

Algorithms

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A course on the study of algorithms.

Definition 0.1 (Algorithm)

An **algorithm** is a procedure for solving a mathematical problem in a *finite* number of steps. It should be

1. finite,
2. correct,
3. efficient

Example 0.1 (GCD of Two Numbers)

Take a look at the following algorithm.

```
1 def gcd(a, b):  
2     if a == b:  
3         return a  
4     elif a > b:  
5         return gcd(a - b, b)  
6     else:  
7         return gcd(a, b - a)  
8  
9 print(gcd(63, 210))
```

1 Complexity

An algorithm, with respect to some inputs \mathbf{n} , will have a runtime that is some function f . We would like a formal way to analyze the asymptotic behavior between two functions.

Definition 1.1 (Complexity)

Given two positive functions f, g ,

1. $f = O(g)$ if f/g is bounded.^a
2. $f = \Omega(g)$ if g/f is bounded, i.e. $g = O(f)$.
3. $f = \Theta(g)$ if $f = O(g)$ and $g = O(f)$.

^aNote that it is more accurate to write $f \in O(g)$, since we consider $O(g)$ a class of functions for which the property holds.