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Personal Notes

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Contents

Chapter 1

Basic Economic Concepts

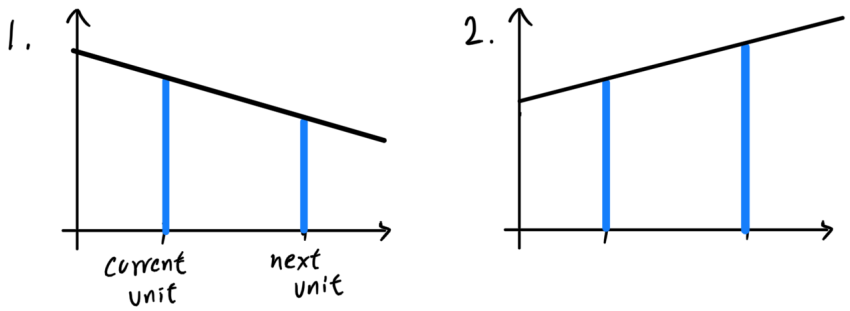
Discrete vs Continuous Models

Note that the models inherently deal with discrete values, but for the sake of simplicity, we represent them as a continuum. This results in two (equivalent) interpretations of the word **marginal**:

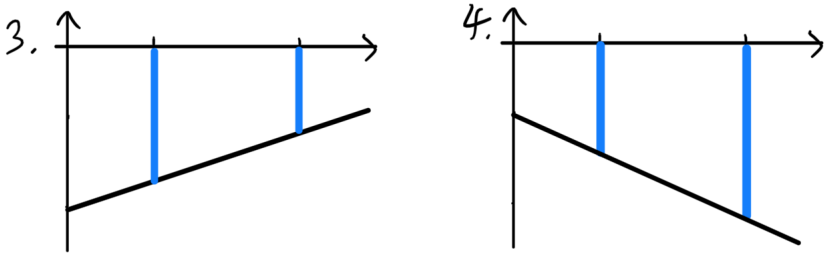
1. Discrete case: marginal value can refer to the additional value gotten from the next unit.
2. Continuous case: marginal value can refer to the derivative of the total value.

Equivalent scenarios between both cases are:

1. Additional value of the next unit is positive but not as positive as the value gotten from the current unit $\iff$ the marginal value has positive values, but is sloping down
2. Additional value of the next unit is positive and is more positive than the value gotten from the current unit $\iff$ the marginal value has positive values and is sloping up

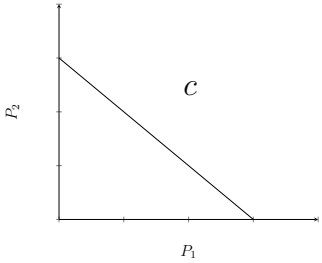


3. Additional value of the next unit is negative but not as negative as the value gotten from the current unit $\iff$ the marginal value has negative values but sloping up
4. Additional value of the next unit is negative and is more negative than the value gotten from the current unit $\iff$ the marginal value has negative values and is sloping down



1.1 High Dimensional Production Possibility Frontiers

The production possibility frontier (PPF) is a very fundamental tool in describing the different possibilities of producing a certain set of outputs. This is usually visualized with a monotonically decreasing (though not necessarily linear) curve in a 2-dimensional plane representing the output production of resources p_1 and p_2. Denoting the set of all quantities of outputs of p_1 and p_2 as P_1 and P_2, respectively, we can visualize the PPF in $P_1 \times P_2$.



This curve is determined assuming that we are given a well-defined quantity of an inherently limited resource. A common example of this is time, or more specifically, the time available to produce the resources p_1 and p_2.

Note that it is conventional in economics to treat P_1, P_2 as continuum, allowing us to draw a "smooth-looking" curve in $P_1 \times P_2$. This greatly simplifies models. But strictly speaking, it should not be assumed that P_1, P_2 are continuum. For example, in the context of producing televisions, it does not make sense to produce 2.5 TVs (or -3 TVs), so the space of outputs for televisions is $\mathbb{N} = \{1, 2, ...\}$. For the technical reader, we start by saying that since production of a certain good or quantity must be positive, P_i must be a subset of $\mathbb{R}^+$. We will treat P_i as having the subspace topology of $\mathbb{R}$ with its Euclidean topology. This means that if P_i is a discrete set (e.g. $P_i = \mathbb{N}$), then it is endowed the discrete topology, implying that any function f is smooth.

Furthermore, by treating time as third "resource," it is not difficult to see that the curve $c \subset P_1 \times P_2$ will change smoothly with respect to the amount of time that we are given. Therefore, letting T be the space representing values of time, we can define a function

$$f : T \longrightarrow P_1 \times P_2$$

such that $f(t_0)$ is the PPF curve existing in $P_1 \times P_2$, given time t_0. We can graph this function f as the set of all points

$$\{(t, p_1, p_2) \mid f(t) = (p_1, p_2)\} \subset T \times P_1 \times P_2$$

which defines a smooth surface, which we will denote S. The level set of S at time $t = t_0$ is then precisely the PPF relating the production of products p_1 and p_2, given t_0 quantity of time.

Let us now generalize this, but first it is important to rigorously distinguish the resources of time and the products themselves. Mathematically speaking, the set T is indistinguishable from the P_is; we can interpret T as a resource that can also be "produced" in the following sense. A point (t_0, a, b) can be interpreted as representing a situation where we produce a quantity of p_1, b quantity of p_2 given that we have t_0 time remaining. It can equivalently be seen as producing a quantity of p_1, b quantity of p_2, and producing t_0 quantity of t (and wasting that time). This allows us to interpret all p_1, p_2, and t as products that can be translated from one to another. But in real-world applications, this distinction is clearly made between what we call *products* that must be generated and *resources* that we can work with (e.g. time, effort, calories, machinery, workforce, etc.).

Now, let us have n different products $p_1, p_2, ..., p_n$ with their respective set of possible quantities being $P_1, P_2, ..., P_n$. Let us also have m resources $t_1, t_2, ..., t_m$ with their respective set of possible quantities being $T_1, ..., T_m$. Then, in the $(m + n)$ dimensional space

$$\prod_{i=1}^{n} P_i \times \prod_{j=1}^{m} T_j$$

we have an $(n + m - 1)$ dimensional surface S defined by the function

$$f : \prod_{j=1}^{m} T_j \longrightarrow \prod_{i=1}^{n} P_i$$

In general, given that we have a specific set of resources

$$t_0 = (t_{01}, t_{02}, ..., t_{0m})$$

$f(t_0)$ outputs a PPF of the n products, which is really a $n - 1$ dimensional surface in the space $P_1 \times P_2 \times ..., P_n$.

This concept of PPFs in higher dimensional spaces of both products and resources can be quite abstract. But we can immediately deduce some obvious properties of the PPF represented by the surface S.

1. $P_i \subset \mathbb{R}^+$ and $T_j \subset \mathbb{R}^+$ for all i, j.
2. We can model S as a graph determined by the equation

   $$f(p, t) = 1$$

   where $p \in \prod P_i$ and $t \in \prod T_j$, and the constant normalized to 1. Restating our descriptions above, given $t = t_0$, let

   $$f_{t_0} \equiv f(\cdot, t_0)$$

   Then, f_{t_0} defines a surface in $\prod P_i$ representing the PPF of the products $p_1, ..., p_n$, with $f_{t_0}(p) = 1$.

3. Given a p_k, and keeping all other variables constant,

$$\frac{\partial}{\partial \alpha} \int_V \left(\{p \mid f_{t_0}(p) \leq 1\} \cap \{p \mid p_k = \alpha\} \right) dV < 0 \tag{1.1}$$

where the integral represents the hypervolume of the set of points in the brackets.

The third property may especially seem like an abstraction, but it merely states that increasing production in product p_k will decrease the capacity (which is the hypervolume, i.e. the PPF) of production for the rest of the products. In the 2-dimensional case, this property is equivalent to the PPF strictly monotonically decreasing.

Note that (1) is a function of α. It is called the *opportunity cost* of producing product p_k given that it has already produced α quantities of p_k. Given resources t_0, we can denote the opportunity cost of producing product p_k for individual A as

$$\mathrm{Opp}_{t_0,A}(p_k)(\alpha) \equiv \frac{\partial}{\partial \alpha} \int_V \left(\{p \mid f_{t_0}(p) \leq 1\} \cap \{p \mid p_k = \alpha\} \right) dV$$

where f is the PPF of individual A. This makes sense intuitively, since (1) represents the instantaneous decrease in the capacity to produce the rest of the products when increasing production of p_k by an infinitesimal amount, from α to dp_k.

Property 3 allows us to apply the implicit function theorem to define each p_i as a function of all the other components of p.

It must be made clear of a couple definitions mentioned in economics. The PPF of the n products and m resources is represented by the set of points

$$\{(p,t) \mid f(p,t) \leq 1\}$$

and we are said to be at a *productive efficiency* if out production is represented by a point in the set

$$\{(p,t) \mid f(p,t) = 1\}$$

Individual A has a *comparative advantage* if the opportunity cost of producing the good is lower than another individual B. That is,

$$\mathrm{Opp}_{t_0,A}(p_k)(\alpha) \leq \mathrm{Opp}_{t_0,B}(p_k)(\alpha)$$

for all α (since $\mathrm{Opp}_A, \mathrm{Opp}_B$ are not necessarily constant).

Let $f_{t_0}(p) = 1$ be the PPF of individual A and $g_{t_0}(p) = 1$ be that of individual B. Then, individual A has an *absolute advantage* over individual B is

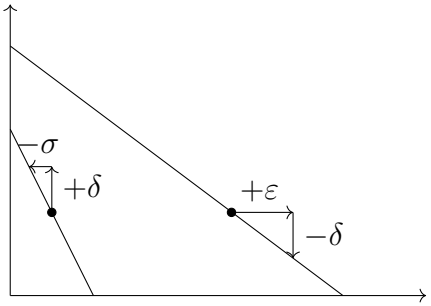
$$S_B \subset S_A$$

where $S_B \equiv \{p \mid g_{t_0}(p) \leq 1\}, S_A \equiv \{p \mid f_{t_0}(p) \leq 1\}$.

