

GEROLAMO CARDANO

The Book on Games of Chance

THE
16TH-CENTURY TREATISE
ON PROBABILITY



Translated by
SYDNEY HENRY GOULD
Foreword by
SAMUEL S. WILKS

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Foreword

Dr. Sydney H. Gould, who has provided this English translation of Cardano's *Liber de ludo aleae* ("The Book on Games of Chance"), and Professor Oystein Ore, who first published the translation together with an excellent commentary on it at the end of his book *Cardano, the Gambling Scholar* (Princeton University Press, 1953), have made valuable contributions to the history of probability by bringing to light this remarkable four-hundred-year-old article. The present publication of the translation itself as a separate booklet makes more widely available a document in the earliest history of probability which merits greater recognition than it has received up to now.

Historians of mathematics such as Libri, Gauraud, Todhunter, Cantor, and Bell have all identified the beginning of the mathematical theory of probability with the Pascal-Fermat correspondence of 1654 concerning the Chevalier de Méré's Problem of Points. This problem can be stated as follows: If the first of two gamblers to receive n points in a game wins, how should the stakes be divided if the gamblers stop playing when one player has x points and the other y ? They have all dismissed Cardano's *Ludo aleae*, a much earlier document, with hardly more than a shrug, describing it merely as a gambler's manual having little or no relevance even for the earliest chapter in the history of the theory of probability.

Cardano wrote his *Ludo aleae* around 1520 while he was student rector at the University of Padua. This makes Cardano's book outdate the Pascal-Fermat correspondence by a little more than one hundred and thirty years. The manuscript remained out of sight until its publication in Latin in 1663, long after Cardano's death in 1576 at the age of seventy-five. It had no direct influence on the development of the theory of probability, since the theory of probability at the time of its publication had reached a stage of sophistication exceeding that of Cardano's work. However, from the point of view of the history of scientific thought, credit must be given to Cardano for a formulation of important ideas concerning probability well over a century before the Pascal-Fermat correspondence.

Cardano was an able man with an extremely wide range of interests. He was a physician, a teacher of mathematics, and an author of books on subjects ranging from medicine and occult practices, to mathematics, astronomy, philosophy, and religion. He was a man of many faults who lived a stormy life. He thrived on controversy. His most notorious controversy in mathematics was with Tartaglia, who accused Cardano of publishing his method of solving the general cubic

equation.

Cardano was an inveterate gambler. As historians of mathematics have pointed out, his *Ludo aleae* is, in essence, a gambler's manual. The odds worked out by Cardano for various events resulting from throwing dice seemed to be of secondary interest to him. He was not overly meticulous in his writing. He made errors in determining odds for certain events, some of which were corrected in later sections of the book, but left standing in the earlier sections! This carelessness is perhaps one of the main reasons why the historians have dismissed the *Ludo aleae* as being of little significance for the history of science.

It is evident from reading Cardano's *Ludo aleae* that he must be credited with an amazing early-sixteenth-century ability to set up what we now call sample spaces associated with the throwing of two dice, three dice, and even four astragals, or knucklebones, and to determine probabilities for the occurrence of various events in these spaces. No recorded evidence seems to exist that anyone before Cardano succeeded in putting odds for the occurrence of chance events into mathematical terms. Cardano determined the probabilities of the various ordinary sums of points (sors), and points as counted in backgammon (fritillus) obtainable by throwing two or three dice. He worked out the probabilities of the five types of events obtainable by throwing four astragals, a game introduced by the ancient Greeks. He dealt with probabilities of some of the simple draws in card games, particularly as practiced in the medieval poker, called primero. He succeeded in determining the probability that a given event with dice occurs in two, three, or any given successive number of throws. Cardano had a good sense of the law of large numbers within the limitations of early-sixteenth-century mathematical science.

The theory of probability, one of the most important branches of modern mathematical science, with applications in almost every field of science and technology, had its origin in gambling, whether reckoned from the Pascal-Fermat correspondence in 1654 or from Cardano's *Ludo aleae* more than a century earlier. But the reader interested in the earliest known beginnings of the theory of probability will find them in this booklet. He will gain further insight on the role of Cardano in the history of probability by reading Professor Ore's commentary on the *Ludo aleae* in his book on *Cardano, The Gambling Scholar*. References to "Ore" in the footnotes (which are by Professor Ore) are to that book.

SAMUEL S. WILKS

Princeton, New Jersey
July, 1960

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1. On Kinds of Games

GAMES depend either on agility of body, as with a ball; or on strength, as with a discus and in wrestling; or on industriously acquired skill, as at chess; or on chance, as with dice and with knucklebones; or on both, as fritillus. Acquired ability may be of two kinds, as in games or in contests. Such a game is primero: for games of cards come under the same name¹ as games of dice, since in ancient times cards were unknown, and for that matter, even the material they are made from. Evidence of this is the fact that men wrote on parchment hides, as of kids for example, and on Egyptian papyrus, on tablets, on wax, and on linden bark. The game is called primero because it holds a primary place among the games of chance, either because of its beauty or because it consists of four primary associations (as it were) which is the number of primary elements² from which we (though not the whole world) are composed. Moreover, it has marvelous varieties.

¹ The same name: *alea*.

² Aristotle's four elements: earth, fire, air, water. Cardano him self did not usually consider fire as an element.

2. On Conditions of Play

ATTENTION must be given to the state of the player and of his opponent, and also to the conditions under which the game is played, such as the amount of money wagered, the place, and the occasion; for the latter is of such importance that permission has been given for gambling at funeral banquets. Thus there is a heading in the law books, "Funeral expenses and games of chance." At other times it has been condemned by the law, as in the Titian and Cornelian Laws.

But in times of great anxiety and grief, it is considered to be not only allowable, but even beneficial. Also, it is permitted to men in prison, to those condemned to death, and to the sick, and therefore the law also permits it in times of grief. For certainly, if any occasion will justify it, none is so worthy of excuse as this one. In my own case, when it seemed to me after a long illness that death was close at hand, I found no little solace in playing constantly at dice.

However, there must be moderation in the amount of money involved; otherwise, it is certain that no one should ever play. As for the excuse made by some that it relieves boredom, this would be better done by pleasant reading, or by narrating tales or stories, or by one of the beautiful but not laborious arts. Among these latter, playing the lute or the virginals, or singing, or composing poetry will be more useful, and for three reasons. First, because such a change from serious business is more praiseworthy than gambling, either since something is produced, as in the case of painting, or because it is according to nature, as in music, or because the man learns something, as in reading or hearing tales or stories. Second, it is not without its element of toil and so does not rob us, against our will, of more time than it should. And time, as Seneca has rightly said in his work on *Length and Shortness of Life*, is the most precious of all things. Third, such employment of leisure is more respectable and does not present a bad example, as gambling does, particularly to one's children and servants. To these facts must be added that gambling arouses anger and disturbs the mind, and that sometimes a quarrel flares up over money, a thing which is disgraceful, dangerous, and prohibited by law. Finally, one cannot gamble alone, whereas the above delights can be enjoyed even when we are by ourselves.

3. Who Should Play and When

So, if a person be renowned for wisdom, or if he be old and dignified by a magistracy or any other civil honor or by a priesthood, it is all the worse for him to play; on the other hand, gambling is proportionately less of a reproach to boys, young men, and soldiers. The greater the amount of money involved, the greater the disgrace; *thus a certain holder of a very high priestly office (namely, a cardinal)* was severely blamed because he played after dinner with the Duke of Milan for a stake of five thousand crowns.

This fault is particularly detestable in princes and is defended by no one except courtiers and flatterers of the prince, who do it either from fear or because they receive gifts if the prince is lucky. In the meantime his subjects are despoiled and the poor are deprived of the aid which has been assigned to them and which is their due. If a man is victorious, he wastes the money won by gambling, whereas if he suffers defeat, then either he is reduced to poverty, when he is honest and without resources, or else to robbery, if he is powerful and dishonest, or again to the gallows, if he is poor and dishonest. One's opponent, too, if he is a man of ill repute or of low station in life who is devoted to the game, is a source of disgrace, and of loss also. For if you play assiduously with men of this kind, you will come away a confirmed gambler; or otherwise you will be the loser because of their greater experience, trickery, and skill.

Your opponent should be of suitable station in life; you should play rarely and for short periods, in a suitable place, for small stakes, and on suitable occasions, as at a holiday banquet. Your opponent might be the king, or a prelate of outstanding character,³ or a relative by blood or by marriage. To play with professional gamblers is most disgraceful, and, as I have said, dangerous. The most respectable place is at home or at the house of a friend, where there can be no public scandal. Lawyers, doctors, and the like play at a disadvantage: for one thing, they appear to have too much leisure; for another, if they win, they seem to be gamblers, and if they lose, perhaps they may be taken to be as unskilful in their own art as in gaming. Men of these professions incur the same judgment if they wish to practice music.

³ Cardano made the acquaintance of the Duke of Milan at play and could count a number of cardinals among his good friends.

4. The Utility of Play, and Losses

As ADVANTAGES from well-managed play we obtain relaxation from anxiety and a pleasure from which we arise ready and eager for serious business; also knowledge of the character of our fellow-citizens for play is, as it were, a *rack* on which anger, greed, and honesty or dishonesty are made clear. For play both produces important evidence and is an actual torturer if the stakes are large. It is also a means of gaining friendship, and many have risen from obscurity because of the friendship of princes formed in play. This is what Cicero meant in his *Philippic* by the words “that fellow-player of yours, condemned for gambling.”

But the losses incurred include lessening of reputation, especially if one has formerly enjoyed any considerable prestige; to this is added loss of time, vain words, including on occasion curses against the gods, the neglect of one's own business, the danger that it may become a settled habit, the time spent in planning after the game how one may recuperate, and in remembering how badly one has played. There are also disputes and often, which is worst of all, provocation to anger; for then a man is carried away into playing for high stakes and into feelings of enmity, so that he no longer has control of his own mind. As a result he throws out large sums of money and may be said to abandon them rather than play for them.

Play is a very good test of a man's patience or impatience. The greatest advantage in gambling comes from not playing at all. But there is very great utility in it as a test of patience, for a good man will refrain from anger even at the moment of rising from the game in defeat.

5. Why I Have Dealt with Gambling

THERE are two reasons why I have considered it fitting for me to deal with gambling: first, on account of its useful features, for, since it is useful, there must also be need for giving a systematic account of that usefulness under various conditions, and pointing out exactly how great it is. Even if gambling were altogether an evil, still, on account of the very large number of people who play, it would seem to be a natural evil. For that very reason it ought to be discussed by a medical doctor like one of the incurable diseases; for in every evil there is a least evil, in every disgrace a least disgrace, in every infamy a least infamy, and similarly in loss of time and fortune. Second, it has been the custom of philosophers to deal with the vices in order that advantage might be drawn from them, as, for example, in the case of anger. Thus it is not absurd for me to discuss gambling, not in order to praise it (for I have known some writers to attempt this, but their efforts are vain and form a bad example; I have read books of this kind) but in order to point out the advantages in it, and, of course, also its disadvantages, in order that the latter may be reduced to a minimum.

6. The Fundamental Principle of Gambling

THE most fundamental principle of all in gambling is simply equal conditions, e.g. of opponents, of bystanders, of money, of situation, of the dice box, and of the die itself. To the extent to which you depart from that equality, if it is in your opponent's favor, you are a fool, and if in your own, you are unjust. With respect to your opponent, if he is more powerful than you, or unscrupulous, or likely to be violent, or a man who hopes to win everything by creating sufficient delay, if he is given to complaint or litigation, or is clamorous, or deceitful, if he can disturb your equanimity by making you afraid or angry, then he is the worst man in the world for you; and to play with him is not to play but simply to lose your money.

