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GROWTH-RATE DETERMINATIONS IN NUTRITION STUDIES WITH THE BACON PIG, AND THEIR ANALYSIS

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Introduction

Some time ago the writer had occasion to analyse statistically the results of a nutrition experiment, in which three different levels of protein content in the food ration were tested on three groups of bacon pigs, which were under experiment from shortly after weaning until they left for the bacon factory at 200 lb. live weight. For full details the paper presenting the results may be consulted (Woodman *et al.*, 1936). In accordance with standard experimental practice the variable examined was the live-weight gain, i.e. the difference, in lb., between the initial and final weights of each pig over the 16-week period of the experiment. The results showed an average drop of about $4\frac{1}{2}$ lb. in live-weight gain from treatment A (ranging from 17·5 to 12·2 % crude protein as the experiment proceeded) to treatment B (22·1–16·9 % crude protein), and a further drop of about 4 lb. from treatment B to treatment C (26·8–21·7 % crude protein). On the analysis of variance test, however, the estimate of treatment variance, with two degrees of freedom, was not significant, and even the "principal effect" of the drop from A to C, with a single degree of freedom, was not significant when isolated. The mean square for this principal effect was obtained directly from the difference of the totals, or means, for A and C, since the increase in protein percentage was linear. It was not that the standard error was abnormally high; in fact the experiment was a very accurate one, the percentage drop in live-weight gain from A to C being 5·7, with a standard error of 2·9; but the effect was small. Considering that the variation in the initial weights of the pigs (who were all of the same age) may have contributed somewhat to the experimental error, this concomitant variable was taken account of by an analysis of co-variance. The result was that the principal effect could now be adjudged significant at the 5 % level ($z=0{\cdot}8561$ with $n_1=1$, $n_2=21$). Slender as the effect was, it was considered worthy of note, the percentage drop from A to C for animals adjusted to have the same initial weight being 5·6, with a standard error of 2·4.

The animals were carefully weighed every week, and yet in the investigation, the results of which have been briefly summarized, no account was taken of the figures other than the initial and final ones, though intervening weight measurements were used, naturally, in fixing the amount of meal to be fed. This raises the point of whether more accurate information may not be gained if the other fifteen observations are brought into the picture. It is not wholly a statistical point, for doubts have been raised in the experimenter's mind whether the figures of live-weight gain are an adequate summary of the information available. For this reason an investigation has been made of the seventeen consecutive weight measurements, at weekly intervals, of the thirty individual pigs constituting the experiment, and the purpose of the present paper is to give the results of this study.

DETERMINATION OF GROWTH CURVE

The weight figures, to the nearest lb., were first plotted on ordinary, and then on logarithmic paper, from week 0 to week 16. This was the period for which observations are available for all pigs. Not all were withdrawn at the end of this time, as some had not reached 200 lb. It was, however, thought desirable to keep the time period the same in all cases. The weight curve was very regular, and showed an upward curvature; since the logarithms showed no sign of linearity, but a definite downward curvature, it was thought best to do the curve fitting on the actual weights. The next stage was to fit a parabola of the second degree to the figures for each pig. As a sample of the calculations involved, the figures for one pig are given (Table I), the method of orthogonal polynomial fitting followed being that of Aitken (1933). The number of the week is represented by x, and the weight (in lb.) by w. In Table I the calculations have been carried as far as needed for the fitting of a (second degree) parabola. The three columns following w are obtained by continuous summation from the bottom upwards, stopping at the result underlined. These underlined numbers are then carried to a column on the left of the coefficients enclosed in a square. These latter are readily calculated, as shown by Aitken, but they are taken from his table for $n=17$, as also the quantities $\Sigma(T_r^2)$ at the foot. The coefficients a_0, a_1 and a_2 are now obtained by cross-multiplying the numbers on the left by the coefficients of each column, row by row, summing and dividing by the number at the foot of the column. It is desirable to note down the numerator before division, because it will be needed later. We are now ready to write down the equation to the parabola. Utilizing the coefficients within the square, it is, in the factorial form appropriate to Aitken's method,

$$W = a_0 + a_1(2x-16) + a_2\left\{\frac{6x(x-1)}{1.2} - 45x + 120\right\}$$

$$= 118{\cdot}12 + 4{\cdot}972(2x-16) + 0{\cdot}0664\left\{\frac{6x(x-1)}{1.2} - 45x + 120\right\}.$$

