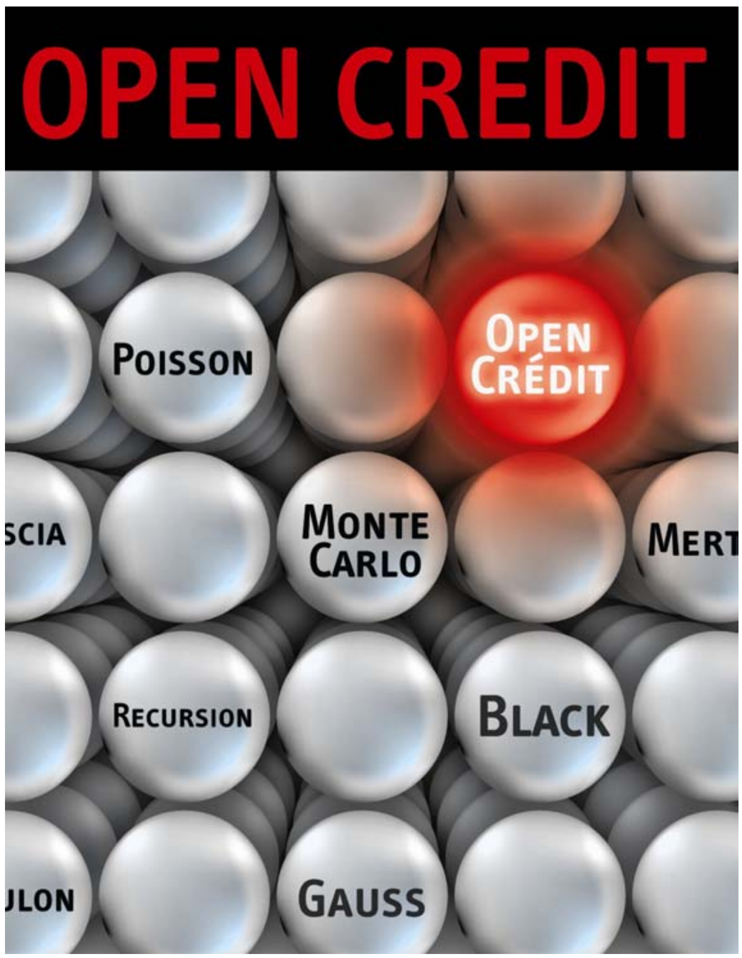
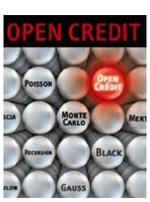
SOCIETE GENERALE
Corporate & Investment Banking

Technical Reference Guide

Release 2007/11/01





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2 Forewords

Transparency and Performance

The objective of OpenCredit is transparency both in terms of model mathematical equations and program implementation. For this purpose, models were implemented in Visual Basic for Application so that investors can actually see how the model work, can audit it if necessary and can replicate it into their own system.
The obvious drawback is low computational speed.

The document is not with the aim to be as rigorous as possible but instead to make mathematical expression easy to intuitively understand.

Drawbacks of current version

- The current implementation of base correlation interpolation discussed in section **Error! Reference source not found.** could be refined.
- Non equal weights and expected recovery rates for CDO squared: For sake of computation time, CDO square model can only take equally weighted inner portfolios of issuers with uniform expected recovery. Authorizing non equal weights and disparate expected recovery levels is feasible by doing recursion with a thin loss granularity, but this will multiply computation times already relatively long in Visual Basic compared to other programming languages.
- American Swap for CDO2: Only European CDO2s are implemented in the current version. This is not much an issue as the difference between American and European CDO2 is close to the same difference on regular CDO.

3 Definitions

NPV, PV (Net Present Value, Present Value): Value as of today, or as of the valuation date of future cash flows.

ZC (Zero Coupon): Also called Discount Factors. For a given maturity, it is the present value of 1 unit of currency to be paid at maturity.