And there is even greater danger to be feared from kibitzers, if they favor your opponent; and so it happens that, if you play in a large crowd of people, you can scarcely avoid folly if they are against you, or else injustice if they are for you. They can injure you in many ways: for example, by giving your opponent open advice and information, which is a twofold evil, since it not only helps their side but also provokes you to anger and disturbs you; for an angry man, as long as he is angry, is simply insane. Others will annoy you by their disorderly talk, even without giving definite information. Some will purposely consult you on serious business; some will even be so impudent as to provoke you to anger by quarreling with you; others will make fun of you in order to make you angry; others, more modest than these, will indicate to your opponent by foot or by hand that the decision he has made is not the right one; others again, a little farther off, will do this with a nod, perhaps with no other purpose, it may be, than to help him by filling your mind with suspicion. Still others will state falsely how the die has fallen; others again will worry you by accusing *you* of such things.

For all these reasons it is of the very greatest advantage to you to have your own supporters if you wish to win unjustly; and to play otherwise in the presence of a crowd is simply to waste your money, not to contend for it; for even if no trickery takes place, suspicion itself disturbs a man and makes him err, so that a suspicious man should not play. There is the same danger when your opponent seizes the die very quickly or moves his counters in such a way that you cannot see what he is doing or what the die has shown; and similarly if he plays with coins which are counterfeit, or debased, or excessively worn by water; or if the

situation is one to cause fear, or if you are in a place of ill repute; and if the die is dishonest or if you are playing with a hanging dice box.⁴

⁴ It is not clear whether the word fritillus here should be translated as dice box or gaming board. See also Ore, p. 111.

7. The Hanging Dice Box and Dishonest Dice

SET the round gaming boards in the middle; if they incline toward your opponent, then the dice box will incline in the opposite direction, and this is unfavorable to you. Similarly, if there is a slope toward you, then the box will be out of plumb in your favor; but if the dice box is not moved, then this does not matter. Similarly, if the board catches the light from the side opposite to you, then this is bad, since it disturbs your mind; on the other hand, it is to your advantage to have the board against a dark background. Again, they say it is of benefit to take up your position facing a rapidly rising moon.

As for the die itself, there are two sorts of danger. In the first place, every die, even if it is an acceptable one, has its favored side either because of its shape, or for some other reason, or by mere chance; and if in this way a large point is changed into a small one, or vice versa, you can readily understand how much of a difference it makes. In the second place, the die may be dishonest either because it has been rounded off, or because it is too narrow (a fault which is plainly visible), or because it has been extended in one direction by pressure on opposite faces. Consequently, it should receive a threefold testing, since there are three pairs of opposite faces which determine the surface to show uppermost. So these matters must be considered very carefully. There are even worse ways of being cheated at cards; these will be noted later on. In everything one must keep a watchful eye and take note of any disparity among the cards.

8. Conditions Under Which One Should Play

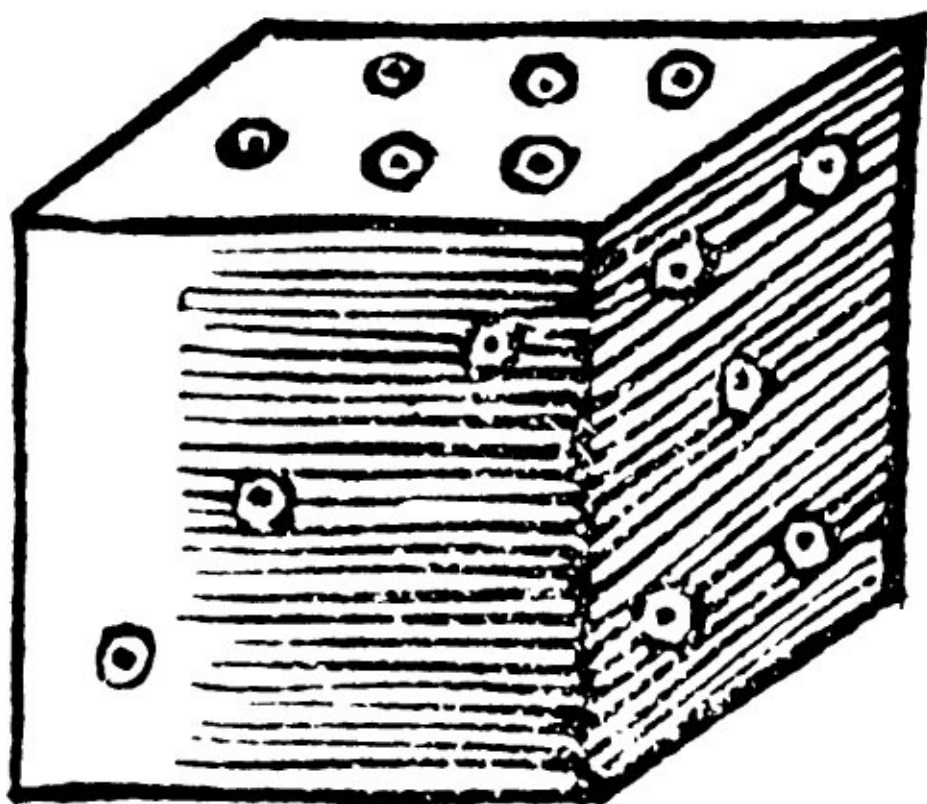
IT IS FOOLISH to play even for small stakes for the mere purpose of being beaten because you do not notice what is going on; but it is also foolish to devote yourself entirely to gambling in order that you may win; for it is inevitable that he who plays more rarely should be less skilful. But should you therefore abandon all study of the arts in order that you may win at dice? And this only for small gain or even with the sole result of losing, and being angry at yourself and an object of scorn to others? And if you should lose a large amount, then certainly the gain to the victor is not so great as the loss to the loser; for usually he gives away a great deal; the man is wasteful and his time is lost; and when the die once falls unluckily for him, his loss may be greater than the net gain of many wins.

So you ought to be more skilful than your opponent, and more experienced, or else you ought to play in such a way that it does not matter how the die falls, as will be the case if you play for small stakes or with your loved ones for a short time after dinner. But if you are determined to play for large stakes, then play constantly with an opponent who is neither more experienced than yourself nor more fortunate, and let the conditions not be unlucky for you. For luck plays a very large part in these matters also.

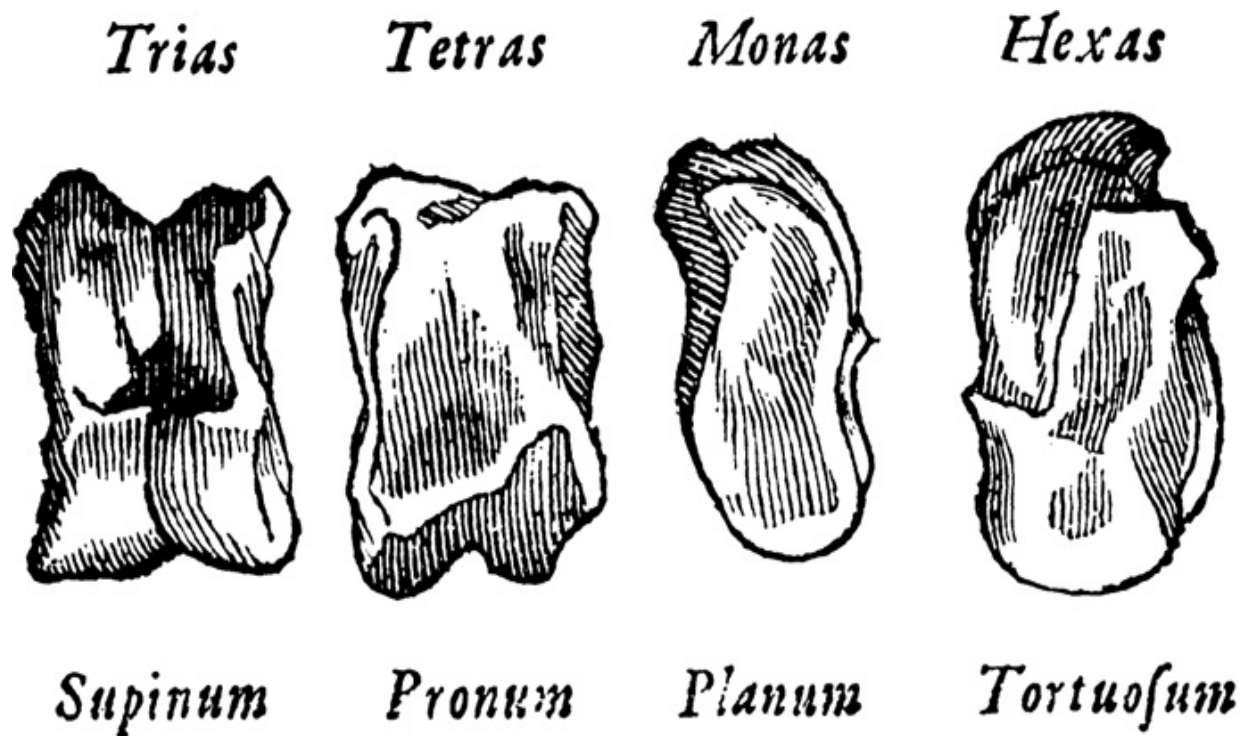
9. On the Cast of One Die

THE TALUS has four faces, and thus also four points. But the die has six; in six casts each point should turn up once; but since some will be repeated, it follows that others will not turn up. The talus is represented as having flat surfaces, on each one of which it lies on its back; it can be seen in this way and is therefore so depicted; in actual practice, however, it is not so, but boys spin it as though it were a spindle and it does not have the form of a die.⁵

One-half of the total number of faces always represents equality;⁶ thus the chances are equal that a given point will turn up in three throws, for the total circuit is completed in six, or again that one of three given points will turn up in one throw. For example, I can as easily throw one, three, or five as two, four, or six. The wagers are therefore laid in accordance with this equality if the die is honest, and if not they are made so much the larger or smaller in proportion to the departure from true equality. But (as I have said) these facts contribute a great deal to understanding but hardly anything to practical play.⁷



27. Tessera. From Frisius, *Arithmeticae Practicae* (1553).



28. Astragals. From Hyde, *Mandragorias*.

⁵ The missing illustrations have been replaced from other sources.

⁶ See explanation in Ore, p. 149.

⁷ The meaning is that no dice games were played with only one die.

10. Why Gambling Was Condemned by Aristotle

BUT when we come to speak of the play itself and of actual gambling, which we will discuss below, I do not hold anyone blameless in this matter except those who play for money because of great grief of mind; and gambling is disgraceful because a man makes gain from his friend against that friend's will. For the case stands thus: gain from those who are both willing and aware is best; next best, is gain from those who are aware and unwilling. To the first class belong lawyers and doctors, and to the second merchants. The third kind is gain from those who, being aware but unwilling, are one's friends, as in gambling. The fourth kind is gain from those who are unwilling and unaware, as in gain by trickery. The fifth kind is gain from those who are unwilling and aware and not one's friends, as in robbery.

Aristotle gives another reason elsewhere when he says (4. *Ethics*, [Chap. 1](#), in *fin.*) that gamblers, thieves, and robbers ply a sordid trade for they traffic in base gain; in fact they do everything for the sake of gain and thereby incur reproach. But thieves at least undergo great risks for their spoils, while gamblers gain from their friends to whom they ought rather to give. So both classes of men whose aim is to make gain where they ought not, traffic in base gain, and all such acquisitive acts are sordid. Moreover, a confirmed gambler is a perjurer and a blasphemer, and is at the same time prodigal and greedy; and if not so already by nature, he soon becomes irascible; he cherishes vain hopes in his idleness and he corrupts the youth.