The equation may, however, be transformed in any way we please. In Fisher's (1936) form, writing $\bar{w}$, g and h for Fisher's constants A, B and C, to avoid confusion with our food treatments, the equation is

$$W = \bar{w} + g(x-\bar{x}) + h\{(x-\bar{x})^2 - (n^2-1)/12\},$$

TABLE I

x	w				W	$w-W$
0	48	2008	—	—	46·5	1·5
1	54	1960	20121	—	53·5	0·5
2	60	1906	18161	111520	60·8	−0·8
3	67	1846	16255	93359	68·6	−1·6
4	76	1779	14409	77104	76·8	−0·8
5	86	1703	12630	62695	85·3	0·7
6	94	1617	10927	50065	94·2	−0·2
7	104	1523	9310	39138	103·6	0·4
8	112	1419	7787	29828	113·3	−1·3
9	124	1307	6368	22041	123·5	0·5
10	134	1183	5061	15673	134·0	0·0
11	144	1049	3878	10612	145·0	−1·0
12	158	905	2829	6734	156·3	1·7
13	170	747	1924	3905	168·0	2·0
14	181	577	1177	1981	180·2	0·8
15	192	396	600	804	192·7	−0·7
16	204	204	204	204	205·6	−1·6

$\Sigma\,(w-W)^2 = 20{\cdot}55$

	a_0	a_1	a_2	W_0	ΔW_0	$\Delta^2 W_0$
	118·12	4·972	0·0664	—	—	—
2008	1	−16	120	46·54	—	—
20121	—	2	−45	—	6·956	—
111520	—	—	6	—	—	0·3984
$\Sigma\,(T_r^2)$	17	1632	69768	$205{\cdot}64 = W_{16}$	—	—

$a_i \Sigma w T_i$	277850·00	D.F.	Sum of squares	Mean square
$2008^2/17 = 237180{\cdot}24$		16	40669·76	—
$8114^2/1632 = 40341{\cdot}30$		15	328·46	—
$4635^2/69768 = 307{\cdot}92$		14	20·54	1·4671

Standard error $= 1{\cdot}211$ or $1{\cdot}025\%$

$$W = 118{\cdot}12 + 4{\cdot}972\,(2x-16) + 0{\cdot}0664\left\{\frac{6x\,(x-1)}{1.2} - 45x + 120\right\}$$

$$= 118{\cdot}12 + 9{\cdot}944\,(x-8) + 0{\cdot}1993\,\{(x-8)^2 - 24\}.$$

where $\bar{x}=8$, the mean week, and n is 17. Thus

$$W = 118{\cdot}12 + 9{\cdot}944\,(x-8) + 0{\cdot}1993\,\{(x-8)^2 - 24\}.$$

The interpretation of g and h is that the former is the average growth rate in lb. per week, while the latter is just half the rate of change of the growth rate in lb. per week per week. $\bar{w}\,(=a_0)$ is, of course, the average weight of the pig in lb. throughout the period.

These calculations were carried out for each pig, but in the case of the one illustrated the additional calculations show what must be done (1) to find the polynomial values W corresponding to the observed w, and (2) to test the significance of the linear and parabolic terms. The calculations are for the most part self-explanatory, but it should be explained that W_0, the value for $x=0$, ΔW_0 and $\Delta^2 W_0$, the first and second differences at this point, are obtained by cross-multiplying a_0, a_1 and a_2 by the coefficients of the rows in the square, and summing. W_{16} is obtained as a check in a similar manner to W_0, but with alternate positive and negative signs in the summation. The sum of squares of residuals, $\Sigma\,(w-W)^2$, furnishes a check on the calculations at the bottom of the table, wherein $\Sigma\,(w-W)^2$ is obtained by repeated subtraction of terms dependent on the mean and the linear and parabolic terms. The number in the numerator here is the numerator of a already referred to. That the linear and parabolic terms are both significant is demonstrated by separate comparisons of the quantities 40341·30 and 307·92, with one degree of freedom each, with the residual mean square 1·4671, having 14 degrees of freedom, by means of the z-test. As a matter of fact, the cubic term is also significant, but not the quartic term, as a continuation of the calculations will show. For present purposes, however, attention has been concentrated, in the agricultural problem, on g and h only.

