Modeling Fee Shifting With Computational Game Theory

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Abstract

Previous computational models of settlement bargaining in litigation have lacked game theoretic foundations. Modern mathematical models, by contrast, generally seek to identify perfect Bayesian Nash equilibria. Algorithmic game theory, however, can be used to compute such equilibria, and this article uses one tool of algorithmic game theory to illustrate how computation and mathematics can serve as complements in modeling settlement bargaining, with each addressing the limitations of the other. It does so by building on a cutting-edge model of settlement bargaining, Dari-Mattiacci and Saraceno (2020), which incorporates both two-sided asymmetric information and fee-shifting. The algorithm, introduced in von Stengel et al. (2002), applies linear programming techniques to the sequence form of the extensive-form game tree, and it computes exact perfect Bayesian Nash equilibria. The algorithm can be applied to optimize among piecewise linear strategies but also can relax the piecewise linearity assumption for a discretized version of the game, with each player receiving one from a finite set of signals and choosing one from a finite set of offers. This makes it straightforward to alter some assumptions in Dari-Mattiacci and Saraceno's model, including that the evidence about which the parties receive signals is irrelevant to the merits and that the party with a stronger case on the merits also has better information. The computational model can also toggle easily to explore cases involving liability rather than damages and can incorporate risk aversion. A significant drawback of the computational model is that bargaining games may have many equilibria, making it difficult to assess whether changes in equilibria associated with parameter variations are causal.

1 Introduction

The literature analyzing the effects of fee shifting confronts a daunting analytic challenge. Settlement bargaining is a two-player asymmetric information game.

The gold standard solution to such a game is a pair of common knowledge strategies that form a perfect Bayesian equilibrium. The perfection requirement, as defined by Fudenberg and Tirole (1991) [10], specializes the general Nash [29] equilibrium criterion in an imperfect information game, by insisting that at no point in the game may a player have any incentive to change the player's strategy. Each player applies Bayesian reasoning to incorporate new information, such as signals of case quality, into estimates of trial outcomes, and the player's probabilistic beliefs are required to be correct given such information. Complicating the challenge of crafting such equilibria, each party might decide not to contest the litigation, one or both parties may be risk averse, players may have asymmetric information, a case may concern liability or damages, and the loser may or may not be required to pay the winner's fees.

Incorporating anywhere near all of these considerations into a single model of settlement bargaining has proven elusive. The settlement bargaining modeler stands before a smorgasbord of potentially critical game features, but faces the admonition to choose no more than a few. The result, Daughety and Reinganum (1993) [6] observed, is a literature that "has grown in a disorganized fashion, resulting in a multitude of models involving different informational endowments and timing structures." This statement remains true nearly three decades later, with the settlement-bargaining-modeling art making unmistakable but limited progress. The earliest models of Landes (1971) [25], Posner (1973) [32], and Gould (1973) [12] had ignored the challenges of Bayesian inference. Later came models, such as Bebchuk (1984) [2] and Polinsky and Rubinfeld (1998) [30], in which one party knows the probability of liability or the amount of damages while the other party knows only the distribution, and Daughety and Reinganum (1994) [8], in which one party has information on liability and the other party, on damages. The latest generation of scholarship, including Friedman and Wittman (2006) [9], Klerman, Lee, and Liu (2018) [23], and Dari-Mattiacci and Saraceno (2020) [5], models two-sided asymmetric information, in which each of the plaintiff and defendant has independent private information about the same issue, for example about the level of damages. The last of these even succeeds at the Herculean task of incorporating fee shifting, but we will see that even it does not escape the curse of dimensionality, adopting a number of restrictive assumptions that make it difficult to assess the generality of its conclusions.

The literature is extraordinarily clever, in both the positive and negative senses of the word. It takes advantage of mathematical assumptions to make otherwise intractable problems tractable. We can expect further progress from relaxing different assumptions, but the goal of developing a single mathematical model that allows exploration of different values of a large number of variables may be unattainable. The literature develops critical intuitions about settlement bargaining, including how changing fee shifting rules may augment or diminish the effectiveness of the litigation system, and review articles, like Katz and Sanchirico (2012) [22], informally integrate various models' conclusions about how fee shifting might affect trial rates or settlement rates with different structures to the litigation game. Even with such reviews, however, it is difficult to generalize about the wisdom of fee shifting, because of the interactivity between trial and settlement rates. If, for example, increased ease of settlement leads to plaintiffs' bringing and defendants' defending more cases, total litigation expenditures in principle could rise.

Scholars have studied settlement bargaining with other methodologies, but these have their own limitations. Empirical analyses are limited to studying the rare cases in which a change in fee-shifting rules occurs, as in the examination by Hughes and Snyder (1995) [18] of a briefly-lived policy experiment in Florida. Laboratory experiments offer another approach, with contributions by Coursey and Stanley (1988) [4], Inglis et al. (2005) [20], Rowe and Vidmar (1988) [35], and Main and Park (2000) [27]. It is not clear, however, whether modest stakes produce results similar to those of real litigation. A final methodology in the literature is computer-based simulation. Priest and Klein (1984) [33], Katz (1987) [21], Hause (1989) [15], and Hylton (1993) [19], were pioneers in using computation, either independently or as complements to formal models. While these articles all include innovations building on the Landes-Gould-Posner model, they share a significant limitation: Unlike the math models, the simulations do not seek perfect Nash equilibria.

It is, however, possible to harness computational power in the quest for perfect Bayesian equilibria, by turning to computational game theory. The settlement bargaining literature has acknowledged the importance of game theoretic concepts, but articles build at most relatively small game trees. Acknowledging that litigation can be viewed as "a particular extensive-form bargaining game," Spier (1994, pp. 202-03) [38] sensibly worries that the results would be sensitive to issues such as "the structure of the asymmetric information." This concern suggests that solvable game theoretic models are insufficiently rich to encapsulate critical aspects of the litigation game. Computational game theory, however, allows for the identification of equilibria in games that could not practically be solved by hand. A subliterature focuses directly on the solution of two-player (and sometimes n-player) general sum games. A litigation game between plaintiff and defendant is general sum, which can be more difficult than a zero-sum game to solve, because the players may transfer wealth not only from defendant to plaintiff, but also from both litigants to lawyers. A useful review of algorithms that can help solve such games is von Stengel (2002) [39].

A publicly available open source software package known as Gambit by McKelvey et al. (2016) [28] features a number of these algorithms. This article, however, applies an algorithm not included in Gambit, specifically an algorithm described in an article in *Econometrica*: von Stengel, van den Elzen, and Talman (2002) [40]. This algorithm, described further below, is guaranteed to produce perfect Bayesian Nash equilibria in a finite game in which the players have perfect recall. Sometimes, these equilibria are pure, with players acting deterministically conditional on the information that they possess, but at other times, they are mixed, with the players randomly choosing at certain moments of the game between or among equally good strategies, each with some nonzero probability. Mixed strategies need not reflect explicit randomization by players; they may be understood as describing balanced populations of litigants who take different approaches, none better than others given opponents' strategies, for reasons exogenous to the model. The authors test their algorithm on games of up to 1,023 nodes. This article pushes the computational limits of the algorithm, applying it to each of a large number of games with up to 16,111 nodes. By separately identifying equilibria corresponding to different information and game structures, we can assess the conditions in which changing fee-shifting rules may increase or decrease the effectiveness of the litigation system, as manifested in accuracy or trial rates, assuming the litigation game is played in equilibrium by rational actors. This approach thus enables modeling of a richer and more diverse litigation environment than any single prior approach.

To illustrate how this approach can complement analytic models, this Article focuses on the model of Dari-Mattiacci and Saraceno (2020), the first article to integrate both two-sided asymmetric information and fee-shifting. In this model, both players know the true quality of the litigation, but the judgment depends not only on this value, but also on the sum of signals independently received by the parties.