Two individuals can engage in *trade* by taking their produced goods and exchanging them for each other in a way such that they produce more in total than what they would have done individually. It is learned in a first-course in economics that *both* A and B can benefit from trade if B has a comparative advantage over A, even if A has an absolute advantage over B. This can be explained in the following way. Given a set amount of

resources t_0 and well defined PPF graphs f, g for A, B, let us single out the points p_A and p_B (with their total production $p_A + p_b$) and attempt to prove that an optimal pair of points (q_A, q_B) exists that maximizes the total quantity gained by producing and trading between A and B. For the reader familiar with basic machine learning, this algorithmic process is similar to the gradient descent algorithm.

Let us focus on the production of one good p_α. Without loss of generality, we can claim that A has a comparative advantage in the production of p_α, and furthermore that there exists a second good p_β in which B has the comparative advantage in. We can (but will not) prove here the latter part of this claim by proving that given a surface S satisfying property 3 above, there cannot exist a surface $\tilde{S}$ (also satisfying prop. 3) such that all of the partial derivatives of S are strictly greater than that of $\tilde{S}$. Since A has a comparative advantage in p_α, we can shift the production of p_α by an infinitesimal amount ε while decreasing the production of p_β by the following infinitesimal amount δ. We can then increase B's production of p_β by that same δ, which will then force us to decrease the production of p_α by σ.



But from our hypothesis that A has a comparative advantage in p_α and B has a comparative advantage in p_β, clearly the ϵ quantity of p_α gained by A is greater than the σ quantity of p_α lost by B. The δ quantity of p_β gained and lost cancels each other out. That is, $(\varepsilon - \delta) + (\delta - \sigma) > 0$. So, by utilizing the comparative advantages of multiple individuals, it is possible to create something out of nothing.

1.2 Supply and Demand

1.2.1 Marginal Value Curves, Demand Curves, and Consumer Surplus

Given a consumer and a certain good $\mathfrak{g}$, we can identify 2 values:

1. The individual's willingness to pay for an additional unit of that good (which can change over time). This can be interpreted as the consumer's "personal value" of $\mathfrak{g}$, which can be monetized to the maximum price that the individual is willing to pay for $\mathfrak{g}$.
2. The actual price of $\mathfrak{g}$ (which is assumed to be constant)

For example, Jason loves bananas and is willing to pay up to \$10 for a banana (personal value) and the store sells them at a price of \$1 per banana.

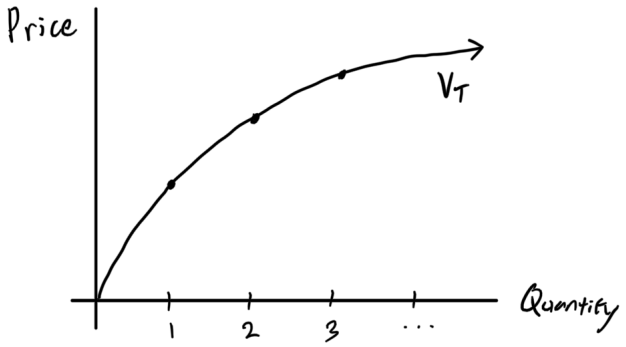
We present a critical (yet not an unreasonable) assumption.

Definition 1.2.1 (Principle of Diminishing Marginal Value)**.** Even though there are exceptions, the marginal value for most goods diminish. That is, the additional value of additional units of $\mathfrak{g}$ decreases.

Example 1. Jason's is willing to buy \$10 of his first banana, but is willing to pay only \$8 for the second banana, \$6 for the third, \$4 for the fourth, \$2 for the fifth, and \$0 for the sixth. This means that after five bananas, he is indifferent to getting an additional banana.

Definition 1.2.2 (Total Value Curve)**.** We can plot the total (monetized) value that each good brings a consumer by graphing the **total value curve**. This can be defined by the function v_T

$$v_T : \big(\text{quantity } q \text{ of } \mathfrak{g}\big) \mapsto \big(\text{total monetized value that buying } q \text{ units brings}\big)$$



When we have 0 units of the good, our total value is 0 since we have nothing. Having 1 unit of the good increases our total value. Having an additional unit of the good increases our total value again, but not as much as the first unit did. The same goes for the third value. We can mathematically describe this by saying that the graph of v_T is *convex*, or *concave down*.

Conventionally, we work a lot more with marginal value curves.

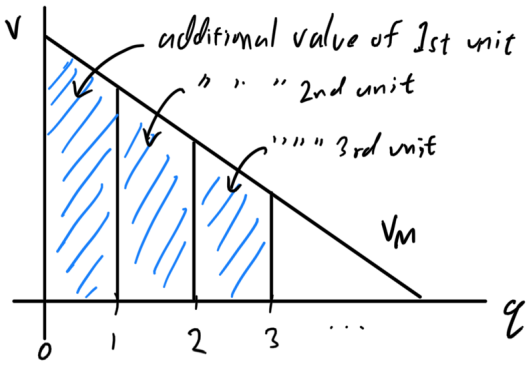
Definition 1.2.3 (Marginal Value Curve)**.** The **marginal value curve**, denoted v_M, is the derivative of the total value curve.

$$v_M = \frac{dv_T}{dq}$$

as the name suggests,

$$v_M : \big(\text{quantity } q \text{ of } \mathfrak{g}\big) \mapsto \big(\text{marginal value of next unit}\big)$$

By the principal of diminishing marginal value, v_M must be monotonically decreasing.



Furthermore, we can see that the area under the curve represents the additional value that each unit has added. That is, each "bar" represents the value of that additional unit. More formally,

$$\int_{i-1}^{i} v_M(q)dq$$

represents the value added by the ith unit of good, and

$$v_T(q) = \int_0^q v_M(x)\,dx$$

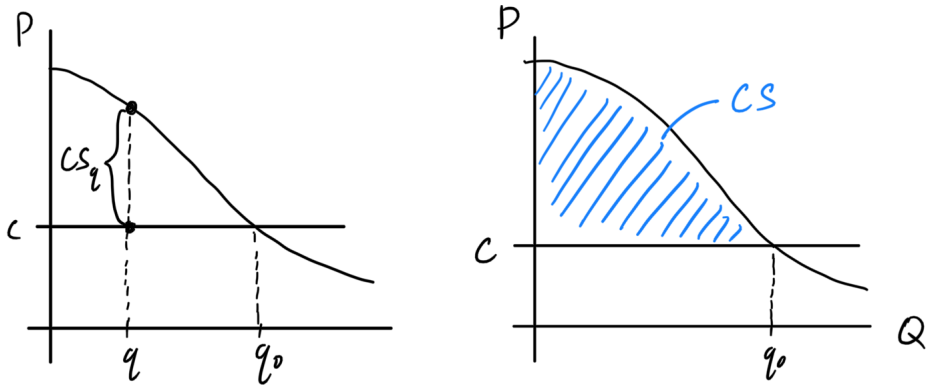
While we get a certain (monetized) marginal value from buying one more unit of $\mathfrak{g}$, we have assumed up to now that we get these for free. This isn't the case, since a unit of a good has a certain price that we pay for. In addition to the marginal value curve, we can graph the **marginal cost curve**, which basically represents the cost of an additional unit of $\mathfrak{g}$ (assumed to be constant). Let's call this value p, which stands for "price."

Definition 1.2.4 (Consumer Surplus)**.** We see that as long as the marginal value is greater than the marginal cost, the consumer would get a net profit. This profit is called the **consumer surplus**. One can see that the consumer surplus is greatest on the first purchase of $\mathfrak{g}$ and then decreases to 0 and negative values with enough purchases. Mathematically, it is defined

$$CS_q = v_M(q) - p$$

Another definition of consumer surplus is the *maximum profit* a consumer could make, given the marginal value and cost. This would be found by summing up all of the individual profits until the point q_0, which represents the point where the marginal profit is equal to the marginal cost. Mathematically, it is defined

$$CS = \int_0^{q_0} v_M(q) - p\,dq$$



Now, we introduce a related function, the demand curve. Note that the marginal value function inputs a quantity q and outputs the marginal value of q (i.e. what the next unit of $\mathfrak{g}$ is worth).

Definition 1.2.5 (Demand Function)**.** The demand function D is a function

D : actual price c of $\mathfrak{g} \mapsto q_0$ wanted to maximize consumer surplus = quantity demanded

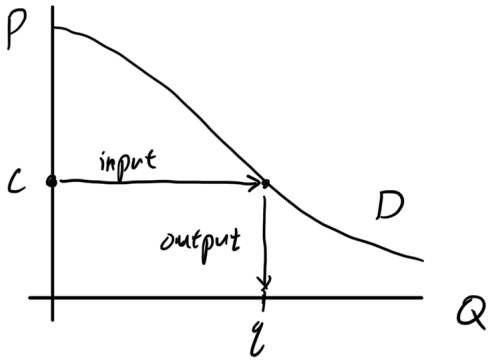
In other words, given the actual price c for a unit of good $\mathfrak{g}$, it is clear that the consumer can buy up to q_0 units of $\mathfrak{g}$ in order to achieve maximum consumer surplus. The quantity of $\mathfrak{g}$ demanded by the consumer would be q_0. Note that

1. the consumer would not want to demand more than q_0 since that would lead to negative value (marginal value is less than cost)
2. the consumer would not want to demand less than q_0 since there is extra room for profit.

Note that both D and v_M represents the same curve in $Q \times P$, but they are inverses

$$D = v_M^{-1}$$

Therefore, we can just think of the demand function as taking in cost and outputting the demanded quantity. This is represented in the **demand curve**.