ANALYSIS OF AVERAGE GROWTH RATE

The figures which now require to be analysed are given in Table II, together with the initial weight w_0, in the order in which the pigs were arranged for feeding, and the sex and feeding treatment are indicated by letters. (H = hog, G = gilt; A = low protein, B = medium protein, C = high protein.) Each pen contained six animals, three hogs and three gilts, all from the same litter. These were allowed to run together, but at feeding time they were segregated into boxes, in the order shown, for individual feeding. The five pens represent five different litters. A further complication of design is that in each pen three animals were selected from the litter which were above average weight at weaning (heavy), while the other three were below average (light). The group of three, to which the three feeding treatments were given, were either all hogs or all gilts, with the exception of pen IV, where the allocation was as indicated in Table II. Since there are five pens, the heavy and light lots are not evened up as regards sex, and this may introduce complications in any sex comparison which may be made. It was not the primary object of the experiment to make these sex comparisons, but evidently any information which can be learnt on this point will be of general interest.

TABLE II

Data for analysis

Pen		Treatment	Sex	w_0	g	h
I	Heavy	A	G	48	9·94	0·199
		B	G	48	10·00	0·146
		C	G	48	9·75	0·136
	Light	C	H	48	9·11	0·139
		B	H	39	8·51	0·154
		A	H	38	9·52	0·209
II	Light	B	G	32	9·24	0·147
		C	G	28	8·66	0·181
		A	G	32	9·48	0·194
	Heavy	C	H	37	8·50	0·144
		A	H	35	8·21	0·119
		B	H	38	9·95	0·178
III	Light	C	G	33	7·63	0·176
		A	G	35	9·32	0·176
		B	G	41	9·34	0·182
	Heavy	B	H	46	8·43	0·171
		C	H	42	8·90	0·155
		A	H	41	9·32	0·176
IV	Heavy	C	G	50	10·37	0·207
		A	H	48	10·56	0·126
		B	G	46	9·68	0·213
	Light	A	G	46	10·98	0·193
		B	H	40	8·86	0·157
		C	H	42	9·51	0·130
V	Light	B	G	37	9·67	0·192
		A	G	32	8·82	0·199
		C	G	30	8·57	0·189
	Heavy	B	H	40	9·20	0·192
		C	H	40	8·76	0·177
		A	H	43	10·42	0·200

w_0 = initial weight at week 0 in lb.
g = average growth rate in lb. per week.
$h = \frac{1}{2}$ (rate of change of growth rate in lb. per week per week).

From the analysis of variance point of view we can evidently eliminate pen differences as irrelevant to the food comparisons we wish to make. We then have the choice between assembling the data in food and heavy-light groups, studying separately the effect of food, the difference between heavy and light, and the interaction between these factors, or arranging the data in food and sex groups, with a similar analysis. Evidently the sex comparison is confounded with the heavy-light comparison in a way that makes it impossible to say that any sex difference found is independent of heavy-light differences and vice versa. Since, however, we may make use of the initial weights in an analysis of co-variance to examine the effects existing in groups of animals with the same initial weight, this complication need not disturb us. We may therefore analyse the data for any particular variable into a pen or litter effect (4 degrees of freedom), a food effect (2 degrees of freedom), a sex comparison (1 degree of freedom), the interaction of food and sex effects (2 degrees of freedom), and the residual or error (20 degrees of freedom). For the growth rate g the analysis is as follows:

TABLE III

Analysis of variance of growth rate

Variation due to	D.F.	Sum of squares	Mean square		
Pens	4	4·8518	1·2130	$z=0{\cdot}5754$	*S*
Food	2	2·2686	1·1343	$z=0{\cdot}5419$	*NS*
Sex	1	0·4344	0·4344		
Interaction	2	0·4761	0·2380		
Error	20	8·3144	0·4157		
Total	29	16·3453			

Standard error per pig = $\sqrt{(0{\cdot}4157)} = 0{\cdot}6194$, or 6·66 % of the mean growth rate per pig (9·304 lb. per week).