Part 1 describes both the game trees to which the algorithm will be applied and the algorithm itself, and it reports statistics on how the size of the game tree affects the running time of the algorithm. Part 2 provides the central results. It begins by scrutinizing the results of two simulations, one in which the American rule applies and one in which the British rule applies, where all other variables are set to baseline values (admittedly values not calibrated to any particular real-world values). This provides some tentative findings, but explores a tiny proportion of the variable space. It then illustrates how a range of values for fee-shifting multipliers and the cost of litigation affect case dispositions (including filing and answer decisions, settlements, and decisions to quit after settlement failure), which in turn affect litigation accuracy and total expenditure levels. Then, the article deviates from baseline parameter values for other variables, demonstrating how changing risk preferences, information endowments, costs structure, and other aspects of game structure may affect outcomes, while continuing to vary cost and fee-shifting levels. Part 3 provides robustness tests of the principal results by considering a much larger number of equilibria identified for a smaller version of the game tree. This permits assessment both of whether the results are sensitive to the granularity of the game tree and whether it matters whether the players play an individual equilibrium, a correlated equilibrium (i.e., when one player plays a particular equilibrium, the other player plays its corresponding strategy) or an average equilibrium (i.e., where one player might be playing one equilibrium while its opponent plays another). Part 4 provides some conclusions and explains why they differ from earlier findings in the literature.

2 Analytical Models of Two-Sided Asymmetric Information

Because this article's goal is to illustrate how computational models can build on limitations of an analytical model, and vice-versa, we will focus on the structure and methodological choices in Dari-Mattiacci and Saraceno (2020). We begin by reviewing the Friedman and Wittman (2006) [9] upon with Dari-Mattiacci and Saraceno build, before summarizing the Dari-Mattiacci and Saraceno model and exploring some of the assumptions inherent in the article.

2.1 Friedman and Wittman's Averaged Signals Model

In the one-sided information models, the structure of bargaining often affects which party receives most of the surplus. Friedman and Wittman avoid this problem by adopting the bargaining protocol of Chatterjee and Samuelson (1983) [3]. In Chaterjee-Samuelson bargaining, the plaintiff and defendant simultaneously submit offers. If the plaintiff's exceeds the defendant's, the case definitively settles at the midpoint; otherwise, bargaining has failed. Costs of trial are borne only in the event of bargaining failure. Friedman and Wittman justify this bargaining structure not on the ground that the protocol is commonly used (it is not), but on the ground that it provides a useful reduced form of a more complicated bargaining process. In contrast to divergent expectations models, with Chaterjee-Samuelson bargaining, a case may go to trial even though there is a social surplus from settlement given the parties' expectations. The reason is that the parties may shade their offers in the hope of deriving a larger portion of the settlement surplus, even at the risk of bargaining failure.

The informational structure is arrestingly simple. The plaintiff observes a signal θ_p drawn from a known distribution, and the defendant independently observes a signal θ_d drawn from the same distribution. The principal results of the paper apply to a "basic litigation model" in which the distribution is the uniform distribution; this extends without loss of generality to any uniform distribution between a lower bound of L and an upper bound of U. In the event that settlement fails, a judgment is entered in the amount of the average $(\theta_p + \theta_d)/2$. Perhaps one can imagine situations in which this might be realistic. For example, the parties might have information about different components of damages in a case in which liability is uncontested, and should trial ensue, the information will be revealed and the judgment will be the sum. But a more intuitively appealing model in most situations would reverse the causality. Signals would depend on the underlying truth to be revealed at judgment, rather than the judgment dependingon signals. Friedman and Wittman cleverly recognize, however, that modeling litigation in this way makes the model tractable.

Friedman and Wittman derive a Nash equilibrium in the basic litigation game. In this equilibrium, the plaintiff will ordinarily offer $\frac{2}{3}\theta_p - 2c + \frac{1}{2}$, and the defendant will ordinarily offer $\frac{2}{3}\theta_d + 2c - \frac{1}{6}$, where c represents each party's trial cost. The word "ordinarily" signals what may seem a mild caveat: Neither party will ever make an offer beyond the range of the other party's possible offers. Thus, the plaintiff's offers are truncated above at $\min(1, 2c + \frac{1}{2})$ and below at $\max(0, 2c - \frac{1}{6})$, while the defendant's offers are truncated above at $\min(1, \frac{7}{6} - 2c)$ and below at $\max(0, -2c + \frac{1}{2})$. Friedman and Wittman do not eliminate the possibility that there might be some nonlinear Nash equilibrium, but they prove that the equilibrium they derive is the unique nontrivial piecewise linear equilibrium. There are also infinitely many trivial equilibria, in which the plaintiff's settlement demands always exceed the defendant's.

Friedman and Wittman's model permits them to focus on the trial rate. They derive a piecewise quadratic formula for the trial rate, and they also examine, in the tradition of Priest-Klein, how the trial rate varies near the midpoint of the decision spectrum. They show that when trial costs are sufficiently low ($c < \frac{1}{6}$), the probability of a trial is higher, the farther the judgment would be from $\frac{1}{2}$, and when trial costs exceed this threshold, the probability of a trial is highest at the $\frac{1}{2}$ point. The intuition is that when costs are high, the parties become more generous, and so the plaintiff's range of offers will fall below the defendant's. The truncations then ensure that cases at the extremes, where either both parties receive a low signal or both parties receive a high signal, are more likely to settle. When trial costs are low, the parties are less generous, and the plaintiff's range of offers will be above the defendant's. Cases at the extremes are then less likely to settle. With the basic litigation game, the $\frac{1}{6}$ cost threshold occurs where the parties' range of offers are equal. Friedman and Wittman also offer a graphical argument that extends to other continuous distributions, though they do not expressly consider the case where liability rather than damages is uncertain.

2.2 Dari-Mattiacci and Saraceno's Evidentiary Signals Model

Dari-Mattiacci and Saraceno (2020) illustrate the challenge of building on Friedman and Wittman by successfully extending the model to fee shifting. The article includes an online appendix with 60 pages of proofs. The difficulty stems from the need to address four principal cases, depending on relative values of parameters, and within these principal cases, to make various calculations that depend on the relative values of other parameters, including in many instances five different formulas for five different ranges of a variable. The resulting product is testimony both to human ingenuity and to endurance, and it makes breakthroughs in our understanding of the effects of fee-shifting with two-sided asymmetric information.

As in Friedman and Wittman, plaintiff and defendant receive signals, now denoted θ_Π and θ_Δ, respectively, and the judgment is an average of the signals. Now, however, both parties have common knowledge of the true merits of the litigation, denoted by q. The signals thus do not serve the function of informing the parties of the true merits, but rather of providing the parties with evidence that they may use to convince the court. The plaintiff's signal θ_Π is drawn from a uniform distribution on the interval $(0, q)$, and the defendant's signal, on the interval $(q, 1)$. Because the defendant's signal can be no less than q, the plaintiff's best possible evidence, where $\theta_\Pi = q$, would convince the court that the judgment must be at least q. Similarly, the defendant's best possible evidence, where $\theta_\Delta = q$, would convince the court that the judgment must be no more than q. But the litigants do not always draw the best possible evidence.

The fee shifting rule that Dari-Mattiacci and Saraceno primarily analyze is triggered based on (1) whether the final judgment is above or below $\frac{1}{2}$ (i.e., which party "wins" in the sense of being awarded more than half of the contested damages), and (2) whether the evidence of the winning party is sufficiently strong. If the judgment is less than $\frac{1}{2}$, then the defendant might be able to shift its costs to the plaintiff, but only if the defendant's signal falls below some threshold, i.e. $\theta_\Delta < t$, where $0 \leq t \leq 1$. Likewise, if the judgment is greater than $\frac{1}{2}$, then the plaintiff might be able to shift its costs to the defendant, but only if the plaintiff's signal exceeds a threshold, i.e. $\theta_\Pi > 1 - t$. An intuition is that if a party wins a case merely because its opponent has produced little evidence, a court will not order fee-shifting; another is that a court will only order shifting of fees when those fees were spent on producing strong evidence. Note that when $t = 0$, fees will never be shifted, so this extreme is the American rule of no fee shifting, and when $t = 1$, fees will always be shifted to a winning party (i.e., to the plaintiff if the final judgment exceeds $\frac{1}{2}$ and to the defendant if the final judgment is less than $\frac{1}{2}$), so that extreme is the English rule of universal fee shifting. The analysis thus effectively allows for a continuum of fee shifting rules.

