and an oppression of the freedom to express challenges to social norms, thereby thwarting social progress. Rogaway finally notes a belief that privacy can enhance security as often as it rubs against it; an example of this could be the use of cryptography to protect a company's data against non-governmental hackers.

Both paradigms have examples for and against them: the FBI's attempts to de-crypt an iPhone owned by a terrorist implicated in the San Bernardino shooting in California could have resulted in the inability to capture other terrorists due to the iPhone's encryption (although this was later hacked using a zero day), whereas the cyberpunk's appreciation for the public use of cryptography could be illustrated by the use of various classical ciphers during the American Civil War by the colonies against a much larger and oppressive government (the United Kingdom [2]); this illustrates the ability of cryptography to maintain a balance of power between governments and individuals, preventing the former from becoming oppressive or detrimental to the latter.

In regards to the protection of individuals from other individuals, it is undeniable that cryptography is a cornerstone to individual privacy by allowing us to communicate with confidentiality, integrity, authenticity, non-repudiation, anonymity and validation.

James B. Coney [13] (Director of the FBI) illustrated a conflict between individual privacy and security within our society, which is greatly contrasted by the cipher-punk movement's view that privacy may enhance security as often as it conflicts with it Rogaway [22].

James B. Coney [13] (Director of the FBI)'s speech assumes a conflict between individual privacy and collective security: the ability for criminals to use encryption requires the usage of equivalently effective security to ensure we can maintain the collective security of the world we live in. As such, James B. Coney argues that individual privacy is a social good, as it's sacrifice provides collective security (on the assumption of a benevolent government).

PLAN: Currently cryptography defends us from over-reaching governments and agencies, but also facilitates an individual's ability to conduct criminal activity.

As such, a conflict between individual privacy and security currently exists. We will then explore whether the usage of encryption may be in the best interest of individuals, and conclude with a conditional response to the stated question.

Two fold argument: - Is cryptography a viable form of privacy? Rogaway [22]'s work on the ethics of cryptography introduces a cipherpunk movement, who believe cryptography is a key tool for protecting individual autonomy threatened by power.

[22] "Cryptography does embed a tendency to empower ordinary people" as it is easily accessible to individuals with minimal resources. This allows individuals to protect themselves against much larger entities who's interests may clash with the individual's, such as governments or corporations.

[22] This can depend on the source of the cryptanalysis: Governmental agencies may fight privacy and right, whereas academic research tends to improve cryptographic software available to the public. Narayanan [20] cites a divergence in cryptographic research between crypto-for-security and crypto-for-privacy, the latter of which has comparatively fallen behind.

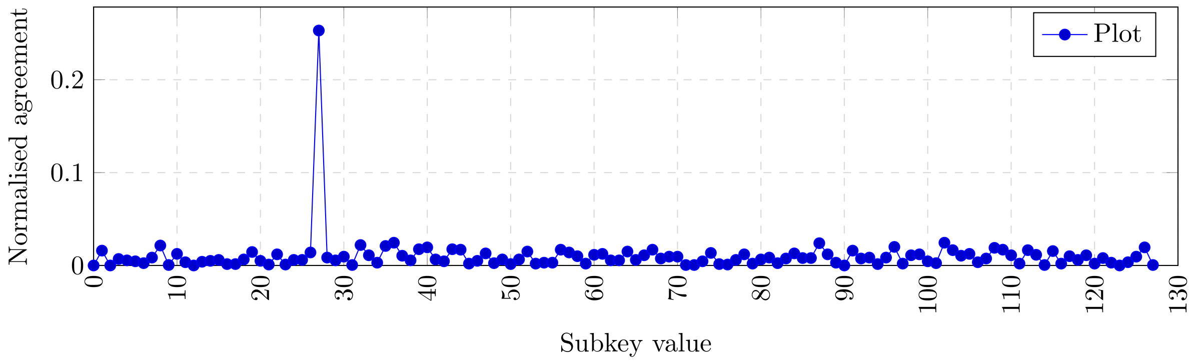
- Is it really feasible to believe the government can't crack all of our encryption anyways? eg: WW2 use of ULTRA without revealing knowledge counter-example: great pirate robert took ages to get caught If true then encryption would only work against other individuals or small organisations without means equivalent to a government's or large corporation.