Comparison of the mean squares for the various effects with that for error is made by the z-test. *S* denotes significance at the 5 % level, and *NS* denotes "not significant". The differences between the average growth rates for the different pens are significant, which shows that different litters have grown at different rates, though this may quite well be due to the fact that their initial weights were very different. The differences between the average growth rates for the food treatments A, B and C are not significant on the 2 degrees of freedom, but if, as in the previous study, we take out the single degree of freedom for the "principal effect", the sum of squares for which is obtained as one-twentieth of the square of the difference between the A and C totals, which are 96·49 and 89·76 respectively, we get the following result:

TABLE III A

	D.F.	Mean square	
Principal effect	1	2·2646	$z=0{\cdot}8876$ *S*
Rest	1	0·0040	
Error	20	0·4157	

In doing this we are taking account of the additional fact that the three food totals, whose variation is given in Table III, are arranged in order of increasing protein percentage in the food. The significance now obtained, combined with the non-significance of the "rest", shows that there has been on the average a linear decrease in the growth rate with increasing protein percentage in the ration, and the result of the experiment may be summarized so far in the following table of average results:

TABLE IV

Summary of results—Growth rate

Food treatment	A	B	C	Mean	Standard error
Lb. per week	9·649	9·288	8·976	9·304	0·1959
%	103·7	99·8	96·5	100·0	2·11

The growth rate has dropped from A to C by 7·2 %, a figure which has a standard error of 3·0.

The interesting thing is that while the accuracy, as represented by the figure of the standard error per pig, is of the same order as that of the live-weight gains of the previous study (6·66 % as compared with 6·51), we have been able to detect a significant effect with the growth rate measurements which was only demonstrated with the live-weight gain figures after account had been taken of initial weights by covariance. This leads us to examine whether the food effect may not be more firmly established in its significance by taking account in the present study of the initial weights, since it may possibly have been considered that we were straining the analysis somewhat in the previous study to demonstrate the significance of what was after all only a small effect, though, of course, it has practical consequences of importance.

Analysis of covariance

The analysis of covariance procedure should by now be reasonably familiar to everyone, so that no apology is needed for presenting the results in a series of tables without lengthy explanation. Reference may be made to Wishart & Sanders (1935) by those requiring such explanation. Analysing the sums of squares and products of the figures for initial weight (w_0) and growth rate (g) in Table II, along the lines of Table III, we have the following table:

TABLE V

Analysis of variance and covariance. Initial weight and growth rate

Variation due to	D.F.	(w_0^2)	(w_0g)	(g^2)	$b=(w_0g)/(w_0^2)$	$(w_0g)^2/(w_0^2)$
Pens	4	605·87	39·905	4·8518		
Food	2	5·40	−0·147	2·2686		
Sex	1	32·03	−3·730	0·4344		
Interaction	2	22·47	3·112	0·4761		
Error	20	442·93	39·367	8·3144	0·08888	3·4989
Total	29	1108·70	78·507	16·3453		
Food + error	22	448·33	39·220	10·5830	—	3·4310
Sex + error	21	474·96	35·637	8·7488	—	2·6739

That the regression of growth rate on initial weight is significant is shown by the following test:

TABLE V A

Test of regression

Variation due to	D.F.	Sum of squares	Mean square
Regression	1	3·4989	3·4989 $z=1·3126$ *SS*
Deviations	19	4·8155	0·2534
Total	20	8·3144	

SS = significant at 1 % level.
Standard error per pig (residual) = 0·5034 or 5·41 %.