Risk-free ZC (Risk-free Zero Coupon): Value of a ZC if payment at maturity is not at risk.

Risky ZC (Risky Zero Coupon): Value of a ZC if payment at maturity is at risk on the default of a company or on excess loss on credit portfolio.

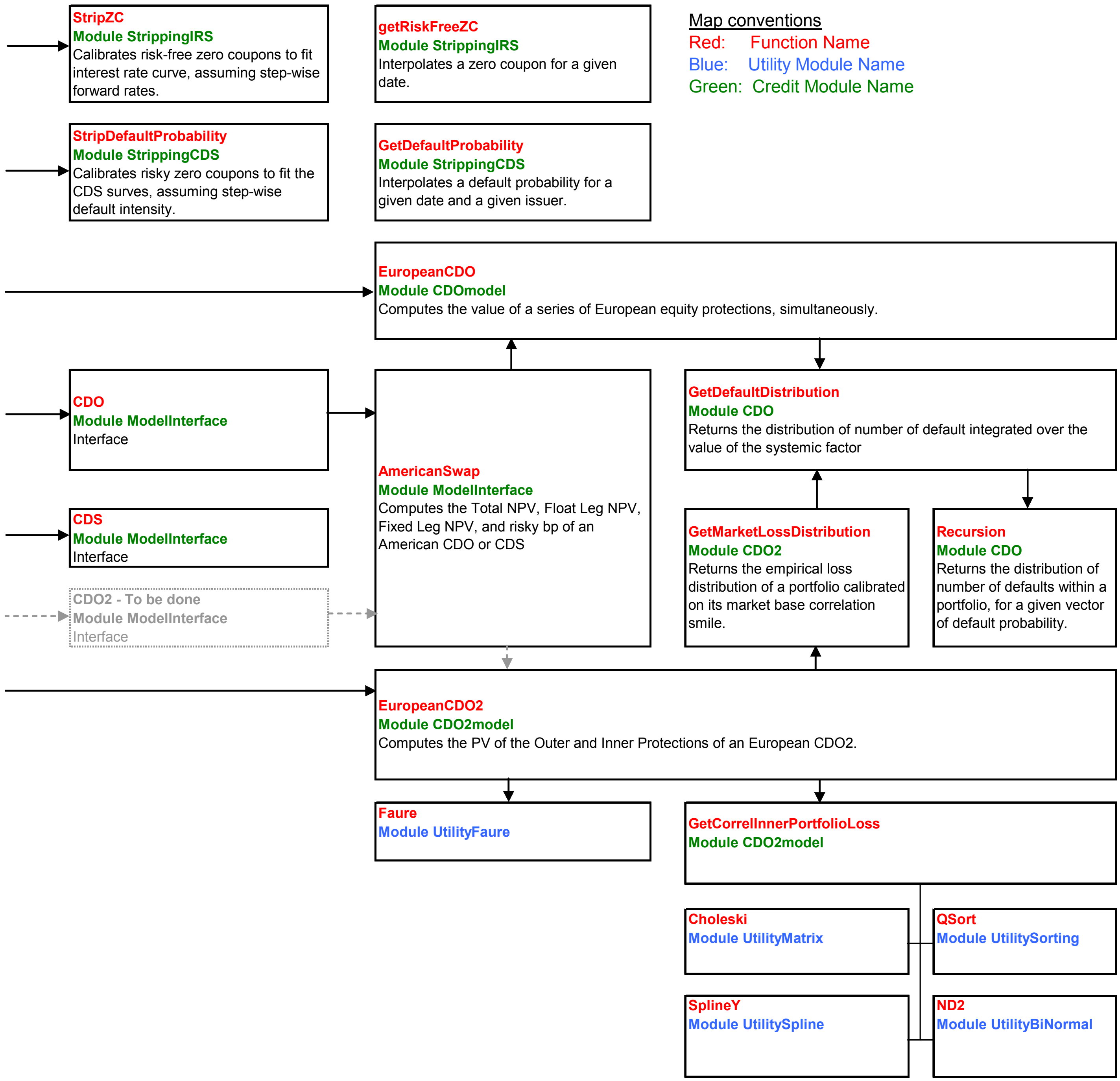
Float Leg: Leg of a CDS or a CDO that pays a floating amount depending on recovery rate (or delivers an obligation against payment of parity) upon the occurrence of a credit event.