- Should the public be allowed privacy at the cost of other social benefits? Rogaway [22] Cipherpunks: "[Will] state and corporate interests eviscerate liberty through electronic surveillance".

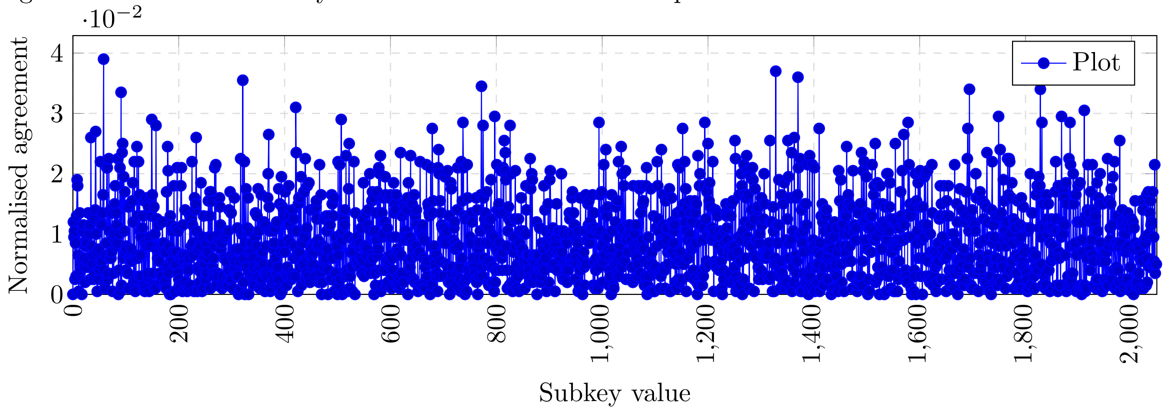
Rogaway [22]: A lack of individual privacy diminishes effective political dissent, causing social progress to be unlikely.

A LFSR agreements

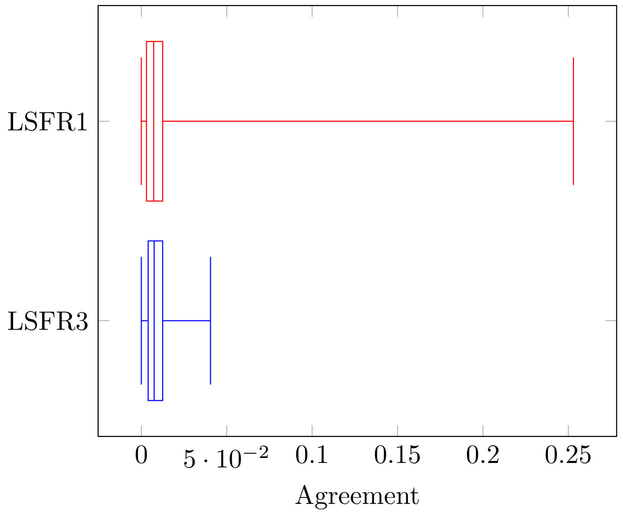
Agreements of 7-bit subkeys in LSFR1 with function output



Agreements of 11-bit subkeys in LSFR2 with function output



Agreements of 13-bit subkeys in LSFR3 with function output (demonstrates the lack of significant difference in agreement between the maximal outlying key and the median in LFSR3, in comparison to LFSR1's outlying key).