To test for the food effect, corrected for initial weight, we proceed as follows, by a process of subtraction of entries in the last column of Table V from the corresponding entries in the (g^2) column:

TABLE V B

Analysis of residual variance—Food

	D.F.	Sum of squares	Mean square
Food + error	21	7·1520	
Error	19	4·8155	0·2534
Difference	2	2·3365	1·1682 $z=0·7641$ *S*

We are thus able to assert, with greater confidence than before, since it is not necessary to split up the 2 degrees of freedom, that the food effect is significant. Correcting the three mean growth rates for variable initial weight by subtracting $b(w_0 - \bar{w}_0)$, where b is the regression coefficient calculated from the error term of Table V, while w_0 is now the *mean* initial weight for treatments A, B or C, $\bar{w}_0$ being the general mean, we may summarise the results as follows:

TABLE VI

Summary of results—Corrected growth rate

Food treatment	A	B	C	Mean	Standard error
Lb. per week %	9·676 104·0	9·235 99·3	9·003 96·8	9·304 100·0	0·1592 1·71

The percentage drop from A to C is unaltered (for in fact the initial weights for these two groups were the same), but the standard error of the three figures is reduced from 2·11 to 1·71, which accounts for the greater significance of the fall.

It will be noted that the gilts are lighter in initial weight than the hogs, but have the higher growth rate, though neither effect is significant. In view, however, of the positive correlation between growth rate and initial weight (from Table V b is 0·0889, corresponding to an r of 0·649), it is of interest to examine whether a significant sex difference emerges after correction for initial weight. The test is as follows, utilizing Table V:

TABLE Vc

Analysis of residual variance—Sex

	D.F.	Sum of squares	Mean square
Sex + error Error	20 19	6·0749 4·8155	 0·2534
Difference	1	1·2594	1·2594 $z = 0·8017$ *S*

The sex effect is now significant at the 5 % level, and correcting for initial weight the mean growth rates for hogs and gilts separately we have the following result:

TABLE VI A

Summary of results—Corrected growth rate

Sex	Hogs	Gilts	Mean	Standard error
Lb. per week	9·092	9·517	9·304	0·1300
%	97·7	102·3	100·0	1·40

The difference between the growth rates for hogs and gilts is 0·425 lb. per week in favour of the gilts, which difference has a standard error of 0·1903, calculated as the square root of

$$0{\cdot}2534\left(\frac{2}{15}+\frac{2{\cdot}0\dot{6}^2}{442{\cdot}93}\right).$$

0·2534 is the error mean square residual, while we are examining the difference between two means of fifteen pigs each. $2{\cdot}0\dot{6}$ is the mean difference in initial weight between hogs and gilts, while 442·93 is the error sum of squares for initial weight from Table V.* On a percentage basis, the difference between hogs and gilts is 4·6, with a standard error of 2·05. The experiment was not specifically designed to examine sex differences in the growth of the pigs, nor is it known how far such differences in growth rate during what is after all only the early part of the normal pig's life (though it is the whole of the life of the pig destined for the bacon factory) are matters of common knowledge. Nevertheless there seems to be little doubt about the effect in the present case.

If pen differences are examined in the same way, it will be found that the residual mean square, after correcting for initial weight, is not significant ($z=0{\cdot}4225$, $n_1=4$, $n_2=19$). This confirms the view that the significant pen differences in growth rate are a consequence of the very different average initial weights at which the different litters entered the experiment, and there is no evidence that rate of growth is a litter characteristic of any particular significance.

Analysis of rate of change of growth rate

A similar analysis to that of growth rate may be carried out on the parabolic term h of the curve fitted to the weight measures. The analysis of variance is shown in Table VII. It is clear, on examination of this table, that the only significant effect is that of sex. This is shown in Table VIII.

* See Wishart and Sanders (1935, p. 54).

