Modeling Fee Shifting With Computational Game Theory

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Abstract

While modern mathematical models of settlement bargaining in litigation generally seek to identify perfect Bayesian Nash equilibria, previous computational models have lacked game theoretic foundations. This article illustrates how algorithmic game theory can complement analytical models. It identifies equilibria by applying linear programming techniques developed in von Stengel et al. (2002) to a discretized version of a cutting-edge model of settlement bargaining, Dari-Mattiacci and Saraceno (2020). This approach makes it straightforward to alter some assumptions in the model, including that the evidence about which the parties receive signals is irrelevant to the merits and that the party with a stronger case on the merits also has better information. The computational model can also toggle easily to explore cases involving liability rather than damages and can incorporate risk aversion. A drawback of the computational model is that bargaining games may have many equilibria, complicating assessments of whether changes in equilibria associated with parameter variations are causal.

1 Introduction

The literature analyzing the effects of fee shifting confronts a daunting analytic challenge. Settlement bargaining is a two-player asymmetric information game. The gold standard solution to such a game is a pair of common knowledge strategies that form a perfect Bayesian equilibrium. The perfection requirement, as defined by Fudenberg and Tirole (1991) [8], specializes the general Nash [21] equilibrium criterion in an imperfect information game, by insisting that at no point in the game may a player have any incentive to change the player's strategy. Each player applies Bayesian reasoning to incorporate new information, such as signals of case quality, into estimates of trial outcomes, and the player's probabilistic beliefs are required to be correct given such information. Complicating the challenge of crafting such equilibria, each party might decide not to contest the litigation, one or both parties may be risk averse, players may or may not have asymmetric information, a case may concern liability or damages, and the loser may or may not be required to pay the winner's fees.

Incorporating anywhere near all of these considerations into a single model of settlement bargaining has proven elusive. The settlement bargaining modeler stands before a smorgasbord of potentially critical game features, but faces the admonition to choose no more than a few. The result, Daughety and Reinganum (1993) [5] observed, is a literature that "has grown in a disorganized fashion, resulting in a multitude of models involving different informational endowments and timing structures." This statement remains true nearly three decades later, with the settlement-bargaining-modeling art making unmistakable but limited progress. The earliest models of Landes (1971) [18], Posner (1973) [24], and Gould (1973) [9] had ignored the challenges of Bayesian inference. Later came models, such as Bebchuk (1984) [1] and Polinsky and Rubinfeld (1998) [22], in which one party knows the probability of liability or the amount of damages while the other party knows only the distribution, and Daughety and Reinganum (1994) [6], in which one party has information on liability and the other party, on damages. The latest generation of scholarship, including Friedman and Wittman (2006) [7], Klerman, Lee, and Liu (2018) [16], and Dari-Mattiacci and Saraceno (2020) [4], models two-sided asymmetric information, in which each of the plaintiff and defendant has independent private information about the same issue, for example about the level of damages. The last of these even succeeds at the Herculean task of incorporating fee shifting, but we will see that even it does not escape the curse of dimensionality, as it must adopt a number of restrictive assumptions that make it difficult to assess the generality of its conclusions.

The literature is extraordinarily clever, in both the positive and negative senses of the word. It takes advantage of mathematical assumptions to make otherwise intractable problems tractable. We can expect further progress from relaxing different assumptions, but the goal of developing a single mathematical model that allows exploration of different values of a large number of variables may be unattainable. The literature develops critical intuitions about settlement bargaining, including how changing fee shifting rules may augment or diminish the effectiveness of the litigation system, and review articles, like Katz and Sanchirico (2012) [15], informally integrate various models' conclusions about how fee shifting might affect trial rates or settlement rates with different structures to the litigation game. Even with such reviews, however, it is difficult to generalize about the wisdom of fee shifting, because of the interactivity between trial and settlement rates. If, for example, increased ease of settlement leads to plaintiffs' bringing and defendants' defending more cases, total litigation expenditures in principle could rise.

Scholars have studied settlement bargaining with other methodologies, but these have their own limitations. Empirical analyses are limited to studying the rare cases in which a change in fee-shifting rules occurs, as in the examination by Hughes and Snyder (1995) [11] of a briefly-lived policy experiment in Florida. Laboratory experiments offer another approach, with contributions by Coursey and Stanley (1988) [3], Inglis et al. (2005) [13], Rowe and Vidmar (1988) [26], and Main and Park (2000) [19]. It is not clear, however, whether the artificial stakes and quick decisions in a lab produce results similar to those of real litigation. A final methodology in the literature is computer-based simulation. Priest and Klein (1984) [25], Katz (1987) [14], Hause (1989) [10], and Hylton (1993) [12] were pioneers in using computation, either independently or as complements to formal models. While these articles all include innovations building on the Landes-Gould-Posner model, they share a significant limitation: Unlike the math models, the simulations do not seek perfect Nash equilibria.

It is, however, possible to harness computational power in the quest for perfect Bayesian equilibria, by turning to computational game theory. The settlement bargaining literature has acknowledged the importance of game theoretic concepts, but articles build at most relatively small game trees. Acknowledging that litigation can be viewed as "a particular extensive-form bargaining game," Spier (1994, pp. 202-03) [28] sensibly worries that the results would be sensitive to issues such as "the structure of the asymmetric information." This concern suggests that solvable game theoretic models are insufficiently rich to encapsulate critical aspects of the litigation game. Computational game theory, however, allows for the identification of equilibria in games that could not practically be solved by hand. A subliterature focuses directly on the solution of two-player (and sometimes n-player) general sum games. A litigation game between plaintiff and defendant is general sum, which can be more difficult than a zero-sum game to solve, because the players may transfer wealth not only from defendant to plaintiff, but also from both litigants to lawyers. A useful review of algorithms that can help solve such games is von Stengel (2002) [29].

A publicly available open source software package known as Gambit by McKelvey et al. (2016) [20] features a number of these algorithms. This article, however, applies an algorithm not included in Gambit, specifically an algorithm described in an article in *Econometrica*: von Stengel, van den Elzen, and Talman (2002) [30]. This algorithm, described further below, is guaranteed to produce exact perfect Bayesian Nash equilibria in a finite game in which the players have perfect recall. Sometimes, these equilibria are pure, with players acting deterministically conditional on the information that they possess, but at other times, they are mixed, with the players randomly choosing at certain moments of the game between or among equally good strategies, each with some nonzero probability. Mixed strategies need not reflect explicit randomization by players; they may be understood as describing balanced populations of litigants who take different approaches, none better than others given opponents' strategies, for reasons exogenous to the model. The authors test their algorithm on games of up to 1,023 nodes. This article pushes the computational limits of the algorithm, applying it to each of a large number of games with up to 16,111 nodes. By separately identifying equilibria corresponding to different information and game structures, we can assess the conditions in which changing fee-shifting rules may increase or decrease the effectiveness of the litigation system, as manifested in accuracy or trial rates, assuming the litigation game is played in equilibrium by rational actors. This approach thus enables modeling of a richer and more diverse litigation environment than any single prior approach.

To illustrate how this approach can complement analytic models, this Article focuses on the model of Dari-Mattiacci and Saraceno (2020), the first article to integrate both two-sided asymmetric information and fee-shifting. In this model, both players know the true quality of the litigation, but the judgment depends not only on this value, but also on the sum of signals independently received by the parties. Part 2 describes the Dari-Mattiacci and Saraceno model, and it identifies a number of assumptions that may be necessary for mathematical tractability but are unrealistic or greatly narrow the model's applicability. Part 3 then presents a discretized version of the Dari-Mattiacci and Saraceno model that can be solved computationally. It also addresses two potential concerns: first, that discretization might itself significantly affect equilibrium strategies, and second, that the game might have multiple equilibria.

Part 4 provides the central results. It first reports accuracy and trial rates for values of litigation quality. These data are consistent with the Dari-Mattiacci and Saraceno model, at least when restricted to parameter values within the range permitted by that model. It then relaxes a number of assumptions of their model by extending the range of permissible litigation quality and litigation quality parameters, disentangling strength of information from litigation quality, allowing the evidence that parties receive to be relevant to the merits, adopting variants on the fee-shifting rules, granting the parties an outside option not to litigate, and featuring risk-averse litigants. The results generally illustrate the importance of Dari-Mattiacci and Saraceno's assumption that the parties' evidentiary signals do not reveal information about the merits. Under this assumption, the English rule tends to improve accuracy, but when the information is about the merits, the English rule generally results in greater inaccuracy. When liability is at issue, however, the English rule appears slightly to reduce inaccuracy. As a further complication, all of these conclusions are sensitive to the definition of accuracy, and an alternative definition that focuses on per-case accuracy instead of average accuracy produces quite different results.

