Quantum Computing

Muchang Bahng

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In quantum mechanics we usually work in a Hilbert space V over field $\mathbb{C}$. A **ket** is of the form $|\, v\rangle$, which mathematically denotes a vector v in V. A **bra** is of the form $\langle f\,|$, which denotes a covector $f \in V^ast$, the dual space. With the usual construction, we say if $|\, m\rangle$ defines a column vector in $\mathbb{C}^n$, then $\langle m\,|$ is its conjugate transpose. Furthermore, let us write the shorthand notation for the classical bit as such:

$$|\, 0\rangle \coloneqq \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |\, 1\rangle \coloneqq \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

When can represent a set of classical bits as the tensor product, which is realized as simply the outer product. For a system of two bits, we have

$$\begin{pmatrix} x_0 \\ x_1 \end{pmatrix} \otimes \begin{pmatrix} y_0 \\ y_1 \end{pmatrix} = \begin{pmatrix} x_0 y_0 \\ x_0 y_1 \\ x_1 y_0 \\ x_1 y_1 \end{pmatrix}$$

We can take our familiar logic gates and represent them as matrix operations. For example, the `NOT` gate, which flips the state of the bit, can be represented as

$$N = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

and the `CNOT` gate, which takes gate, which takes two bits and flips the state of the second bit if the first is a $|0\rangle$ and does nothing otherwise, can be represented as

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \implies \begin{cases} C\left(\begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right) = C \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |01\rangle \\ \ldots \\ C\left(\begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right) = C \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |10\rangle \end{cases}$$