B Stream.py

```
class Lfsr:
    # Initialisation method for a LFSR
    def __init__(self, length, taps, initial_val):
        self.taps = []
        self.length = length
        for tap in taps:
            self.taps.append(tap)
        self.reg_val = initial_val

    # Calculates the tap value for the current LFSR's state
    def calc_tap(self):
        val = self.reg_val[self.taps[0]]
        for x in self.taps[1::]:
            val = (val ^ self.reg_val[x])
        return val

    # Calculate the tap value, shift the register and return the
    # value outputted by the register
    def shift(self):
        new_val = self.calc_tap()
        self.reg_val = [new_val] + self.reg_val
        return self.reg_val.pop()


# Implements the boolean function combining LFSR outputs
def combine_lfsr_outputs(out_1, out_2, out_3):
    val = str(out_1) + str(out_2) + str(out_3)
    val = int(val, 2)
    sub_arr = [1, 1, 0, 1, 0, 0, 1, 0]
    return str(sub_arr[val])


# Static instantiation of every LFSR
arr0 = [1, 1, 0, 0, 0, 0, 1]
l0 = Lfsr(7, [5, 6], arr0)

arr1 = [0, 1, 1, 1, 1, 0, 0, 1, 1, 1, 1]
l1 = Lfsr(11, [8, 10], arr1)

arr2 = [1, 1, 0, 0, 1, 0, 0, 0, 1, 0, 1, 0, 0]
l2 = Lfsr(13, [7, 10, 11, 12], arr2)


def main():
    # Shift and combine output of LFSR's for 25 bits
    out = ""
    for x in range(25):
        out_0 = l0.shift()
        out_1 = l1.shift()
        out_2 = l2.shift()
        out += combine_lfsr_outputs(out_0, out_1, out_2)
    print "\nChallenge␣output:␣" + str(out)


if __name__ == "__main__":
    # execute only if run as the entry point into the program
    main()
```

C Walsh transform of combining function

Output	L000	L001	L010	L011	L100	L101	L110	L111
000	1	1	1	1	1	1	1	1
001	1	−1	1	−1	1	−1	1	−1
010	1	1	−1	−1	1	1	−1	−1
011	1	−1	−1	1	1	−1	−1	1
100	1	1	1	1	−1	−1	−1	−1
101	1	−1	1	−1	−1	1	−1	1
110	1	1	−1	−1	−1	−1	1	1
111	1	−1	−1	1	−1	1	1	−1
Walsh-Hadamard values:	0	0	0	0	−4	4	−4	−4