Now the Christians tolerate gambling, though it was condemned by the ancients; but they do not allow cursing, and so they prosecute the lesser evils. As for the fact that this evil takes its rise among princes, they, of course, wish to allow themselves everything; and so, as I have said, gambling is a very great evil in the state. Nevertheless, in times of great fear and sorrow, when even the greatest minds are much disturbed, gambling is far more efficacious in counteracting anxiety than a game like chess, since there is the continual expectation of what fortune will bring. Nor does it claim a man's whole attention as strictly as chess does. At such a time physical games are harmful and dangerous.

11. On the Cast of Two Dice

IN THE CASE of two dice, there are six throws with like faces, and fifteen combinations with unlike faces, which when doubled gives thirty, so that there are thirty-six throws in all, and half of all these possible results is eighteen. As for the throws with unlike faces, they occur in pairs in the eighteen casts of equality, so that equality for such a throw consists of nine casts; and this reasoning gives eighteen casts for equality for a throw (1,1); for in that number of casts this throw can appear and not appear with equal probability; and similarly for the throws (2,2) and (3,3).

But the throw (1,2) can turn up in two ways, so that for it there is equality in nine casts; and if it turns up more frequently or more rarely, that is a matter of luck.

The number of throws containing at least one ace is eleven out of the circuit of thirty-six; or somewhat more than half of equality; and in two casts of two dice the number of ways of getting at least one ace twice is more than $1/6$ but less than $1/4$ of equality.

In three casts of two dice the number of times that at least one ace will turn up three times in a row falls far short of the whole circuit, but its turning up twice differs from equality by about $1/12$. The argument is based upon the fact that such a succession is in conformity with a series of trials and would be inaccurate apart from such a series.

Moreover, a repeated succession, such as favorable points occurring twice, arises from circuits performed in turn; for example, in 3,600 casts, the equality is $1/2$ of that number, namely, 1,800 casts; for in such a number of casts the desired result may or may not happen with equal probability. So the whole set of circuits is not inaccurate, except insofar as there can be repetition, even twice or three times, in one of them. Accordingly, this knowledge is based on conjecture which yields only an approximation, and the reckoning is not exact in these details; yet it happens in the case of many circuits that the matter falls out very close to conjecture.

12. On the Cast of Three Dice

THROWS with three alike are the same, except in one respect, as the throws with two alike in the preceding chapter; thus there are 6 of them. The number of different throws of three dice with doublets and one different point is 30, and each of these occurs in 3 ways, which makes 90. Again, the number of different throws with three different faces is 20, each of which occurs in 6 ways, which makes 120. Thus the circuit of all of them will be 216 and equality will be 108.

I shall set down, by way of example, some simple terms, together with the number of various ways in which they can occur. With respect to these simple terms, then, in the case of a doublet and one different, there are 6 possible values for the like faces and 5 for the odd point; so, since there are 6 faces, there will be 30 varieties of throws. Also, each of them can be varied in three ways, making 90. But the 20 cases with three unlike faces, since each of them can be varied in 6 ways, will make 120. Thus a triplet throw is the $1/108$ part of equality, while any doublet and one different point, since it can be varied in 3 ways, will be the $1/36$ part of equality. This is just as in the case of two dice where a doublet is $1/18$ of equality, and so occurs once in 18 throws, which is $1/6$ of 108. Therefore, in comparison with the former, it will occur three times as often.

Such, then, is the law for doublets, as we shall say, or for the calculation of the stake in this case. But two distinct faces, as (1,2), we shall distinguish thus: that, if an ace is adjoined, it will be done in three ways, and if a deuce, then in the same number of ways, therefore already 6. Moreover, it happens in 4 other ways: but the latter are varied, each one, in 6 different ways; therefore there will be 24, so that with the remaining 6 there will be 30.

But 35 to 36 has the proportion of equality to 108 when compared to 12, therefore almost $1/6$ but not quite.

But three different faces, as (1,2,3), bear to the number of equality exactly the same proportion as in doublets for two dice. Moreover, the single faces themselves in one die have a proportion $1/3$ of equality; since, therefore, there are three dice, they will obtain the proportion of equality, so that out of the 216 possible results, each single face will be found in 108 and will not be found in as many. The reasoning is exactly the same for one face in the case of two dice; the latter is equal to the whole circuit in three throws and to equality in half that number.

13. On Composite Numbers Up to Six and Beyond and for Two and Three Dice

IN THE CASE of two dice, the points 12 and 11 can be obtained respectively as (6,6) and as (6,5). The point 10 consists of (5,5) and of (6,4), but the latter can occur in two ways, so that the whole number of ways of obtaining 10 will be 1/12 of the circuit and 1/6 of equality. Again, in the case of 9, there are (5,4) and (6,3), so that it will be 1/9 of the circuit and 2/9 of equality. The 8 point consists of (4,4), (3,5), and (6,2). All 5 possibilities are thus about 1/7 of the circuit and 2/7 of equality. The point 7 consists of (6,1), (5,2), and (4,3). Therefore the number of ways of getting 7 is 6 in all, 1/3 of equality and 1/6 of the circuit. The point 6 is like 8, 5 like 9, 4 like 10, 3 like 11, and 2 like 12.

In the game Fritillus 11 cases are to be added, because the number can be shown on a single die. Thus, the point 2 is obtained in 12 different ways, which is 2/3 of equality and 1/3 of the circuit. The point 3 is obtained in 13 different ways, 4 in 14, 5 in 15, which is 10/12 of equality and 5/12 of the whole circuit, and 6 in 16, which is very close to equality.

TABLES FOR CASTS WITH TWO AND THREE DICE

Sors with two dice

2,12 : 1
5, 9 : 4

3,11 : 2
6, 8 : 5

4,10 : 3
7 : 6

Equality by 18

Sors and Fritillus with three dice

<i>Sors</i>	<i>Fritillus</i>	
3,18 : 1	3 : 115	
4,17 : 3	4 : 120	
5,16 : 6	5 : 126	
6,15 : 10	6 : 133	Circuit 216
7,14 : 15	7 : 33	Equality 108
8,13 : 21	8 : 36	
9,12 : 25	9 : 37	
10,11 : 27	10 : 36	
	11 : 33	
	12 : 26	

Remaining points as in Sors

13 : 21
14 : 15

.

Single point has 108 cases
Two points have 111 cases

In the case of three dice there will be three faces. The point 3 exceeds equality in Fritillus, in Sors it is represented only by 3 aces, that is, by 1/108 of equality. In Fritillus the point 4 can be obtained in 120 ways; in Sors it is 1/36 of equality, or 1/72 of the circuit. The point 5 arises from a double ace or a double deuce; it will therefore be 1/18 of equality in Sors; but in Fritillus, it can be obtained in 126 ways, that is, more than equality by 1/6. The point 6, however, in Sors can be obtained in 10 ways, namely, 3 deuces, 2 aces with a 4, and (3,2,1). So in Fritillus there will be these same possibilities and in addition those which are made with two dice, namely (3,3), (1,5), and (4,2); these are 15 and with the 10 others they become 25, so that there are 133. The point 7 can be obtained in Sors in 15 ways; in Fritillus it already ceases to be the half or to have equality. Therefore, there are only these same possibilities and those which are produced by two dice, and there are (1,6), (2,5), and (4,3); therefore they are 18. The sum is 33, less than a third of equality. But the point 8 in Sors occurs in 21 ways; in Fritillus it is 1/3 of equality, namely 36. The point 9 in Sors occurs in 25 ways,

and in Fritillus in 12 more, therefore 37 in all. The point 10 in Sors occurs in 27 ways and in Fritillus in 9 more, consequently 36; the other numbers correspond in turn in Sors, as you see in the table. But in Fritillus the point 11 occurs in only 33 ways and 12 in 26. The rest of the numbers above 12 are equal to those which are in Sors.

14. On Combined Points

WE must enter upon a consideration of this sort in the case of two dice, because the ace occurs in 11 casts, and the deuce likewise, and the trey and so on for each of them, but the ace or deuce do not occur in 22 casts but only in 20. For the ace occurs in 11 and the deuce in 9 more. Thus, if the trey is added, there will not be 29, nor 31, but 27, and in general numbers of casts are those given in the table. Thus, if all the casts are collected, there are 36; for since this is the complete circuit, it is necessary that in every throw some face should occur because they complete the number of the circuit.

	TWO DICE	TOTAL
<i>Cases for 1 point</i>	11	11
<i>Additional for: 2</i>	9	20
3	7	27
4	5	32
5	3	35
6	1	36

If, therefore, someone should say, “I want an ace, a deuce, or a trey,” you know that there are 27 favorable throws, and since the circuit is 36, the rest of the throws in which these points will not turn up will be 9; the odds will therefore be 3 to 1. Therefore in 4 throws, if fortune be equal, an ace, deuce, or trey will turn up 3 times and only one throw will be without any of them; if, therefore, the player who wants an ace, deuce, or trey were to wager three ducats and the other player one, then the former would win three times and would gain three ducats; and the other once and would win three ducats; therefore in the circuit of 4 throws they would always be equal. So this is the rationale of contending on equal terms; if, therefore, one of them were to wager more, he would strive under an unfair condition and with loss; but if less, then with gain. Similarly, if the 4 be included, there will be 32 favorable throws, and the number of remaining throws will be only 4. Therefore, the player will place a stake eight times as great as his opponent, because the proportion 32 to 4 is eightfold, and similarly for the other cases, nor is there any need for an argument based on the average.

Consequently, we can make the corresponding statements in the other cases. Thus if we want (1,1) or (2,2), there are 2 favorable throws; and since there are

34 other throws, the odds are 17 to 1. Thus, also, if we want a (1,1), the odds are 35 to 1, and in fact all the above rules ought to be reduced to this one principle, as can be discerned from the equality of the wagers. For example, in order that an ace may turn up, since there are 11 favorable throws, the odds will be 25 to 11, a little more than 2 to 1.

The same reasoning is to be observed in the case of three dice, both for simple and for combined faces, and let us state, from the above, that there are 108 casts of the ace ([Chap. 12](#) in fine). Therefore, it will be necessary to find 6 terms, of which the largest shall be 108, and the rest shall be equidistant from this number and from each other, and such that all together they make up 216, as you see in the table:

	THREE DICE	DIFFERENCE
<i>Cases for 1 point</i>	91	30
<i>Additional for: 2</i>	61	24
3	37	18
4	19	12
5	7	6
6	1	
	—	
	216	

For the ace does not have a number of favorable throws equal to half the whole circuit, but the odds are 91 to 125, or very nearly, if inverted, 25 to 18, greater therefore than 4 to 3. So the player who has wagered that an ace will not turn up will gain to the extent to which in 7 turns he has his throw free of it, and if he wagers 4, he will still gain those 3. In the same way we must enter upon the consideration in the remaining cases. It is evident that with two dice the increments are equal. But for three they have an equal excess, as is clear from the table.

Other questions must be considered more subtly, since mathematicians also may be deceived, but in a different way. I have wished this matter not to lie hidden because many people, not understanding Aristotle, have been deceived, and with loss. So there is one general rule, namely, that we should consider the whole circuit, and the number of those casts which represents in how many ways the favorable result can occur, and compare that number to the remainder of the circuit, and according to that proportion should the mutual wagers be laid so that one may contend on equal terms.

But if two casts are necessary, we shall multiply them in turn, and the remainders for those numbers in turn, and if three are necessary, or four, we shall

do the same, and then we shall have to make the comparison in accordance with the numbers thus obtained. Thus, if it is necessary for someone that he should throw an ace twice, then you know that the throws favorable for it are 91 in number, and the remainder is 125; so we multiply each of these numbers by itself and get 8,281 and 15,625, and the odds are about 2 to 1. Thus, if he should wager double, he will contend under an unfair condition, although in the opinion of some the condition of the one offering double stakes would be better. In three successive casts, therefore, if an ace is necessary, the odds will be 753,571 to 1,953,125, or very nearly 5 to 2, but somewhat greater.