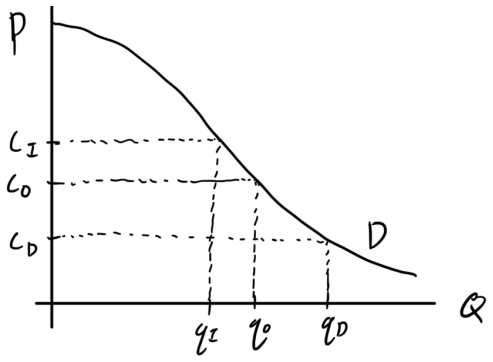
Theorem 1.2.1 (Law of Demand). The **law of demand** states that for a consumer, the quantity q demanded for a certain good is inversely proportional to the price p of that

good, assuming all else being equal. That is,

$$\begin{aligned} \text{increase in price} &\implies \text{decrease in quantity demanded} \\ \text{decrease in price} &\implies \text{increase in quantity demanded} \end{aligned}$$

In simple terms, we can interpret this law as such. If the price for a certain good goes down, the consumers, with the same money, have more buying power to buy more quantities. Hence, the quantity demanded increases.

Proof. This is quite easy to see with our current tools. Given a price p_0 and $q_0 = D(p_0)$, say the cost increases to p_I. Since the demand curve (interpreted as a marginal value curve) is unchanged, the marginal returns net of marginal cost decreases, lessening the consumer surplus CS_q for each q. But if CS_q is lessened for all q, then it would approach 0 faster as q increases, meaning that quantity demanded q_I would decrease. The same logic can be applied to proving that a decreased cost leads to an increase in quantity demanded.



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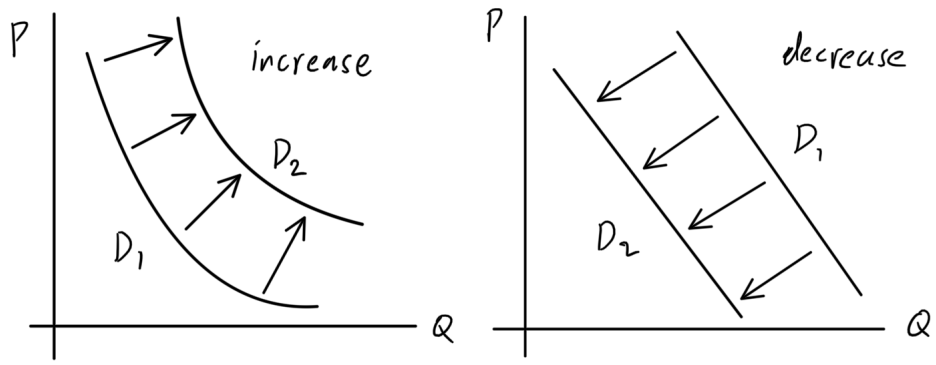
Note that all else being equal, if the cost changes, the quantity demanded changes inversely. However, there are external factors that can shift the entire demand curve.

Definition 1.2.6 (Shift in Demand)**.** External forces can change our needs for certain goods, increasing or decreasing their demand. An increase in demand is represented by an upwards/rightwards shift of the demand curve. This can be interpreted equivalently as:

1. more units demanded at a given price, or
2. a higher willingness to pay for each unit (i.e. higher marginal value)

A decrease in demand is represented by a downwards/leftwards shift of the demand curve. This can be interpreted equivalently as

1. less units demanded at a given price, or
2. a lower willingness to pay for each unit (i.e. lower marginal value)



Note that there is a difference between a change is demand and a change in quantity demanded.

1. A change in demand refers to the shift in the entire demand curve
2. A change in the quantity demanded refers to the movement of a point up and down the demand curve

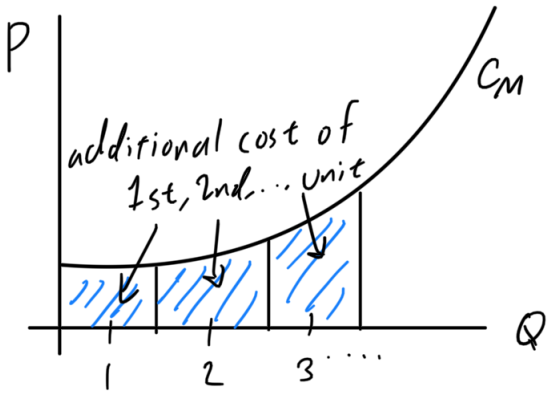
1.2.2 Marginal Cost Curve, Supply Curve, Producer Surplus

The relationship between the marginal cost and the supply is similar to that of the marginal value and the demand. The functions are inverses of each other, and represent the same graph on $P \times Q$.

Definition 1.2.7 (Marginal Cost Curve)**.** The **marginal cost**, denoted c_M, is the additional cost that it takes to produce/supply one additional unit of a good $\mathfrak{g}$. That is,

$$c_M : \big(\text{quantity } q \text{ of } \mathfrak{g}\big) \mapsto \big(\text{marginal cost of next unit}$$

Marginal cost tends to increase; that is, the more units that are supplied, the more it will cost to produce an additional unit.



Just like how consumer surplus is the marginal value net marginal (actual, constant) cost, we have an analogous definition for producer surplus.

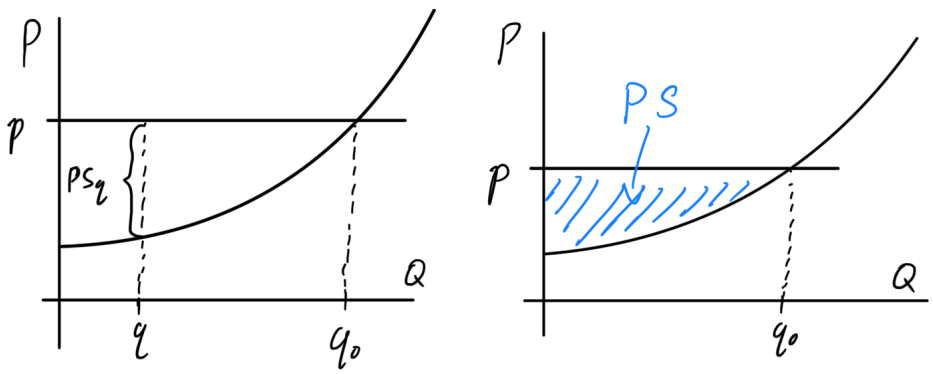
Definition 1.2.8 (Producer Surplus, Profit)**.** Given the marginal cost of good $\mathfrak{g}$, a supplier would like to sell it to consumers at a constant price c (analogous to how a consumer buys $\mathfrak{g}$ at price p).

The **producer surplus** is the profit/loss gotten from selling a unit of good $\mathfrak{g}$ at price p net the marginal cost. As long as the price p is greater than the marginal cost, then the supplier would get a net profit. Due to the monotonicity of the marginal cost function, the producer surplus is greatest on on the first supply of $\mathfrak{g}$, and then decreases to 0 and negative values with enough supplies. Mathematically, it is defined (on the qth supply)

$$PS_q = p - c_M(q)$$

Another definition of the producer surplus is the **maximum profit** a supplier could make. This would be found by summing up all of the individual profits until the point q_0, which represnts the point where the marginal profit is equal to the amrginal cost. Mathematically, it is defined

$$PS = \int_0^{q_0} p - c_M(q)\, dq$$



Definition 1.2.9 (Supply Function)**.** The supply function S is a function

$$S : \text{actual price } c \text{ of } \mathfrak{g} \mapsto q_0 \text{ wanted to maximize producer surplus} = \text{quantity supplied}$$

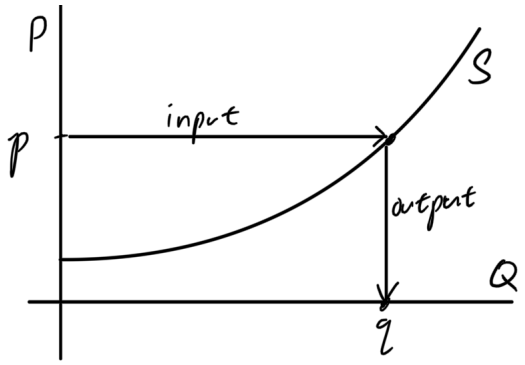
In other words, given the actual price c for a unit good $\mathfrak{g}$, it is clear that the supplier can produce up to q_0 units of $\mathfrak{g}$ in order to achieve maximum consumer surplus. The quantity of $\mathfrak{g}$ supplied by the consumer would be q_0. Note that

1. the supplier would not want to supply more than q_0 since that would lead to a loss in money (since the marginal cost is greater than the price)
2. the supplier would not want to supply less than q_0 since there is extra room for profit.

Note that both S and c_M represent the same curve in $Q \times P$, but they are inverses

$$S = c_M^{-1}$$

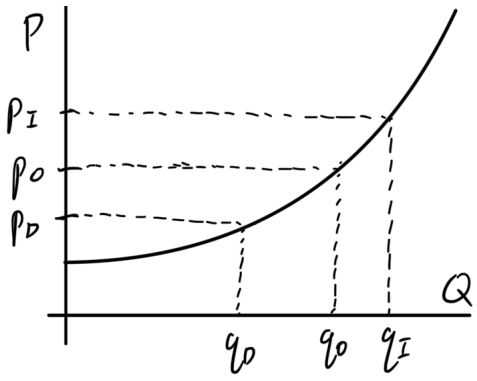
Therefore, we can just think of the supply function as taking in cost and outputting the demanded quantity. This is represented in the **supply curve**.



Theorem 1.2.2 (Law of Supply). The **law of supply** states that for a consumer, the quantity q supplied for a certain good is proportional to the price p of that good, assuming all else equal. That is,

$$\begin{aligned} \text{increase in price} &\implies \text{increase in quantity supplied} \\ \text{decrease in price} &\implies \text{decrease in quantity supplied} \end{aligned}$$

In simple terms, we can interpret the law as such: Producers are willing to offer more of a product for sale on the market at higher prices by increasing production as a way of increasing profits.