D Streamattack.py

```python
class Lfsr:

    def __init__(self, length, taps, initial_val):
        self.taps = []
        self.length = length
        for tap in taps:
            self.taps.append(tap)
        self.reg_val = initial_val

    # Calculate the tap output for the current register value
    def calc_tap(self):
        val = self.reg_val[self.taps[0]]
        for x in self.taps[1::]:
            val = (val ^ self.reg_val[x])
        return val

    # Calculate the tap, push the value to the front of the register, and pop the last
        value
    def shift(self):
        new_val = self.calc_tap()
        self.reg_val = [new_val] + self.reg_val
        return self.reg_val.pop()

    def set_reg(self, new_reg_val):
        self.reg_val = new_reg_val

    def get_taps(self):
        return self.taps

    def get_length(self):
        return self.length


# Implements the given boolean function
def combine_lfsr_outputs(out_1, out_2, out_3):
    if out_1 == 0 and out_2 == 0 and out_3 == 0:
        return "1"
    elif out_1 == 0 and out_2 == 0 and out_3 == 1:
        return "1"
    elif out_1 == 0 and out_2 == 1 and out_3 == 0:
        return "0"
    elif out_1 == 0 and out_2 == 1 and out_3 == 1:
        return "1"
    elif out_1 == 1 and out_2 == 0 and out_3 == 0:
        return "0"
    elif out_1 == 1 and out_2 == 0 and out_3 == 1:
        return "0"
    elif out_1 == 1 and out_2 == 1 and out_3 == 0:
        return "1"
    elif out_1 == 1 and out_2 == 1 and out_3 == 1:
        return "0"
    else:
        return ""


# Execute the shift registers and return their combined output for the requested number
    of bits
def test_combined(lfsr1, lfsr2, lfsr3, num_vals):
    out = ""
    for x in range(num_vals):
        out_1 = lfsr1.shift()
        out_2 = lfsr2.shift()
        out_3 = lfsr3.shift()
        out += combine_lfsr_outputs(out_1, out_2, out_3)
    return out


# Convert a decimal value to a binary array of given length
def dec2bin(length, key_val):
    val = [int(c) for c in '{0:b}'.format(key_val)]
    while len(val) < length:
        val.insert(0, 0)
    return val
```

```
# Brute force a single LFSR by finding a subkey with maximal agreement
def crack(lfsr, data):
    length = lfsr.get_length()
    lfsr_max_key = (2 ** length)
    data_len = len(data)
    max_agreement_val = -1.0
    max_agreement_ind = -1
    for x in range(0, lfsr_max_key):
        # Reset LFSR with hypothetical subkey
        lfsr.reg_val = dec2bin(lfsr.length, x)
        output = []
        for y in range(0, data_len):
            output.append(lfsr.shift())
        # Calculate normalised agreement
        agreement = abs(0.5 - calc_agreement(data, output))
        if agreement > max_agreement_val:
            # Store best subkey so far
            max_agreement_val = agreement
            max_agreement_ind = x
    # Return best subkeys
    return max_agreement_ind, max_agreement_val


# Crack an LFSR using a known LFSR (lfsr2) and it's subkey (key_2)
def crack_with(lfsr, lfsr2, key_2, data):
    length = lfsr.get_length()
    lfsr_max_key = (2 ** length)
    data_len = len(data)
    max_agreement_val = -1.0
    max_agreement_ind = -1
    for x in range(1, lfsr_max_key):
        key_val = dec2bin(length, x)
        lfsr.reg_val = key_val[0:length]
        lfsr2.reg_val = key_2
        output = []
        for y in range(0, data_len):
            # XOR output of cracking LFSR with output from second LFSR (which has an
                independently checked key)
            output.append(lfsr.shift() ^ lfsr2.shift())
        agreement = abs(0.5 - calc_agreement(data, output))
        if agreement > max_agreement_val:
            # Store best subkey so far
            max_agreement_val = agreement
            max_agreement_ind = x
    # Return best subkeys
    return max_agreement_ind, max_agreement_val


# Calculate agreement of output from a single or multiple LFSR (xor'd together) with an
    expected output
def calc_agreement(data1, data2):
    length = len(data1)
    agreements = 0
    for x in range(0, length):
        if data1[x] == data2[x]:
            agreements += 1
    return float(agreements) / float(length)


# Convert a binary array into a decimal value
def bin2dec(arr):
    return int("".join([str(x) for x in arr]), 2)


arr = [0, 1, 0, 1, 1, 0, 0]
l = Lfsr(7, [5, 6], arr)
arr1 = [0, 1, 0, 0, 0, 1, 0, 1, 0, 1, 1]
l1 = Lfsr(11, [8, 10], arr1)
arr2 = [0, 0, 0, 1, 0, 0, 1, 1, 0, 1, 0, 0, 0]
l2 = Lfsr(13, [7, 10, 11, 12], arr2)

lsrs = [l, l1, l2]


def main():
    # Read ciphertext from file and convert to binary array
    with open('stream.txt', 'r') as myfile:
        data_raw = myfile.read().replace('\n', '').replace('␣', '')
        data = [int(c) for c in data_raw]

        key_l = crack(l, data)[0]
        print "L0:␣" + str(key_l)

        key_l1 = crack_with(l1, l, dec2bin(l.length, key_l), data)[0]
        print "L1:␣" + str(key_l1)

        print crack_with(l2, l, dec2bin(l.length, key_l), data)
        key_l2 = crack_with(l2, l, dec2bin(l.length, key_l), data)[0]
        print "L2:␣" + str(key_l2)


if __name__ == "__main__":
    # execute only if run as the entry point into the program
    main()
```