15. On an Error Which Is Made About This

BUT this reasoning seems to be false, even in the case of equality, as, for example, the chance of getting one of any three chosen faces in one cast of one die is equal to the chance of getting one of the other three, but according to this reasoning there would be an even chance of getting a chosen face each time in two casts, and thus in three, and four, which is most absurd. For if a player with two dice can with equal chances throw an even and an odd number, it does not follow that he can with equal fortune throw an even number in each of three successive casts.

But when he produces an odd number on the first cast or on the second or third cast in one trial, he can be deceived in his reckoning by the eight various results. Therefore, in comparisons where the probability is one-half, as of even faces with odd, we shall multiply the number of casts by itself and subtract one from the product, and the proportion which the remainder bears to unity will be the proportion of the wagers to be staked. Thus, in 2 successive casts we shall multiply 2 by itself, which will be 4; we shall subtract 1; the remainder is 3; therefore a player will rightly wager 3 against 1; for if he is striving for odd and throws even, that is, if after an even he throws either even or odd, he is beaten, or if after an odd, an even. Thus he loses three times and wins once.

And for 3 successive casts we shall multiply 3 by itself; the result is 9; take away one; the remainder is 8. The proportion of 8 to 1 is valid for 3 successive casts, and for 4 casts this proportion is 15 to 1, and for five, 24 to 1. Thus, in 125 casts, there will be only five sequences of throws in their proper places, that is, with the same number of consecutive even throws, so that they may begin with an odd number either in the first or the sixth or the eleventh place. For otherwise there would be more than five consecutive even throws. All this reasoning can be demonstrated from the number of places in each of the circuits mentioned, although in the cases involving multiplication it seems to be false. For example, if we consider the number 20, the odds become 7,999 to 1. Yet it can scarcely be believed that in 7,999 throws a player can cast an even number twenty times in a row. But we have demonstrated this; because, although in 8,000 performed only once, this reasoning can deceive us, still in an infinite number of throws, it is almost necessary for it to happen; for the magnitude of the circuit is the length of time which shows forth all forms.

We turn now to cases where the probability is not $1/2$; for example, in the

above-mentioned case where we want an ace, deuce, or trey on either of two dice, the odds are 3 to 1. In order to make the study easier, we shall consider one die only. We therefore take a talus with four faces, having an even number on one face and an odd number on the other three, and we investigate the wagers for successive appearances of an odd face. Let us give to the odd faces the names *a*, *b*, and *c* and to the even one *d*, and in the following investigation let there be 4 columns, as you see in the table.

A	B	C	D
a	a	a	a
b	b	b	b
c	c	c	c
d	d	d	d

Let the first column be written under *A* and the second under *B* and let the second cast be combined with the first so that there will be 16, of which 9 will be odd on both faces and the other 7 will contain an even face.

Similarly, for a die with 3 faces, 2 of them being odd, there will be only 4 odd results and the other 5 will be even, and if there are 5 faces and 4 of them are odd, there will be 16 odd results out of the 25 possibilities, and the other 9 will be even.

In all these cases the whole number is multiplied into itself and similarly the number of similar faces into itself, and the latter number is compared to the remainder, and similarly, if there are only 3 faces and 2 of them are favorable to us, we shall multiply the whole number into itself, and then that number into itself, and this part will be compared to the remainder. Thus, in the case of one die, let the ace and the deuce be favorable to us; we shall multiply 6, the number of faces, into itself: the result is 36; and two multiplied into itself will be 4; therefore the odds are 4 to 32, or, when inverted, 8 to 1.

If three throws are necessary, we shall multiply 3 times; thus, 6 multiplied into itself and then again into itself gives 216; and 2 multiplied into itself and again into 2, gives 8; take away 8 from 216: the result will be 208; and so the odds are 208 to 8, or 26 to 1. And if four throws are necessary, the numbers will be found by the same reasoning, as you see in the table; and if one of them be subtracted from the other, the odds are found to be 80 to 1.

All this is for one die only, but the same reasoning holds for two and three

dice also, as we shall show by an example. Let the favorable cases be the ace, deuce, or trey, occurring in three successive casts. As already mentioned, the number of the circuit is 36 and the number of favorable throws is 27. We multiply 36 into itself 3 times; the result is 46,656. Multiply 27 into itself 3 times; the result will be 19,683. Subtract the smaller from the larger; the remainder will be 26,973, which gives the odds when compared to 19,683. These odds, namely, the ratio of the remainder to the smaller number, are greater than 4 to 3 and less than 3 to 2.

Similarly, it has been stated that with three dice any one face, whichever one it may be, has, taken by itself, 91 favorable cases in the whole circuit of 216. Therefore, if that face is required three times in a row, we shall multiply the whole circuit, and the result is 9,324,125. When the latter number is divided by the smaller of the above numbers, namely, 753,571, we get the odds determining the stake to be wagered, namely, a little greater than 12 to 1. From this it is clear that any other reasoning is not satisfactory, but that this reasoning is generally true.

16. On Card Games

IF I SHOULD wish to speak about all of them, it would be an endless task, but there are two kinds, those with planning and those without such planning. There is a difference from play with dice, because the latter is open, whereas play with cards takes place from ambush, for they are hidden.

Primero is the noblest of all. It consists of four cards as far as difference of kind is concerned: for among the French, Spanish, Germans, and Italians cards are of four different suits; there are thirteen cards of each suit and so fifty-two in all. They run from one to ten in the sequence of numbers. Then there is the king together with the jack, and the French have a queen and the Italians a knight.

In primero the cards eight, nine, and ten are removed: the king, the jack, and the queen or knight are each worth 10. From the two to the five, 10 is added, so that they are worth 12, 13, 14, and 15; and the six and the seven are tripled, so that the six is worth 18 and the seven 21, while the ace is worth 16. So the greatest total (on three cards of the same suit) consists of the seven, the six, and the ace, and they are 21, 18, and 16 or 55 in all. But if the number of these cards is completed by adding the five, then all these cards of the same suit make up the number 70.

Now there are two kinds of primero. In one the greater number wins, and this number is different according to the nature of the hands; in the other the smaller number wins; and in this latter kind, which is very little in use, the smallest number to be obtained on cards of various suits is 20, while in primero it is 40 and in fluxus 42. But the hands are the same as in the former kind and keep the same order relative to one another.

There are five kinds of bids: *numerus*, *primero*, *supremus*, *fluxus*, and *chorus*.

Numerus occurs when two or three cards are of the same suit. The least possible result here is 20, which is made from two minor cards, which they call face cards because they have depicted on them the human face (as one says) of the king, the queen (or the knight, as the case may be), and the jack. The greatest *numerus* is 54: it consists of three cards, the seven, the six, and the five.

The second type of hand is *primero*, when all the cards are of different suits, and, whatever it may be, it surpasses every *numerus*. In it the least number is 40 (as I have said), and the greatest is 81.

This *primero*, however great, is surpassed by *supremus*, i.e. 55, when three cards of the same suit are seven, six, and one, which make up 55.

In the fourth place is *fluxus*, which consists of four cards of the same suit and surpasses primero and supremus. Here the smallest number (as I have said) is 42 and the greatest is 70.

The fifth type of hand is like primero but has all its cards of the same denomination, as four sixes or sevens, or four kings. But three kings and one queen do not make a *chorus*, although all are valued at the same number, namely, 10. This hand surpasses all the preceding ones. In it the greatest number wins and this greatest number is 84. If the face cards are equal, as four kings and four jacks, the four kings do not win, since they are not superior in value, but that player who is nearest to the dealer on the right. This is the general rule for all equal numbers in equal bids.

The cards are dealt around twice, two at a time and not singly. It is not permissible to count diverse bids as more than the greatest of these, but supremus can be considered as primero when another has bid primero. Also chorus can always be concealed for primero and for fluxus when another has announced it.

But there is one evil custom which ought to be condemned: some players show chorus, then cast away a card at a certain moment and later receive cards in exchange and wish to remain with chorus, but this procedure is too contentious and offers occasion for fraud in many ways. For that reason, luck ought to be enjoyed as it comes; for if it had wished to favor the player in the exchange of cards, it would not have shown itself in the matter beforehand. This rule ought not to be disregarded.

There are three cases in which one customarily calls for new cards:

First when you have the lower point, for instance, your opponent has 45 and you have 36, so that with any card among the two [in your suit] you will be the winner.

Secondly, if your opponent should have the point 40 but you have three cards of different suits, then you will only be able to win by primero.

Thirdly, if he should have primero, or the supreme point, or even only a higher point but in such a way that your three cards have a lower value, then it will be necessary for you to complete a fluxus in order to win.

It is customary to remove a part of each pot, namely, one-half or thereabouts, and the remainder is left to chance in order that it may belong to the victor.

By such reasoning the sharing in the first case will be made on an equal basis (unless from the discarded cards one knows that there remain fewer or more than one should expect in general: for if ten cards should remain to be distributed, and in these there remain five, or only one, one would have to argue differently). In the primero the share ought to be one-half to twice, and in fluxus one-third to

three times.

But I shall tell what should be done rather than what is done since the customs vary in different countries. Thus, in order that the conditions should be fair, the amount removed should be divided as follows:

In the first case according to the amount by which this equality is preferable to the one already holding two cards; for the two other cards occupy two different suits, but the four cards should be so that an encounter occurs. But if there is some more subtle reasoning, it is now fitting to pass over it, since in matters of this kind practical use alone should be regarded.

But in the second case one player has already received two shares, the other player one, as we said. Out of four turns he would lose in three cases and win only in one so that in one circuit he would lose all, which is therefore the condition. But players are moved by the reasoning that, if two cards were awaited, it would be suitable to take equal shares, therefore out of half of two cards, half of the share. It is necessary to take up the reasoning as follows: out of four cases in one circuit he would lose three and win one, so that he would lose half of that which he has. Therefore he will necessarily lose half of his stake while the opponent gains as much. Thus he should receive only $1/4$ of the removed part and the opponent $3/4$.

But in fluxus, since there are three cards lacking from that suit among those to be dealt, it is necessary to withdraw five shares of which the one of the two who has three cards wins one; rather, he will receive only two and his fellow player nine.

Holding one card to primero is preferable on the basis of the $1/3$ share we already mentioned. Therefore, I conclude that the scales are so balanced that we favor those who play with the more unfavorable fortune.

In the case of primero, the one who expects it should accept two shares, the other five. But in the case of fluxus he should have one to the other's five shares, or rather one share to the other's four and this is nearer to the true reasoning.

However, care must be taken that this is not done arbitrarily, but either always or never, for otherwise you will play under most unfavorable conditions. For the opponent may be able to recognize the card from the outside and so, if it should be favorable to you or to him, he may not accept the proposal; and he will by no means do so if it should be favorable to you and not to him. But if it is required to accept the conditions, let it be done before the card is requested since then the cards are drawn from below and not from the top.

17. On Frauds in Games of This Kind

CARDS have this in common with dice, that what is desired may be got by fraud: the most contemptible kind is that which is backed up by the sword; a second kind has to do with recognition of the cards—in its worst form it consists of using marked cards, and in another form it is more excusable, namely, when the cards are put in a special order and it is necessary to remember this order. Such players are accustomed, when they know where the desired card is, to keep it on the bottom and to deal out others, which chance alone would not call for, until they get the suppressed card for themselves. But the other players in the first-mentioned class carry out very dangerous frauds which are worthy of death, as in fact the latter is also, but it is more concealed. Those, however, who know merely by close attention what cards they are to expect are not usually called cheats, but are reckoned to be prudent men. As for those who use marked cards, some mark them at the bottom, some at the top, and some at the sides. The first kind are marked quite close to the bottom and may be either rough or smooth or hard; the second are marked with color and with slight imprints with a knife; while on the edges cards can be marked with a figure, a rough spot, with interwoven knots or humps, or with grooves hollowed out with a file. Some players examine the appearance of a card by means of mirrors placed in their rings. I omit the devices of kibitzers—the organum, the consensus, and the like.⁸

Certainly, in view of the small amount of pleasure provided by gambling and the rarity with which it favors our wishes, there are so many difficulties in it, and so many possibilities of loss, that really there is nothing better than not to play at all. There are also some who smear the cards with soap so that they may slide easily and slip past one another. This was the trick practiced upon me by the well-known Thomas Lezius of Venice,⁹ patrician, when in my youth I was addicted to gambling.