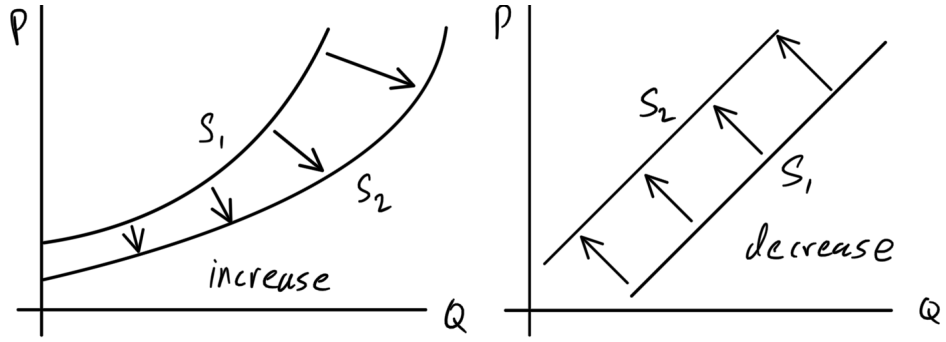


Definition 1.2.10 (Shift in Supply)**.** An increase in supply is represented by a rightwards/downwards shift of the supply curve. This can be interpreted equivalently as:

1. more units available at a given price, or
2. a lower price for the supply of the same number of units

A decrease in supply is represented by a leftwards/upwards shift of the supply curve. This can be interpreted equivalently as:

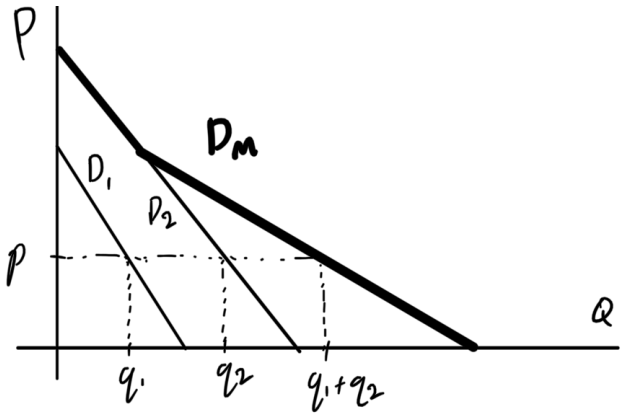
1. less units available at a given price, or
2. a higher price for the supply of the same number of units



1.2.3 Market Demand and Supply

While we have talked about individual supply and demand, we can extend this to talk about the supply and demand of the market, which refers to a population of individuals.

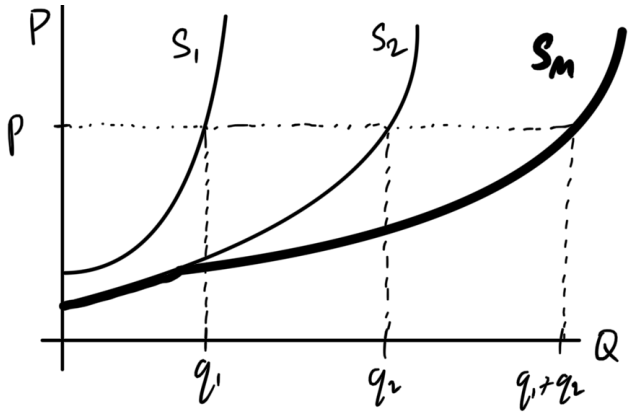
Definition 1.2.11 (Market Demand)**.** For a certain good $\mathfrak{g}$, the market demand curve of $\mathfrak{g}$ can be found by taking the **horizontal sum** of the individual market demand curves; it is a horizontal sum because we are summing up the quantities demanded at every price level.



Mathematically speaking, let there be N consumers $I_1, \ldots, I_N$ in a market. At a price level of p_0, if consumer I_i demands quantity $D_i(p_0)$ of $\mathfrak{g}$ (where D_i is the demand function of the ith consumer), then the demand of the market $\mathcal{M}$ at the price p_0 is

$$D_{\mathcal{M}}(p_0) = \sum_{i=1}^{N} D_i(p_0)$$

Definition 1.2.12 (Market Supply)**.** We also take the horizontal sum of the market supply curves of every supplier in the market.



Mathematically, let there be M suppliers $I_1, \ldots, I_M$ in a market. At a price level of p_0, if supplier I_i produces quantity $S_i(p_0)$ of $\mathfrak{g}$ (where S_i is the supply function of the ith supplier), then the supply of the market $\mathcal{M}$ at the price p_0 is

$$S_{\mathcal{M}}(p_0) = \sum_{i=1}^{M} S_i(p_0)$$

Substitutes and Complements

The supplies and demands for some goods can influence those of other, related ones.

Definition 1.2.13 (Normal, Inferior Goods)**.** We can categorize goods as such, based on income. Usually, with an increase in income, people's demands rise for goods.

1. Goods whose demand does not increase with income are known as **inferior goods** (e.g. bus fares). They get substituted for better quality goods.
2. Goods whose demand increases with income are known as **normal goods**.

Definition 1.2.14 (Complements)**.** For a given good $\mathfrak{g}$, a **complementary good** $\mathfrak{h}$ is a good whose appeal increases with the popularity of its complement. We can define them more specifically within the context of demand and supply:

1. If the price of $\mathfrak{h}$ decreases, then the quantity demanded for $\mathfrak{h}$ increases. This causes the demand of complementary good $\mathfrak{g}$ to increase. Similarly, the price of $\mathfrak{h}$ increasing causes the quantity demanded to decrease, resulting in the demand of $\mathfrak{g}$ to decrease.
2. If the price of $\mathfrak{h}$ increases, then the quantity supplied for $\mathfrak{h}$ increases. This causes the supply of complementary good $\mathfrak{g}$ to increase. Similarly, the price of $\mathfrak{h}$ decreases causes the quantity supplied to decrease, resulting the supply of $\mathfrak{g}$ to decrease.

Definition 1.2.15 (Substitutes)**.** For a given good $\mathfrak{g}$, a **substitute** $\mathfrak{h}$ is a good whose appeal decreases with the popularity of its substitute. We can define the more specifically within the context of demand and supply:

1. If the price of $\mathfrak{h}$ decreases, then the quantity demanded for $\mathfrak{h}$ increases. This causes the demand of substitute $\mathfrak{g}$ to decrease. Similarly, the price of $\mathfrak{h}$ increasing causes the quantity demanded to decrease, resulting in the demand of $\mathfrak{g}$ to increase.

2. If the price of $\mathfrak{h}$ increases, then the quantity supplied for $\mathfrak{h}$ increases. This causes the quantity supplied of substitute $\mathfrak{g}$ to decrease. Similarly, the price of $\mathfrak{h}$ decreasing causes the quantity supplied to decrease, resulting in the quantity supplied of $\mathfrak{g}$ to increase.

1.2.4 Equilibrium

Definition 1.2.16 (Surplus)**.** When the price is such that the quantity supplied of a good or service exceeds the quantity demanded, some sellers are unable to sell because fewer units are purchased than are offered. This condition is called a **surplus**.

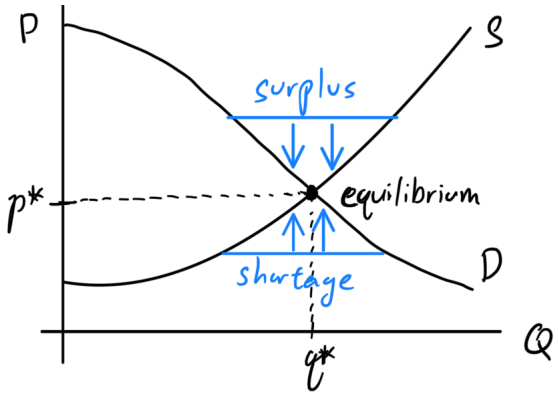
The sellers who fail to sell have an incentive to sell at a lower price, which will put a downwards pressure on prices, leading them to fall. This price cutting reduces the surplus until the quantity supplied equals the quantity demanded.

Definition 1.2.17 (Shortage)**.** When the price is low enough that the quantity demanded exceeds the quantity supplied, a **shortage** exists.

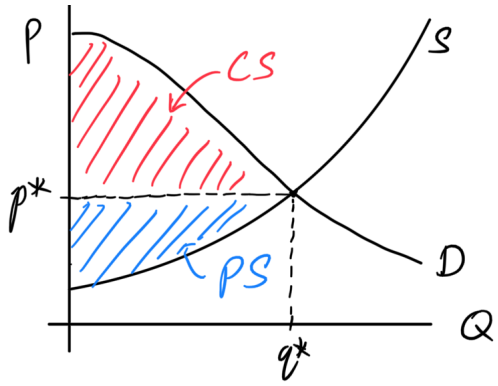
In this case, some buyers fail to purchase, and these buyers have an incentive to accept a slightly higher price in order to be able to trade. This puts an upward pressure on prices, leading them to rise. This price rise reduces the shortage until the quantity supplied equals the quantity demanded.

Definition 1.2.18 (Equilibrium)**.** When the pressure for higher prices balances the pressure for lower prices, or when the quantity supplied is the same as the quantity demanded, the state is said to be in **equilibrium**.

Equilibrium is **efficient**; that is, it maximizes the gains from trade, assuming that the only people affected by any given transaction are the buyers and sellers.