Overall, the results are sufficiently nuanced and sensitive to model specification that the exercise highlights the difficulty of reaching conclusions about the effects of fee-shifting rules that are applicable to a wide range of models. The results are too numerous to be discussed in any more than cursory detail here, but an Online Repository, available at https://github.com/mbabramo/Fee-shifting-article/, contains over 110,000 files, including summary data, heatmaps, logs and the resulting equilibria for each of 35,350 model permutations, plus scatterplots illustrating variables such as accuracy measures, trial rates, and settlement bids for various permutations of trial costs, case quality, and fee shifting. In addition, the Online Repository includes two appendices, one providing a brief introduction to the von Stengel et al. algorithm and the other applying the algorithm to a nondiscretized version of the algorithm, an approach that allows for an arguably truer replication of some of Dari-Mattiacci and Saraceno's results, but only where those results do not include discontinuities in the parties' bid functions. A separate code repository at https://github.com/mbabramo/ACESim4/tree/feeshifting contains the complete source code used to generate results, and a ReadMe.txt file provides information about how to replicate the results.

2 Analytical Models of Two-Sided Asymmetric Information

Because this article's goal is to illustrate how computational models can build on limitations of an analytical model, and vice-versa, we will focus on the structure and methodological choices in Dari-Mattiacci and Saraceno (2020). We begin by reviewing Friedman and Wittman (2006) [7], the model upon with Dari-Mattiacci and Saraceno build, before exploring the Dari-Mattiacci and Saraceno model and its assumptions.

2.1 Friedman and Wittman's Averaged Signals Model

In the one-sided information models, the structure of bargaining often affects which party receives most of the surplus. Friedman and Wittman avoid this problem by adopting the bargaining protocol of Chatterjee and Samuelson (1983) [2]. In Chaterjee-Samuelson bargaining, the plaintiff and defendant simultaneously submit offers. If the plaintiff's exceeds the defendant's, the case definitively settles at the midpoint; otherwise, bargaining has failed. Costs of trial are borne only in the event of bargaining failure. Friedman and Wittman justify this bargaining structure not on the ground that the protocol is commonly used (it is not), but on the ground that it provides a useful reduced form of a more complicated bargaining process. With Chaterjee-Samuelson bargaining, a case may go to trial even though there is a social surplus from settlement given the parties' expectations. The parties shade their offers in the hope of claiming as much of the settlement surplus as possible, even at the risk of bargaining failure.

The informational structure is arrestingly simple. The plaintiff observes a signal θ_p drawn from a known distribution, and the defendant independently observes a signal θ_d drawn from the same distribution. The principal results of the paper apply to a "basic litigation model" in which the distribution is the uniform distribution; this extends without loss of generality to any uniform distribution between a lower bound of L and an upper bound of U. In the event that settlement fails, a judgment is entered in the amount of the average $(\theta_p + \theta_d)/2$. Perhaps one can imagine situations in which this might be realistic. For example, the parties might have information about different components of damages in a case in which liability is uncontested, and should trial ensue, the information will be revealed and the judgment will be the sum. But a more intuitively appealing model in most situations would reverse the causality. Signals would depend on the underlying truth to be revealed at judgment, rather than the judgment depending on signals. Friedman and Wittman cleverly recognize, however, that modeling litigation in this way makes the model tractable.

Friedman and Wittman derive a Nash equilibrium in the basic litigation game. In this equilibrium, the plaintiff will ordinarily offer $\frac{2}{3}\theta_p - 2c + \frac{1}{2}$, and the defendant will ordinarily offer $\frac{2}{3}\theta_d + 2c - \frac{1}{6}$, where c represents each party's trial cost. The word "ordinarily" signals what may seem a mild caveat: Neither party will ever make an offer beyond the range of the other party's possible offers. Thus, the plaintiff's offers are truncated above at $\min(1, 2c + \frac{1}{2})$ and below at $\max(0, 2c - \frac{1}{6})$, while the defendant's offers are truncated above at $\min(1, \frac{7}{6} - 2c)$ and below at $\max(0, -2c + \frac{1}{2})$. Friedman and Wittman do not eliminate the possibility that there might be some nonlinear Nash equilibrium, but they prove that the equilibrium they derive is the unique nontrivial piecewise linear equilibrium. There are also infinitely many trivial equilibria, in which the plaintiff's settlement demands always exceed the defendant's.

Friedman and Wittman's model permits focus on the trial rate. They derive a piecewise quadratic formula for the trial rate, and they also examine, in the tradition of Priest-Klein, how the trial rate varies near the midpoint of the decision spectrum. They show that when trial costs are sufficiently low ($c < \frac{1}{6}$), the probability of a trial is higher, the farther the judgment would be from $\frac{1}{2}$, and when trial costs exceed this threshold, the probability of a trial is highest at the $\frac{1}{2}$ point. The intuition is that when costs are high, the parties become more generous, and so the plaintiff's range of offers will fall below the defendant's. The truncations then ensure that cases at the extremes, where either both parties receive a low signal or both parties receive a high signal, are more likely to settle. When trial costs are low, the parties are less generous, and the plaintiff's range of offers will be above the defendant's. Cases at the extremes are then less likely to settle. With the basic litigation game, the $\frac{1}{6}$ cost threshold occurs where the parties' range of offers are equal. Friedman and Wittman also offer a graphical argument that extends to other continuous distributions, though they do not expressly consider the case where liability rather than damages is uncertain.

2.2 Dari-Mattiacci and Saraceno's Evidentiary Signals Model

Dari-Mattiacci and Saraceno (2020) illustrate the challenge of building on Friedman and Wittman by successfully extending the model to fee shifting. The article includes an online appendix with 60 pages of proofs. The difficulty stems from the need to address four principal cases, depending on relative values of parameters, and within these principal cases, to make various calculations that depend on the relative values of other parameters, including in many instances five different formulas for five different ranges of a variable. The resulting analysis is testimony both to human ingenuity and to endurance, and it makes breakthroughs in our understanding of the effects of fee-shifting with two-sided asymmetric information.

As in Friedman and Wittman, plaintiff and defendant receive signals, now denoted θ_Π and θ_Δ, respectively, and the judgment is an average of the signals. Now, however, both parties have common knowledge of the true merits of the litigation, denoted by q. The signals thus do not serve the function of informing the parties of the true merits, but rather of providing the parties with evidence that they may use to convince the court. The plaintiff's signal θ_Π is drawn from a uniform distribution on the interval $(0, q)$, and the defendant's signal, on the interval $(q, 1)$. The plaintiff's best possible evidence, where $\theta_\Pi = q$, would convince the court that the judgment must be at least q. Similarly, the defendant's best possible evidence, where $\theta_\Delta = q$, would convince the court that the judgment must be no more than q.

The fee shifting rule that Dari-Mattiacci and Saraceno primarily analyze is triggered based on (1) whether the final judgment is above or below $\frac{1}{2}$ (i.e., which party "wins" in the sense of being awarded more than half of the contested amount), and (2) whether the evidence of the winning party is sufficiently strong. If the judgment is less than $\frac{1}{2}$, then the defendant might be able to shift its costs to the plaintiff, but only if the defendant's signal falls below some threshold, i.e. $\theta_\Delta < t$, where $0 \leq t \leq 1$. Likewise, if the judgment is greater than $\frac{1}{2}$, then the plaintiff might be able to shift its costs to the defendant, but only if the plaintiff's signal exceeds a threshold, i.e. $\theta_\Pi > 1 - t$. An intuition is that if a party wins a case merely because its opponent has produced little evidence, a court will not order fee-shifting; another is that a court will only order shifting of fees when those fees were spent on producing strong evidence. Note that when $t = 0$, fees will never be shifted, so this extreme is the American rule of no fee shifting, and when $t = 1$, fees will always be shifted to a winning party (i.e., to the plaintiff if the final judgment exceeds $\frac{1}{2}$ and to the defendant if the final judgment is less than $\frac{1}{2}$), so that extreme is the English rule of universal fee shifting. The t parameter allows for a continuum of fee shifting rules.

This information structure enables Dari-Mattiacci and Saraceno to derive the offers that the parties will make. They prove that each party's offer function is a best response to its opponent's offer function and thus that a Bayesian Nash equilibrium exists. They also derive formulas for settlement amounts, along with identification of the ranges of parameters values where such settlements occur, and accordingly of the litigation rate. They prove that the litigation rate depends only on c (now representing the combined trial cost of the two parties) and is thus independent of both case quality q and the fee-shifting rule t. This produces the surprising conclusion that the litigation rate is the same under both the American and the English rule. Finally, they offer a calculation of litigation accuracy, and they prove that when costs are below a certain threshold, the English rule produces more accuracy than the American rule, while the reverse is true when costs are above a certain threshold. The stylized fact that litigation is cheaper in England may thus help explain the choice of rule in each country.

2.3 Assumptions in Dari-Mattiacci and Saraceno's Model

The Dari-Mattiacci and Saraceno model many assumptions, often driven, quite reasonably, by the demands of mathematical tractability. It is difficult to develop strong intuitions about whether they matter. In identifying these assumptions, we create a series of challenges for a computational model that aspires to assess the robustness of the analytical model.

Parameter values

Balanced true merits The true merits variable is constrained so that $\frac{1}{3} \leq q \leq \frac{2}{3}$. The reason for this constraint is that with more extreme values of q, the increasingly one-sided nature of asymmetric information leads the pure strategy equilibria derived by the authors to break down. A computational model ideally would be able to find an equilibrium with relatively extreme quality values.