Now, in general, gambling is nothing but fraud and number and luck. Against fraud the one remedy is to beware of men of deceitful mind; for, just as a good man cannot be a cheat, so a cheat cannot be a good man. When you suspect fraud, play for small stakes, have spectators, shuffle the cards instead of merely collecting them, and if another collects them without shuffling, he is acting fraudulently. Have your own cards, and if others send out to buy cards, let them buy from men you can trust. Examine them inside and out and edgewise; touch the corners; if they are rough, or too smooth, or hard, or uneven, do not play; for

before you can recognize what is wrong, your opponents will perhaps ruin you. Finally, let no one examine the cards in private. In *primaria* (which is also called *primero*), it is customary to uncover the cards from the back and from above as little as possible so that the kibitzers cannot see anything; a great part of the art appears to consist in this, and players boast about their skill in this respect.

Considerable perplexity arises on the following point. Since prestidigitators are capable of such admirable feats, why is it that they are usually unlucky at cards? It would seem reasonable that, just as they are able to deceive us with balls, pots, and coins, they should also be able to do it with cards and so invariably come out winners. But the condemned Spaniard was ordered (in fact, the prohibition, they say, was on pain of death) not to play, seeing that he could at will produce four cards that make chorus¹⁰ either by deceiving the eye or by making the exchange by quickness of hand; for we must assign to either of these a prodigious art of prestidigitation. But Franciscus Soma of Naples¹¹ (about whom I have spoken elsewhere, at the beginning of book 18, *De Subtilitate*) could change cards so quickly that nothing more wonderful could be imagined.

⁸ The “organ” was a loose floor board under the table on which one player rested his foot, receiving signals from an accomplice who moved the board slightly, usually by pulling a string from another room. The “consensus” is an unknown device; the word itself means complicity or collusion.

⁹ Thomas Lezius was probably one of Cardano’s gambling companions during the time he lived in the town of Sacco.

¹⁰ Chorus: four of a kind.

¹¹ In chapter eighteen of *De Subtilitate* the man’s name is given as Soma. For his card tricks see Ore, p. 133.

18. Customary Conventions in Primero

IN PRIMERO certain conventions are considered customary. It is not permitted to draw from the discards: that is considered a fraud. Additions are therefore made gradually from the deck. If the point is simple, or even if it is superior, they change cards once. If primero or fluxus is present when a player has divulged his hand, then that player is master of the lot.

Those who admit four of a kind, or in other words chorus, do not admit the supremus in order that the pot may be increased, and those who admit the supremus do not admit chorus in order that it may be permissible to increase the stake. He who, not having announced his primero or fluxus, shall have increased the deposit, except when purposely changing cards, loses his deposit; but if he has not increased it, he is compelled at the will of the others to change cards; if one has too little and the others more, then those who so wish can contend separately beyond that which is least. And even though that third player should win, still they contend for the remainder just as if they were playing alone among themselves.

If anyone places a wager from the beginning, then if someone accepts it, the others are absolved. If no one does, then the last player from him who places is compelled to stake. If he wishes to bet more than the first amount, anyone who wishes may accept the condition, or else refuse it, but for the first bet (as I have said) the last player is held responsible. If no one bets, they are compelled to exchange one or two cards according to their judgment. There are those who play for equal stakes in the case of two cards and the game is a mixture, at least according to my opinion, of the French Geleus and primero.

If anyone wins with the greater point, he is compelled to show another card. If he does not do so, he loses his deposit because he could have fluxus. If he professes to have primero and his fellow-player refuses to increase the deposit, he is compelled to show primero. For he might have been able to deceive him without primero by inviting him to play for a greater deposit. Similarly, if he invites a bet on the basis of a point, he is compelled to show two different cards and one of the same suit as one of them, in order that nobody may suspect him of having fluxus or primero.

19. On the Diversity of Points or Numbers in Primero

THE HIGHEST simple point after the greatest of all, which is the supremus, is 54, the lowest is 20, and therefore the mean is 37. However, on account of the supremus, 38 is considered the mean, but it cannot be made with two cards; therefore 39 is taken to be the mean, and if you have the superior position on the right of the dealer, it will be called the superior mean.

But in primero, when it cannot be beaten by supremus, nor by anything else except chorus, the highest is 78. But the highest without restriction is 84 and the lowest is 40. Therefore the mean without restriction is 59, or rather 62 if you are at the right. For 78 also on the left can be beaten by a similar hand, but 79 can be beaten in no way except by chorus. For chorus wins over everything and supremus may beat primero; but for one or the other of them the liberty of increasing the bets is taken away. Again in fluxus the lowest number is 42 and the greatest 70, so that the mean is 61. Let us therefore reason correctly: since the mean number from the beginning is 37, the greatest is 39 and the least is 34. They are made up from the highest cards combined in three ways, and they can make up the supreme point with the expectation of one card. Therefore, since there are 102 points in the other eight cards, and, in the game of primero 37 is the mean point of two (high) cards, the mean point is 50 in three cards, and of the rest twice 13, so the mean point will be 63. Thus, the same thing happens in fluxus as in primero; moreover, many varieties occur, both of the above-mentioned sort, of which greater or smaller varieties may appear, and of the sort which I shall mention.¹²

¹² The bids in primero were divided into high and low, and Cardano tries to explain how the dividing points may have been selected in each case.

For the point bids everything above and including 39 should be considered a superior point. To arrive at this figure Cardano recalls that 20 and 54 were the two extremes for the points and their mean is 37. Here the supreme point with the value 55 is not considered to be a point, but if this is done the mean becomes 37V6 and thus 38 should be the starting figure for the high points. However, as Cardano argues, 38 cannot occur on two cards, therefore 39 is preferred.

In the case of the primero Cardano has to perform a similar juggling to make his argument fit the rule. When the chorus or four of a kind is counted as primero, the maximum is 84 and the minimum 40, giving the mean 62; thus superior primero should start at 63. This appears to be the correct figure, at least for the person in the lead to the right of the dealer. If the chorus is not counted, he arrives at the mean 59 based on the top value 78. However, this maximal value of 78 is hard to reconcile with the fact that 3 sevens and a six give a prime value of 81, while 3 sevens and an ace give 79. The bid of 78 consists of 2 sevens and 2 sixes.

Cardano's mean rule is still harder to fit in the case of the flush. The extremes are 42 and 72 as stated

previously and Cardano first gives the mean erroneously as 61 instead of 57. However, he notices immediately that this does not lead to the conditions he wishes and he resumes his reasoning on a different basis.

We recall that when play begins each player has two cards and there may be considerable passing and drawing before one of them obtains cards which he considers sufficiently promising for an opening bid. In case he goes for a flush two high cards in the same suit would be the most desirable hand for making a *vada* ("it goes," the start of play). This would be either seven, six or seven, ace or six, ace; each of them can produce the supreme point with a single additional card, as Cardano observes. The corresponding point values are 39, 37, and 34. He assumes that a player opens with the middle high pair, a seven and an ace with the point value 37. There remain eight cards in the same suit and their point values add up to 102. Thus the average value for each of the remaining cards is $12\frac{1}{2}$ which Cardano rounds off to 13. He computes correspondingly that on three cards the average is $37 + 13 = 50$ and for a flush $37 + 13 + 13 = 63$ which is the same figure as for a *primero* and evidently the middle value he wishes to establish.

20. On Luck in Play

IN THESE MATTERS luck seems to play a very great role, so that some meet with unexpected success while others fail in what they might expect; so that the above reasoning about the mean does not apply. For this mean is composed of extremes, not as in lawsuits, and valuations, and the like. For it is agreed by all that one man may be more fortunate than another, or even than himself at another time of life, not only in games but also in business, and with one man more than another and on one day more than another.

If anyone should throw with an outcome tending more in one direction than it should and less in another, or else it is always just equal to what it should be, then, in the case of a fair game there will be a reason and a basis for it, and it is not the play of chance; but if there are diverse results at every placing of the wagers, then some other factor is present to a greater or less extent; there is no rational knowledge of luck to be found in this, though it is necessarily luck.

However, astrologers make claims for themselves; yet I have never seen an astrologer who was lucky at gambling, nor were those lucky who took their advice. The reason is as follows: although, as in all matters of chance, it happens that they occasionally make the right forecast, nevertheless, if they have guessed right (for when they do not guess right, they must lose) then immediately afterward they go very badly wrong, since they become venturesome and lose more from one mistake than they win from forecasting correctly four times in a row. For the path into error is always steeper, and the loss is greater than the gain.

And I have seen others who make their decision with the help of geomancy; but this is an unstable vanity and dangerous unless very moderate use is made of its deceptive help. As for the fact that some of them do make forecasts, this is to be explained on the ground that others make still more frequent mistakes. For since there is necessarily some inequality in this respect, one man will be right more often and another less. These statements appear likely to our reason.

Yet I have decided to submit to the judgment of my readers what happened to me in the year 1526 in the company of Thomas Lezius, the patrician of Venice, leaving it to each reader to form his own opinion. I had just duly resigned from the office of rector of the scholars in the University of Padua on the third of August, and now I was journeying with Hieronymus Rivola, a scholar from Bergamo, on a certain night of the same month toward Venice. We were playing

a game (called Bassette) and I won all the money he had. Then he asked me to play with him on credit, if I am not mistaken, up to two or three aurei, and I won again. Then, finally, he wanted to carry it on endlessly, but I refused. He promised to pay what he owed me within three days; but he did not come.

Then he chanced to meet me and said that he would come to pay the money on Saturday (which was the day of the Nativity of the Virgin) and promised to take me to a beautiful prostitute. At that time I was just completing my twenty-fifth year, but I was impotent. Nevertheless, I accepted the condition; there was not a word about the game. He came on the day agreed; and in that year the festival of the Blessed Virgin was on Saturday. He took me to the home of Thomas Lezius; there was no Thais there, but a bearded man with a young servant. No money was paid but we played with marked cards. I lost to him all the money which he owed me, and he reckoned it as part of his debts just as though he had given it to me. I lost about twenty-five aurei or even a few more which I had, and played on, giving my clothes and my rings as security.

I returned home in sadness (as was natural), especially since there was no hope of getting money from home because uprisings and plots were raging at Milan. And so (and now I tell the truth, there being no reason why I should lie) I contrived for myself a certain art; I do not now remember what it was, since thirty-eight years have passed, but I think it took its rise in geomancy, by which I kept in mind on up to twenty-four plays all the numbers whereby I should win and all those whereby I should lose; by chance the former were far more numerous than the latter, even in the proportion (if I am not mistaken) of seven to one; and I do not recall now in what order these were against me.

But when I saw that I could not safely hold more numbers in my memory, I admonished my young servant, whose name was Jacob, that when he saw I had won back my clothes and my money he was to call me. I threatened that if he did not do it I would beat him severely. He promised and we went. As the game went on I won and lost in all the plays just as I had foreseen and after the third play I realized that there was no trickery or deceit about it. They laid down money freely and I accepted the wagers, but he was delighted by the example of the previous day and also on account of the marked cards (as I have said).

Thus his thoughts were inflamed by his youthful ardor; but the result was otherwise, for, on those plays in which I saw (as it were, without any real foreknowledge) that I would win, I did not reject any amount of money and made large bets of my own, and in the other cases, where I knew he would win, I refused if he was the first to wager, and wagered very meagerly myself: thus the result was that within twenty plays I regained my clothes, my rings, and money and also what he had added besides. As for the clothes, the rings, and a collar for

the boy, I sent them home piecemeal. Out of the total number there remained four deals; I played and won, and also came out victor in a few deals which were not contained in the number.