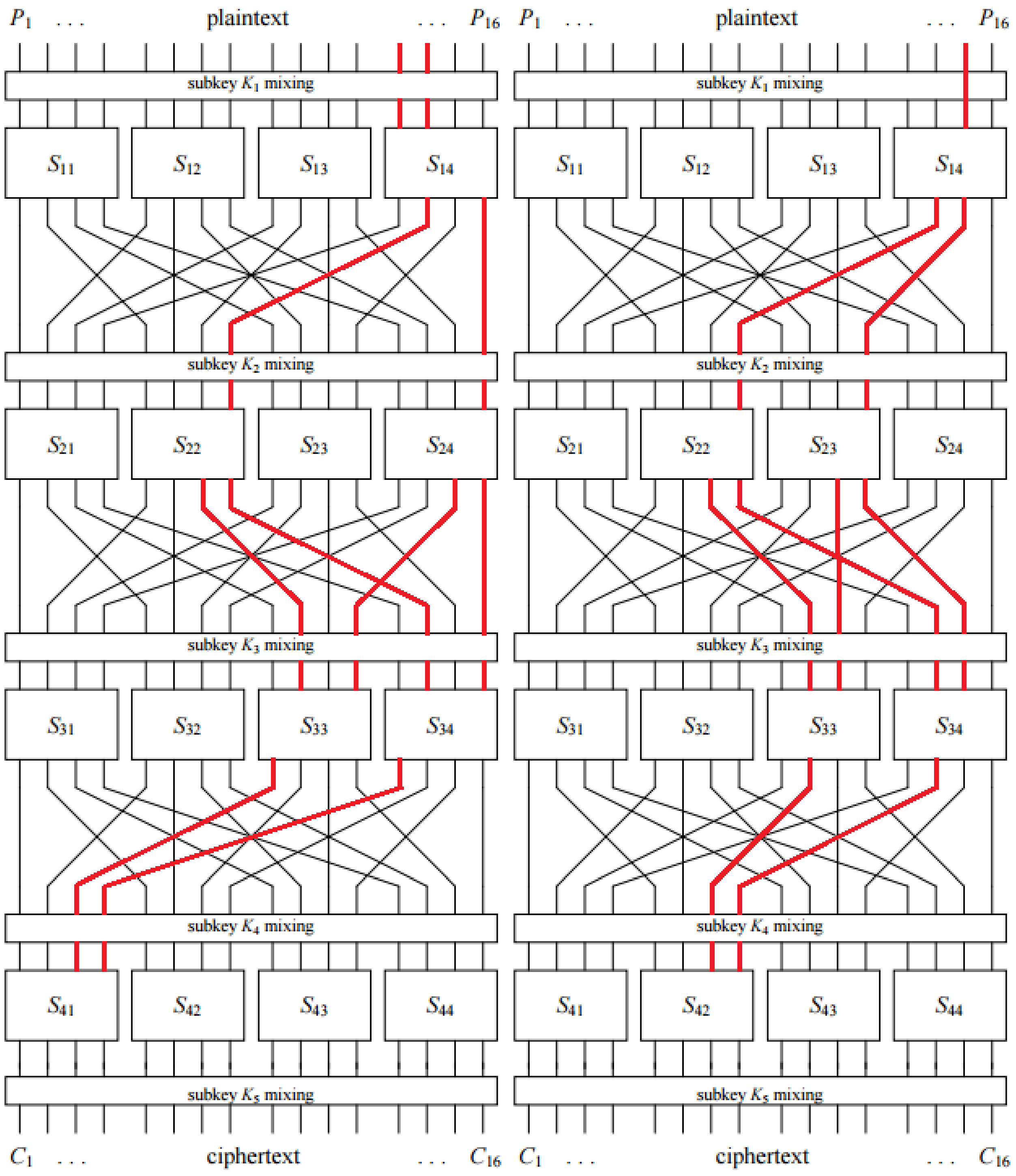
E Block.py

```
# Convert a decimal number into a chunked 16 bit binary number, outputted in an array of
      chunks of 4 bits
def dec2bin_chunks(key_val):
    val = "{0:b}".format(key_val)
    while len(val) < 16:
        val = "0" + val
    n = 4
    return [val[i:i + n] for i in range(0, len(val), n)]


# Convert a decimal to a binary string of minimum length
def dec2bin(val, length):
    out = "{0:0b}".format(val)
    while len(out) < length:
        out = "0" + out
    return out


# Convert a binary value to a decimal
def bin2dec(val):
    return int(val, 2)


# Execute an s_box substitution on a 4 bit input value
def substitute(in_val):
    arr = [0x4, 0x0, 0xC, 0x3, 0x8, 0xB, 0xA, 0x9, 0xD, 0xE, 0x2, 0x7, 0x6, 0x5, 0xF, 0
        x1]
    return arr[int(in_val)]


# Execute an s_box submission for all 16 bits
def do_substitution(val):
    bin_vals = dec2bin_chunks(val)
    sub_vals = [dec2bin(substitute(bin2dec(x)), 4) for x in bin_vals]
    sub_vals = "".join(sub_vals)
    return bin2dec(sub_vals)


# Executes a permutation of the 16 bit value according to the provided network structure
def permute(val):
    a = list(dec2bin(val, 16))
    permuted_val = [a[0], a[4], a[8], a[12], a[1], a[5], a[9], a[13], a[2], a[6], a[10],
         a[14], a[3], a[7], a[11],
                    a[15]]
    return bin2dec("".join(permuted_val))


# xor a value with a subkey
def apply_subkey(val, subkey):
    return val ^ subkey


# Execute a single round of subkey mixing, substitution and optionally permutation
def do_round(val, subkey, last):
    val = apply_subkey(val, subkey)
    val = do_substitution(val)
    if not last:
        val = permute(val)
    return val


# Executes all 4 rounds of the algorithm
def do_4_rounds(val, subkeys):
    for x in range(0, 4):
        print "␣"
        val = do_round(val, subkeys[x], x > 2)
        print "Round:␣{0}␣Val:␣{2}␣Subkey:␣{1}␣Last:␣{3}".format(x, subkeys[x], val, x >
             2)

    val = apply_subkey(val, subkeys[4])
    return val


def main():
    subkeys = [4132, 8165, 14287, 54321, 53124]
    print do_4_rounds(13571, subkeys)


if __name__ == "__main__":
    main()
```