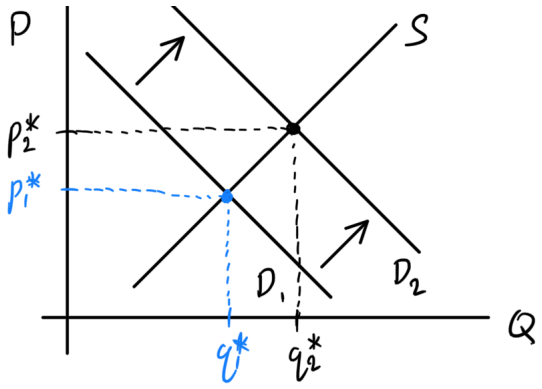
Definition 1.2.19 (Producer, Consumer Surplus at Equilibrium)**.** Given that the market is at equilibrium with equilibrium price p^*, the consumer and producer surplus can be identified.



We can just think of the demand and supply curves as the marginal value and marginal cost, respectively. Given the price p^* for the good, the consumers are willing to buy up to quantity q^*, and the suppliers are willing to supply quantity q^*.

1.2.5 Changes in Supply and Demand

Definition 1.2.20 (Increase in Demand)**.** An increase in demand will lead to the demand curve D_1 shifting upwards/rightwards to a new demand curve D_2.



The equilibrium quantity demanded would increase from q_1^* to q_2^*, while the equilibrium price would increase from p_1^* to p_2^*. Indeed, this makes sense. Interpreting the demand curve as a marginal value curve, we can see that the increase in demand increases the marginal value of the good. Therefore, consumers are willing to pay *more money* (increased price) for *more units* of the good (increased quantity demanded).

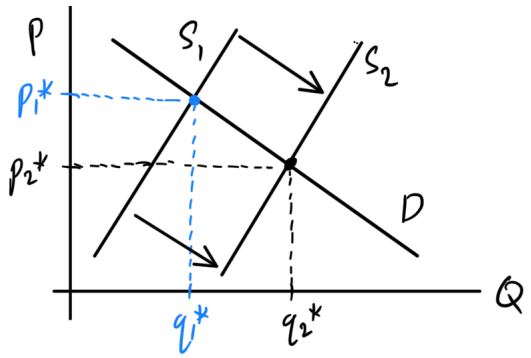
Example 2 (Stocks). Marathon Digital Holdings (MARA) sold for \$19.66 (closing price) on May 13, 2021. Where does this price come from? People place orders on stocks all the time, wanting to sell it at high prices and buy at low prices. When looking at the supply and demand curve of MARA stocks, we see that \$19.66 is the equilibrium price of a MARA stock.

Furthermore, there are thousands of people placing orders to sell MARA at prices higher than \$19.66 and to buy MARA at prices lower than \$19.66. This causes a discrepancy in these prices, and these prices aren't fulfilled until these orders meet the equilibrium.

External forces, such as earnings reports and news, affect the demand of MARA stocks all the time, which fluctuates the demand curve and therefore affecting the equilibrium

point. A positive Q1 earnings report could increase the demand for MARA, increasing the equilibrium price (i.e. the market price) to, say \$21.11. As the demand curve goes up, all orders placed that sell MARA at prices below \$21.11 are fulfilled.

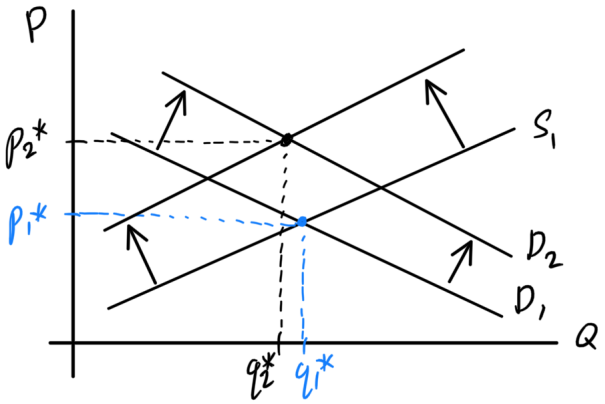
Definition 1.2.21 (Increase in Supply)**.** An increase in supply will lead to the supply curve S_1 shifting downwards/rightwards to a new demand curve S_2.



The equilibrium quantity demanded would increase from q_1^* to q_2^*, while the equilibrium price would decrease from p_1^* to p_2^*. Indeed, this makes sense. Interpreting the supply curve as a marginal cost curve, we can see that the increase in supply decreases the marginal cost of the good. Therefore, suppliers can produce *more units* of the good (increased quantity supplied) at a *lower cost* (decreased price).

Decreases in supply and demand can be interpreted with the same logic as above.

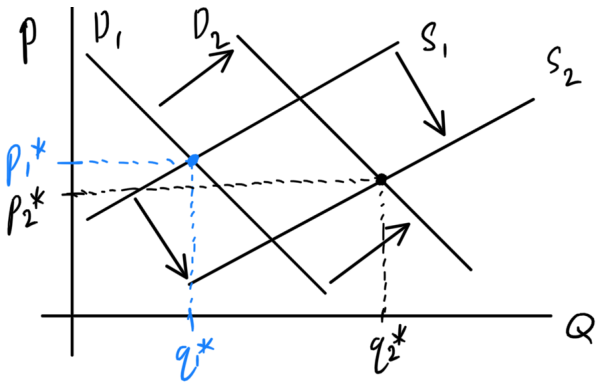
Example 3 (Demand Increases, Supply Decreases). If the demand increases from D_1 to D_2 and supply decreases from S_1 to S_2 as shown below, our equilibrium point moves directly up.



The increased demand increases the marginal value of the good, and the decreased supply increases the marginal cost of the good. Both of these changes drive the price up. The demand change increases the quantity demanded, while the supply change decreases the quantity supplied.

Example 4 (Demand Increases, Supply Increases). If both the demand and supply increases, the quantity demanded will doubly increase while the price will hover around the

same.

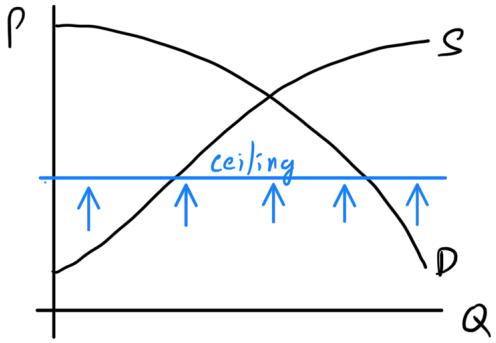


The increased demand and supply increases the marginal value while decreasing the marginal cost. Both contribute to the increase in quantity demanded/supplied. The demand change increases the price, while the supply change decreases the price.

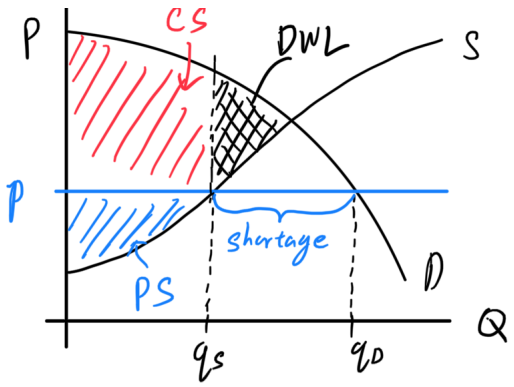
1.2.6 Price Controls, Quotas

If the equilibrium price for a certain good or service is not sustainable, the government could intervene by setting a price control.

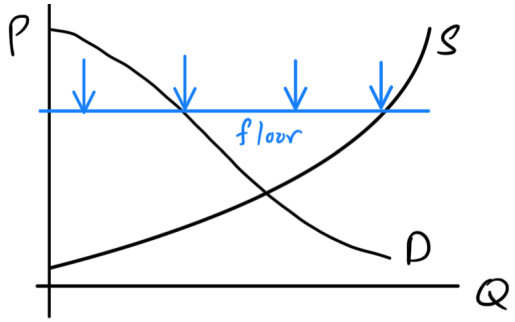
Definition 1.2.22 (Price Ceilings)**.** If the price is too high (e.g. rent), then the government could set a **price ceiling** that limits how high a certain price can be for that good.



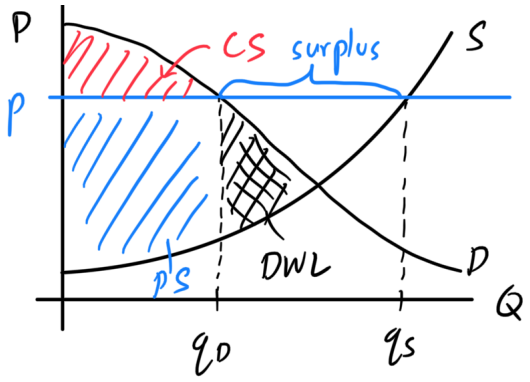
Note that a price ceiling could cause shortages, inefficiency, and black markets. More specifically, since the actual price p of the good is lower than the equilibrium price p^*, the quantity demanded increases to q_D while the quantity supplied decreases to q_S. Therefore, since suppliers are only willing to sell q_S units of good (due to price), this leaves extra demand (a shortage). That is, the consumer and producer surplus can only extend up to quantity q_S (and not up to q^*). This can hurt both parties since they are restricted from maximizing their surplus/profit, and the wasted opportunity cost is called the **deadweight loss**.



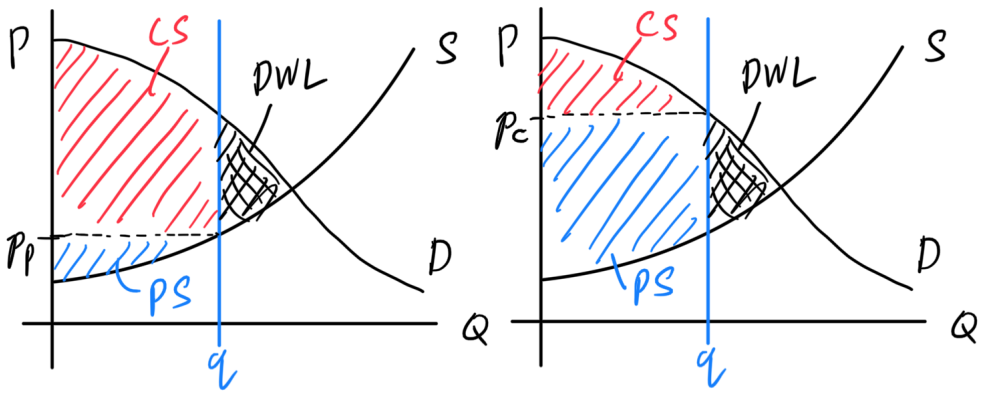
Definition 1.2.23 (Price Floors)**.** If the price is too low (e.g. minimum wage), then the government could set a **price floor** that limits how low a certain price can be for that good.