TABLE VII

Analysis of variance of rate of change of growth rate

Variation due to	D.F.	Sum of squares	Mean square
Pens	4	0·0034835	0·0008709
Food	2	0·0012578	0·0006289
Sex	1	0·0030603	0·0030603 $z=0·8060$ *S*
Interaction	2	0·0008078	0·0004039
Error	20	0·0122093	0·0006105
Total	29	0·0208187	

Standard error per pig $= \sqrt{(0·0006105)} =$ 0 02686, or 15·63 % of the mean, 0·1719.

TABLE VIII

Summary of results—Rate of change of growth rate

Sex	Hogs	Gilts	Mean	Standard error
$\frac{1}{2}$ {lb./(week)^2}	0·1618	0·1820	0·1719	0·00638
%	94·1	105·9	100·0	3·71

Not only have the gilts shown a higher average growth rate than the hogs (when corrected for initial weight), but they now show a higher rate of change of growth rate, i.e. there is a greater degree of curvature in the growth figures. The difference in favour of the gilts is 11·8 %, with a standard error of 5·25.

Finally we may examine the rate of change figures in relation to initial weight. The table is as follows:

TABLE IX

Analysis of variance and covariance. Initial weight and rate of change of growth rate

Variation due to	D.F.	(w_0^2)	(w_0h)	(h^2)	$b=(w_0h)/(w_0^2)$	$(w_0h)^2/(w_0^2)$
Pens	4	605·87	−0·18386	0·0034835		
Food	2	5·40	0·0117	0·0012578		
Sex	1	32·03	−0·3131	0·0030603		
Interaction	2	22·47	−0·1293	0·0008078		
Error	20	442·93	0·10186	0·0122093	0·00023	0·0000234
Total	29	1108·70	−0·5127	0·0208187		

$r_{w_0h}=0·0438$ *NS*

That the regression, however, of rate of change of growth rate on initial weight is not significant is shown by the following test:

TABLE IX A

Test of regression

Variation due to	D.F.	Sum of squares	Mean square
Regression	1	0·0000234	0·0000234 *NS*
Deviations	19	0·0121859	0·0006414

This being so, we are not likely to add to the information already obtained by examining the various effects when corrected for initial weight. No improvement, for example, is shown in the significance of the sex comparison. The downward trend shown in the figures of rate of change of growth rate with increasing protein percentage in the ration, while suggestive of what may be happening, is definitely not significant, even if the principal effect, with 1 degree of freedom, be isolated from the remainder.

DISCUSSION

By considering the actual weekly figures of the weights of the thirty pigs given over to this nutrition experiment, we have been able to demonstrate the significance of the fall in average growth rate with increasing protein percentage in the ration, and the sex difference in favour of the gilts in the rate of change of the growth rate, without making any allowance for initial weight. This contrasts with the previous study where only live-weight gain was considered. When the figures of mean growth rate are corrected for initial weight, the significance of the food effect is stronger, and a sex difference in favour of the gilts emerges as significant. Not only is the taking into consideration of the initial weights valuable from the point of view of reaching such conclusions, but it seems to be necessary to do so if we are to disentangle the sex comparison from the heavy-light comparison with which it is to some extent confounded by the design adopted for the experiment.

Were the decisions reached by separate examination of the growth rate and change of growth rate figures not so clear-cut, it might be necessary to take these figures (g and h of Table II) together in a simultaneous analysis of variance and covariance, and reach a single test of significance of the effect of food (or of sex) on both simultaneously, after the manner suggested by Bartlett (1934). The method outlined in this paper of calculating a number of quantities to express the growth of the pigs would seem, in fact, to be well adapted to this method of analysis, since we are seeking the effect of the food ration on growth, which is expressed by both of the variables g and h (and possibly by the cubic term as well). Not only so, but the fact that it is desirable to take initial weights into account suggests that Bartlett's method should be applied to the partial variables derived from g and h when w_0 is held constant, and a test of significance derived in the same sort of way as in the usual covariance analysis. We have, in fact, a case of multiple dependent variables, with one independent variable, a special case of the kind envisaged by Day & Fisher (1937). This point is not pursued in the present paper, but is commended to the attention of investigators.

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