Low or moderate cost Dari-Mattiacci and Saraceno follow Friedman and Wittman in implicitly assuming that the cost variable is not so high that the plaintiff's untruncated offer range is entirely below the defendant's untruncated offer range. The truncation functions defined by Friedman and Wittman are undefined, because when their c is sufficiently high, they instruct that the plaintiff's offers should be truncated above at 1 and below at a number greater than 1, and similarly the defendant's offers are truncated below at 0 and above at a number less than 0. With sufficiently high costs, there will be many Nash equilibria; the parties will be determined not to go to trial, but neither party would deviate from any positive allocation of the surplus from settlement. Literal application of the Dari-Mattiacci and Saraceno formula, however, would lead to both players truncating their bids rather than choosing any of these equilibria.

In a subtle way, the Dari-Mattiacci and Saraceno cost assumption is more restrictive than Friedman and Wittman's. A computational model can be used to assess whether parties' strategies form an equilibrium, and with sufficiently high costs, the bid functions identified by Dari-Mattiacci and Saraceno in some cases do not form equilibria. This can be traced in part to a complication in what Dari-Mattiacci and Saraceno call Case 4B. They implicitly assume that the bid functions that they derive would each contain a discontinuity, but if $6c(1-q) > 1$, the plaintiff's bid function consists only of a single line segment. For example, the code at https://dotnetfiddle.net/5XNcYl illustrates that for the parameters $t = 0.8, q = 0.4, c = 0.3$, the plaintiff's strategy cannot be a best response, because, given the defendant's presumed strategy, the plaintiff would be slightly better off with a bid function in which it always bids one-third of its normalized signal. It establishes this by calculating the players' utility at each of 1,000,000 points with the recommended and alternative plaintiff strategy. In correspondence, Dari-Mattiacci and Saraceno have acknowledged this complication and that their model implicitly assumes that c is not too high. This is a reasonable assumption, but a challenge for the computational model is to overcome it.

Structural constraints

Piecewise linearity Dari-Mattiacci and Saraceno explicitly assume a linear relationship between the parties' signals and their offers, but allow for discontinuities at points where fee-shifting would change. Because fee-shifting depends partly on the quality of the evidence possessed by the winning party, a litigant will know whether it will be entitled to fee-shifting if it wins, and the signal values at which this fact changes are points at which Dari-Mattiacci and Saraceno are able to break the problem down into smaller pieces. Piecewise linearity thus allows for explicit modeling of the effects of changes in a fee-shifting rule, but because it is unclear how restrictive this assumption is, it is a prime candidate for relaxation in a computational model.

Asymmetric information quality equivalence Recall that the plaintiff receives a signal in the range $(0, q)$ and the defendant, in $(q, 1)$. As a consequence, when $q > \frac{1}{2}$, the plaintiff's signal has a greater potential effect than the defendant's, and when $q < \frac{1}{2}$, the reverse is true. The single variable q thus serves two, independent functions in the model: one is to represent the "true merits" of the case, while the other is to represent the degree of information asymmetry. This greatly increases the tractability of the model, and plausibly it allows for consideration of both issues related to accuracy and issues related to information asymmetry. The problem, though, is that the issues are necessarily conflated; where a case is at an extreme of the probability distribution, there is always high information asymmetry. There is no obvious reason to believe that true merits should generally track information asymmetry in this way. The question thus arises whether the results would be the same if the model allowed independent variation of true merits and information asymmetry.

Signals independent of true merits Dari-Mattiacci and Saraceno refer to the signals that the parties receive as "evidence," but there is a paradox: The parties are assumed to know the true merits of the case (q) and indeed use this information in constructing their offer functions. Thus the variance in the signals that each party may receive has nothing to do with the merits. The signal informs each party only about the party's likely ability to persuade the judge about the true merits. The judge does not know the true merits, but is trying to guess the true merits. The higher q, the higher the parties' signals will tend to be, so the judge's strategy is reasonable, even if non-Bayesian. But the result is that from the perspective of the parties, for whom q is fixed, the randomness in case outcomes has to do only with who is lucky in finding promising evidence.

This point can be more clearly seen in a transformation of the model that Dari-Mattiacci and Saraceno offer. They note that the signals θ_Π and θ_Δ can be mapped one-to-one onto signals from 0 to 1, which they label z_Π and z_Δ. These signals are thus independent signals from a unit uniform distribution, and the θ signals can be derived from them according to the formulas $\theta_\Pi = qz_\Pi$ and $\theta_\Delta = q + (1-q)z_\Delta$. This highlights that the θ signals result from commingling the true merits of the case and the random uniform distribution draws. With these transformations, the judgment depends on the following formula:

$$J(z_\Pi, z_\Delta) = \frac{1}{2}q + \frac{1}{2}(qz_\Pi + (1-q)z_\Delta) \tag{1}$$

As this presentation makes clear, the decision is half based on the true merits of the case, independent of any evidence presented by the parties. The other half is a weighted average of the parties' uniform distribution draws, with the weights equal to q, here representing the degree of information asymmetry. The only reason that this makes sense from the perspective of the judge is that the judge does not observe z_Π and z_Δ directly. In essence, the parties have asymmetric information about their persuasive abilities. This approach is convenient in a mathematical model that seeks, as theirs does, to measure accuracy. It is considerably easier (though still extraordinarily difficult) to measure outcomes relative to the constant q than it would be relative to a function of q and the parties' signals.

A challenge for the computational model is to assess whether results about accuracy continue to obtain when the true merits are defined to be inclusive of the parties' normalized signals. On this formulation, q would represent knowledge that the parties share about the true merits, and the z signals represent private information about the true merits. When the judge adds these together according to the above formula, the judge obtains not only the judgment, but also what could be considered the true merits. This is thus a conceptual reformulation with no implications for which cases settle. It requires only an alteration of the definition of accuracy.

Game structure

Fee-shifting structure Fee shifting in Dari-Mattiacci and Saraceno's model depends not only on which party wins more than half of the judgment at trial, but also on the quality of the evidence produced by the winning party. This is mathematically convenient, because each party knows the quality of its own evidence and thus whether fee-shifting will occur for any given value of the opponent's signal and any value of t. An alternative approach would be for fee-shifting to depend on both parties' evidence. Indeed, Dari-Mattiacci and Saraceno explicitly consider fee-shifting based on the margin of victory. If we redefine t to represent the margin-of-victory parameter, then if $\theta_\Pi + \theta_\Delta < t$, the plaintiff must pay the defendant's fees, and if $\theta_\Pi + \theta_\Delta > 2 - t$, then the defendant must pay the plaintiff's fees. In this regime, if $t = 0$, no fee shifting occurs (the American rule), and if $t = 1$, fee shifting always occurs absent an evenly split judgment (the English rule); thus, the margin-of-victory approach converges with the other approaches at the extremes. Dari-Mattiacci and Saraceno explicitly calculate the parties' offers under this approach, but they do not prove their results related to accuracy. This raises the question whether their accuracy results are robust to the alternative specification. One might also imagine other fee-shifting rules, such as a simple rule in which a party always occurs when the party wins half of the judgment but the proportion of fees shifted may vary from 0 to 1.

Damages issue Dari-Mattiacci and Saraceno explicitly describe their model as one in which the parties are arguing about how to divide a disputed asset, such as in a case of divorce, and they point out that without loss of generality, this can be extended to a judicial determination of damages between some minimum and maximum value. An extension would be to consider cases where liability is at issue, i.e. where the plaintiff will receive 1 if $\theta_\Pi + \theta_\Delta > 1$ and 0 otherwise. This should be trivial in a computational model, which need only transform the judgment values in particular cases.

Contestation Dari-Mattiacci and Saraceno's model assumes that the plaintiff always files the lawsuit and that the defendant always answers. Shavell (1982) [27] and others have highlighted that some potential litigants may choose not to litigate. In their Online Appendix, they offer some informal considerations of how fee shifting might affect filing. They reason, but do not explicitly prove, that the greater the fee-shifting parameter t, the more often the plaintiff will not file or the defendant will not contest litigation. This could affect the settlement bids that the parties make. Thus, we cannot be sure that an equilibrium in the original model will remain an equilibrium when the parties are allowed not to contest the litigation.

Risk neutrality The plaintiff and defendant are assumed to be risk neutral. Incorporating risk aversion into the model would likely add considerable challenge, though the argument could still proceed in case-by-case fashion. Incorporating risk aversion is virtually costless to a computational model, requiring only the transformation of the parties' utilities in any game outcome.

Accuracy definition Dari-Mattiacci and Saraceno define inaccuracy in their appendix as "the square distance between the expected outcome E_t and the merits q." We have already explored how we might reconceive the definition of the merits so that all evidence is counted as part of the true merits, so let us continue moving from right to left in this definition.