This information structure enables Dari-Mattiacci and Saraceno to derive the offers that the parties will make. They prove that each party's offer function is a best response to its opponent's offer function and thus that a Bayesian Nash equilibrium exists. They also derive formulas for settlement amounts, along with identification of the ranges of parameters values where such settlements occur, and accordingly of the litigation rate. They prove that the litigation rate depends only on c (now representing the combined trial cost of the two parties) and is thus independent of both case quality q and the fee-shifting rule t. This produces the surprising conclusion that the litigation rate is the same under both the American and the English rule. Finally, they offer a calculation of litigation accuracy, and they prove that when costs are below a certain threshold, the English rule produces more accuracy than the American rule, while the reverse is true when costs are above a certain threshold. The stylized fact that litigation is cheaper in England may thus help explain the choice of rule in each country.

2.3 Assumptions in Dari-Mattiacci and Saraceno's Model

The Dari-Mattiacci and Saraceno model adopts a number of assumptions. Many of these assumptions appear to be driven, quite reasonably, by the demands of mathematical tractability, and it is difficult to develop strong intuitions for whether they matter. In identifying these assumptions, we create a series of challenges for a computational model that aspires to assess the robustness of the analytical model.

Structural constraints

Piecewise linearity Dari-Mattiacci and Saraceno explicitly assume a linear relationship between the parties' signals and their offers. They allow, however, for discontinuities in the linear relationship. In this sense, the strategies they model are similar to those of Friedman and Wittman, and indeed Dari-Mattiacci and Saraceno similarly truncate the parties' strategies. The assumption is somewhat stronger, however, in that Friedman and Wittman demonstrated that the piecewise linear strategies they derived would be a Nash equilibrium even when nonlinear strategies are possible. On the other hand, Dari-Mattiacci and Saraceno allow for additional discontinuities at points where fee-shifting would change. This is central to the design of their model and the thrust of their analysis. Because fee-shifting depends partly on the quality of the evidence possessed by the winning party, a litigant will know whether it will be entitled to fee-shifting if it wins, and the signal values at which this fact changes are points at which Dari-Mattiacci and Saraceno are able to break the problem down into smaller pieces. Piecewise linearity thus allows for explicit modeling of the effects of changes in a fee-shifting rule, but because it is unclear how restrictive this assumption is, it is a prime candidate for relaxation in a computational model.

Asymmetric information quality equivalence Recall that the plaintiff receives a signal in the range $(0, q)$ and the defendant, in $(q, 1)$. As a consequence, when $q > \frac{1}{2}$, the plaintiff's signal has a greater potential effect than the defendant's, and when $q < \frac{1}{2}$, the reverse is true. The single variable q thus serves two, independent functions in the model: one is to represent the "true merits" of the case, while the other is to represent the degree of information asymmetry. This greatly increases the tractability of the model, and plausibly it allows for consideration of both issues related to accuracy and issues related to information asymmetry. The problem, though, is that the issues are necessarily conflated; where a case is at an extreme of the probability distribution, there is always high information asymmetry. There is no particular reason to believe that true merits should generally track information asymmetry in this way. The question thus arises whether the results would be the same if the model allowed independent variation of true merits and information asymmetry.

Parameter values

Balanced asymmetric information Meanwhile, the true merits variable is constrained so that $\frac{1}{3} \leq q \leq \frac{2}{3}$. The reason for this constraint is that with more extreme values of q, the increasingly one-sided nature of asymmetric information leads the pure strategy equilibria derived by the authors to break down. This highlights once again the problematic nature of asymmetric information quality equivalence, because it means that the authors not only cannot model situations with relatively high information asymmetry, but also that they cannot model situations in which the true merits of a case are near the extremes of the probability distribution. Perhaps a computational model might be able to find an equilibrium with relatively extreme quality values and/or with relatively extreme information asymmetry, and this could help extend the understandings provided by the model.

Low or moderate cost Dari-Mattiacci and Saraceno follow Friedman and Wittman in implicitly assuming that the cost variable is not so high that the plaintiff's untruncated offer range is entirely below the defendant's untruncated offer range. The truncation functions defined by Friedman and Wittman are undefined, because when their c is sufficiently high, they instruct that the plaintiff's offers should be truncated above at 1 and below at a number greater than 1, and similarly the defendant's offers are truncated below at 0 and above at a number less than 1. With sufficiently high costs, there will be many Nash equilibria; the parties will be determined not to go to trial, but neither party would deviate from any positive allocation of the surplus from settlement. Literal application of the Dari-Mattiacci and Saraceno formula, however, would lead to both players truncating their bids and would not choose any of these equilibria.

In a subtle way, the Dari-Mattiacci and Saraceno cost assumption is more restrictive than Friedman and Wittman's. As we will see shortly, a computational model can be used to assess whether parties' strategies form an equilibrium, and with sufficiently high costs, the bid functions identified by Dari-Mattiacci and Saraceno in some cases do not form equilibria. This can be traced, at least in part, to a complication in what Dari-Mattiacci and Saraceno call Case 4B. They implicitly assume that the bid functions that they derive would each contain a discontinuity, but if $6c(1-q) > 1$, the plaintiff's bid function consists only of a single line segment. It can be shown, for example, that for the parameters $t = 0.8, q = 0.4, c = 0.3$, the plaintiff's strategy cannot be a best response, because, given the defendant's presumed strategy, the plaintiff would be very slightly better off with a bid function in which it always bids one-third of its normalized signal. In correspondence, Dari-Mattiacci and Saraceno have acknowledged this complication and that their model implicitly assumes that c is not too high. This is a reasonable assumption, but a challenge for the computational model is to overcome it.

Risk neutrality The plaintiff and defendant are assumed to be risk neutral. Incorporating risk aversion into the model would likely add considerable challenge, though the argument could still proceed in case-by-case fashion. Incorporating risk aversion is virtually costless to a computational model, requiring only the transformation of the parties' utilities in any game outcome.

Game structure

Fee-shifting structure Fee shifting in Dari-Mattiacci and Saraceno's model depends not only on which party wins more than half of the judgment at trial, but also on the quality of the evidence produced by the winning party. This is mathematically convenient, because each party knows the quality of its own evidence and thus whether fee-shifting will occur for any given value of the opponent's signal and any value of t. An alternative approach, however, would be for fee-shifting to depend on both parties' evidence. Indeed, Dari-Mattiacci and Saraceno explicitly consider fee-shifting based on the margin of victory, defined by a parameter m, where $0 \leq m \leq 1$. With this approach, if $\theta_\Pi + \theta_\Delta < m$, then the plaintiff must pay the defendant's fees, and if $\theta_\Pi + \theta_\Delta > 2 - m$, then the defendant must pay the plaintiff's fees. In this regime, if $m = 0$, no fee shifting occurs (the American rule), and if $m = 1$, fee shifting always occurs absent an evenly split judgment (the English rule); thus, the margin-of-victory approach converges with the other approaches at the extremes. Dari-Mattiacci and Saraceno explicitly calculate the parties' offers under this approach, but they do not prove their results related to accuracy. This raises the question whether their accuracy results are robust to the alternative specification. One might also imagine other fee-shifting rules, such as a simple rule in which a party always occurs when the party wins half of the judgment but the proportion of fees shifted may vary from 0 to 1.

Damages vs. liability Dari-Mattiacci and Saraceno explicitly describe their model as one in which the parties are arguing about how to divide a disputed asset, such as in a case of divorce, and they point out that without loss of generality, this can be extended to a judicial determination of damages between some minimum and maximum value. An extension would be to consider cases where liability is at issue, i.e. where the plaintiff will receive 1 if $\theta_\Pi + \theta_\Delta > 1$ and 0 otherwise. For example, they might generalize the model to an arbitrary cumulative distribution function mapping $\theta_\Pi + \theta_\Delta$ onto the judgment, but this would add considerable challenge. Once again, this should be trivial in a computational model, which need only transform the judgment values in particular cases, either to 0 or 1 or based on some other distribution.

Signal variance independent of true merits Dari-Mattiacci and Saraceno refer to the signals that the parties receive as "evidence" of the true merits of the case, but there is a paradox: The parties are assumed to know the true merits of the case (q) and indeed use this information in constructing their offer functions. Thus the variance in the signals that each party may receive has nothing to do with the merits. Given the fixed value of q, whether the plaintiff receives a signal slightly above 0 or slightly below q tells the plaintiff nothing about the true merits. What receipt of the signal accomplishes is to inform the plaintiff about the plaintiff's likely ability to persuade the judge about the true merits. The judge does not know the true merits, but is trying to guess the true merits. The higher q, the higher the parties' signals will tend to be, so the judge's strategy is reasonable, even if non-Bayesian. But the result is that from the perspective of the parties, for whom q is fixed, the randomness in case outcomes has to do only with who is lucky in finding promising evidence.