Fixed Leg: Leg of a CDS or a CDO that pays a fixed spread until the occurrence of default.

European CDS/CDO (European Credit Default Swap): Credit derivatives where float leg payment is made at maturity as opposed to immediate payment upon credit event.

American CDS/CDO (American Credit Default Swap): Credit derivatives where float leg payment is made immediately upon credit event.

4 Function Library Map



5 Calibration of Risky Zero Coupon

Most credit models take the Risky Zero Coupon Curve as an input.
This section describes how risky ZCs are stripped from CDS curves.
Throughout this section, we consider CDS with a notional amount of 1 unit of currency.

VB Function: StripDefaultProbability
Excel Sheet: CDS Strip

Assumptions:

- Deterministic recovery rate equal to expected recovery rate.
- No account of holidays and week-ends.
- No correlation between interest rate and credit.

5.1 Float Leg

We call float leg, the protection leg of a CDS that pays a floating amount depending on the recovery upon the occurrence of a credit event, as opposed to the fixed leg which pays fixed credit spread until default.

The net present value (NPV or PV) of the floating leg of a standard CDS (also called American CDS) of maturity T is given by:

$$C_{Am}^{float}(T) = C_{Euro}^{float}(T) + \int_0^T f(0,t) C_{Euro}^{float}(t) dt$$

where:

- $C_{Euro}^{float}(t)$ is the PV of the European float leg of maturity t, that pays 1 minus the recovery rate at t if a credit event occurred before t.
- $f(0,t)$ is the instantaneous forward rate at time t obtained from risk-free zero coupons $B(0,t)$ by:

$$f(0,t)dt = 1 - \frac{B(0,t+dt)}{B(0,t)}$$

Assuming that T is an integer multiple N of months, we integrate by monthly increments:

$$C_{Am}^{float}(T) = C_{Euro}^{float}(T) + \sum_{i=1}^{N} C_{Euro}^{float}(t_i)\left(1 - \frac{B(0,t_i)}{B(0,t_{i-1})}\right)$$

where $t_0 = 0$, i.e. today.

The last term is simply the difference between the float leg of the American CDS and the float leg of the European CDS. With the notation $\Delta C_{Am-Euro}^{float}(T) = \sum_{i=1}^{N} C_{Euro}^{float}(t_i)\left(1 - \frac{B(0,t_i)}{B(0,t_{i-1})}\right)$, we have:

$$C_{Am}^{float}(T) = C_{Euro}^{float}(T) + \Delta C_{Am-Euro}^{float}(T)$$

Assuming no correlation between default and interest rate, the European float leg is given by:

$$\mathbf{C_{Euro}^{float}(T) = Pr\,ob(\tau_{default} \leq T) \times (1-R) \times B(0,T)}$$

where
$B_{risky}(0,T)$ is the risky ZC that pays 1 at time T only if no default occurs before T

R is the recovery rate.

This gives: $C_{Euro}^{float}(T) = (1-R) \times \left(B(0,T) - B_{risky}(0,T)\right)$

For sake of simplicity, we omit the recovery term in the following.

We finally obtain 2 equations implemented in Visual Basic:

$$C_{Am}^{float}(T) = B(0,T) - B_{risky}(0,T) + \Delta C_{Am-Euro}^{float}(T)$$

For computation efficiency, $\Delta C_{Am-Euro}^{float}(T)$ is computed by recursion:

$$\Delta C_{Am-Euro}^{float}(T) = \Delta C_{Am-Euro}^{float}(T - 3months) + \sum_{i=N-2}^{N} \left(B(0,t_i) - B_{risky}(0,t_i)\right)\left(1 - \frac{B(0,t_i)}{B(0,t_{i-1})}\right)$$

5.2 Fixed Leg

As long as no default occurs, spread is paid quarterly on a Act/360 basis
If and when a default occurs, accrued spread is paid at time of default in exchange for the receipt of the float leg amount.