Note that a price floor could cause surpluses, inefficiency, and wasted resources. More specifically, since the actual price p of the good is higher than the equilibrium price p^*, the quantity demanded decreases to q_D while the quantity supplied increases to q_S. Therefore, since consumers are only demanding up q_D units of good (due to price), this leaves extra supply (a surplus). That is, the consumer and producer surplus can only extend up to quantity q_D (and not up to q^*). This can hurt both parties since they are restricted from maximizing their surplus/profit, leading to **deadweight loss**.



Definition 1.2.24 (Quotas)**.** A **quota**, or **quantity control**, is an upper limit on the quantity of some good that can be bought or sold.



Since the quantity is limited, we have a bit of an instability. As long as the price is at or below p_C consumers are willing to demand q quantity of the good, and as long as the price is at or above p_P, the producers are willing to supply q quantity of the good. Because of this quota, there is both a downward and upward pressure on the price of the good, but it will always remain in between p_P and p_C. However, the deadweight loss of not being able to increases consumer/producer surplus from the quota is still there.

1.2.7 Elasticities

In general, elasticity refers to the rate at which a value is affected by a change in another value. Technically, the **x-elasticity of y** measures the fractional response of y to a fractional change in x.

Definition 1.2.25 (Price-Elasticity of Demand)**.** For a certain good $\mathfrak{g}$, its **elasticity** refers to how sensitive the quantity demanded is to a change in its price. That is,

1. If a price change of $\mathfrak{g}$ leads to a greater change in quantity demanded, then $\mathfrak{g}$ is said to be **elastic**.
2. If a price change of $\mathfrak{g}$ leads to a smaller change in quantity demanded, then $\mathfrak{g}$ is said to be **inelastic**.

It makes sense, then, to define the price elasticity of demand as

$$\frac{\partial q}{\partial p} = \frac{\partial(\text{quantity demanded})}{\partial(\text{price})}$$

However, this is sensitive to unit changes in the price or quantity, so to represent this in a unit-free formula

$$\varepsilon = \frac{\%\text{ change in quantity } q}{\%\text{ change in price } p},$$

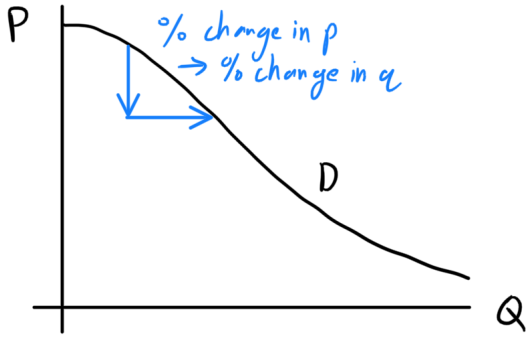
we measure the *fractional* response of q to the *fractional* response of p. That is,

$$\varepsilon = -\frac{dq/q}{dp/p} = -\frac{p}{D(q)}\frac{dD(p)}{dp} = -\frac{p\,D'(p)}{D(p)}$$

The negative sign is to account for the negative monotonicity of the demand curve (to make ε positive). If

1. $|\epsilon| > 1$, then it is elastic and q changes more than p.

2. $|\epsilon| = 1$, then it is unit elastic and q changes like p.
3. $|\epsilon| < 1$, then it is inelastic and q changes less than p.

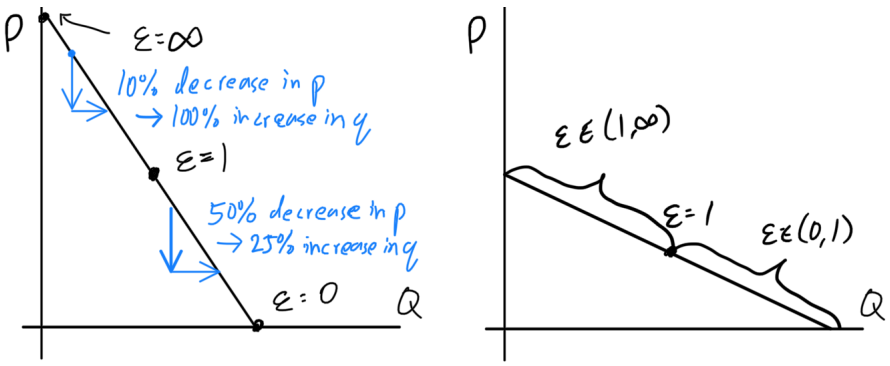


Example 5. For example, suppose prices of a certain good rise by 1%.

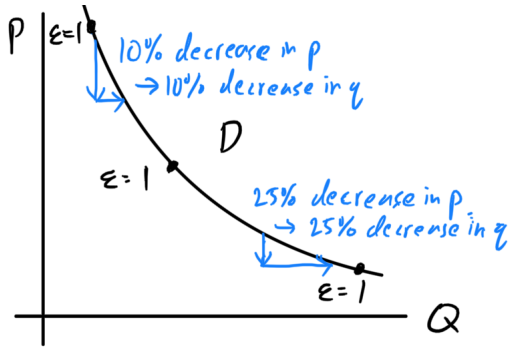
1. If $\varepsilon = 0.5$, then the quantity demanded will fall by 0.5%.
2. If $\varepsilon = 1$, then the quantity demanded will fall by 1%.
3. If $\varepsilon = 2$, then the quantity demanded will fall by 2%.

Demand Curves with Various Elasticities

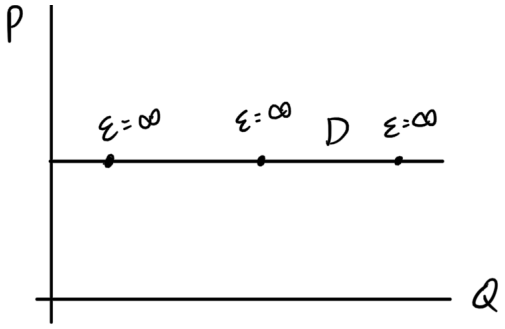
Definition 1.2.26 (Linear Demand Curve)**.** Counterintuitively, a linear demand curve has a varying elasticity. When the price is high (and quantity demanded low), the elasticity is high; that is, a small decrease in price from the current price would add a significant portion of the current quantity demanded. Right at the midpoint, we would have unit elasticity. As the point moves further down, it would become more and more inelastic since a small decrease in price from the current price would add an even less significant portion of the current current quantity demanded.



Definition 1.2.27 (Constant Elasticity Demand Curve)**.** A demand curve with constant elasticity looks like a rational function.

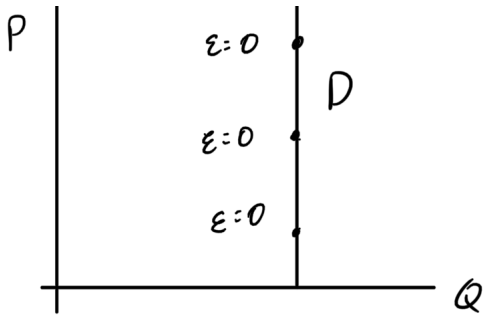


Definition 1.2.28 (Perfectly Elastic Demand Curve)**.** If the elasticity $\varepsilon = \infty$, then the quantity demanded has an infinite response to price change. This is called **perfect elasticity**. A small increase in the price of a good will cause the quantity demanded to drop to 0.



An apple company can try to increase the price of apples from \$0.50/kg to \$0.55/kg. Since there are other companies selling apples for a cheaper price, consumers will not purchase apples from this company, dropping the quantity demanded to 0. Note that in this scenario, we are talking about the demand curve representing the apple company and its customers, not the entire apple market.

Definition 1.2.29 (Perfectly Inelastic Demand Curve)**.** If the elasticity $\varepsilon = 0$, then the quantity demanded has no response to a price change. This is called **perfect inelasticity**. Even a large increase in the price of a good will not cause the quantity demanded to change at all.



For example, insulin shots, despite their price, are needed regardless by people with diabetes. Therefore, an increase in the price of insulin shots will not change the need (i.e. the quantity demand) for these shots by the diabetics.

Definition 1.2.30 (Price Elasticity of Supply)**.** For a certain good $\mathfrak{g}$, its **elasticity of supply** refers to how sensitive the quantity supplied is to a change in its price. That is,

1. If a price change of $\mathfrak{g}$ leads to a greater change in quantity supplied, then $\mathfrak{g}$ is said to be **elastic**.
2. If a price change of $\mathfrak{g}$ leads to a smaller change in quantity demanded, then $\mathfrak{g}$ is said to be **inelastic**.

Intuitively, it seems that

$$\frac{dq}{dp} = \frac{d(\text{quantity supplied})}{d(\text{price})}$$

is the definition, but in actuality we measure the fractional response of q to the fractional response of p.

$$\eta = \frac{\%\text{ change in quantity } q}{\%\text{ change in price } p}$$

That is,

$$\eta = \frac{dq/q}{dp/p} = \frac{p}{S(p)} = \frac{p\,S'(p)}{S(p)}$$

1.3 Producer Theory

Definition 1.3.1 (Firm as a Production Function)**.** At the most basic level, a **firm** is an entity that transforms things into other, more valuable things, a process known as **production**. We can interpret the firm as a production function f

$$f(\text{inputs}) = \text{output}$$

Interpreting each input as $x_1, x_2, \ldots, x_n$ and output as y, we can model a firm buying x_1 amount of input 1, x_2 amount of input 2, ..., x_n amount of input n and producing an amount y of the output.

$$f(x_1, x_2, \ldots, x_n) = y$$

Ideally, firms would utilize a set of technologies for transforming things and then use this transformation to maximize net profits.