The definition of E_t is complex, involving double integrals over both costs and the parties' signals. The essence is that it is a measure of the expected outcome of a dispute, taking into account both the settlements and the trials. The outcome in the event of trial that they calculate is represented by G, which "captures both the decision on the merits and fee shifting." For example, if the judgment is for 0.45 and the plaintiff pays costs of 0.10 to the defendant, then $G = 0.35$. The inclusion of fee shifting costs reflects that imposition of fee shifting not only affects settlement negotiations, but also affects the amount that the plaintiff must pay to the defendant at trial. It is reasonable to view the difference between the expected value of G and the value of q as a measure of accuracy, but the question remains whether conclusions about accuracy would be robust to alternative specifications.

Outcome expectation The specification chosen focuses on the expectation of settlement or trial results, rather than on the actual result in particular cases. It is a comparison of the expectation of the result with the true merits, not a measure of the error. If, for example, there are two scenarios in which the correct result based on the true merits would be for the defendant to pay the plaintiff 0.50, and in one scenario the defendant pays 0 and in the other scenario the defendant pays 1, then this measure would count the legal system as perfectly accurate. Because the parties are risk-neutral, they would be indifferent between receiving perfectly accurate results and results that are correct on average. An alternative approach, especially important risk-averse parties but conceptually useful even for risk-neutral parties, would measure inaccuracy within each case rather than based on an average across cases. Easier said than done, of course. In the analytical model, this would require moving a minus q term and a squared term within the double integrals in the current E_t definition.

Accounting for costs The accuracy measure also ignores the pre-fee shifting costs that the parties pay. Dari-Mattiacci and Saraceno note "that the plaintiff receives $G - \frac{c}{2}$ and the defendant pays $G + \frac{c}{2}$." Imagine a case with very high costs and no fee shifting, where each party spends a million dollars and the court arrives at precisely the correct conclusion that the defendant owes the plaintiff 50 cents. From this definition's perspective, this outcome counts as a perfectly accurate result. That is a plausible definition of accuracy, but one that offers no comfort to the parties. An alternative definition of accuracy would consider any amounts actually spent at trial, for example counting the outcome from the plaintiff's perspective as $G - \frac{c}{2}$. A similar definition could measure accuracy from the defendant's perspective. Either of these two approaches captures three distinct aspects of costs: (1) Costs impact settlement negotiations; (2) when trial occurs, costs are deadweight losses to society at large; and (3) costs may reduce (or perhaps in some cases increase) the accuracy of adjudication viewed as a black box from the perspective of each individual litigant. Although it is reasonable for Dari-Mattiacci and Saraceno to define accuracy entirely independently of cost, an interesting question is whether any conclusions based on this definition will extend to definitions that incorporate costs.

Squared accuracy Finally, one might quibble about the use of a squared term rather than an absolute value. Admittedly, it is conventional to measure (in)accuracy using the ℓ_2 norm rather than the ℓ_1 norm. The convention reflects the dominance of ordinary least squares regression over least absolute deviation regression, but that dominance stems as least in part from the greater tractability of the former. Portnoy and Koenker (1997) [23] note that computational power mitigates this advantage, and that an advantage of the ℓ_1 norm is that it is more robust to outliers. Because the Dari-Mattiacci and Saraceno definition compares the outcome expectation with q, its results extend to the ℓ_1 norm. The computational results here will be reported using the ℓ_1 norm, because interpretation is more intuitive, because this will make it more straightforward to compare different accuracy measures, and because this will not change any results about when accuracy rises or falls.

3 Litigation as an Extensive Form Game

This section describes the discretization of the Dari-Mattiacci and Saraceno evidentiary signals game into an extensive form game, in which each player receives one of a finite set of signals and must choose one of a finite number of bids. It then acknowledges the objection that there might be many equilibria to any parameterization of the game by running the von Stengel et al. algorithm with 5,000 different random initializations, producing almost 200 different equilibria. This result highlights that when assessing the effect of changing parameters, the algorithm must be executed over a large number of parameters so that multiple families of equilibria can be identified if they exist.

3.1 A Discretized Evidentiary Signals Game

In the discretized model, each party receives a discrete signal from a set of n_S possible signals and then must make a discrete offer from $n_{\mathscr{B}}$ available bids. For legibility, Figure 1 illustrates a highly simplified version of this game for $n_S = 2$ and $n_{\mathscr{B}} = 2$, but we will generally use $n_S = 10$ and $n_{\mathscr{B}} = 10$, which produces a game tree consisting of 11,111 nodes. The circles identify the players (Chance, Plaintiff, or Defendant), as well as information set numbers. The plaintiff does not observe the signal received by the defendant and vice-versa. Thus, for example, at each of the four points in the tree labeled as “D1,” the defendant has the same information set, in which it has received the signal 1 (corresponding to $z_\Delta = 0.25$) instead of the signal 2 (corresponding to $z_\Delta = 0.75$). Thus, the defendant must assign the same move probabilities over its two alternative offers (corresponding to 0.25 and 0.75) at each of these points. The diagram illustrates an equilibrium identified by the algorithm for this simple game, in which each player is always aggressive, with the plaintiff demanding 0.75 and the defendant offering only 0.25. Under the Chaterjee and Samuelson bargaining protocol, this does not result in a settlement.

The strategies permitted with this approach are in some ways more constrained and in some ways less constrained than the strategies in Dari-Mattiacci and Saraceno. The analytical model restricts the litigants from playing nonlinear strategies between the points that produce discontinuities in their proferred equilibria. The computational model may allow for nonlinearities between the critical points, but only at the 10 discrete signal values. A further restriction of the computational model is that offers can be made only at 10 discrete points. Thus, analytical and computational approaches both impose admittedly arbitrary constraints on what strategies are permissible. Many articles in the settlement bargaining literature impose tighter constraints, such as with models where a party receives one of two signals instead of one of ten.

An important question is whether the differences between these restrictions and Dari-Mattiacci and Saraceno’s significantly affect the equilibria. One can reason that these assumptions do not seem critical to game dynamics, but it is difficult to be sure. There is no way to test whether the piecewise linearity assumption is driving the analytical model’s key conclusions without some other

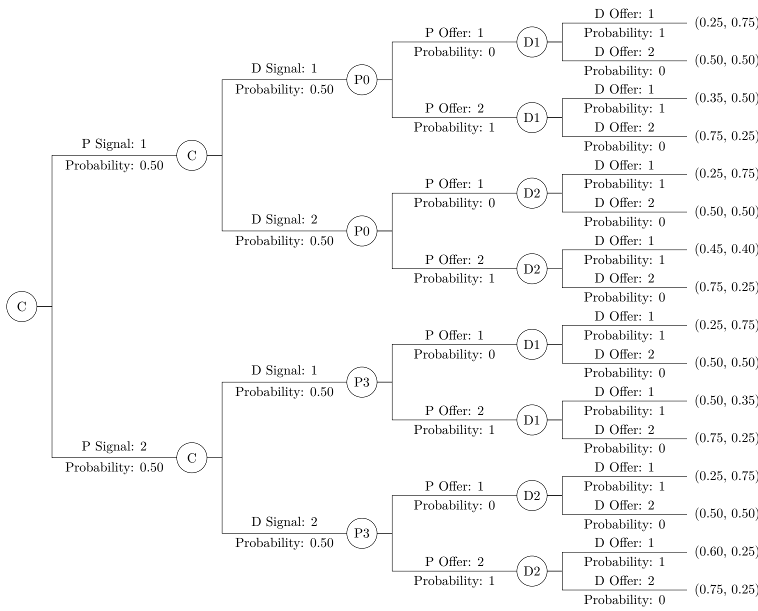

Figure 1: The game tree with $n_S = 2$ and $n_{\mathscr{B}} = 2$
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model that relaxes those assumptions. Fortunately, the computational model can be such a model. Later, we will use the computational model to calculate equilibria employing the discretization assumption. But we can also use computation to measure the extent to which, for any given parameters, a discretized version of the Dari-Mattiacci and Saraceno model deviates from equilibrium.

This assessment requires two steps. First, we calculate each player's offer value at the discrete signal values, and then identify the discrete offer value nearest that value. This produces discretized versions of the bid functions derived by Dari-Mattiacci and Saraceno. Second, we determine each player's best response to the other player's strategy using an algorithm detailed in Lanctot (2013, p. 141) [17] and use these to calculate the exploitability of those strategies. That is, let $\sigma_{\mathscr{p}}$ represent the strategy of player $\mathscr{p}$ at the discretized equilibrium, let $\sigma_{-\mathscr{p}}$ represent the strategy of that player's opponent, and let $\mathscr{U}_{\mathscr{p}}(\sigma_{\mathscr{p}}, \sigma_{-\mathscr{p}})$ represent the expected return of player $\mathscr{p}$ given these strategies. Now, let $\sigma_{\overline{\mathscr{p}}}$ represent the best response of player $\mathscr{p}$ to $\sigma_{-\mathscr{p}}$. We then define exploitability of player $-\mathscr{p}$'s strategy $\mathscr{E}_{-\mathscr{p}} = \mathscr{U}_{\mathscr{p}}(\sigma_{\overline{\mathscr{p}}}, \sigma_{-\mathscr{p}}) - \mathscr{U}_{\mathscr{p}}(\sigma_{\mathscr{p}}, \sigma_{-\mathscr{p}})$. Allowing P and D to represent, respectively, the plaintiff and defendant in the discretization of the Dari-Mattiacci and Saraceno equilibrium, we define overall exploitability $\mathscr{E} = \frac{\mathscr{E}_P + \mathscr{E}_D}{2}$, i.e. the average of the amounts that each player can improve its score by changing its strategy holding the other's strategy constant. By definition, at a Nash equilibrium, overall exploitability is zero, and so $\mathscr{E}$ provides a measure of how much discretization moves the strategies from Nash equilibrium.