With respect to the k^{th} spread due for the 3 months period T_{k-1} to T_k, payments are:

- Payment at T_k, if there is no default before T_k
- Payment at τ_{def} of the accrued spread if $T_{k-1} < \tau_{def} \leq T_k$

Let's note $S(T_k)$ the PV of the coupon due for that period for a normalized spread of 100%.

$$S(T_k) = \left(1 - \Pr ob(\tau_{def} < T_k)\right)(T_k - T_{k-1}) \times B(0,T_k) + \int_{t=T_{k-1}}^{T_k} (t - T_{k-1}) \times B(0,t) \times d\Pr ob(\tau_{def} < t)$$ where

$\dfrac{d\Pr ob(\tau_{def} < t)}{dt}$ is the default probability density.

$$S(T_k) = (T_k - T_{k-1}) \times B_{risky}(0,T_k) + \int_{t=T_{k-1}}^{T_k} (t - T_{k-1}) B(0,t) f(t) dt$$

We integrate by monthly increment.

At last the total PV of the fixed leg is the sum of the PV of each future spread payments:

$$C^{fixed}(T_k) = \sum_{i=1}^{k} S(T_k)$$

5.3 PV of Am CDS

At last the total PV of the American CDS of maturity T_k is given by

$$PV(T_k) = (1-R) C_{Am}^{float}(T_k) - spread \times C^{fixed}(T_k)$$

5.4 Bootstrapping

The PV of American CDS of maturity T_k is function of all risky ZCs up to T_k.

Assuming default is driven by a Poisson event with a default intensity $\lambda(t)$, the non default probability before time T is given by:

$$1-\Pr ob(\tau_{default} \leq T) = \exp\left(-\int_0^T \lambda(t)dt\right)$$

curve Point Step-Up Default Intensity method

The CDS curve is given only for P points at maturities $T_{k=1\ldots P}$, and we assume $\lambda(t)$ constant between each point.

The stripping function calibrates the risky ZC so that the PV of American CDS are null. (The PV of a CDS traded at 50 bps if the market is still at 50bps is 0)

It proceeds iteratively by starting with ZC=1 at t=0. Assuming that all the risky ZC for up to T_{k-1} are known.

$\lambda(t)$ being constant between 2 points of the curve, ZCs between T_{k-1} and T_k are given by:

$$B_{risky}(0,T_{k-1}+j/12) = B_{risky}(0,T_{k-1}) + \left(\frac{B_{risky}(0,T_k)}{B_{risky}(0,T_{k-1})}\right)^{j/12 \big/ (T_k - T_{k-1})}$$

As such, once all risky ZCs up to T_{k-1} are known, the PV of the CDS of maturity T_k is only a function of $B_{risky}(0,T_k)$.

We solve for $B_{risky}(0,T_k)$ such that $PV(T_k)=0$ with a standard Newton algorithm which converges extremely rapidly, starting by a good guess value.

Quaterly Step-Up Default Intensity method

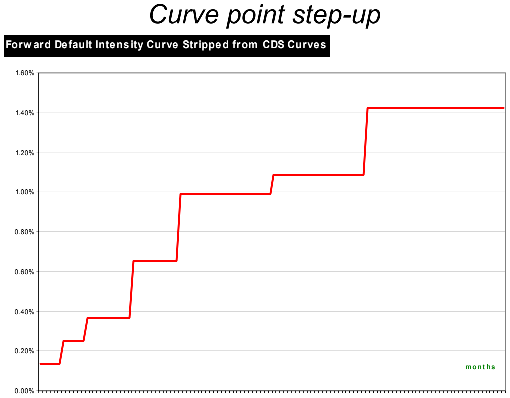
The methodology remains nearly identical except that we assume $\lambda(t)$ constant during a maximum period of 3month.