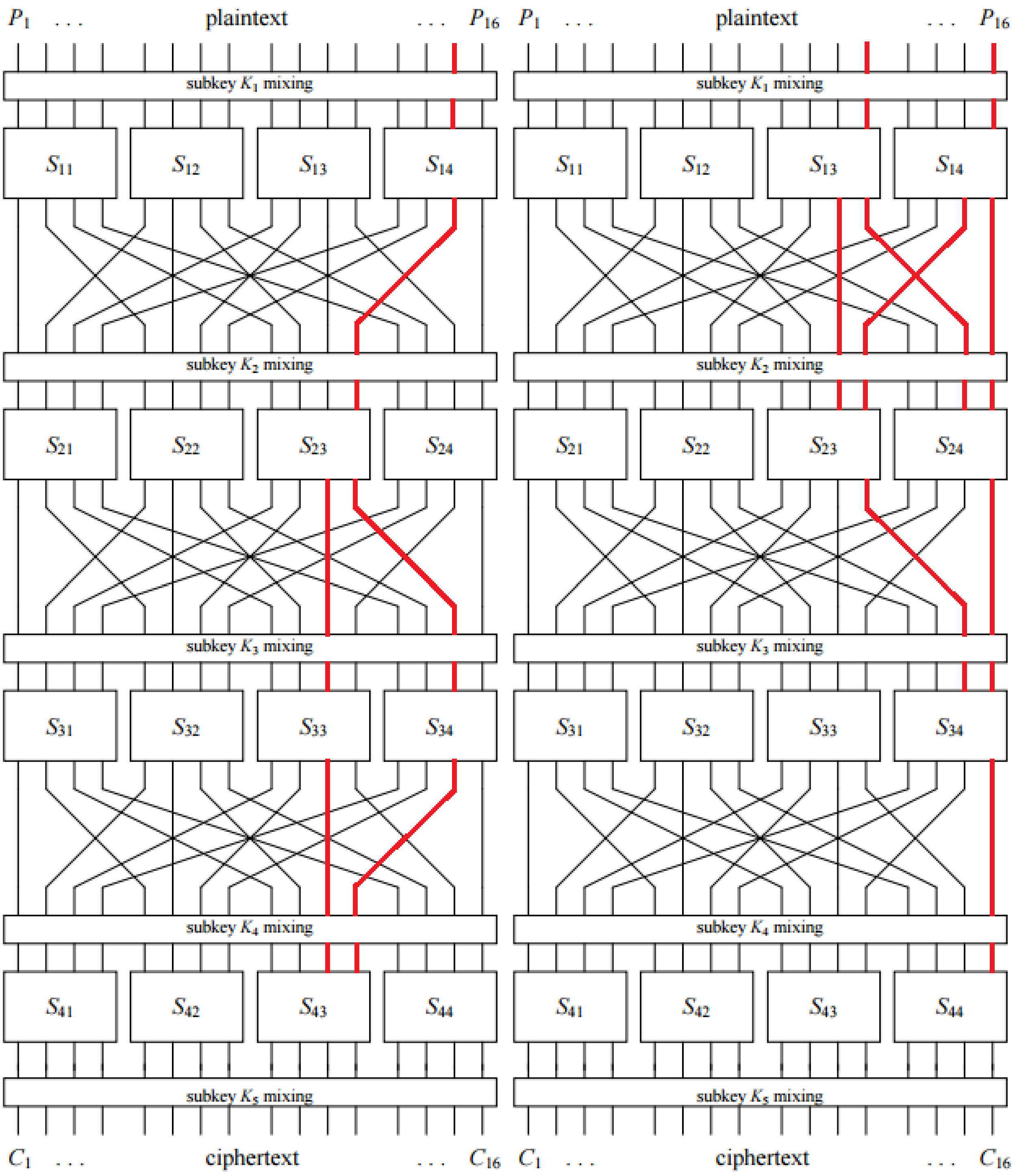
F Difference distribution table

Output Difference	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	8	2	2	0	0	0	0	0	0	0	0	2	2
2	0	0	4	2	2	0	0	0	2	4	0	0	0	0	0	2
3	0	4	0	0	0	0	0	4	0	0	4	0	4	0	0	0
4	0	0	0	0	0	0	4	0	0	0	2	6	2	2	0	0
5	0	0	0	2	0	2	0	0	8	2	0	0	0	0	0	2
6	0	0	4	0	4	0	0	0	2	2	0	0	0	0	2	2
7	0	4	0	0	0	0	0	4	0	0	2	2	2	2	0	0
8	0	0	0	0	2	2	0	0	2	2	0	0	0	0	8	0
9	0	2	0	0	0	0	2	0	0	0	2	4	0	6	0	0
10	0	2	0	0	0	0	2	4	0	0	2	0	4	2	0	0
11	0	0	4	2	2	0	0	0	0	2	0	0	0	0	2	4
12	0	0	4	2	0	6	0	0	2	0	0	0	0	0	2	0
13	0	2	0	0	0	0	6	0	0	0	0	2	2	4	0	0
14	0	2	0	0	0	0	2	4	0	0	4	2	2	0	0	0
15	0	0	0	0	4	4	0	0	0	4	0	0	0	0	0	4

G Paths through substitution-permutation network


P_1 . . . plaintext . . . P_{16}
subkey K_1 mixing
S_{11} S_{12} S_{13} S_{14}
subkey K_2 mixing
S_{21} S_{22} S_{23} S_{24}
subkey K_3 mixing
S_{31} S_{32} S_{33} S_{34}
subkey K_4 mixing
S_{41} S_{42} S_{43} S_{44}
subkey K_5 mixing
C_1 . . . ciphertext . . . C_{16}
P_1 . . . plaintext . . . P_{16}
subkey K_1 mixing
S_{11} S_{12} S_{13} S_{14}
subkey K_2 mixing
S_{21} S_{22} S_{23} S_{24}
subkey K_3 mixing
S_{31} S_{32} S_{33} S_{34}
subkey K_4 mixing
S_{41} S_{42} S_{43} S_{44}
subkey K_5 mixing
C_1 . . . ciphertext . . . C_{16}

The diagrams above demonstrate the path through the substitution-permutation network undertaken to crack the first 8 bits of the subkey, using the differential pairs `(12, 12288)` and `(13, 768)` respectively.



P_1 ... plaintext ... P_{16}
subkey K_1 mixing
S_{11} S_{12} S_{13} S_{14}
subkey K_2 mixing
S_{21} S_{22} S_{23} S_{24}
subkey K_3 mixing
S_{31} S_{32} S_{33} S_{34}
subkey K_4 mixing
S_{41} S_{42} S_{43} S_{44}
subkey K_5 mixing
C_1 ... ciphertext ... C_{16}

The diagrams above demonstrate the path through the substitution-permutation network undertaken to crack the last 8 bits of the subkey, using the differential pairs `(2, 48)` and `(17, 1)` respectively.