This point can be more clearly seen in a transformation of the model that Dari-Mattiacci and Saraceno offer. They note that the signals θ_Π and θ_Δ can be mapped one-to-one onto signals from 0 to 1, which they label z_Π and z_Δ. These signals are thus independent signals from a unit uniform distribution, and the θ signals can be derived from them according to the formulas $\theta_\Pi = qz_\Pi$ and $\theta_\Delta = q + (1-q)z_\Delta$. This highlights that the θ signals result from commingling the true merits of the case and the random uniform distribution draws. With these transformations, the judgment depends on the following formula:

$$J(z_\Pi, z_\Delta) = \frac{1}{2}q + \frac{1}{2}(qz_\Pi + (1-q)z_\Delta) \tag{1}$$

As this presentation makes clear, the decision is half based on the true merits of the case, independent of any evidence presented by the parties. Meanwhile, the decision is half based on a weighted average of the parties' uniform distribution draws, with the weights equal to q. Recall that q represents the degree of information asymmetry. Thus, in effect, half of the judge's decision is based on the true merits and half of the judge's decision depends on a weighted average of signals that are entirely independent of the true merits. The only reason that this makes sense from the perspective of the judge is that the judge does not observe z_Π and z_Δ directly. The Dari-Mattiacci and Saraceno model could be realistic in some contexts, in which the parties have asymmetric information about their persuasive abilities, independent of the merits. In any event, their approach may be necessary in a mathematical model that seeks, as theirs does, to measure accuracy. It is considerably easier (though still extraordinarily difficult) to measure outcomes relative to the constant q than it would be relative to a function of q and the parties signals.

A challenge for the computational model is to assess whether results about accuracy continue to obtain when the true merits are defined to be inclusive of the parties' normalized signals. On this formulation, q would represent knowledge that the parties share about the true merits, and the z signals represent private information about the true merits. When the judge adds these together according to the above formula, the judge obtains not only the judgment, but also the true merits. This is thus a conceptual reformulation with no implications for which cases settle. It requires only an alteration of the definition of accuracy.

Accuracy definition Dari-Mattiacci and Saraceno define inaccuracy in their appendix as "the square distance between the expected outcome E_t and the merits q." We have already explored how we might reconceive the definition of the merits, so let us continue moving from right to left in this definition.

The definition of E_t is complex, involving double integrals over both costs and the parties' signals. The essence is that it is a measure of the expected outcome of a dispute, taking into account both the settlements and the trials. The outcome in the event of trial that they calculate is represented by G, which "captures both the decision on the merits and fee shifting." For example, if the judgment is for 0.45 and the plaintiff pays costs of 0.10 to the defendant, then $G = 0.35$. The inclusion of fee shifting costs reflects that imposition of fee shifting not only affects settlement negotiations, but also affects the amount that the plaintiff must pay to the defendant at trial. It is reasonable to view the difference between the expected value of G and the value of q as a measure of accuracy, but the question remains whether conclusions about accuracy would be robust to alternative specifications.

Outcome expectation The specification chosen focuses on the expectation of settlement or trial results, rather than on the actual result in particular cases. It is a comparison of the expectation of the result with the true merits, not a measure of the error. If, for example, there are two scenarios in which the correct result based on the true merits would be for the defendant to pay the plaintiff 0.50, and in one scenario the defendant pays 0 and in the other scenario the defendant pays 1, then this measure would count the legal system as perfectly accurate. Because the parties are risk-neutral, they would be indifferent between receiving perfectly accurate results and results that are correct on average.

An alternative measure, which arguably would be more appropriate at least for risk-averse parties, would aggregate the distance between the actual outcome and the ideal outcome in each case. In more technical terms, instead of calculating a measure of inaccuracy that is a function of E_t, the authors might have calculated a measure of expected inaccuracy in which the inaccuracy is calculated within each case rather than based on an average across cases. Easier said than done, of course. In the analytical model, this would require moving a minus q term and a squared term within the double integrals in the current E_t definition.

Accounting for costs The accuracy measure also ignores the pre-fee shifting costs that the parties pay. Dari-Mattiacci and Saraceno note "that the plaintiff receives $G - \frac{c}{2}$ and the defendant pays $G + \frac{c}{2}$." Imagine a case with very high costs and no fee shifting, where each party spends a million dollars and the court arrives at precisely the correct conclusion that the defendant owes the plaintiff 50 cents. From this definition's perspective, this outcome counts as a perfectly accurate result. That is a plausible definition of accuracy, but one that offers no comfort to the parties. An alternative definition of accuracy would consider any amounts actually spent at trial, for example counting the outcome from the plaintiff's perspective as $G - \frac{c}{2}$. A similar definition could measure accuracy from the defendant's perspective. Either of these two approaches captures three distinct aspects of costs: (1) the costs impact settlement negotiations; (2) when trial occurs, the costs are deadweight losses to society at large; and (3) costs may reduce (or perhaps in some cases increase) the accuracy of adjudication viewed as a black box from the perspective of each individual litigant. Although it is reasonable for Dari-Mattiacci and Saraceno to define accuracy entirely independently of cost, an interesting question is whether any conclusions based on this definition will extend to definitions that incorporate costs.

Squared vs. absolute value Finally, one might quibble about the use of a squared term rather than an absolute value. Of course, it is conventional to measure (in)accuracy using the ℓ_2 norm rather than the ℓ_1 norm. The convention reflects the dominance of ordinary least squares regression over least absolute deviation regression, but that dominance stems as least in part from the greater tractability of the former. Portnoy and Koenker (1997) [**portnoykoenker**] note that computational power mitigates this advantage, and that an advantage of the ℓ_1 norm is that it is more robust to outliers. Because the Dari-Mattiacci and Saraceno definition compares the outcome expectation with q, any results on accuracy necessarily extend to the ℓ_1 norm. The computational results here will be reported using the ℓ_1 norm, because interpretation is more intuitive and because this will make it more straightforward to compare different accuracy measures.

3 Litigation as an Extensive Form Game

This section describes an algorithm that can be used to find equilibria in any extensive form game. It illustrates that this approach can come close to replicating the simpler Dari-Mattiacci and Saraceno results by applying the algorithm to a model in which each party selects a slope for its bid function and a truncation. The nmore flexible approach, however, relaxes the assumption of piecewise linearity by discretizing each player's signals and offers.

3.1 The von Stengel, van den Elzen, and Talman Algorithm

In principle, any two-player extensive form game with a finite number of strategies can be presented in strategic form, with the parties' payoffs embedded in a matrix. The row player may use a pure strategy and select a single row from the matrix or a mixed strategy, a probability distribution over the matrix rows. The column player analogously chooses a column or a probability distribution over matrix columns. Nash (1951) famously proved that every strategic game involving a finite number of players has at least one equilibrium, now called a Nash equilibrium, in which neither player could increase the player's payoff by switching to a different strategy given the opponent's strategy. Moreover, given a game in strategic form, any equilibrium in pure strategies can be identified relatively easily by the algorithm of iterative elimination of dominated strategies. If a cell of the matrix exists where the column player's utility is greater than in any other cell in its row and the row player's utility is greater than in any other cell in its column, then that cell represents a Nash equilibrium.

This approach is insufficient to find mixed strategy equilibria. The problem of finding Nash equilibria, however, can be converted into a linear programming problem. The reader with a strong interest in the algorithm applied here should read von Stengel (2002) for a comprehensive overview, including proofs that it produces equilibria. The reader interested solely in legal ramifications may skip this section altogether. A virtue of using computational game theory to identify equilibria is that one may choose to treat algorithms as black boxes, particularly because all equilibria found in this article were computationally verified.