Between two points for which CDS curve is given, we assume a quarterly step-up default intensity with a constant drift between T_{k-1} and T_k.

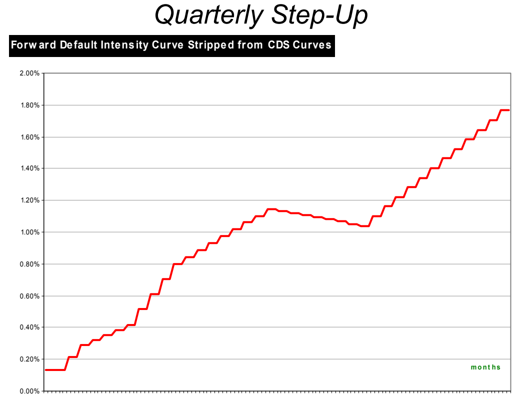
This implementation results in smoother CDS curves and CDS interpolation.

Illustration: comparison of default intensity depending on step-up period

Curve point step-up



Quarterly Step-Up



CDS and CDO valuations and durations will vary depending on method used.

5.5 Quanto CDS

A formal derivation is beyond the scope this document, only the formula used to calculate quanto CDS is described below. Quanto CDS are priced through a quanto default probability which can be expressed as

$$QDP(T) = N\left(N^{-1} \circ DP(T) - \rho\sigma\sqrt{T}\right)$$

where

DP is the default probability
σ Is the FX volatility
ρ Is the correlation between FX and time to default
QDP is the quanto default probability

6 CDO

6.1 Loss Distribution at time T under the Gaussian Copula Model

VB Functions: Recursion, GetDefaultDistribution

OpenCredit computes prices for non equally weighted portfolios and/or non uniform recovery rates per issuer.

For the sake of simplicity, the following equations are given assuming an equally weighted portfolio with uniform recovery rate. Doing so, one can calculate the probability of having k defaults.

In case of a non equally weighted portfolio and/or different recovery rates, the potential loss per issuer (i.e. the product of the issuer notional and 1 minus the recovery rate) is approximated with a multiple of a given loss unit. Once the loss unit is defined, the model calculates the probability of each loss unit multiple. The choice of the loss unit will determine the price precision (each potential loss being approximated by a multiple of that loss unit) and the calculation time (a lower loss unit would lead to an increased number of probabilities being calculated).

OpenCredit embeds an algorithm that computes a loss unit being the greatest common factor of all potential losses.

- Example of portfolio having equally weighted names (notional 1 per name) but with different recovery rates (20% and 40%). In that case, the potential loss is either 0.8 or 0.6 and the loss unit is 0.2.

The user may decide to define his own loss unit amount (for instance, for a portfolio having recovery rates of 30% for most of the issuers and 99.90% recovery rates for the rest, which implies a loss unit of 0.001 and dramatically increases the number of loss unit multiples to be calculated).

Consider a portfolio of equally weighted exposure on N issuers and assume that expected recovery rates are uniform.
Let n be the notional amount of exposure per issuer.

The portfolio credit loss $L(T)$ at time T is given by:

$L(T) = (1-R)n\sum_{k=1}^{N}\delta_k$ where δ_k =1 if issuer k defaulted, 0 otherwise

Without loss of generality we assume R=0 and n=1.

After stripping CDS Curves, we know the default probability of each issuer. In order to know the distribution of the loss $L(T)$, we need to know the joint-distribution of defaults.

Given the probability of default of issuer k, $p_k(T)$, default happens if

$N(g_k) < p_k(T)$ or $g_k < N^{-1}\left(p_k(T)\right)$

where g_k is a Gaussian variable of mean 0 and standard deviation 1, and N the cumulative normal distribution.

