

Tensor-Train Diffusion Models

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1 Abstract

In this work, we explore the application of fixed, low-rank tensor-train to Denoising Probabilistic Diffusion Models. We show the parametric noise can be modeled using tensor-trains and basis functions. We will also provide details on how the model can be trained using Riemannian Optimization algorithm for fixed-rank tensor-trains. The main objective is to develop a more efficient DDPM with respect to memory and training-time.

2 Background

2.1 Denoising Diffusion Probabilistic Models (DDPM)

DDPM models are one of the state-of-the-art generative models [?]. Given a random variable $\mathbf{x} \in R^D$ where the log-likelihood function $\mathbf{x} \in p_\theta(\mathbf{x})$ should be maximized.

In DDPM, the training happens in two phases:

Forward Phase In this phase, a set of intermediate latent variables are generated \mathbf{x}_t $t = 0, 1, 2, \dots, T$, where \mathbf{x}_0 is the input data and $\mathbf{x}_T \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$ $\mathbf{0} \in R^D, \mathbf{I} \in R^{D \times D}$.

Given the following set of constants:

$$\begin{aligned} \beta_t &\leq \beta_t \leq 1 \\ \alpha_t &= 1 - \beta_t \\ \bar{\alpha}_t &= \prod_{i=1}^T \alpha_i \end{aligned} \tag{1}$$

2.2 Tensor-Trains

2.3 Tensor-Train optimization

3 Model

3.1 Architecture

3.2 Optimization

4 Experiments

4.1 Plan

1. Isolation experiments for parametric noise model
2. Use a ResNet, Unet for training such data (for validation)
3. Apply TT opt to isolated data coming from different distributions (the effect of distribution on tt opt)
4. The effect of rank and number of cores,
5. The effect of opt algorithm parameters
6. Drawing a loss landscape for the original DDPM and TT ones
7. Analyzing the convergence of each optimization method
8. Analyzing the number of computations with each method
9. Analyze the memory footprint for each

convergence analysis examples <https://arxiv.org/abs/2208.05314> <https://openreview.net/pdf?i=38484>
<https://www.igpm.rwth-aachen.de/Download/reports/pdf/IGPM423.pdf> <https://www.jstor.org/stable/2344444>
<https://youtu.be/4WDedazTV4?si=ksqwnfZMYNFU595> <https://www.cis.upenn.edu/~cis6100/Wirth-optim-Riemann.pdf> <https://arxiv.org/abs/1712.09913>

- 4.2 Analysis of Gradient Descent Optimization with Neural Networks Model**
- 4.3 Optimization with Gradient Descent for Tensor-Train Model**
- 4.4 Optimization with Alternating Linear Scheme for Tensor-Train Model**
- 4.5 Optimization with Riemannian Gradient Descent for Tensor-Train Model**

Why ? OPTIMIZATION METHODS ON RIEMANNIAN MANIFOLDS AND THEIR APPLICATION TO SHAPE SPACE <https://www.uni-muenster.de/AMM/num/wirtl>
 "Even if an embedding is known, one might hope that a Riemannian optimization method performs more efficiently since it exploits the underlying geometric structure of the manifold. For this purpose, various methods have been devised, from simple gradient descent on manifolds [25] to sophisticated trust region methods [5]."

4.6 Results