Stellar astrophysics

Eddington Luminosity

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Assume a cloud around a star of mass M and luminosity L.

1. **Find a mass-luminosity condition for ejecting the cloud**

   As seen in class, the radiation force per unit mass can be written as $f_r = \frac{\kappa}{c} F_r$, where κ is the opacity, c the speed of light and F_r is the flux of the star. In order to eject the cloud, this force should be greater than the gravitational force felt per unit mass by a particle of the envelope. In spherical mass, if the mass of said envelope is neglected, and only the inner mass is taken into account as the whole star, then, independently of the inner mass distribution $f_G = G\frac{M}{R^2}$, where G is the unniversal gravitation constant, M the mass of the star and R the radius of the star.

   Therefore, substituting the flux by luminosity over surface, the condition is:

   $$f_r > f_G \quad \Rightarrow \quad \frac{\kappa}{c}\frac{L}{4\pi R^2} > G\frac{M}{R^2}$$
   $$\frac{L}{M} > \frac{4\pi G c}{\kappa} \tag{1}$$

2. **Find the terminal velocity of the cloud if it starts at rest at a distance R**

   The energy per unit mass of the envelope will be the energy due to the radiation pressure and the energy due to the gravitational potential of the inner star:

   $$u = u_r + u_g = \frac{\kappa}{c}\frac{L}{4\pi R} - G\frac{M}{R} \tag{2}$$

   Note that $f = -\nabla u$.

   The terminal velocity will be achieved when all the potential energy is converted to kinetic energy:

   $$\frac{1}{2}v_t^2 = u = \frac{\kappa}{c}\frac{L}{4\pi R} - G\frac{M}{R} v_t^2 = \frac{2GM}{R}\big(\frac{\kappa L}{4\pi c G M} - 1\big)$$

   where all energies continue being expressed as energy per unit mass of the envelope and v_T^2 is the terminal velocity. Nothe that as the terminal velocity is onlu defined when the condition in part 1 is defined, Eq. 3 will always have a real solution.

3. **Assume that the opacity per unit mass is κ. Find the maximum luminosity the object can have without ejecting the outer layers (assume pure hydrogen completely ionized). This luminosity is called the Eddington luminosity**

   The maximum luminosity an object can have can be easily found from part 1. Particularly, in the case of completely ionized hydrogen, $\kappa = \sigma_T/m_H$ being σ_T the Thompson scattering cross section and m_H the mass of the hydrogen atom, any luminosity fulfilling the following condition will imply the ejection of the envelope:

   $$L > \frac{4\pi G M c}{\kappa} = \frac{4\pi G M m_H c}{\sigma_T}$$

   Therefore, the maximum luminosity before ejecting the envelope is:

   $$L_{EDD} = \frac{4\pi G M m_H c}{\sigma_T}$$

   which simplifies the expression of the terminal velocity in cases where $L > L_{EDD}$ to:

   $$v_t^2 = \frac{2GM}{R}\big(\frac{L}{L_{EDD}} - 1\big)$$

In the case of the Sun, its Eddington luminosity is:

$$L_\odot = 4 \times 10^{33} \text{ erg/s} \; ; \; M_\odot = 2 \times 10^{33} \text{ g} \; ; \; G = 6.674 \times 10^{-8} \text{ cm}^3\text{g}^{-1}\text{s}^{-2}$$

$$m_H = 1.6733 \times 10^{-24}\text{g} \; ; \; c = 2.99792458 \times 10^{10}\text{cm/s} \; ; \; \sigma_T = 6.65 \times 10^{-25} \text{ cm}^2$$

$$L_{EDD} = 1.265 \times 10^{38} \text{ erg/s} = 3.16 \times 10^5 L_\odot$$