Definition 1.3.2 (Types of Firms)**.** There are 4 major types of firms created in law:

1. A **proprietorship** is a firm owned by a single individual (the proprietor). The sole owner has *unlimited liability* for the company and pays personal income taxes on profits earned. They are easy to establish and dismantle due to a lack in government regulations and requirements.
2. **Partnerships** share profits (equally or unequally).
3. A **corporation** is a legal entity that is separate and distinct from its owners. It offers *limited liability*, meaning that shareholders may take part in the profits through dividends and stock appreciation but are not personally liable for the company's debts. However, it is costly (in money and time) to organize. Some types of corporations include:

(a) An **S corporation** (usually for smaller businesses with fewer than 100 shareholders) has the benefits of a corporation while being taxed as a partnership (i.e. no federal corporate taxes, only individual taxes).

(b) A **limited liability corporation (LLC)** is a hybrid between a partnership and a corporation. They provide their owners with limited liability but like a partnership, LLCs' profits are taxed as part of the owners' personal income.

4. **Non-profit firms** is prohibited from distributing a profit to its owners. Religious operations, academic associations, environmental groups, hospitals, private universities are all organized as non-profit firms. They are not taxed by the government, given that they are engaged in government-approved activities. However, commercial profits (e.g. gift shops) are taxed.

Production Functions

Definition 1.3.3 (Cobb-Douglas Production Function)**.** The **Cobb-Douglas production function** is one simple example model of a firm's production, defined

$$f(x_1, x_2, \ldots, x_n) = a_0 x_1^{a_1} x_2^{a_2} \ldots x_n^{a_n}$$

The constants $a_1, a_2, \ldots, a_n$ are positive, generally less than 1. For example, with two goods, capital K and labor L, Cobb-Douglas can be expressed as

$$f(K, L) = a_0 K^a L_b$$

Definition 1.3.4 (Fixed-Proportions Production Function)**.** The **fixed proportions** production function is defined

$$f(x_1, x_2, \ldots, x_n) = \min\{a_1 x_1, a_2 x_2, \ldots, a_n x_n\}$$

This function has the property that adding an input beyond a necessary level does no good. For example, the productive value of having more than one shovel per worker is quite low.

Definition 1.3.5 (Inputs as Perfect Substitutes)**.** Two inputs K and L are **perfect substitutes** in a production function f is they enter as a sum. That is, if

$$f(K, L, \ldots, x_n) = g(K + cL, x_3, \ldots, x_n)$$

for some constant c. In other words, with an appropriate scaling of the units of one of the variables, all that matters is the sum of the two variables, not the individual values.

Definition 1.3.6 (Marginal Product)**.** The **marginal product of an input** is just the partial derivative of the production function with respect to that input. That is, the marginal product of input x_i is

$$\frac{\partial f}{\partial x_i}$$

Example 6 (Marginal Product in Cobb-Douglas Function)*.* In the two dimensional Cobb-Douglas production function with inputs K and L (and constant A):

$$f(K, L) = AK^\alpha L^\beta$$

the marginal product of capital K is

$$\frac{\partial f}{\partial K}(K, L) = \alpha A K^{\alpha-1} L^{\beta}$$

Assuming that $0 < \alpha, \beta < 1$ (which is true in most cases), the marginal product of capital increases in the amount of labor, and decreases in the amount of capital. Indeed this is, since for example, an extra computer is very productive in a situation with lots of workers and few computers, but not so productive in a situation where there are lots of computers and few people to operate them.

The **value of the marginal product** of an input is basically just the marginal product times the price of the output. It is basically the value added by an additional unit of an input. If the value of the marginal product exceeds the cost of the input, then it is profitable to use more of that input.

We now introduce the notions of short-term and long-term factors.

Definition 1.3.7. Given the inputs $x_1, x_2, \ldots, x_n$ of a firm's production function,

1. if an input x_i can be adjusted (changed) quickly, it is called a **short-run factor**. Examples include cheap labor (McDonald's workers), simple equipment (shovels).
2. if an input x_i can be adjusted (changed) slowly, it is called a **long-run factor**. Examples include very skilled labor (engineers) and expensive equipment (passenger aircraft).

1.3.1 Profit Maximization

Definition 1.3.8 (Profit)**.** Consider a production function

$$y = f(x_1, x_2, \ldots, x_n)$$

where the marginal cost of renting/buying unit x_i is ω_i. Suppose p is the price of the output y. Then, the **profit** π is just the price of the total outputs minus the cost of renting/buying all the inputs, which can be represented by the function $\mathcal{C}$. That is, given inputs $x_1, x_2, \ldots, x_n$, the profit of the firm is:

$$\pi = p\, f(x_1, \ldots, x_n) - \mathcal{C}(x_1, \ldots, x_n)$$

However, in most cases the cost is expected to increase linearly with each input, giving the equation

$$\pi = p\, f(x_1, \ldots, x_n) - \sum_{i=1}^{n} \omega_i x_i$$

Using multivariate calculus, solving for when the Hessian matrix $H\pi$ is 0 will find the vector $(x_1, \ldots, x_n)$ that maximizes π.

This function representation of firms gives us lots of flexibility in solving optimization problems. For example, given a firm with production function

$$f : \mathbb{R}^n \longrightarrow \mathbb{R},\ y = f(x_1, x_2, \ldots, x_n)$$

say that due to a lack of time, there is a limit on the amount of capital you can get that is constrained by the function

$$g(x_1, x_2, \ldots, x_n) \leq c$$

for some constant c. (For example, you may have two inputs capital K and labor L. It takes an hour to get additional capital and 2 hours to get additional labor. If you have 24 hours to get the capital, the constraint equation would be $K + 2L \leq 24$). Then f can be easily optimized using the method of Lagrange multipliers (constrained to c).

Shadow Value

Definition 1.3.9 (Shadow Value)**.** Given inputs $x_1, \ldots, x_n$, assume that the input x_i for some i in $1, \ldots, n$ cannot be adjusted on the short-run (e.g. a factory can't be built immediately). This creates a constraint on the profit available to the entrepreneur. Since x_i is fixed, there is no direct value of x_i because we can interpret the firm as a function without that one input; that is, as

$$f : \mathbb{R}^{n-1} \longrightarrow \mathbb{R},\ f(x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n)$$

But in the case that this constraint was relaxed (that is, if we could easily change the quantity of this input), we can measure its value. This is called the **shadow value**. That is, the shadow value of an input is the increase in profit associated with it.

Example 7 (Shadow Value of Capital). Consider the capital/labor input firm. The profit of the firm is:

$$\pi = pF(K, L) - rK - \omega L$$

Given a fixed capital K_0, let L^* be the function of K such that $L^*(K_0)$ is the value of labor that maximizes π. In other words, given a fixed value of capital $K = K_0$, the maximum value of π is

$$\pi\big(K_0, L^*(K_0)\big) = pF\big(K_0, L^*(K_0)\big) - rK_0 - \omega L^*(K_0)$$

But assuming that we *can* change this value (relaxing the constraint of $K = K_0$), we can view the above as a function of K.

$$\pi\big(K, L^*(K)\big) = pF\big(K, L^*(K)\big) - rK - \omega L^*(K)$$

By taking the derivative of this with respect to K, we can see how a (hypothetical) change in K will change the potential profit generated by the firm. The derivative below assumes that we can change L accordingly.

$$\frac{d\pi(K, L^*(K))}{dK} = p\frac{dF(K, L^*(K))}{dK} - r - \omega\frac{dL^*(K)}{dK}$$

This ideal scenario where L is changed accordingly can be shown by the hypothetical growth curve below:

However, if L cannot be changed accordingly, we can view the case as such: given that we are at fixed capital K_0 and optimized level of labor $L^*(K_0)$, we can calculate the shadow

value of K by keeping $L^*(K_0)$ constant while hypothetically changing K. Then we would relax the constraint $K = K_0$ (but not $L^*(K_0)$)

$$\frac{d\pi(K, L^*(K_0))}{dK} = \frac{d}{dK}\Big(pF\big(K, L^*(K_0)\big) - rK - \omega L^*(K_0)\Big) = p\frac{dF(K, L^*(K))}{dK} - r$$

This in essence tells us the potential profit that can be generated by the firm by hypothetically increasing the capital *with the current level of labor* $L^*(K_0)$. This scenario is shown by the straight line curve:

Over the long run, the firm can adjust both the capital and the labor. That is, let L^{**} and K^{**} be the labor and capital, respectively, that maximizes the profit π. This can be found using regular optimization methods in multivariate calculus.

Example 8 (Cobb-Douglas). The Cobb-Douglas production function has profit

$$\begin{aligned}\pi &= pF(K, L) - rK - \omega L \\ &= pAK^{\alpha}L^{\beta} - rK - \omega L\end{aligned}$$

Optimizing π gives the solutions

$$L^{**} = \left(\frac{Ap\alpha^{\alpha}\beta^{1-\alpha}}{r^{\alpha}\omega^{1-\alpha}}\right)^{\frac{1}{1-\alpha-\beta}}, \text{ and } K^{**} = \left(\frac{Ap\alpha^{1-\beta}\beta^{\beta}}{r^{1-\beta}\omega^{\beta}}\right)^{\frac{1}{1-\alpha-\beta}}$$

While these solutions may appear complicated, they make sense. They are proportionally dependent on the output price p, and the input prices r and ω shows straightforward relationships too.

Types of Costs

We define 5 types of cost functions.