H Blockattack.py

```
# S-box
s_box = [0x4, 0x0, 0xC, 0x3, 0x8, 0xB, 0xA, 0x9, 0xD, 0xE, 0x2, 0x7, 0x6, 0x5, 0xF, 0x1]

# Generate the reverse s-box to avoid searching through entire array for every
    substitution
reverse_s_box = []
for x in range(16):
    reverse_s_box.append(s_box.index(x))


# Convert a decimal value to an array of 4-bit binary chunks
def dec2bin_chunks(key_val):
    val = "{0:b}".format(key_val)
    while len(val) < 16:
        val = "0" + val
    n = 4
    return [val[i:i + n] for i in range(0, len(val), n)]


# Convert a decimal value to a binary value padded to length
def dec2bin(val, length):
    out = "{0:0b}".format(val)
    while len(out) < length:
        out = "0" + out
    return out


# Convert a binary value to decimal
def bin2dec(val):
    return int(val, 2)


# Execute a reverse s_box substitution for a single 4 bit value
def reverse_substitute(in_val):
    return reverse_s_box[in_val]


# Execute a reverse s_box substitution for the full 16 bit value
def do_reverse_substitution(val):
    bin_vals = dec2bin_chunks(val)
    sub_vals = [dec2bin(reverse_substitute(bin2dec(x)), 4) for x in bin_vals]
    sub_vals = "".join(sub_vals)
    return bin2dec(sub_vals)


# Method used to calculate XOR profile for all differential pairs
def calculate_xor_profiles():
    maxs = 0
    differential_pairs = []
    for x in range(0, 15):
        for y in range(0, 15):
            s = 0
            for z in range(0, 15):
                if (s_box[z] ^ s_box[z ^ x]) == y:
                    s += 1
            if not (x == 0 and y == 0):
                if maxs < s:
                    maxs = s
            differential_pairs.append((x, y, s))
    differential_pairs.sort(key=lambda x: x[2], reverse=True)
    print differential_pairs


# Method to find most agreeable subkey for a given s-box, plaintext difference and
    ciphertext difference pair
def crack_section_subkey(plain_diff, cipher_diff, shift):
    mask = 0xF << shift
    vals = {}
    subkeys = []
    with open('block.txt', 'r') as myfile:
        for line in myfile:
            (key, val) = line.split()
            vals[int(key)] = int(val)
        for subkey in range(0, 15):
            prob = 0
            for plaintext in vals:
                cipher1 = vals[plaintext]
                cipher2 = vals[plaintext ^ plain_diff]
                # Shift subkey to correct index, xor to cipher text and mask to find
                    agreement over only those 4 bits
                cipher1 = (cipher1 ^ (subkey << shift)) & mask
                cipher2 = (cipher2 ^ (subkey << shift)) & mask

                cipher1 = do_reverse_substitution(cipher1)
                cipher2 = do_reverse_substitution(cipher2)
                if (cipher1 ^ cipher2) == (cipher_diff & mask):
                    prob += 1
            prob = float(prob) / len(vals)
            subkeys.append((subkey, prob))
    subkeys.sort(key=lambda x: x[1], reverse=True)
    return subkeys[0]


def main():
    crack = crack_section_subkey(12, 12288, 12)
    print "Bits␣1..4:␣value:␣␣␣␣{0},␣␣␣␣prob:␣␣{1}".format(crack[0], crack[1])
    crack = crack_section_subkey(13, 768, 8)
    print "Bits␣5..8:␣value:␣␣␣␣{0},␣␣␣␣prob:␣␣{1}".format(crack[0], crack[1])
    crack = crack_section_subkey(2, 48, 4)
    print "Bits␣9..12:␣value:␣␣␣{0},␣␣␣␣prob:␣␣{1}".format(crack[0], crack[1])
    crack = crack_section_subkey(17, 1, 0)
    print "Bits␣13..16:␣value:␣␣{0},␣␣␣␣prob:␣␣{1}".format(crack[0], crack[1])


if __name__ == "__main__":
    main()
```

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