Figure 2 illustrates the results. The outer horizontal c axis represents trial cost. The outer vertical q axis represents litigation quality. All of the later simulations will be executed with $q \in \{\frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6}\}$, but we omit the row for $q = \frac{2}{3}$ where it is identical or symmetric to the row for $q = \frac{1}{3}$, and we omit the extreme values of q because they are outside the bounds assumed by Dari-Mattiacci and Saraceno. (Diagrams including these rows, both for this and for other truncated diagrams produced later, are available in the Online Repository.) Each mini-graph represents the measurement of exploitability $\mathscr{E}$ for each value of the fee-shifting threshold t in $\{0, \frac{1}{100}, \frac{1}{50}, \frac{3}{100}, ..., 1\}$. We can see that for $c \leq \frac{1}{8}$, exploitability is very close to zero; overall all of these cases, the average $\mathscr{E}$ value is 0.00047. This indicates that the discretization changes the strategic dynamics of the game very little. With higher trial costs, considerably higher exploitability values (up to 0.067) can be seen in Figure 2. This highlights Dari-Mattiacci and Saraceno's implicit assumption, noted earlier, that c is not too high.

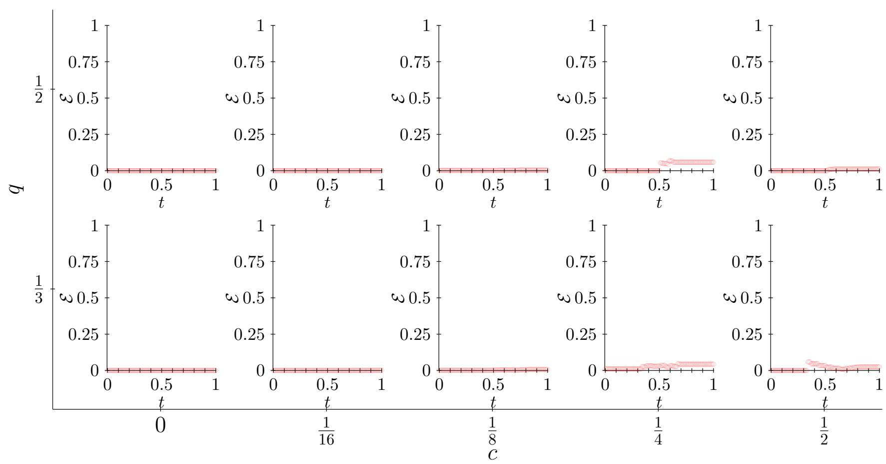

Figure 2: Exploitability of discretized Dari-Mattiacci and Saraceno strategies

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Using the discretization of the Dari-Mattiacci and Saraceno strategies, we can calculate outcome variables. Results on accuracy are illustrated in Figure 3. The accuracy A is defined analogously to Dari-Mattiacci and Saraceno, i.e. by comparing each party's expected outcome (taking into account settlements and judgments, including fee shifting, and not taking into account each party's own legal costs) to the true merits q, though for easier visual interpretability, the absolute value instead of the square of the difference is shown. This illustrates, as Dari-Mattiacci and Saraceno prove, that for relatively low costs, the English rule produces at least as good accuracy (relative low A) in comparison to the American rule. This does not reflect that the reverse is true with relatively high costs, again presumably due to their implicit assumption governing costs. Note also that accuracy is very high when $q = \frac{1}{2}$. This does not indicate that each case produces the correct result, but that the Dari-Mattiacci and Saraceno accuracy measure is based on averages across cases, and with $q = \frac{1}{2}$, the cases in which the plaintiff receives too much exactly balance those in which the plaintiff receives too little.

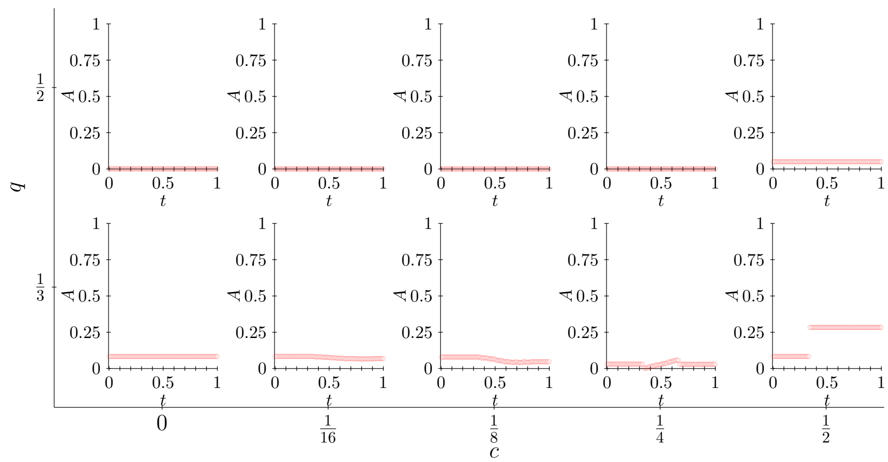

Figure 3: Accuracy with discretized Dari-Mattiacci and Saraceno strategies

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Figure 4, meanwhile, illustrates the analytical model's conclusions regarding the trial rate L. As Dari-Mattiacci and Saraceno show, and as is reflected here at least with $c \leq \frac{1}{8}$, trial rates are invariant to the degree of fee shifting and the quality of the true merits, though small wobbles are observable here as a result of discretization. Also as expected, trial rates fall as the cost of trial rises. Note that trial rates are relatively high, perhaps in part because of the assumption of risk neutrality.

The accuracy and trial results, it is worth emphasizing, are simply calculations based on discretizations of Dari-Mattiacci and Saraceno's constructed equilibria, not equilibria calculated by application of the von Stengel et al. algorithm. These measurements, however, provide a baseline that we can compare to results from equilibria calculated by that algorithm. We will thus be able to assess whether exact, perfect equilibria calculated by the algorithm produce similar results in the discretized version of the original additive evidence game and in variations generated by relaxing various assumptions of the Dari-Mattiacci and Saraceno model.

3.2 Equilibria for a single set of parameters

Which equilibrium of a game the von Stengel et al. algorithm identifies may depend on the initialization of the information sets. We ran the algorithm 5,000 times for a single set of parameter values representing the middle of the values we are exploring, i.e. $t = \frac{1}{2}, q = \frac{1}{2}, c = \frac{1}{8}$, and identified 194 distinct

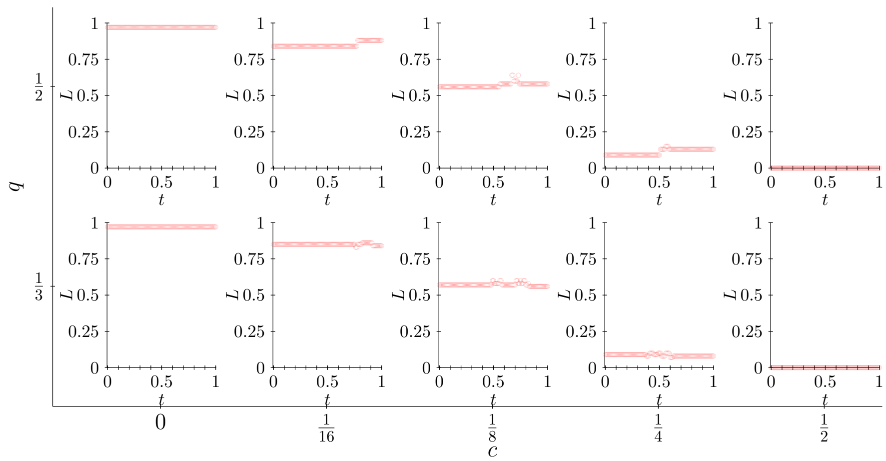

Figure 4: Trial with discretized Dari-Mattiacci and Saraceno strategies

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equilibria. Figure 5 illustrates for each player an average strategy in which each equilibrium strategy is played with equal probability, without regard to whether the player's opponent chooses the corresponding equilibrium strategy. For example, when receiving a signal corresponding to $z_\Pi = 0.05$, the plaintiff makes an offer $\mathscr{B}_P = 0.35$ approximately 8% of the time.