The joint distribution of defaults is driven by a joint distribution of the g_k, $f(g_1,...,g_N)$, where f is called a Copula function.

The widely used Gaussian Copula model for CDO simply relies on the assumption:

- the simplest choice for the Copula f is the multivariate normal distribution:
  $f(g_1,...,g_N) = N(g_1,...,g_N,[\rho])$, where $[\rho]$ is a correlation matrix.

One could use a full correlation matrix for the g_k . However this is not a practical choice as:

- The concept of correlation of 2 Gaussian variables that drive default is not intuitive and the correlation can not be directly measured from historical data. So it would presumptuous to believe it is possible to fill the matrix precisely.
- Using any type of correlation matrix means that pricing must be done by very time consuming Monte Carlo simulations and it is not practical for daily operations.
- It is possible to demonstrate that simplified forms of matrix are good enough to obtain a good representation of the distribution of $L(T)$

Instead we assume that the market is globally driven by a systemic factor f .

$$g_k = \beta_k f + \sqrt{1-\beta_k^2}\,\varepsilon_k$$

where f is Gaussian, β_k is the *classical* beta of g_k with respect to f : the correlation between g_k and f equals β_k .

ε_k are independent Gaussian variables AKA idiosyncratic shocks.

Under this assumption the correlation matrix between the g_k has a regular form:

$$[\rho] = \begin{pmatrix} 1 & \beta_1\beta_2 & . & . & \beta_1\beta_N \\ \beta_1\beta_2 & 1 & . & . & . \\ . & . & 1 & \beta i\beta j & . \\ . & . & . & 1 & \beta_{N-1}\beta_N \\ \beta_1\beta_N & . & . & \beta_{N-1}\beta_N & 1 \end{pmatrix}$$

f given, issuer k defaults if $\beta_k f + \sqrt{1-\beta_k^2}\,\varepsilon_k < N^{-1}\left(p_k(T)\right)$ or $\varepsilon_k < \dfrac{N^{-1}\left(p_k(T)\right) - \beta_k f}{\sqrt{1-\beta_k^2}}$

Therefore, the default probability of k, knowing the value of f is:

$$p_k^f \equiv P(\tau_{def}^k < T\big|_f) = N\left(\frac{N^{-1}\left(p_k(T)\right) - \beta_k f}{\sqrt{1-\beta_k^2}}\right)$$

Besides, f being given, the default of issuers are independent.

The **Recursion function** computes the distribution of losses knowing f recursively, adding the exposure on each issuer 1 by 1.

It starts with an empty portfolio such that the probability of no loss is 100% and the probability of loss from 1 to N is null.

$$\Pr ob^0\left(L(T)=0\right)\Big|_f = 100\%$$

$$\Pr ob^0\left(L(T)=i\right)\Big|_{f,\, i=1\ldots N} = 0\%$$

Adding the first issuer, probabilities become:

$$\Pr ob^1\left(L(T)=0\right)\Big|_f = 100\% \times (1-p_k^f)$$

$$\Pr ob^1\left(L(T)=1\right)\Big|_f = (1-p_k^f)$$

$$\Pr ob^1\left(L(T)=i\right)\Big|_{f,\, i=2\ldots N} = 0\%$$

Assuming that the vector of probability $\Pr ob^{k-1}\left(L(T)=i\right)\Big|_{f,\, i=0\ldots N}$ after the addition of issuer k-1 is known, we add issuer k and obtain the new probability vector:

For $i \neq 0$: $\Pr ob^k\left(L(T)=i\right)\Big|_f = (1-p_k^f)\Pr ob^{k-1}\left(L(T)=i\right)\Big|_f + p_k^f \Pr ob^{k-1}\left(L(T)=i-1\right)\Big|_f$

$$\Pr ob^k\left(L(T)=0\right)\Big|_f = (1-p_k^f)\Pr ob^{k-1}\left(L(T)=0\right)\Big|_f$$

Once the distribution of the loss is known for a given f, function **GetDefaultDistribution** integrates over f yielding the unconditional loss distribution:

$$\Pr ob\left(L(T)=i\right) = \int_{f=-\infty}^{+\infty} \Pr ob\left(L(T)=i\right)\Big|_f \varphi(f)\,df$$

where $\varphi(f) = \dfrac{e^{-f^2/2}}{\sqrt{2\pi}}$ is the normal density.