Still, we will provide an introduction to the algorithm for the reader with an intermediate level of interest. Let the payoffs for players 1 and 2, respectively, be represented by the $M \times N$ matrices A and B, whose entries are positive; any game with negative payoffs can be scaled to an equivalent such game. Player 1's strategy can thus be represented by a vector $x \in \mathbb{R}^M$ whose components sum to 1, and Player 2's strategy can be represented by a vector $y \in \mathbb{R}^N$ whose components have the same property. Using matrix notation, if $E = [1, ..., 1] \in \mathbb{R}^{1XM}$ and $F = [1, ..., 1] \in \mathbb{R}^{1XN}$, then $Ex = 1$, $Fy = 1$, and $x, y > 0$. Player 1's strategy is called a "best response" to a strategy y if it maximizes the expected payoff $x^T Ay$ subject to the $Ex = 1$ constraint, and similarly a strategy y maximizing $x^T By$ subject to the $Fy = 1$ constraint is a best response to x. The strategy pair (x, y) forms a Nash equilibrium if each is a best response to the other. To find a Nash equilibrium, one can take advantage of a linear programming principle known as duality. Under strong duality, if there is an optimal solution to a "primal LP," then a dual LP has the same optimal solution. In the dual LP, the variables and constraints are reversed and the objective (maximization or minimization) is inverted, relative to the primal LP. Thus, the primal problem of maximizing $x^T Ay$ subject to $Ex = e$ (where $e = 1$) has a dual problem of finding u that minimizes $e^T u$ subject to $E^T u - Ay \geq 0$. A similar dual minimization problem can be used to represent player 2's constrained maximization of $x^T By$.

In von Stengel (2002), Theorem 2.4 shows that (x, y) form a Nash equilibrium if and only if, for some u and v, all of the following hold:

$$\begin{aligned}
x^T(E^T u - Ay) &= 0 \\
y^T(F^T v - B^T x) &= 0 \\
Ex &= e \\
Fy &= f \\
E^T u - Ay &\geq 0 \\
F^T v - B^T x &\geq 0 \\
x, y &\geq 0
\end{aligned}$$

These conditions collectively define a mixed linear complementarity problem (LCP). The word "complementarity" refers to the fact that because x, y, A, and B are nonnegative, the first line implies that x and $E^T u - Ay$ cannot have a positive component in the same position. The same holds for y and $F^T v - B^T x$. The best known algorithm for finding a solution to a linear complementarity problem is that of Lemke and Howson (1964) [26].

Lemke and Howson is inadequate to the settlement bargaining problem, however, because the bimatrix game will be too large. When an extensive form game is converted to a matrix, the matrix must contain a separate row for all permutations of the choices that the row player may make at each information set, and similarly a column for each permutation of the column player's information set choices. For example, if each player has 10 different information sets (perhaps corresponding to 10 different possible signals), and may make 10 different moves at each information set, then $10^(10)$ rows or columns would be needed for that player. Several different scholars, the earliest being Romanovskii (1962) [34], offer a way around this dilemma. The trick, as explained by Koller, Megiddo, and von Stengel (1996) [24], is to craft a matrix in which the rows or columns represent not a player's pure strategies, but instead the sequences that the player may play at any stage of the game. In the above example, there would be 101 sequences per player (10 for each information set plus an empty sequence). If we inserted an opportunity by each player to quit after receiving its signal, then there would be 121 sequences per player (the above, plus decisions to quit or not at each of the 10 signals). Some sequences may be incompatible, because it may be impossible for a player to play a certain information set given a move by another; thus, the relevant matrix includes a zero in such cells.

The von Stengel, van den Elzen, and Talman (2002) algorithm adapts the "sequence form" approach by combining techniques used in von Stengel (1996) with developments in van den Elzen and Talman (1999), which adapts the Lemke-Howson algorithm to ensure perfect equilibria. The result is an algorithm that produces perfect, Nash equilibria. The requirement of perfection would not be needed if players committed to strategies at the outset of the game. But in litigation, such commitments do not occur. Thus, even if an equilibrium meets the Nash criterion, meaning that neither player will have an incentive at the outset of the game to change its strategy choice given the opponent's strategy choice, it may be imperfect, if a player might have an incentive to deviate in its strategy choice after learning new information. The solution concept of perfect Bayesian equilibrium, a term in use since at least the 1960s, has various formal definitions in the literature, including that of Fudenberg and Tirole (1991) [10], and is the imperfect information analogue of the subgame perfection equilibrium concept proposed by Selten (1965) [36].

The algorithm is initialized by setting every information set to a fully mixed behavior strategy, that is a strategy in which each action has some positive probability of being played. A data structure called a tableau is initialized in turn based on these values. This tableau encodes not only action probabilities, but also the relationships among information sets, specifically which information sets may follow other information sets in sequence. The initialization is designed so that the equations reflected by the tableau reflect a "basic feasible solution" to some of the equations in the linear complementarity problem, albeit ordinarily not a solution that corresponds to a perfect Nash equilibrium. The algorithm then proceeds through a series of pivoting steps, which represent linear algebraic manipulations in which variables are added to or removed from the list of non-zero variables known as the basis. When the variable initially added to the basis eventually leaves the basis, a Nash equilibrium is guaranteed.

The algorithm is not without its limitations. First, the algorithm is not guaranteed to produce all Nash equilibria. One may attempt to obtain multiple equilibria by choosing different initializations of the information sets. Second, to ensure that an equilibrium is reached, the algorithm must be performed using exact arithmetic with rational numbers, rather than floating point arithmetic. Even given the generous amount of storage allowed by modern 64-bit computer architectures, floating point operations involve rounding, and that may prevent the algorithm from converging. The use of exact arithmetic is cumbersome, however, given the frequent need to calculate greatest common factors of integers with many digits. Third, the algorithm is roughly linear in the size of the game tree, not in the size of the set of player sequences. The game tree grows exponentially as more moves are added to the game.

3.2 A Near Replication

To replicate Dari-Mattiacci and Saraceno as closely as possible, we must restrict each player's bid function to consist of piecewise linear segments. Yet, there are an infinite number of possible combinations of piecewise linear segments, and so we must adopt some restrictions. In theory, we could, for example, require each player to choose one of 100 starting values and one of 100 ending values for each of 10 signal ranges; for example, plaintiff might bid 0.47 at $z = 0.5$ and 0.92 at $z = 0.6$, representing one such segment. But, for each player, this would produce $(100 \times 100)^{10}$ possible strategies, and the game tree would be the square of this in size.

For this approach to be feasible, the strategies must be severely constrained. For this preliminary exercise, we thus allow each of the plaintiff and defendant (denoted P and D) to choose an offer $\mathscr{B}$ line with a slope $\mathscr{m}$ in $\{\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, 1\}$ and a minimum value of $\mathscr{b}$ in $\{0.02, 0.06, 0.10, \ldots, 0.98\}$. Because truncations are critical to the Dari-Mattiacci and Saraceno equilibria, each player also chooses a portion $\mathscr{t}$ of this line to truncate in $\{0, \frac{1}{9}, \frac{2}{9}, \ldots, 1\}$. Thus, $\mathscr{B}_P = \max(\mathscr{m}_P \cdot z + \mathscr{y}_P, \mathscr{t}_P)$ and $\mathscr{B}_D = \min(\mathscr{m}_D \cdot z + \mathscr{y}_D, \mathscr{t}_D)$. Each player may in effect choose from $25 \times 4 \times 10 = 1,000$ strategies, so the game tree consists of 1,000,000 final nodes. At each of these nodes, for each of 10,000 combinations of the players' signals, the players' utilities are calculated by determining based on the corresponding strategies, whether the case settles and the resulting outcome of the game.

Figure 1 illustrates a result of the algorithm, in this case for parameters $q = 0.4$, $c = 0.2$, and $t = 0.4$. The panels on the left represent the plaintiff's strategies; on the right, the defendant's. The results of the computational model, on the bottom, are reasonably similar to the equilibrium for these parameters identified in the analytical model of Dari-Mattiacci and Saraceno, on the top, though hardly an exact match. This result is an exact equilibrium in the game as defined with these restrictions, but it is only an approximate equilibrium in the original Dari-Mattiacci and Saraceno game, which does not impose these restrictions.