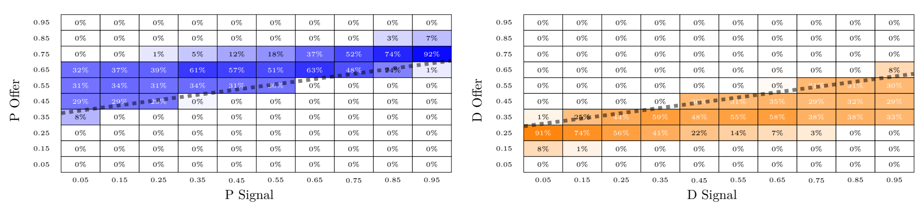

Figure 5: Average strategies across 194 equilibria for $t = \frac{1}{2}, q = \frac{1}{2}, c = \frac{1}{8}$

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Although there are a large number of equilibria, they cluster near the equilibrium derived by Dari-Mattiacci and Saraceno, represented by dotted lines, though most of the equilibria reflect somewhat more aggressive game play. Some of the equilibria differ from others in ways that have no bearing on the game outcome. For example, if plaintiff chooses $\mathscr{B}_P \geq 0.75$ for some signal, settlement will never occur, and so an offer of 0.75 is functionally equivalent to an offer of 0.85. Of the 194 equilibria identified, 106 are pure strategy equilibria while 88 reflect mixed strategies, such as one in which given a particular signal, a party made one offer with exact probability $\frac{4{,}164}{15{,}277}$ and another with probability $\frac{11{,}113}{15{,}227}$. Correlated strategies over the 194 equilibria would by definition also be exact Nash equilibria. The average strategy of the exact equilibria in Figure 5 is not itself an exact equilibria, but is reasonably close, with exploitability

$(E) = 0.0027$.

It will not be practical to seek out 5,000 equilibria for each of the 35,350 parameter values explored in this article and the Online Repository, as this exercise took nearly three and a half hours of computer time. But our later results will continue to reflect initialization of each information set to a random determination of a fully mixed strategy for each player, and we will run the algorithm for every value of $t \in \{0, \frac{1}{100}, \frac{1}{50}, \frac{3}{100}, ..., 1\}$. It will thus be possible to identify different families of equilibria that arise for very close values of t.

4 Results

This section aggregates results from the discretized additive evidence game as defined by Dari-Mattiacci and Saraceno and then progressively relaxes assumptions of their model identified in Section 2.3.

4.1 Aggregating results over parameter values

Figures 3 and 4 calculated accuracy and trial results using the strategies constructed by Dari-Mattiacci and Saraceno. Figures 6 and 7 show the analogous results calculating discretized equilibria with the von Stengel et al. algorithm. More detail is available for each of the 2,525 permutations of parameter values executed under these assumptions in the Online Repository folder Detailed results : Original.

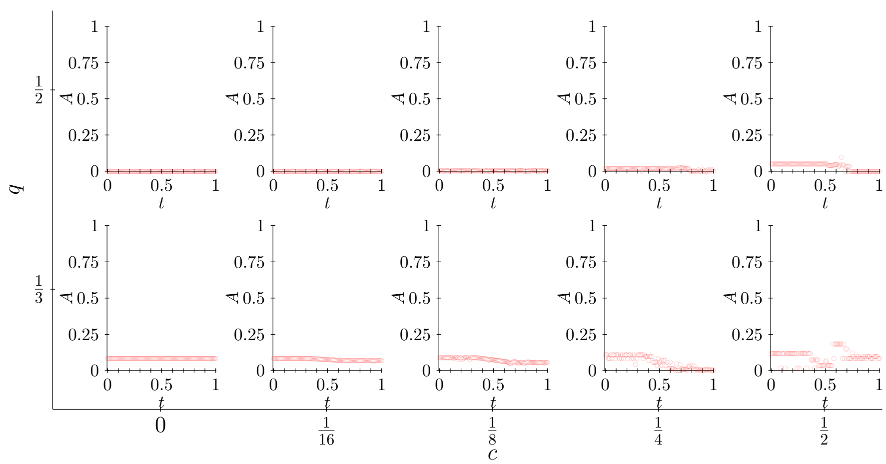

Figure 6: Accuracy with calculated equilibria

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These results are generally close to the results corresponding to Dari-Mattiacci and Saraceno's constructed equilibria, at least for $c \leq \frac{1}{8}$. The shapes of the accuracy curves are very similar, reflecting that their results concerning the better accuracy of the English rule with low trial costs extend to the computational

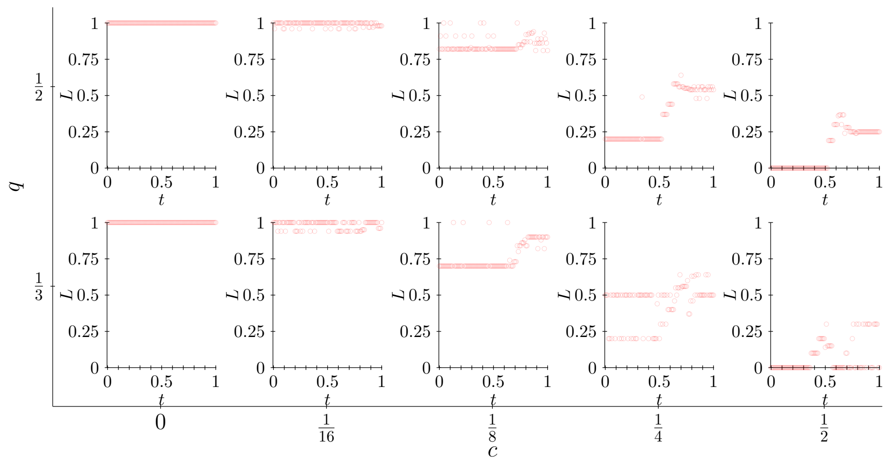

Figure 7: Trial rates with calculated equilibria

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model. Trial rates in the computational model are a bit higher, and multiple equilibria are apparent. The most notable difference is that with sufficiently high costs, the English rule appears to increase trial rates. For example, with $q = \frac{1}{3}$ and $c = \frac{1}{4}$, some equilibria with around half of cases being litigated exist at all fee shifting levels, but equilibria with much lower trial rates are apparent only with relatively low levels of fee shifting. We must, however, acknowledge a limitation of the computational model here. The absence of evidence of equilibria with lower trial rates does not prove that no such exact equilibria exist, only that the algorithm did not identify them over a range of fee shifting values with different randomized initial conditions.

4.2 Relaxing assumptions

The analysis so far has endeavored to track Dari-Mattiacci and Saraceno as closely as possible. Use of the computational model, which discretizes signals and allows for different bid for each signal, reinforces their central results while clarifying the importance of their implicit assumption regarding trial costs. The computational model's potential lies not so much in replication, but in changing critical assumptions. Section 2.3 identified a number of restrictive assumptions in Dari-Mattiacci and Saraceno's model. Though justified by the need for tractability, these assumptions either seem unrealistic or apply to some but not all cases. In this subsection, we will identify how various assumptions may be relaxed.

The model in Figures 6 and 7 already relaxes three assumptions: balanced asymmetric information, low or moderate cost, and piecewise linearity. Although only q values permitted by the Dari-Mattiacci and Saraceno model are shown, a broader range of such values is included in the Online Repository. Figures 6 and 7 illustrate a broader range of costs than Dari-Mattiacci and

Saraceno's implicit costs assumption allows, and discretizing signals and bids inherently relaxes piecewise linearity.

With the computational model, we can also replace the assumption of asymmetric information quality equivalence with an assumption of equal information strength. (The Online Repository also includes models in which the plaintiff has one-fourth of the available private information.) This assumption avoids the conflation of the true merits with the degree of information asymmetry. The judgment is thus computed as follows:

$$J(z_\Pi, z_\Delta) = \frac{1}{2}q + \frac{1}{4}(z_\Pi + z_\Delta) \tag{2}$$

As in Equation 1, half of the judgment depends on q, but the remainder of the judgment no longer weights the parties' private signals by q and $1-q$.

With the change embodied in Equation 2, it also becomes straightforward to remove the assumption that signals are independent of the true merits with an assumption that they form part of the merits. That is, q now represents shared information about half of the merits, and J in Equation 2 now represents the merits. In other words, the normalized information signal that each party has about how the court will rule is now evidence of the merits rather than just information that may sway the court even though it has nothing to do with the merits. Because this is a change only in interpretation, it does not require any additional computation. It merely requires a different baseline for calculating accuracy. (We will consider other changes to the accuracy definition below.) A possible limitation of this assumption is that it ignores that trial judgments themselves may reflect noise or bias, but it seems less plausible that parties would have asymmetric information about noise than that they would have asymmetric information about the merits. We leave the task of modeling judicial error and shared information about judicial bias to future work.

The computational model also makes it straightforward to change the fee-shifting rule. First, we consider the margin-of-victory fee shifting for which Dari-Mattiacci and Saraceno calculate equilibria but do not offer results regarding accuracy. Second, we also consider what we will call "ordinary fee shifting." With this approach, the party that wins more than half of the judgment receives the benefit of fee-shifting, but the proportion of the winning party's legal fees paid by the other party is multiplied by t. Note that t serves a different function in the original Dari-Mattiacci and Saraceno definition than in these alternatives.

