Analysis of Markovian Population Models
Dissertation Defense

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Motivation

This is your first presentation!

Markovian Population Models

Semantics

- ▶ counting agents / population size
- ▶ continuous time
- ▶ exponential jump times / CTMC dynamics

Markovian Population Models

Stationary Distribution

Markovian Population Models
Moment Dynamics

- ▶ Moment formula
- ▶ Moment figure (distribution vs. Moments and SSA)
- ▶ Analytic integration and resulting martingale process

Bounding Mean First-Passage Times
Martingale Process and Linear Moment Constraints

▶ expected occupation time and exit measures (in relation to expectation of the martingale)

Bounding Mean First-Passage Times
Moment Matrices and Semi-Definite Programs

- ▶ semi-definite moment constraints (positive variance as example)
- ▶ hint at localizing matrices

Bounding Mean First-Passage Times
Results and Practical Issues

- ▶ moment stiffness, re-scaling issue
- ▶ some examples

Bounding Mean First-Passage Times

Hausdorff Constraints and Linear Programs

- ▶ linear constraints possible if domains (time and space) are finite
- ▶ 1D visualization of Hausdorff constraints

Linear Control Variates
Using Correlated RVs with Known Expected Value

- ▶ segue: use the same martingale constraints to enhance MC estimation
- ▶ use correlations between target RV and martingales (linear regression, i.e. control variates)

Linear Control Variates
Finding Efficient Sets of Control Variates

- ▶ time-weighting has a large influence on the correlation
- ▶ Infinitely many possibilities (cost needs to be controlled though)
- ▶ Alg. 1: Tighten an initial proposal set
- ▶ Alg. 2: Re-sample promising candidates

Linear Control Variates

Results

- best example?

State-Space Aggregation

Treating Hyper-Cubes of States as One

Stationary Distribution

Finite-Space Projection

Stationary Distribution

Iterative Refinement Algorithm

Bridging Problem

Dynamical Analysis Under Initial *and* Terminal Constraints

Conclusions and Future Directions

Bibliography

Foster-Lyapunov Functions

Local Augmentation of Foster-Lyapunov Functions

Control Variates in General

Control Variates Selection Algorithm 1

Control Variates Selection Algorithm 2

Semi-definite programming