Still, by combining a number of such parameter sets, one can perform a

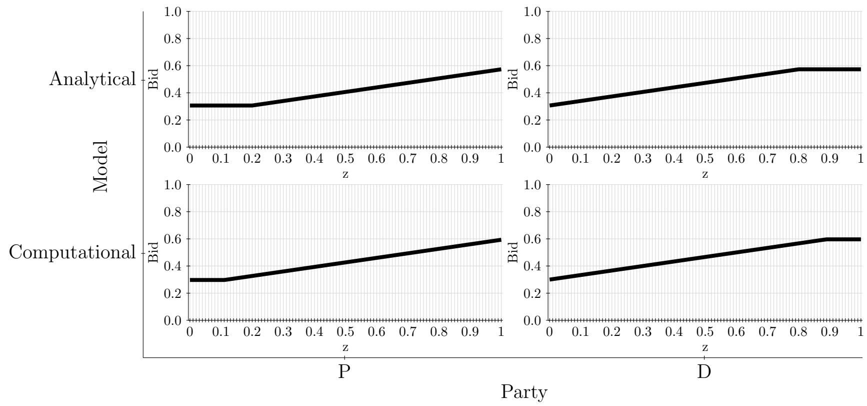

Figure 1: Attempted replication with $q = 0.4$, $c = 0.2$, and $t = 0.4$

task similar to that of Dari-Mattiacci and Saraceno, observing how changes in parameters affect the equilibrium. The results of a number of such observations are available in the Online Supplement to this article in the "DMS Replication" folder, corresponding to parameter values that are multiples of 0.1 that meet Dari-Mattiacci and Saraceno's assumptions regarding q and c and for which in their analytical model neither party's strategy consists of more than a single line segment. The results in Figure 1 are typical, though there are some surprises, like Figure 2, in which the algorithm calculates a mixed strategy equilibrium, with the relative darkness of each of the parties' strategies corresponding to the probability that a party will play them. Though it seems unlikely that any individual would play such a strategy, a mixed strategy can be interpreted as a set of pure strategies that might be played by different litigants, where neither litigant knows the other litigant's type. Given the significance of mixed strategies in games like chicken that bear some resemblance to settlement bargaining, the ability of the algorithm to find mixed strategies is a strength of the computational approach, but it complicates any attempt at replication.

Although these results illustrate that the algorithm can approximately replicate some analytical results, they may be disappointing. The most interesting results of Dari-Mattiacci and Saraceno occur in cases in which the strategies are discontinuous, and we have not even attempted to replicate those cases. The reason that the algorithm is so constrained in this preliminary analysis is that it does not fully take advantage of the sequence form. Each party has a limited number of information sets, in many of which it considers a large number of possibilities (such as which of 25 minimum values to select) that in turn dictate the party's strategy over a range of signals. The result is not much better than could be achieved by applying iterated elimination of dominated strategies to the full bimatrix game. Still, the exercise is sufficiently simple that one might wonder why the settlement bargaining literature has eschewed even that well known technique. This approach is more flexible than a simple linear model,

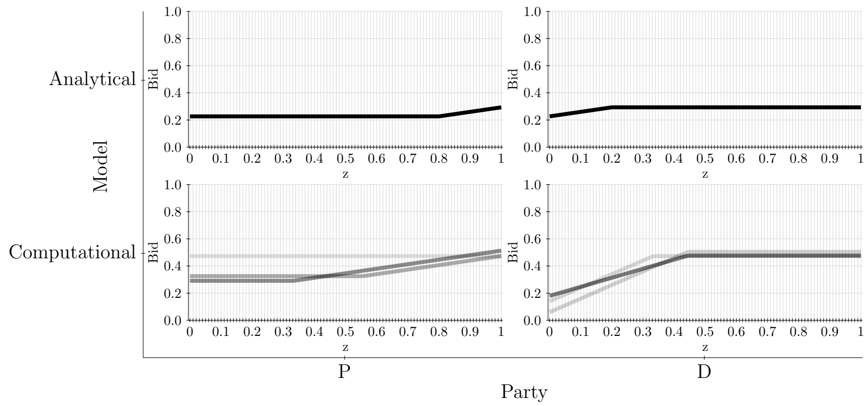

Figure 2: Attempted replication with $q = 0.4$, $c = 0.3$, and $t = 0.6$

and it can be applied to a wide range of game specifications that are analytically intractable. Still, a more fruitful strategy is to discretize the model, so that each player has separate information sets based on the signals that it receives.

3.3 A Discretized Evidentiary Signals Game

In the discretized model, each party receives a discrete signal from a set of n_S possible signals and then must make a discrete offer from $n_{\mathscr{B}}$ available offers. For legibility, Figure 3 illustrates a highly simplified version of this game for $n_S = 2$ and $n_{\mathscr{B}} = 2$, but we will generally use $n_S = 10$ and $n_{\mathscr{B}} = 10$, which produces a game tree consisting of 11,111 nodes. The circles identify the players (Chance, Plaintiff, or Defendant), as well as information set numbers. The plaintiff does not observe the signal received by the defendant and vice-versa. Thus, for example, at each of the four points in the tree labeled as "D1," the defendant has the same information set, in which it has received the signal 1 (corresponding to $z_\Delta = 0.25$) instead of the signal 2 (corresponding to $z_\Delta = 0.75$). Thus, the defendant must assign the same move probabilities to its two alternative offers (corresponding to 0.25 and 0.75) at each of these points. The diagram illustrates an equilibrium identified by the algorithm for this simple game, in which each player is always aggressive, with the plaintiff demanding 0.75 and the defendant offering only 0.25. Under the Chaterjee and Samuelson bargaining protocol, this does not result in a settlement.

The strategies permitted with this approach are in some ways more constrained and in some ways less constrained than the strategies in Dari-Mattiacci and Saraceno. The analytical model restricts the litigants from playing nonlinear strategies between the points that produce discontinuities in their proferred equilibria. The computational model may allow for nonlinearities between the critical points, but only at the 10 discrete signal values. A further restriction of the computational model is that offers can be made only at 10 discrete points. Thus, analytical and computational approaches both impose admittedly arbi-

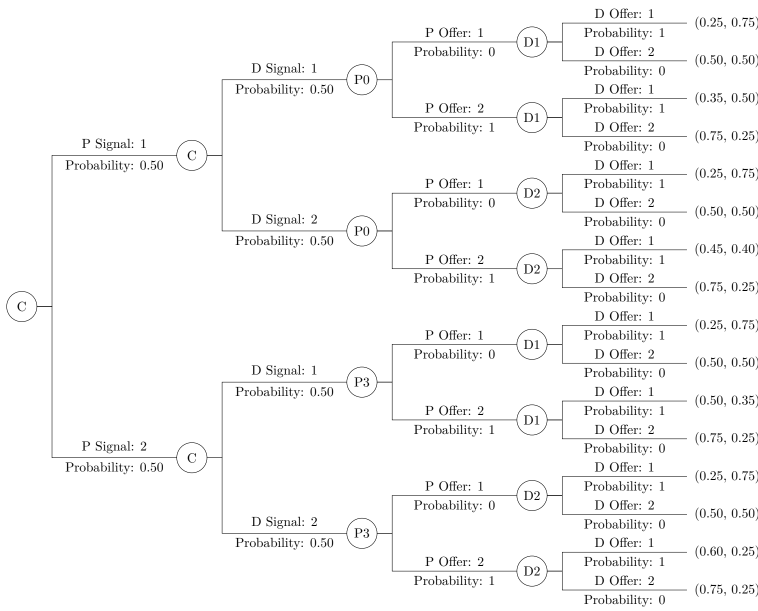

Figure 3: The game tree with $n_S = 2$ and $n_{\mathscr{B}} = 2$

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trary constraints on what strategies are permissible. This is, of course, not unusual, and many articles in the settlement bargaining literature impose tighter constraints, such as in models where a party receives one of two signals instead of one of ten.

An important question is whether these differing restrictions significantly affect the equilibria. One can reason that these assumptions do not seem critical to game dynamics, but it is difficult to be sure. There is no way to test whether the piecewise linearity assumption is driving the analytical model's key conclusions without some other model that relaxes those assumptions. Fortunately, the computational model can be such a model. Later, we will use the computational model to calculate equilibria employing the discretization assumption. But we can also use computation to measure the extent to which, for any given parameters, a discretized version of the Dari-Mattiacci and Saraceno model deviates from equilibrium.