The computational model also enables easy toggling between a damages (or asset allocation) issue, as in Dari-Mattiacci and Saraceno, and a liability issue. This requires only changing the pre-fee-shifting payoffs so that in cases in which a plaintiff wins a judgment for greater than $\frac{1}{2}$, the plaintiff receives a payoff of 1, and in all other cases, the plaintiff wins 0. (This reflects an assumption that the plaintiff bears the burden of proof, an issue that is irrelevant in Dari-Mattiacci and Saraceno's model, where the probability that $J = \frac{1}{2}$ is zero.)

A more challenging task for the computational model is to allow the parties to decide sequentially whether to file and answer. If the plaintiff does not file, the plaintiff receives 0, and if the plaintiff files but the defendant does not answer, the plaintiff receives 1. Not filing or not answering avoids the payment of trial costs. This redefinition changes the game tree. Figure 8 shows a portion of the game tree in the simplifed game tree in which $n_S = 2$ and $n_{\mathscr{B}} = 2$. Note that something like this branch appears in the game tree for each combination of plaintiff and defendant signal. We also make a further change, allowing $n_{\mathscr{B}} = 12$, with the highest and lowest bids defined, respectively, to correspond to situations in which the plaintiff or the defendant refuses to negotiate; without this change, a party would never refuse to file or answer, because it could always guarantee a settlement at a slightly more favorable value. With these changes, the game tree contains 16,111 nodes instead of 11,111.

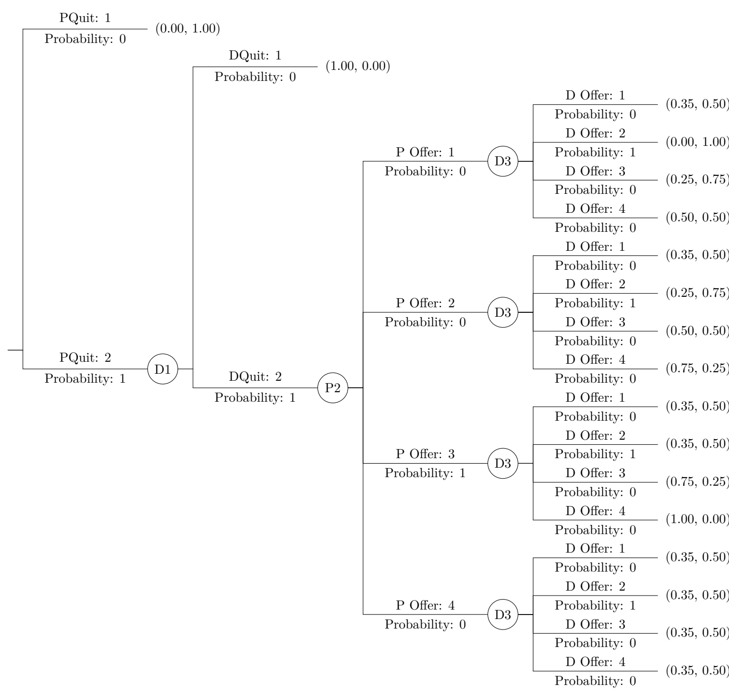

Figure 8: A portion of the simplified game tree with file and answer decisions

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The assumption of risk neutrality can be replaced with an assumption of mild risk aversion by modifying the parties' payoffs in the game, so that utility is a nonlinear function of wealth. Specifically, let $U = -e^{-\alpha W}$, where the parameter $\alpha = 1$ and W, representing each party's initial wealth, is set at 10. These parameters can be modified to adjust risk aversion.

Table 1 identifies the various models to be analyzed. Model A represents the original model, while Model B adds the assumption of equal information strength. Model C reflects the same equilibrium as Model B but calculating accuracy relative to a baseline that includes the parties' private information. Models D and E build on these assumptions to test margin-of-victory and ordi-

Assumption	A	B	C	D	E	F	G	H
Broad costs range	✓	✓	✓	✓	✓	✓	✓	✓
Broad quality range	✓	✓	✓	✓	✓	✓	✓	✓
Discretized signals	✓	✓	✓	✓	✓	✓	✓	✓
Equal information strength		✓	✓	✓	✓	✓	✓	✓
Signals form part of merits			✓	✓	✓	✓	✓	✓
Margin-of-victory fee shifting				✓				
Ordinary fee shifting					✓			
Liability issue						✓	✓	✓
Contestation choice							✓	✓
Risk aversion								✓

Table 1: Assumptions used in subsequent models

nary fee shifting, respectively. Models F, G and H return to the same fee-shifting rule as Dari-Mattiacci and Saraceno, incorporating all of the same assumptions as Model C, but with liability at issue instead of damages. In Models G and H, the parties may choose whether to contest litigation, and in Model H only, the parties are mildly risk averse.

4.3 Results

Figure 9 aggregates the results. Each row represents one of the models described above. The (in)accuracy values are averages of the values calculated for each $q \in \{\frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6}\}$. Because Models C through H count the parties' signals within the merits, with $c = 0$, because all cases go to trial, there is by definition no inaccuracy. As c rises, however, the English rule tends to increase inaccuracy in these models instead of decreasing it, revealing the importance of this assumption. With higher costs, the parties' offers will track their shared information on q more closely, but this effect is much more pronounced with the English rule. (This can be observed in diagrams contained in the Online Repository showing average plaintiff and defendant offers.) Results correspondingly are less influenced by the parties' private information. When q represents the true merits, the English rule therefore tends to improve accuracy, but when the parties' private information is about the merits rather than simply representing noise, the English rule tends to increase inaccuracy.

Changing the definition of fee shifting, as represented by Models D and E, does not alter the fundamental pattern. The most significant difference from Model C is that inaccuracy increases gradually over the range of t values. With the Dari-Mattiacci and Saraceno definition, given low values of t, fee shifting will be rare, occurring only where a party wins and its own evidence is very strong. With these other models of fee-shifting, it will be relatively rare for a party to be able to rule out the possibility of fee-shifting.

The increase in inaccuracy with the English rule does not generally manifest when liability is at issue. Indeed, with sufficiently high costs and no risk aversion (in Models F and G), there appears to be a slight reduction in inaccuracy with the British rule. The explanation can be traced in part to a greater incidence of trial in the winner-take-all liability context. With relatively high trial costs, trial rates are higher in Model F than in Model C. Because all trials by definition produce no inaccuracy in these models, the increase in trial rates tends to improve accuracy, all else equal. Risk aversion, meanwhile, drives trial rates back down in Model H, and this tends to increase inaccuracy.

Another important phenomenon in Models G and H is that the parties may decide not to file or not to answer. Unsurprisingly, parties are more likely to give up (a) when q indicates that they have a bad case; (b) when t is relatively high; and (c) when the parties are risk averse. (This is evident in diagrams of quit rates available in the Online Repository.) Decisions not to contest litigation will tend to improve accuracy. Usually, the party that decides not to contest will be the party that would lose, and so, by avoiding a settlement, the accurate result is achieved.

There are many nuances that Figure 9 and the summaries above do not capture. Interestingly, for example, with $c = \frac{1}{16}$ and $q = \frac{1}{6}$, the plaintiff in Model G will decide not to file with a probability of approximately 0.7, yet the plaintiff will always file with $c = \frac{1}{8}$ and $q = \frac{1}{6}$ if $t < \frac{1}{2}$. When the plaintiff does file with $c = \frac{1}{16}$, the case always goes to trial, but with $c = \frac{1}{8}$, the case will sometimes be settled. One can see here the intricate dance between the file decision and the parties decisions whether to contest litigation. Because higher costs make settlement more likely, they may lead a party to be more willing to contest litigation in the first place. Once t is sufficiently large, the plaintiff will quit at greater rates, because it recognizes that the defendant will have greater resolve in settlement negotiations. Granted, even this is a simplified story. In an article of this length, we cannot account for every strategic nuance revealed by the computational results, and any textual explanation will not fully describe the dynamics of changes in the Nash equilibrium with changes in parameters.

We will conclude by using a different definition of accuracy for all of Models A-H. This definition considers a plaintiff's *net* recovery, taking into account costs and fee-shifting. Suppose, for example, that the court would award a judgment of 0.65 but the plaintiff receives only 0.55, either as a settlement or on net at trial after each party pays its fees and fee-shifting is resolved. Under this definition, which focuses on the danger of undercompensation, an inaccuracy error of 0.10 is logged. If the plaintiff receives more than the court would award at trial, that will count as 0 inaccuracy, but it would lead to an error in a similar measure of accuracy focusing on excessive payments by defendants and thus on concerns of overdeterrence. Figure 10 shows the plaintiff-centered measure, but the defendant-centered measure would show a mirror of these effects, given that the game is entirely symmetrical.

Even with risk neutrality, which measure better captures social welfare depends on factors such as whether parties might be able to predict the errors before they engage in actions with the potential to lead to litigation. A comparison of Figures 9 and 10 illustrates how the definitions differ. First, inaccuracy levels generally appear higher with the undercompensation inaccuracy measure, highlighting that low levels of inaccuracy on our earlier measures may correspond to situations in which some plaintiffs receive way too much while others receive way too little. Second, Model A, the computational version of Dari-Mattiacci and Saraceno's original model, demonstrates improvements in accuracy with greater fee shifting in Figure 9 but not in 10. Third, Models F, G and H, corresponding to a winner-take-all liability issue, illustrate that the danger of undercompensation inaccuracy becomes much greater with high costs. The English rule appears to reduce undercompensation inaccuracy in these models, while raising undercompensation inaccuracy in other models. Whatever the merits of the fee-shifting rule, Figure 10 illustrates a far more malign view of the implications of high trial costs than Figure 9.