He was already perturbed and full of admiration, since he saw that in all the plays in which we played for high stakes I came out the victor, and in those in which he won I myself wagered little and when he wished to wager a great deal I refused. So (he said) I believe some demon is advising you, or that you know by some enchantment what is going to happen. What happened after that I remember that I have narrated elsewhere.¹³

But now I return to the question at issue; if anyone should say that my Genius was advising me, although apparently he had never yet revealed himself, I will not dispute it; yet, there must have been some art by which the Genius himself had this foreknowledge. On this point (as I have said) I leave the decision to others.

Now I think it worthy of consideration that this fortune of mine seems to have been something greater than mere chance, since we see in it a beginning, an increase, and a certain continuance so that certain remarkable things happen, as for instance that two aces occurred twice when defeat could not otherwise be brought about, and other things of this sort. We would also see a decline and then very often a change, and then great calamity or great good fortune, and other things in the same way. In view of all this I should think we ought to decide that there is something in this, although we do not know the law which connects the parts. It is as though you were fated in advance to be enriched or despoiled; especially seeing that from this there can follow something more important, as it happened to the man who, on leaving a game after losing all his money, injured the image of the Blessed Virgin with his fist. He was arrested and condemned to be hanged.

But whether the cause of that luck, be it in the conjunction of the stars or in the construction of a certain order of the universe, can affect the cards, which are considered bad or good only according to the conventions of men (since they signify nothing of themselves), is so worthy of doubt that it is easier to find a cause of this fact without that purpose than with it; without it the matter can well be reduced to chance, as in the constitution of the clouds, the scattering of beans, and the like.

¹³ De Vita *Propria*, Chapter 20. See Ore, p. 129.

21. On Timidity in the Throw

FOR THIS REASON it is natural to wonder why those who throw the dice timidly are defeated. Does the mind itself have a presentment of evil? But we must free men from error; for although this might be thought true, still we have a more manifest reason. For when anyone begins to succumb to adverse fortune, he is very often accustomed to throw the dice timidly; but if the adverse fortune persists, it will necessarily fall unfavorably. Then, since he threw it timidly, people think that it fell unfavorably for that very reason; but this is not so. It is because fortune is adverse that the die falls unfavorably, and because the die falls unfavorably he loses, and because he loses he throws the die timidly. So, since the timid throwing of the die is fourth in order while the unfavorable falling is second, the fourth cannot be the cause of the second, neither the proper cause nor a concomitant, but rather the other way around. Therefore, the timid throw, provided that the die is honest and he does not seize it in a way suggested by his opponent, cannot be the cause of the unlucky number: it can happen for other reasons.

22. On the Twofold Division of Games

SOME GAMES use dice, that is, the play is open, and others cards, that is, the play is concealed. Each kind is again subdivided, since some games consist solely of chance, especially dice, and also primero and fluxus in cards. But some join to chance the art of play, as *fritillus* in dice and in cards *taro*, *ulcus*, *triumphus* and the like. Therefore it is established that games consist either of luck alone, or of luck and art, and since each kind can be open or concealed, it is clear that there are four fundamental kinds of games among those which consist in luck, and not in bodily strength or agility.

23. On Card Games in Which There Is Occasion for Trained Skill

SINCE here we exercise judgment in an unknown matter, it follows that the memory of those cards which we have deposited or covered or left should be of some importance, and in certain games it is of the greatest importance, as in *trappola*, the Venetian game. In this game the threes, fours, fives, and sixes are removed. In the four suits there are sixteen of these, so that thirty-six cards are left. They are dealt out, five at a time and then four; in the case of two players eighteen cards are dealt and the same number left in a pile; if the first player is satisfied with his cards, he retains them, and if the second player is dissatisfied with his, he exchanges them and receives the nine top cards on the pile; and if these please him, then he himself keeps them, but if not, he exchanges them again with the second pile.

So you see how much depends on memory, judgment, skill in avoiding deception with due regard for safety. Many players, then, although they remember well, do not avoid the stratagems of their adversary carefully enough, or do not play with foresight, or too timidly, or as though they were angry.

Therefore when I had settled in the town of Sacco I was delighted by this game in a wonderful way; from it I saw the beginning of all good fortune. For by careful attention I brought it about that I was always mindful of all the cards which I had discarded. But art is of very great importance in the play. For the player who gets the last card has six points; if it is a two, it is worth 25, the six points being due to its being the last card. The ten, since it is the smallest, unless it is alone, cannot fail to be beaten: but in last position it is doubled, and so is worth 20, and when 6 is added to this, it is worth 26; if this happens twice, that is, if the two is in both the last and next to the last position, it is worth 52, namely, 26 each; but if there are three twos they are worth 78, namely, 3 times 26. Since the three last cards are all twos, it would mean 104 if the 52 were doubled. But this is not right; for then it would be necessary that the last should be worth 26, the last together with the next to the last worth 78, and the third last, together with the other two, worth 234, which no one admits. But they agree on a custom or rule of this sort that the two also, if it is the first card, or among the rest, should not be beaten, since the adversary would lack cards of that suit. It is worth 10, but among some it is worth 12. The jack is worth 3 points, the queen (or the knight) 4, the king 5, the one they call *luneta*, as being supreme in its suit

(and it conquers the king and all other cards), is worth 6; three lunetas before the play are worth 12, three twos are worth 10, three kings, or knights (or queens), or jacks are worth 6, and it is not necessary to say, except in the case of lunetas, what suit they are; but all of these, together with the two, are called by the name of face cards. But it is necessary for three to be alike and of one suit, as three twos, or three kings. It is not necessary that we should name the lunetas or face cards at the beginning of the game, but only before we show any of those three, or before we play, or in playing, provided we have not collected any of those; if any player gets all the cards, this is called *cucus*¹⁴ and the whole score is doubled.

When I had reduced the memory of these cards to the knowledge of one word, I learned to include in this fashion in a single word many other things as well; and thence by practicing this invention a whole text and all that was contained in it. And after that passages were found and derived from authors, which led to extemporaneous declaiming.

But I return to the play of cards; seeing that this kind of game was most artificial, since it would depend on a fixed procedure and forced arrangements, I omitted all exercise of that kind of divination for many reasons. First of all, I betook myself to Padua and thence to Sacco and so I had no further opportunity for the game; for that type of chance does not have any place where skill is mixed into the game. Secondly, I was afraid that it might ruin me through being overconfident. Moreover, I said that, if it is deceptive because of the demon, it is thereby contrary to law, but if it depends on chance, then it is foolish to trust it. Also I shrank from a game which was condemned by the law. Moreover, I thought it foolish to wish to contend so absurdly when I could play under more profitable or safer conditions.

¹⁴ The word means jackdaw.

24. On the Difference Between Play with Cards and Play with Dice

CARD GAMES differ from dice games even when these require skill, because play with dice depends more on judgment of future events; mostly, to be sure, on the success of one's opponent but also on one's own success, while play with cards requires only judgment of one's present holdings and of one's opponent's. To conjecture about the present is more the part of a prudent man skilled in human wisdom; but to conjecture about the future, although it is another kind of guessing, not as to what will be, but what we may rightly count on, is nevertheless the part rather of a divine man, or of an insane one, for the melancholy are given to prophesy. For in play with dice you have no certain sign, but everything depends entirely on pure chance, if the die is honest. Whatever there may be in it beyond unfounded conjecture and the arguments given above should be put down to blind chance.

But in cards, apart from the recognition of cards from the back there are a thousand other natural and worthy ways of recognizing them which are at the disposal of a prudent man. In this connection the game of chess surpasses all others in subtlety. It is subjected little or not at all to the arbitrariness of chance. Similarly, exercise with weapons surpasses everything else in usefulness, play with balls surpasses in healthfulness, trappola in charm, primero in beauty of invention and variety, sanctius by the greatness of the stakes, fritillus in attentive competition with little fatigue, tarochi in the passing of time, cricones in dignity, and triumphus in prudence and imitation of human life. So it is more fitting for the wise man to play at cards than at dice and at triumphus rather than at other games; so it is agreed (but it is not in use) that this is a sort of midway game played with open cards, very close to the game of chess. It has an end when nothing further can be done and every game makes its own end. It is played with nine cards (for this is a satisfactory number) and is the mean between the great and the small; when the cards are placed on the table one begins to play, as one is accustomed to do with hidden cards. Since this is a most ingenious game, I am very much surprised that it has been neglected by so many nations.

25. On Card Games

THUS card games do not consist in expectation of future events except when cards are being exchanged. There are several kinds but it is not possible to enumerate them. For what Horace said about words can suitably be said about games: it will be sufficient to distinguish them by their kind and type, both simple and composite. First, therefore, are the kinds of games which depend solely on the arbitrariness of chance, and, secondly, those requiring attention and skill.

Dependent on chance only are *primero* and the *fluxus* which is played with three cards; the French type is the most beautiful; for an ace with a face card, or with a deuce, makes 21, which is the highest point among the other nations generally. But among the French, if another one is present, they make 21V2. But three points surpass even fluxus and there is a *supremum* which can be considered either as a fluxus or as a point. Also, 20, together with a one or another card, makes 20V2, and so they have 20, 20½, 21, 21½, and 22, which (as I have said) is superior to fluxus. Then comes *fluxada*; and *silerium*, which, in my opinion, is the same as 40 and 31. *Bassette* is the game of desperate men; then there is the game of *cricones*, which belongs to chance, and to skill belong *triumphi*, *triumfeti*, and variations of these, a variation of *tarochi*, *one hundred*, *trappola*, *romfa*, *scaltara*, and many others.

There is another division according to their peculiar differences, since some games involve judgment of the cards, whereas others do not but involve the drawing of cards. Some games by the number of cards, some by the estimation, some by the consequences, some by a fixed reckoning of the place or position, some by some other property, make up the whole number of types by which games may be classified as being superior or inferior.

26. Do Those Who Teach Also Play Well?

PERHAPS someone will quite rightly ask whether the same people who know these rules also play well or not. For it seems to be a different thing to know and to execute, and many who play very well are very unlucky. The same question arises in other discussions. Is a learned physician also a skilled one? In those matters which give time for reflection, the same man is both learned and successful, as in mathematics, jurisprudence, and also medicine, for very rarely does the sick man admit no delay.

But in those matters in which no time is given and guile prevails, it is one thing to know and another to exercise one's knowledge successfully, as in gambling, war, dueling, and commerce. For although acumen depends on both knowledge and practice, still practice and experience can do more than knowledge. Also a certain physical acquaintance is of greater value in those matters where there is need of special knowledge, as in the appraisal of gems, paintings, and the recognition of counterfeit or genuine money.

So there are three elements in the case, not all of equal importance, namely, physical nature, acumen, and quickness. So it was right for Hannibal to make fun of the philosopher who had never seen a battle line and was discoursing on war. Thus it is with all games which depend on the arbitrariness of fortune, either entirely or together with skill, when they are played rapidly and time is not given for careful thought. Since physical games, depending on agility of hand or sharpness of eye, are subject to training, it is not surprising that to know is one thing and to exercise one's knowledge, and exercise it rightly, is another. In certain matters, as in military affairs, knowledge joined to practice is of great value, but not practice joined to knowledge; for what is the principal thing in each matter ought to have precedence and be the greater.

27. Is There Some Element Apart from Skill Which Plays a Role in the Exercise of Skill?

SINCE it is necessary, in addition to practice, which gives us experience, to have agility, quickness, keen senses, knowledge, and method, it remains to ask whether there is anything beyond all these things which contributes to victory in games. And I mean even where you have plenty of time for reflection; for that is the most important element, together with the judgment which arises from it.