Integration is done numerically by well known Guassian Quadrature (refer for instance to http://www.efunda.com/math/num_integration/num_int_gauss.cfm)

$$\Pr ob\left(L(T)=i\right) = \sum_k \Pr ob^N\left(L(T)=i\right)\Big|_{f_k} \varphi(f_k)\omega_k$$

where f_k and ω_k are Gauss Hermite abscissas and weights

For information only: Relation between correlation of Gaussian variables and default event correlation at T under the Gaussian Copula Model

We compute the correlation of $\delta_j(T)$ and $\delta_k(T)$ under the Gaussian Copula Model.

By definition: $\rho_{event}(T) = E(\delta_j\delta_k)/\sqrt{E(\delta_j^{\,2})E(\delta_k^{\,2})}$

Therefore $\rho_{event}(T) = N_2\left(N^{-1}(p_j(T)), N^{-1}(p_k(T)), \beta_j\beta_k\right) \Big/ \sqrt{p_j(T)p_k(T)}$

Where N_2 is the bivariate normal distribution.
Although this is not used in pricing model, this relationship is useful when looking at event correlation levels used by rating agencies.

For information only: There are several concepts of correlation for credits.

- Correlation of default events over a time period T: This is the correlation of $\delta_j(T)$ and $\delta_k(T)$, the default indicators (1 or 0) of issuers j and k from 0 to T. Note that talking about a correlation of occurrence of events is meaningless, one must always speak about occurrence over a certain timeframe.
- Correlation of default times: τ_j and τ_k being the default time of issuers j and k, this is the correlation of τ_j and τ_k.
  One can show that the correlation of default-time under the Gaussian model is very close to the correlation of the Gaussian variables.
- Correlation of credit spreads: This is a notion that is very far from other correlations. Spreads of 2 issuers can be very correlated with little default correlation.
- Correlation of asset under the Merton model: Under the Merton model, an issuer defaults when the value of its assets reaches a certain level with respect to its total or net debt. There are several ways to implement such model. When the asset value follows a Black Scholes diffusion, it appears that correlation of asset and correlation of Gaussian Copula are almost equal. When the asset value is derived from the stock value which follows a Black Scholes, then correlation of Gaussian Copula are close to correlation of stock but not equal as assumed by some market participants.
  In any case one can only rely on stock correlation if one believes that the Merton model reflects the reality, which is a reasonable assumption for very high yield issuers but less convincing for high grades.
- Correlation of stocks.
- Correlation of Gaussian Copula: As described in section 6.1.

6.2 European CDO

The payoff of an European CDO is: $\left(L(T)n(1-R)-K_1\right)^+ - \left(L(T)n(1-R)-K_2\right)^+$

where $(x)^+ = \dfrac{x+|x|}{2}$ (equal to x if x>0, equal to 0 if x<0)

This is the payoff of a call strike K_1 on the portfolio loss, less a call strike K_2.

The PV of the call strike K_1 is given

by $CDO_{Euro}^{float}(T,K_1) = B(0,T)\sum_{i=0}^{N}\Pr ob\left(L(T)=i\right)\left(i\times n(1-R)-K_1\right)^+$

Implementation is straightforward in function **Function EuropeanCDO.**

6.3 Efficient computation of Greeks

We would like to know by how much the price of a CDO changes if the spread of one issuer changes or if its beta against the systemic factor changes.

One can proceed by simply shifting the spread curve of that issuer, re-compute the PV of the CDO and observe the difference from the original PV.
This would however require N times as much computation time as for the price.

It is more efficient to compute the sensitivity of the loss distribution against the default probability of each issuer as follows.