5 Conclusion

The results reinforce that many conclusions of Dari-Mattiacci and Saraceno's elegant model are robust to alternative specifications. Greater trial costs increase settlement rates, but have relatively limited effects on the values at which settlements are struck. At least with relatively low trial costs, the English rule tends to improve accuracy, measured as an average across all cases. These conclusions do not hinge on assumptions such as balanced asymmetric information, asymmetric information quality equivalence, or the independence of signals from the true merits. These conclusions suggest that continued development of these models may be fruitful. At the same time, the computational model illustrates the importance of both the definition of accuracy and whether a case concerns damages or liability. The English rule does not clearly improve per-case accuracy when damages is at issue, but it does appear to produce substantial benefits when liability is at issue. All of this analysis, of course, is within the framework of the additive evidentiary signals framework. An important direction for future work is to endogenize the occurrence of disputes, with the parties receiving signals of the facts, instead of the merits representing the addition of signals from an exogenous uniform distribution.

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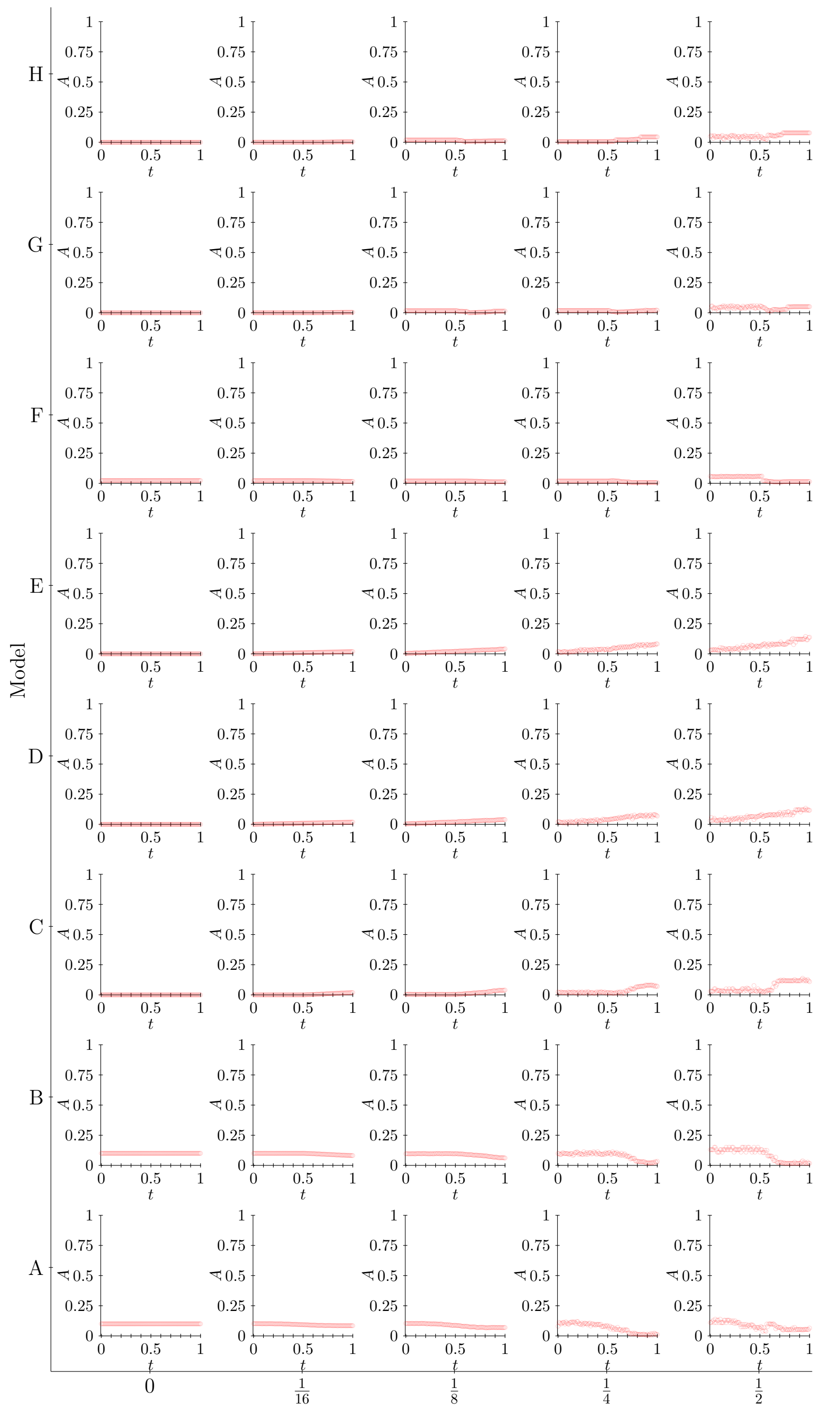

Figure 9: Overall accuracy averaged across values of q

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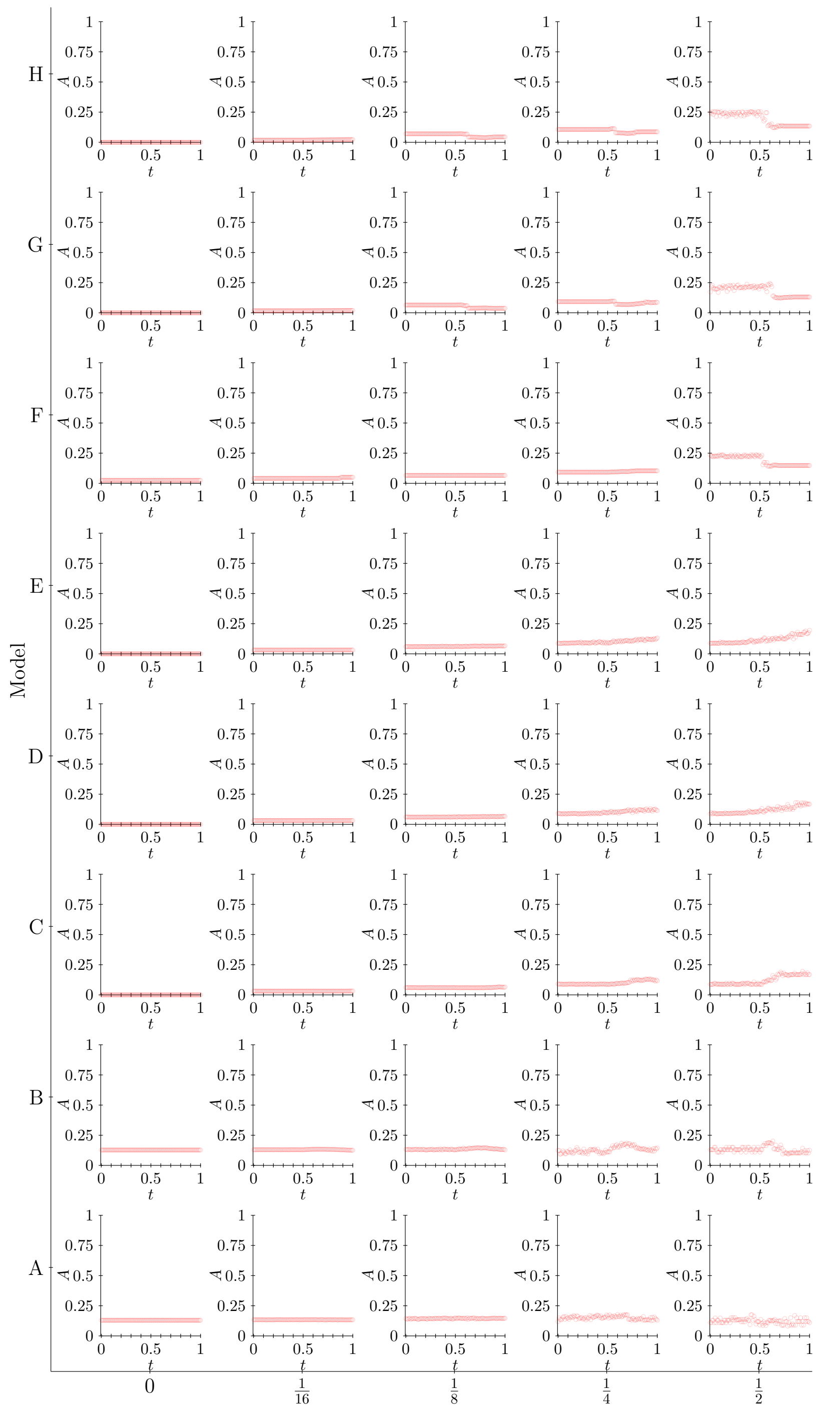

Figure 10: Undercompensation inaccuracy averaged across values of q
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