Just remember the ridiculous story that was told about the knot which loosened for the man who had been hanged; thus some people have one thing and other people other things to report. There are stones which, as they claim, increase the skill of those who wear them and I know that the pearl is among them, and similarly other stones which increase boldness, as those which are called nicolos.¹⁵

However, there is no question here of these or of similar things, but (as we have said) we must discuss the question of luck. So, just as there is change of persons, ages, years, months, days, and hours, so change varies fortune; thus, in everything that changes, there is necessarily something, since it has been agreed that it changed. As for instance: if I were to be defeated tomorrow and win the day after, there would necessarily be something of such a sort as tomorrow and the day after tomorrow; perhaps someone will say that there is nothing except time, which changes or can change fortune, nor is this reasoning absurd, since the variety of time depends on fate, and other things not so.

If, then, it is to be or not to be, how can it be changed by amulets? This same argument demonstrates that neither a doctor nor a barber nor a war lord ought to heal better, or cut hair, or manage warfare better if they were without these things. Even if this should be true, since it is so absurd and contrary to human opinion and reasoning, I do not see how it could be admitted.

Let us therefore examine what this luck is and on what principle it depends; certainly it seems to me to be a disposition of affairs in accordance with or adverse to the will or plan of a man; so that no matter how you act, the matter turns out well or badly, or agrees with human plans or does not agree. Good fortune is twofold, like force and guile in human affairs, and it may suit our plan or deceive us. For if I stay at home or do not stay, it can turn out very badly either way; but whatever happens, we are subject to the authority of the Prince. So it is also in games. Therefore, there are two kinds of happenings and non-

happenings, one of them absolute and the other relative to plan or judgment,

These matters are treated in the books about fate, but as far as the present question is concerned, it is sufficient to say here that fortune is changed by some principle, as a plan is changed, sometimes openly, as when I should go to war and experience a different fortune than if I should go on a journey or remain at home, but sometimes secretly as in gambling, as when I should shake more or less, and a different point should come forth with each different impetus.

Therefore, fortune works secretly in a twofold way, either by changing the plan from which action results or else by means of an event which is fortuitous in comparison with the existing circumstances. For example, I have an enemy, and his bad luck is thus part of my plan; but if his power should be increased by the marriage of his daughter, this pertains to luck and has nothing to do with my plan. Moreover, although the principle of the first of these two, namely, luck, is not clear, nevertheless it is beyond doubt.

We must therefore think that there is some method in each, and that it is possible to be blessed or to be unfortunate, in just the same manner as those who play while drunk, or angry, or in fear, or suspicion, things which are familiar to everybody.

But of the other method there is also some secret principle. To these matters belong amulets, witchcraft, and the like, and just as in each case (as they say) the sword fits its own sheath and the foot its own shoe, so the hour, the day, the year, and the place must fit; so also in this question, what will make one man happy will make another wretched. But most important seems to be judgment and foresight because it provides far-reaching counsel. For the other things admit ordinary treatment, namely, things which are evident and which extend to one throw or even to two.

¹⁵ The nicolo was the popular name for the onyx. It was supposed to promote audacity and steadfastness.

28. On Far-reaching Plans, Judgment, and Procedure [in Backgammon]

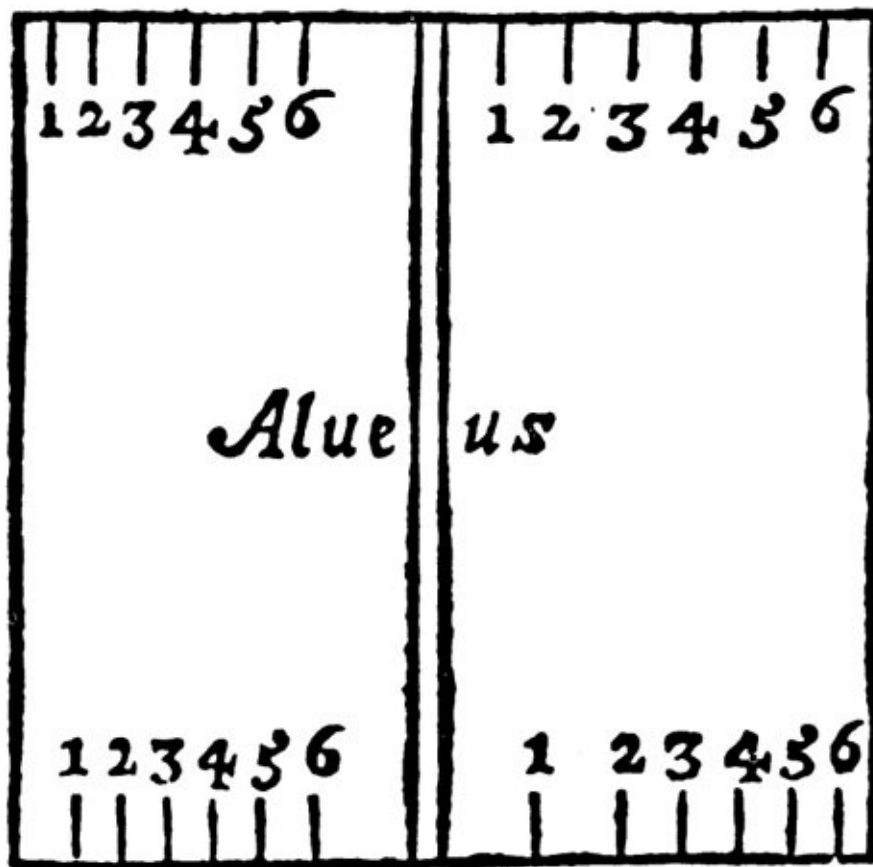
OF THE GREATEST IMPORTANCE in gambling is foresight and judgment pertaining to many throws, as on the board matters are so arranged that there is a twofold way to our goal, the one that we advance more rapidly, the other that we retard our adversary; we advance more rapidly by occupying better positions, and more quickly, if on the contrary, we retard our adversary either in order that he may have worse positions and fewer or that he may fall behind in entering the last row.

Since, therefore, the ways are as stated, we advance as far in each of them as we can. But we must take care that we apply ourselves to the advance to a greater extent, but not entirely, since this can scarcely be done, seeing that, in order to proceed more quickly, you must take care to be safe from the incursions of your adversary and to impede him. Again two aims arise, whether you ought to proceed deliberately or, on the other hand, whether you ought to lie in ambush. Therefore, you have already four goals: to proceed easily into the best positions; not to be open to hindrance; to hinder him; and to lie in safe ambush if the occasion should arise to hinder him. But there are two ways to hinder him: either to block him off from his position or from the passage to it; or else to cast out his counters, so that they are forced to return to their original positions.

For you know that there are four tables on the board, each of which contains six points. But we could also foresee hindrance beforehand, so that there are six aims. The obstacle also should be either in several or in special places. The special places are directly opposite and where he is forced to transpose to his disadvantage. But to foresee these things far in advance calls for a certain skill which no art can teach in general. Or rather one can certainly teach it, but nobody can be of such judgment as to acquire it.

Therefore, I shall first teach it by one example, in order that the approach to all these matters may be easier. Suppose that you despair of victory because you are delayed, let us say, by a fourth part of the throws; for it is always necessary to refer the matter to what is left, and although the proportion may be the same, still, the closer it is to the end of the game, the more difference it makes. Therefore, you must consider whether your disadvantage is small and can be repaired by a change of luck, for then the game is not to be conceded. Again, if you are closer to the end, although you can and ought to concede defeat, still you

will not do so. Again, if you are farther away from the end but the opponent's men have already gone far in advance in comparison with your own, you ought not to contemplate such a great step. Therefore, when your hopes are far inferior to his and the game has not proceeded very far, and your men have not progressed to points from which they cannot by any means be extracted since they are not ahead of his, then it is necessary to think about returning them and blocking some point. When you have finished deliberating, it is desirable to find a means to concentrate entirely on this business and try to bring back the men which have advanced. Moreover, this should sometimes be done aggressively, which is both deceptive and dangerous, and sometimes slyly (though of course it can be perceived), a procedure which, as it is a matter of greater art, is also safer and more certain.



29. A backgammon board. From *De Ludo Aleae*.

Having entered in this way upon general considerations, it is necessary to persevere. For this reason one must keep one's eyes open in this sort of business and especially consider whether one ought to act or not, and, if so, then whether openly or not. This judgment and counsel, this providence, both in details and in

general, is deeper than human nature. It is like the prospect of the sky and of infinity. In general, however, higher points are usually blocked with greater advantage, for fear that the adversary may proceed, but the medium ones, like those which they call homes, are dissolved.

29. On the Character of Players

EVEN evil has its own laws, as in the case of robbers and pirates. There are some who with many words drive both themselves and others from their proper senses. I omit speaking of those who, as though they were insane, hurl invectives against the gods. Certain people are so contentious that they provoke others to such anger that they forget everything. Finally there are those who crack jokes to their adversaries; others are silent, which would be best if they did not demand the same from their adversaries.

There is a golden mean in everything; let a man play such that first of all he does not forget the opening moves and those plays which contribute toward victory, that he is not angry, that he does not provoke anger, that he is not afraid, that he does not talk nonsense, that he does not vex a partner, especially when one is losing. Let him remember himself, his adversary, the bystanders, the place. If the adversary is not of this type, it is better not to play. For as a little bile in a great deal of honey and a little bit of rottenness in a large mixture of pleasant things can do far more to produce vomiting than all the rest can do to produce delight, so a blameworthy adversary can, like a brandished sling, drive far away all the pleasure of the game and all the peculiar delights derived from it. That taciturnity which verges on utter silence is too hard and severe, since it is a greater pleasure to talk without playing than to play without talking.

30. On Games of Chance Among the Ancients

THE ANCIENTS had a game of chance which they called *tesserae*, although *alea* has usurped the common name of all games which depend on chance. The inventor (as they say) was Palamedes while the Trojan war was in progress. For they sat, according to the testimony of all, around that city for ten years, so that it was devised in order to relieve the tedium. Those objects are properly called *tesserae* which others call cubes, with six faces and the same number of points. It was forbidden in the Roman period except during the Saturnalia; whence Martial (Lib. 14.1) “and the slave shakes the dice box without fear of the aedile, since he sees so close at hand the freezing of the ponds/” Nevertheless the cube seems to differ in some way from the tessera according to the statement of Vitruvius. The cube is an object of six sides with equal width of square plane faces. On whatever side it falls when it is thrown, and in good condition, it has a solid resting place; and the same is true of the *tesserae* which those players throw who play on a board; and they are called falls or throws among the Greeks. Our authority for this statement is Eustathius,¹⁶ according to Caelius Calcagninus,¹⁷ from whom we take this chapter and the following one.

It is agreed that they have six planes, of which number the tessera also consists. The same Eustathius has told us that on each tessera the individual points on the opposite faces should add up to the same number, namely, seven, as six is opposed to one, and five to two, and four to three. This is done with the idea of making it easier to detect falsification, if anyone should make a false die, by duplicating the one and leaving out some other number. This is an especially disgraceful act, particularly if during the play the dice are thrown trickily by sleight of hand while the adversary’s attention is distracted.

So it was our duty to add this chapter of review, not so much for the sake of the ancient customs as for giving information by means of which, when understood, something more useful could be derived. For as this was designed to remove the greatest fraud, it offers an occasion for a lesser one. For those who throw the points they want, by agility of hand, do so in three ways. First, in such a way that one die rides over the other, or so that one die rides over the two others if there are three of them: that is the best method and is called “to ride over” in colloquial speech, generally, “to change the die.” A second method is more deceptive, and consists in letting the die cling to one of the sides or to two sides (of the board) and it is called in colloquial speech “spuntonum.”¹⁸ The

third is not very safe, but is on the other hand very deceptive, when the die is thrown straight with such an impetus and such a number of points exposed above that it is probable that the point which we wish will come uppermost.