We have described in section 6.1 how to build the loss distribution for a portfolio of N entities, by recursion, adding one entity at a time, the value of the systemic factor being given.
Note that the order by which entities are added to the portfolio has obviously no impact on the final loss distribution of the portfolio.
We recall the process here.

We start with 0 entity, add them one by one and obtain the final loss distribution as:

For any i, $\Pr ob^{N}\left(L(T)=i\right)\Big|_{f} = (1-p_N^f)\Pr ob^{N-1}\left(L(T)=i\right)\Big|_{f} + p_N^f \Pr ob^{N-1}\left(L(T)=i-1\right)\Big|_{f}$

Using the notation: $\Pr ob^{N-1}\left(L(T)=-1\right)=0$

To simplify the equations, let's note:

$$P(N,i,f)=\Pr ob^{N}\left(L(T)=i\right)\Big|_{f}$$

We derive the probability against p_N^f as $\dfrac{\partial P(N,i,f)}{\partial p_N^f} = P(N-1,i-1,f)-P(N-1,i,f)$

This simply is the difference of the probability of i-1 defaults and i defaults for a portfolio where issuer N was excluded.

We can remove that entity (here again the order of entities does not matter) from the portfolio by the reverse process of the recursion:

$$P(N-1,i,f)=\left(P(N,i,f)-p_N^f P(N-1,i-1,f)\right)\Big/(1-p_N^f)$$

Doing so, we are able to compute the sensitivity of the loss distribution against the probability of default of each issuer, the systemic factor f being known.

<u>Sensitivity against a parallel shift of the spread curve</u>

Function StripDefaultProbability described in section 5 computes the sensitivity of the default probability p_k of each issuer k, against a parallel shift of its spread curve.

For the new value of p_k, one can compute the new value of $p_k^{f_u}$ for each systemic factor f_u used in the Gaussian quadrature integration. And such the change of the loss distribution as:

$$\Delta P(N,i)\Big|_{p_k+\Delta p_k} = \sum_u \frac{\partial P(N,i,f_u)}{\partial p_k^f}\left\{N\left(\frac{N^{-1}\left(p_k(T)+\Delta p_k\right)-\beta_k f_u}{\sqrt{1-\beta_k^2}}\right)-p_k^{f_u}\right\}\varphi(f_u)\omega_u$$

With this, one can compute the change of PV for the CDO and its ratio over the change of PV of the CDS on issuer k so as to know the notional amount of CDS that needs to be dealt to hedge the delta of the CDO.

<u>Sensitivity against beta</u>

In the same fashion, without any further computation, the change of loss distribution against a change of the beta of entity k is given by:

$$\Delta P(N,i)\Big|_{\beta_k+\Delta\beta_k} = \sum_u \frac{\partial P(N,i,f_u)}{\partial p_k^f}\left\{N\left(\frac{N^{-1}\left(p_k(T)\right)-(\beta_k+\Delta\beta_k)f_u}{\sqrt{1-(\beta_k+\Delta\beta_k)^2}}\right)-p_k^{f_u}\right\}\varphi(f_u)\omega_u$$

7 American Swaps

American CDS and CDO calculations can be derived from an integration of European credit derivatives with the float leg given by:

$$C_{Am}^{float}(T) = C_{Euro}^{float}(T) + \int_0^T f(0,t) C_{Euro}^{float}(t) dt$$

And the fixed leg (for unit spread = 100%) by:

$$S(T_k) = (T_k - T_{k-1}) \times B_{risky}(T_k) + \int_{t=T_{k-1}}^{T_k} B(t) \times (t - T_{k-1}) \times d\,\mathrm{Pr}\,ob(\tau_{def} < t)$$

This is already done in **Function AmericanSwap**, by computing European float legs at several dates.
Hence computation time for an American float leg is a multiple of the computation time for European float legs.

Note that the function returns PV in the amount of currency, not in % of the notional amount of either portfolio or tranche.