This assessment requires two steps. First, we calculate each player's offer value at the discrete signal values, and then identify the discrete offer value nearest that value. This produces discretized versions of the bid functions derived by Dari-Mattiacci and Saraceno. Second, we determine each player's best response to the other player's strategy using an algorithm detailed in [CITE] and use these to calculate the exploitability of those strategies. That is, let $\sigma_{\mathscr{p}}$ represent the strategy of player $\mathscr{p}$ at the discretized equilibrium, let $\sigma_{-\mathscr{p}}$ represent the strategy of that player's opponent, and let $\mathscr{U}_{\mathscr{p}}(\sigma_{\mathscr{p}}, \sigma_{-\mathscr{p}})$ represent the expected return of player $\mathscr{p}$ given these strategies. Now, let $\sigma_{\overline{\mathscr{p}}}$ represent the best response of player $\mathscr{p}$ to $\sigma_{-\mathscr{p}}$. We then define exploitability of player $-\mathscr{p}$'s strategy $\mathscr{E}_{-\mathscr{p}} = \mathscr{U}_{\mathscr{p}}(\sigma_{\overline{\mathscr{p}}}, \sigma_{-\mathscr{p}}) - \mathscr{U}_{\mathscr{p}}(\sigma_{\mathscr{p}}, \sigma_{-\mathscr{p}})$. Allowing P and D to represent, respectively, the plaintiff and defendant in the discretization of the Dari-Mattiacci and Saraceno equilibrium, we define overall exploitability $\mathscr{E} = \frac{\mathscr{E}_P + \mathscr{E}_D}{2}$, i.e. the average of the amounts that each player can improve its score by changing its strategy holding the other's strategy constant. By definition, at a Nash equilibrium, overall exploitability is zero, and so $\mathscr{E}$ provides a measure of how much discretization moves the strategies from Nash equilibrium.

Figure 4 illustrates the results. The outer horizontal c axis represents trial cost. The outer vertical p axis represents litigation quality. All of the later simulations will be executed with $q \in \{\frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6}\}$, but we omit the row for $q = \frac{2}{3}$ where it is identical or symmetric to the row for $q = \frac{1}{3}$, and we omit the extreme values of q because they are outside the bounds assumed by Dari-Mattiacci and Saraceno. (Diagrams including these rows, both for this and for other truncated diagrams produced later, are available in the Online Repository.) Each mini-graph represents the measurement of exploitability $\mathscr{E}$ for each value of the fee-shifting threshold t in $\{0, 0.01, ..., 1\}$. We can see that for $c \leq \frac{1}{8}$, exploitability is very close to zero; overall all of these cases, the average $\mathscr{E}$ value is 0.00047. This indicates that the discretization changes the strategic dynamics of the game very little. With higher trial costs, considerably higher exploitability values (up to 0.067) can be seen in Figure 4. This is likely due to Dari-Mattiacci and Saraceno's implicit assumption, noted earlier, that c is not too high.

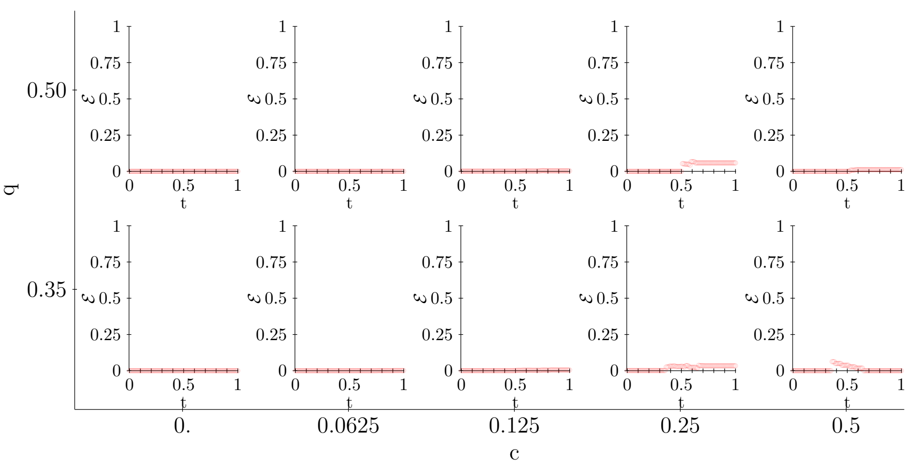

Figure 4: Exploitability of discretized Dari-Mattiacci and Saraceno strategies

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Using the discretization of the Dari-Mattiacci and Saraceno strategies, we can calculate outcome variables. Results on accuracy are illustrated in Figure 5. The accuracy A is defined analogously to Dari-Mattiacci and Saraceno, i.e. by comparing each party's expected outcome (taking into account settlements and judgments, including fee shifting, and not taking into account each party's own legal costs) to the true merits q, though for easier visual interpretability, the absolute value instead of the square of the difference is shown. This reflects, as Dari-Mattiacci and Saraceno prove, that for relatively low costs, the English rule produces at least as good accuracy (relative low A) in comparison to the American rule. This does not reflect that the reverse is true with relatively high costs, again presumably due to their implicit assumption governing costs. Note also that accuracy is very high when $q = \frac{1}{2}$. This does not indicate that each case produces the correct result, but that the Dari-Mattiacci and Saraceno accuracy measure is based on averages across cases, and with $q = \frac{1}{2}$, the cases in which the plaintiff receives too much exactly balance those in which the plaintiff receives too little.

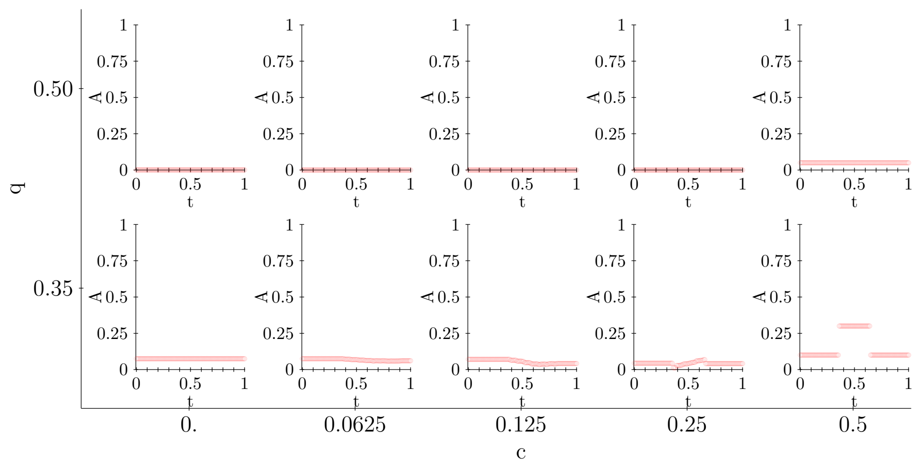

Figure 5: Accuracy with discretized Dari-Mattiacci and Saraceno strategies

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Figure 6, meanwhile, illustrates the analytical model's conclusions regarding the trial rate L. As Dari-Mattiacci and Saraceno show, and as is reflected here at least with $c \leq \frac{1}{8}$, trial rates are invariant to the degree of fee shifting and the quality of the true merits, though small wobbles are observable here as a result of discretization. Also as expected, trial rates fall as the cost of trial rises. Note that trial rates are relatively high, perhaps in part because of the assumption of risk neutrality.

The accuracy and trial results, it is worth emphasizing, are simply calculations based on discretizations of Dari-Mattiacci and Saraceno's constructed equilibria, not equilibria calculated by application of the von Stengel et al. algorithm. These measurements, however, provide a baseline that we can compare to results from equilibria calculated by that algorithm. We will thus be able to assess whether exact, perfect equilibria calculated by the algorithm produce similar results in the discretized version of the original additive evidence game and in variations generated by relaxing various assumptions of the Dari-Mattiacci

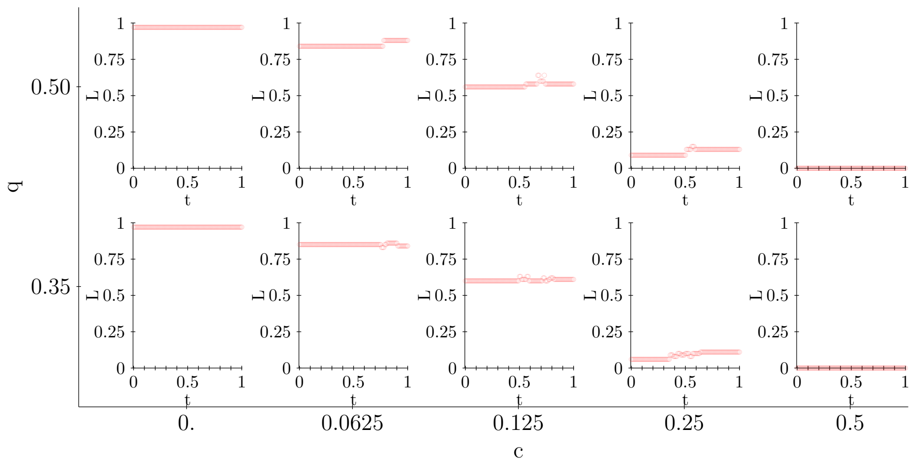

Figure 6: Trial with discretized Dari-Mattiacci and Saraceno strategies

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and Saraceno model.