In all these matters it is of the utmost help that the cheat should know how to throw the point in a certain position. Thus, it is not only of assistance to know that the six is opposite to the one and that the other points are in circuit, but also it is necessary to know exactly where, for instance, when the two and five are on the heads, the six and the one are on the sides (in figure, [Chap. 9](#)). So, against all these tricks they have devised the ore from the similitude of the fish. For it seems to devour the tesserae as the ore devours other smaller fish. Persius (*Saty.* 3): “not to be deceived by the neck of the narrow ore.” Pomponius the poet of Bologna: “while contemplating the ore I have lost the little dice.”

But the children inserted nuts into the ore so that the gambling object which he names is not the same as this one, although similar to it. Horace calls this *pyx* a *pyrgus*, thereby using a Greek word when he says (*Saty.* 9, *Sermon.* 2), “put the tali in the pyrgus.” For they put in it not only tali but also tesserae; it is in constant use at Bologna but not at Milan. Martial calls it a *turricula*, whence in the *Apophoreta* on the turricula: “the wicked hand seeks to gather up and cast the tali; if it casts through me, it always produces what it wishes.” This game of tesserae is modified so as to be played with the fritillus (for that is not the pyrgus but a gaming board; you may take it so, I do not contend about words); then you would say that with the board (if that is acceptable) the game is very similar to human life and, as it were, a modified example of what is stated in the *Adelphi* of Terence (Act 4, Scene 7): “The life of man is like when you play with the tesserae: if the best throw does not turn up, then whatever does turn up by chance, you must use to the best advantage.” Moreover they played with two and with three tesserae; with two, for example, in the poem of Martial in the *Apophoreta* (14, 17): “Here for me the tessera is counted twice with a six.” On the other hand, three are mentioned in the Greek proverb “either three sixes or else three ones.”¹⁹

In our time also, games with the gaming board have become better known with three dice, *e.g.* *sperainum*, *Speraia*, and *speraionum*, from which they also call the board itself *sperainum*; but in the colloquial language they call it *sbaraid* and *sbaraionum*, and similarly the little board *sbarainum*. Calcagninus thinks that *sperainum* is derived from “spargere” (to scatter), and *sbarainum* is that which would scatter. However that may be, all these games would be played with three tesserae; but we use two tesserae in a different way in *sbarainum*, since we always suppose the third die to be a six.

But *sbaraia* with three tesserae we call *sbaraionum*, as well as that game itself

with three tesserae, but in it anyone may double the throws. This is the game of Princes and does not require thought; sbarai is ingenious; sbarainum occupies an intermediate place; it is more in use because sbarai is longer but sbarainum depends mostly on luck. There are three other rather celebrated games with two tesserae, namely, *tocadigium*, of which there are two kinds, the smaller, which depends on luck, and the greater, which requires far-seeing judgment; and there is also *canis Martius*, and tables, which are games requiring a moderate degree of skill; but *canis Martius* requires an outstanding intelligence.

In tables five men are placed in the last point from your right, and two in the first from your left; three in the second from your right and five in the sixth from your left, and your adversary places the same number in the corresponding places directly opposite to these. The most important consideration is to hinder the passage of your adversary's men and to strike them off. There is also another game which they call *Minoretum*, and there are two varieties of this, as of *tocadigium*, namely, the greater and the smaller; in the latter it is not possible to reenter with a man which has been struck off, and the game is lost just as in *tocadiglia* when the pieces have been taken. But in greater *tocadiglia*, when this happens it is nevertheless up to the player's judgment whether he wishes to continue.

While the game of tesserae did not exceed the number of three, the game of *tali* had four, as will be seen below. So Martial says in the *Apophoreta*: "I, the tessera, would not wish to be equal in number to the *tali*, provided only the value of the throw should often be greater for me than for the *tali*." Ovid, moreover, tells us that the tesserae consisted of ivory, although in our times they are made of any kind of bone: "whether he shall play and throw the ivory numbers with his hand." In our times I have seen them made of rock crystal with gold points; tesserae are called by our contemporaries *Dati* because, in ancient times, they were accustomed to say that they played *datatim* (giving them to each other in turn), whence Plautus in the *Curculio*, "those slaves of the fine gentlemen who play *datatim* in the street"; and Pomponius, "when I played *datatim* with you in bed"; and Quintilian seems also, as that same Calpurnius avers, to have said, "remembering with what die he had gone wrong he returned to the one with which he had played"; but when he speaks about a game of twelve *scrupi* he seems to have referred the word *datus* to a game, but not to a game with dice. The cube (as I have said), they called *Monas* and sometimes *asinus*. Moreover, Midas was fortunate in his throws, whence the proverb,²⁰ "Midas, who is the best adviser in tesserae."

But this was only in one kind of game, very frequent and much practiced, as three sixes in the throw amongst us (2. *de arte amand.*): for this is superior to all

the other numbers because it consists of three similar faces, and the highest at that; but of the beautiful games which are in use, there is only sperainum, or sbarainum with two dice, which is without the six.

¹⁶ Eustathius, archbishop of Thessalonica (d. A.D. 1193), author of a famous commentary on Homer, where some of the ancient Greek games are discussed.

¹⁷ Celio Calcagnini, *De Talomm ac Tesserarum*. . . . See Bibli ography, Ore, p. 179.

¹⁸ The word means halberd or spear.

¹⁹ Meaning: “highest luck or deepest despair.”

²⁰ The Midas throw may possibly have been the same as the Venus.

31. On Play with Knucklebones

KNUCKLEBONES were also made of ivory, as Martial says in the *Apophoreta*, “nor do the six and the one shake my ivory” I wished to say this because, although the talus is part of the back part of the leg of an animal, and in particular of a horned animal, those which were in use were nevertheless not natural tali but were made out of ivory or other material to resemble them, and were made smaller in order that they also, in the manner of the tesserae, might be enclosed in the pyrgus and be thrown out, and might not, by being placed together, fall according to the thrower’s wish; it is agreed that tali and the Greek word *astragali* mean the same thing; Lysander used to say that children ought to be deceived with astragali and men with oaths. The saying was worthy of a Lacedaemonian, that is, of the worst kind of man; for nothing was more vile than that country which even Aristotle shunned, though he endured the Athenians, in whom there were innumerable disadvantages.

The talus is found in the joint of horned animals where the tibia joins the foot. It has six surfaces like all other bodies which are not round, on account of their three dimensions, since in each dimension there are two boundaries. But two of them are so small that it seems to have only four; one of them is humped up since it is curved, and on the opposite side it is concave and on each side it has a square, as it were, except that it narrows a little; on these are the one and opposite it the six, and then the three and four; and these are the four numbers with which the game with tali is played.

They call the one *Chios* and the six *Cous*, and when the one is compared with the six, the former is considered unlucky and the latter lucky. But when they are compared with the Venus, they are both unlucky. Whence Suetonius says, in the epistle about Augustus, “for when the tali have been thrown, if a player has thrown the dog or the six on a talus, they put a denarius in the middle for each, and then all of them were taken by the player who had thrown a Venus.” Therefore, it is clear that the dog is the one, both because it is opposed to the six and because of those words about each talus.

So let us now consider composite cases: first, there may be four alike, as four aces, then a total of 12 with three alike, as (1,1,1,3), (1,1,1,4), and (1,1,1,6); and also (6,6,6,1), (6,6,6,3), or (6,6,6,4). But there are 18 cases of two alike. Thus all the throws of the knucklebones are 35. Among all these the most fortunate is the Venus, which consists of the dice presenting the natural position of the numbers,

namely, one, three, four, and six, which is unique in knucklebones. But if it be compared to the total, it can happen in 24 ways, so that the four cases of all alike cannot happen more frequently:

<i>All different</i>	1
<i>Four alike</i>	4
<i>Three alike</i>	12
<i>Two alike</i>	18
SUM	355

Therefore, this one throw alone, which we call Venus, will happen 6 times as often as a throw with all alike. The three alike have 48 throws, but from the 6 throws two-alike and two-alike, since they can vary in 6 ways, there are 36 throws. The rest vary in 12 ways; therefore, there will be 144 throws; altogether, then, 256, of which number the number of similar ones, that is, four equal, is the 1/64 part:

$$4 + 24 + 36 + 48 + 144 = 256$$

But for the Venus the 24 cases is about 1/11, that is, it will happen that the Venus is thrown more often. Next to the Venus the most favorable throw was the *Stesichorius*, giving a total of 8, consisting of two ones and two threes. But it occurred in 6 cases only, wherefore it was considered among the rarest throws.

There was also the *Euripidean* throw, with four fours-it is counted 40, falsely, as some would say, because it cannot reach that sum. But that the Venerean throw was actually such is very clear from Martial, "when the tali have fallen for you with no face the same, you will say that I have given you great gifts." This is in the game for money; but the tali were also used to prophesy fate and the Venus was considered favorable and the dogs especially unfavorable, whence Propertius, "When I too was seeking Venus with favorable tali, the damned dogs always leaped out." Thus also in choosing the king at banquets according to that passage of Horace, "nor will you appoint by lot the arbiters of the wine"; and also this passage, "whom Venus names as arbiter of drinking." There was also another way, among children, of playing with tali, with an arbitrary number of them, just as they do now with little balls of lead, not contending for money but for the tali themselves. There is a story of this sort about Cupid, who won all the tali of Ganymede; these stories are well known but show the usual Greek levity. There are also other names for the throws, such as *Voltorius* and *Basilicus*, as Plautus says in the *Curculio*:

*He invokes Planesius,
He throws four voltorii.
I seize the tali;
I invoke my kindly nurse Heres;
I throw Basilicus.*

He describes everything very well with one exception, namely, the four voltorii, that is, the fours; for they are long and thin and black and interpricked with white; this was the best point; but the parasite threw basilicus, which is probably the same as Venus; but on that basis I think it was the four sixes, since they strove for the highest point, not for the Venuses and the dogs; or if you wish, like the basilicus and the Venus, the vultures were sixes.

You will say, were not three sixes superior to four vultures? I reply no, since they were not alike, as I have also said in the game of tesserae about the throws. It would be permitted to bring up more cases here than with tesserae, since the total is 256 and of three tesserae 216, although the variety is more in the tesserae, namely, 56, but in the tali (as has been said) it is only 35.²¹

²¹ Cardano here has the misprint 36.

32. Conclusion of the Work

BUT IN GENERAL it is to be observed, both for dice and for knucklebones, that since both of these complete the circuit in as many throws as there are faces, the die in six and the astragal in four, so in any one throw of any number of either dice or astragals, even if there should be a hundred of them, each makes up all possibilities in the same way. And so, if the total number shown by them be divided by the number of planes or faces, we get the average number.

Thus, in the case of six dice, one of which has only an ace on one face, and another a deuce, and so on up to six, the total number is 21, which, divided by 6, the number of faces, gives $3\frac{1}{2}$ for one throw. However, the rule is not general; but in the case of most throws, it will turn out less than 3 on account of the excess of the greater numbers, which have to be computed.

But in the case of astragals, in order to determine the average number, add the greatest to the least, that is 4 and 24; the result is 28; half of this, 14, is the average number. So, when there is more than one ace, that throw is called the dog, because whatever the other dice may be, the throw cannot exceed the average number; from such a power of impeding, therefore, all throws with more than one ace are called the dog.

The same reasoning applies to finding the average number also in the case of three dice; for since the greatest is 18 and the least 3, added together they make 21, half of which is $10\frac{1}{2}$, which is the average number.

By these pleasures it is permitted to relax the mind with play, in turmoils of the mind, or when our labors are light, or in great tension, or as a method of passing the time. A reliable witness is Cicero, when he says (*De Oratore*, 2): “men who are accustomed to hard daily toil, when by reason of the weather they are kept from their work, betake themselves to playing with a ball, or with knucklebones or with dice, or they may also contrive for themselves some new game in their leisure.”

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