4 Results

This section first identifies multiple equilibria with a single set of parameters. It then aggregates results from the additive evidence game as defined by Dari-Mattiacci and Saraceno over many different sets of parameters. Finally, it progressively relaxes assumptions of their model that are identified above.

4.1 Equilibria for a single set of parameters

Which equilibrium of a game the von Stengel et al. algorithm identifies may depend on the initialization of the information sets. We ran the algorithm 5,000 times for a single set of parameter values representing the middle of the values we are exploring each, i.e. $t = \frac{1}{2}, q = \frac{1}{2}, c = \frac{1}{8}$, and identified 194 different equilibria. Figure 7 illustrates for each player a strategy in which each of these is played with equal probability. For example, when receiving a signal corresponding to $z_\Pi = 0.05$, the plaintiff makes an offer $\mathscr{B}_P = 0.35$ approximately 8% of the time.

Although there are a large number of equilibria, they cluster near the equilibrium derived by Dari-Mattiacci and Saraceno, represented by dotted lines, though some of the equilibria reflect somewhat more aggressive game play. Some of the equilibria differ from others in ways that have no bearing on the game outcome. For example, if plaintiff chooses $\mathscr{B}_P \geq 0.75$ for some signal, settlement will never occur, and so an offer of 0.75 is functionally equivalent to an offer of 0.85. Of the 194 equilibria identified, 106 are pure strategy equilibria while 88 reflect mixed strategies, but many are mixed strategy equilibria, such

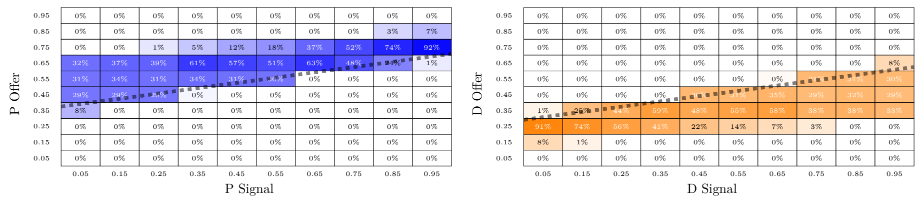

Figure 7: Average strategies across 194 equilibria for $t = \frac{1}{2}, q = \frac{1}{2}, c = \frac{1}{8}$

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as one in which given a particular signal, a party made one offer with exact probability $\frac{4{,}164}{15{,}277}$ and another with probability $\frac{11{,}113}{15{,}227}$. Correlated strategies over the 194 equilibria would by definition also be exact Nash equilibria. The average strategy in 7 is not an exact equilibria, but is close, with exploitability $(E) = 0.0027$.

It will not be practical to seek out 5,000 equilibria for each parameter value for each variation of the model in the remainder of the paper, as this exercise took nearly three and a half hours of computer time. But our later results will continue to reflect initialization of each information set to a random determination of a fully mixed strategy for each player, and we will run the algorithm for every value of $t \in 0, 0.01, ..., 1$, so it will be possible to identify different families of equilibria that arise for very close values of t.

4.2 Aggregating results over parameter values

Figures 5 and 6 calculated accuracy and trial results using the strategies constructed by Dari-Mattiacci and Saraceno. Figures 8 and 9 show the analogous results calculating discretized equilibria using the original rules of the additive evidence game. This is a snippet of the aggregated results. For each of the 1,515 permutations of parameter values executed under these assumptions, the Online Repository includes files with the exact equilibria calculated, heatmaps illustrating the parties' strategies, reports elaborating various outcome variables, and logs of program execution. Such files are also available for each of the assumptions to be explored later in this article.

These results are generally close to the results corresponding to Dari-Mattiacci and Saraceno's constructed equilibria, at least for $c \leq \frac{1}{8}$. The shape of the accuracy curves are very similar, reflecting that their results concerning the better accuracy of the English rule extend to the computational model. Trial rates in the computational model are a bit higher, and multiple equilibria are apparent. The most notable difference is that with sufficiently high costs, the English rule appears to increase trial rates. For example, with $q = \frac{1}{3}$ and $c = \frac{1}{4}$, some equilibria with around half of cases going to trial exist at all fee shifting levels, but equilibria with much lower trial rates are apparent only with relatively low levels of fee shifting. We must, however, acknowledge a limitation of the computational model here. The absence of evidence of equilibria with lower trial rates

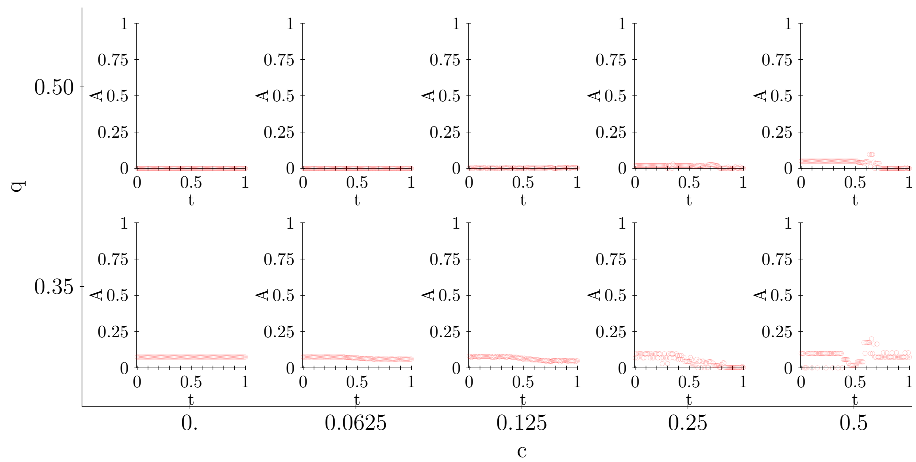

Figure 8: Accuracy with calculated equilibria

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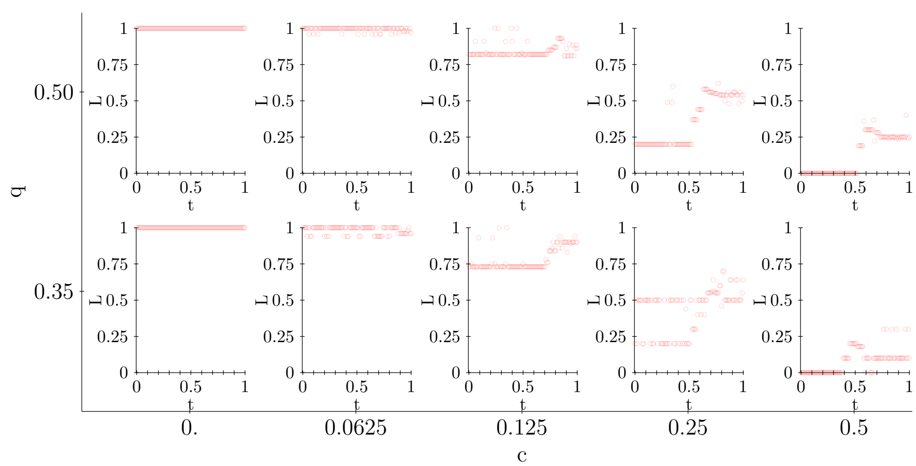

Figure 9: Trial rates with calculated equilibria

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does not prove that no such exact equilibria exist, only that the algorithm did not identify them over a range of fee shifting values with different randomized initial conditions.

4.3 Relaxing assumptions

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