Definition 1.3.10 (Short-Run Total Cost)**.** Suppose that given a firm's production function f with inputs $x_1, x_2, \ldots, x_n$ and output y, with $\mathcal{C}$ the cost function such that

$$\pi = f - \mathcal{C}$$

Now, assume that you want to produce at least quantity q of the output. Then, the **short-run total cost of producing** q is

$$\text{SRTC}(q) \equiv \min_{x_1,\ldots,x_n} \{\mathcal{C}(x_1, \ldots, x_n) \mid F(x_1, \ldots, x_n) \geq q\}$$

In other words, it is the minimum cost of producing at least q outputs. Furthermore, we can account for units that are not adjustable in the short run. Suppose there are k inputs $x_{j_1}, \ldots, x_{j_k}$ that are not adjustable in the short-run. Then, the **short-run total cost of producing** q**, given the quantity** $x_{j_1} = x^*_{j_1}, \ldots, x_{j_k} = x^*_{j_k}$, is

$$\text{SRTC}(q \,|\, x_{j_i} = x^*_{j_i} \;\forall\, i = 1, \ldots, k) = \min_{x}\{\mathcal{C}(x) \mid F(x) \geq q, x_{j_i} = x^*_{j_i} \;\forall\, i = 1, \ldots, k\}$$

where x represents the vector $(x_1, \ldots, x_n)$. More simply put, $\text{SRTC}(q \,|\, x_i)$ is merely just $\text{SRTC}(q)$ restricted to the level set formed by the intersections of the k hyperplanes $x_{j_1} = x^*_{j_1}, \ldots, x_{j_k} = x^*_{j_k}$.

Definition 1.3.11 (Short-Run Marginal Cost)**.** The **short-run marginal cost of producing** q is just the derivative of $\text{SRTC}(q)$.

$$\text{SRMC}(q) = \frac{d}{dq}\,\text{SRTC}(q) = \text{SRTC}'(q)$$

That is, it is the additional cost incurred if we were to produce one more unit of the output ($q+1$ outputs). Similarly, the **short-run marginal cost of producing** q**, given the quantity** $x_i = x_{i0}$, is

$$\text{SRMC}(q \,|\, x_{j_i} = x^*_{j_i}\ \forall\, i = 1, \ldots, k) = \text{SRTC}'(q \,|\, x_{j_i} = x^*_{j_i}\ \forall\, i = 1, \ldots, k)$$

It is the additional cost incurred if we were to produce one more unit of the output, given that we cannot change input $x_i = x_{i0}$.

Definition 1.3.12 (Short-Run Average Cost)**.** The **short run average cost of producing** q is simply obtained by dividing the short-run total cost by the quantity produced.

$$\text{SRAC}(q) = \frac{\text{SRTC}(q)}{q}$$

As the name suggests, it is the average (minimum) cost of producing quantity q outputs. If we are restricted in input $x_i = x_{i0}$, then

$$\text{SRAC}(q \,|\, x_{j_i} = x^*_{j_i}\ \forall\, i = 1, \ldots, k) = \frac{\text{SRTC}'(q \,|\, x_{j_i} = x^*_{j_i}\ \forall\, i = 1, \ldots, k)}{q}$$

Definition 1.3.13 (Short-Run Average Variable Cost)**.** Assume that we have a fixed costs of operations

$$x_{j_1} = x^*_{j_1}, x_{j_2} = x^*_{j_2}, \ldots, x_{j_k} = x^*_{j_k}$$

Then, the **short-run average variable cost of producing** q eliminates the fixed costs of operation to get the average short-run cost. For simplicity of notation, we will shorten

$$\left(x_{j_i} = x^*_{j_i}\ \forall\, i \in [1, k]\right) \implies \left(x_{j_i} = x^*_{j_i}\right)$$

and assume that there are k restricted costs. Then, we have

$$\text{SRAVC}(q \,|\, x_{j_i} = x^*_{j_i}) = \frac{\text{SRTC}(q \,|\, x_{j_i} = x^*_{j_i}) - \text{SRTC}(0 \,|\, x_{j_i} = x^*_{j_i})}{q}$$

We can just imagine $\text{SRAVC}(q)$ as calculating the short-run average cost, given that an external party paid for all fixed costs of operations. This means that if there are no fixed costs, then

$$\text{SRAVC} = \text{SRAC}$$

Example 9. Given a firm production function F with inputs capital K and labor L, let capital K be non-adjustable on the short run, set at $K = k$ and let the cost function be

$$\mathcal{C}(K, L) = rK + \omega L$$

In order to find the SRTC(q), we must minimize $\mathcal{C}$, so the constraint $F(K,L) \geq q$ is really satisfied with equality $F(K,L) = q$. But since $K = k$ is a constant, the equation

$$F(k,L) = q$$

which maps $l \mapsto F(k,l) = q$, also determines a mapping from q to L by the implicit function theorem. We will call this function L_S, defined such that

$$F\big(k, L_S(q)\big) = q$$

The short run total cost of producing q, given $K = k$, is

$$\text{SRTC}(q \,|\, K = k) = rk + \omega L_S(q)$$

The implicit function theorem also states that since $F(k,\cdot)$ and $L_S(\cdot)$ are inverses,

$$\left.\frac{dL_S(q)}{dq}\right|_{F=q} = \frac{1}{\frac{\partial F}{\partial L}\big(k, L_S(q)\big)}$$

Therefore,

$$\text{SRMC}(q \,|\, K = k) = \text{SRTC}'(q \,|\, K = k) = \frac{d}{dq}(rk + \omega L) = \omega \left.\frac{dL}{dq}\right|_{F=q} = \frac{\omega}{\frac{\partial F}{\partial L}\big(k, L_S(q)\big)}$$

Then,

$$\text{SRAC}(q \,|\, K = k) = \frac{\omega}{q\frac{\partial F}{\partial L}\big(k, L_S(q)\big)}$$

Finally,

$$\text{SRAVC}(q \,|\, K = k) = \frac{\text{SRTC}(q \,|\, K = k) - \text{SRTC}(0 \,|\, K = k)}{q} = \frac{\omega L_S(q \, K = k)}{q}$$

Definition 1.3.14 (Long-Run Total Cost)**.** On the long run, all inputs can vary, meaning that the long-run cost doesn't need to be conditioned on the x_{j_i}'s. Therefore, **long-run total cost of producing** q is

$$\text{LRTC}(q) = \min_x \{\mathcal{C}(x) \mid F(x) \geq q\}$$

Definition 1.3.15 (Long-Run Marginal Cost)**.** The **long run marginal cost of producing** q is the derivative of the long run total cost.

$$\text{LRMC}(q) = \text{LRTC}'(q)$$

Definition 1.3.16 (Long-Run Average Cost)**.** The **long-run average cost** is

$$\text{LRAC}(q) = \frac{\text{LRTC}(q)}{q}$$

It is clear that since all costs are variable, we don't have to deal with constraints and therefore the LRAC is the same as the long run average variable cost.

Example 10 (Cobb-Douglas). For the Cobb-Douglas production function $F(K,L) = AK^\alpha L^\beta$, with $\alpha + \beta < 1$, with $K = k$ fixed in the short-run but not on the long run, we have

$$\begin{aligned}
\mathrm{SRTC}(q \mid K = k) &= rK + \omega\left(\frac{q}{AK^\alpha}\right)^{\frac{1}{\beta}} \\
\mathrm{SRAVC}(q \mid K = k) &= \omega\frac{q^{(1-\beta)/\beta}}{(AK^\alpha)^{1/\beta}} \\
\mathrm{SRMC}(q \mid K = k) &= \omega\frac{q^{(1-\beta)}}{\beta}\beta(AK^\alpha)^{1/\beta} \\
\mathrm{LRTC}(q \mid K = k) &= \left(\left(\frac{\alpha}{\beta}\right)^{\frac{\beta}{\alpha+\beta}} + \left(\frac{\beta}{\alpha}\right)^{\frac{\alpha}{\alpha+\beta}}\right)\omega^{\frac{\beta}{\alpha+\beta}} r^{\frac{\alpha}{\alpha+\beta}} \left(\frac{q}{A}\right)^{\frac{1}{\alpha+\beta}}
\end{aligned}$$

Dynamic Firm Behavior

We will describe how a firm that produces output y responds to price changes of y. Assume that the price of a unit of y is p. Then, the firm earns profits

$$\pi = pq - \mathcal{C}(q \mid K = k)$$

where $\mathcal{C}(q \mid K = k)$ is the total cost of producing given that the firm currently has capital k. To maximize profits, the firm chooses quantity q_s such that

$$0 = p - \mathcal{C}'(q \mid K = k) \implies p = \mathcal{C}'(q \mid K = k)$$

i.e. when the price equals marginal cost. Obviously, this is a good strategy only if producing a positive quantity is desirable, that is, if

$$pq_s - \mathcal{C}(q_s \mid K = k) \geq p \cdot 0 - \mathcal{C}(0 \mid K = k)$$

This can be written as

$$p \geq \frac{\mathcal{C}(q_s \mid K = k) - \mathcal{C}(0, K = k)}{q_s} = \mathrm{SRAVC}(q_s \mid K = k)$$

This means that the firm will start producing products provided that its price exceeds the average variable cost.

Theorem 1.3.1. The profit maximizing firm produces the quantity q_s where price equals marginal cost, provided price is as large as minimum average variables cost. If price falls below the minimum average variable cost, the firm shuts down.

Let us draw the supply curve of the firm; that is, the curve that represents the choice of the firm (on the quantity it supplies) as a function of the price (the thick line).

Therefore, if the price is above the minimum average variable cost (AVC), the firm shuts down since the average revenue (not accounting for fixed costs) it makes on producing is less than the marginal cost. When price is above the minimum average variable cost, the marginal cost gives the quantity supplied by the firm.

We can also talk about the average total cost,

1.3.2 Perfect Competition Dynamics

1.3.3 Investment

Definition 1.3.17 (Time Value of Money)**.** The promise of $1 in the future is not worth $1 today, since having the $1 right now allows us to invest it immediately to get returns. The **present value** of a dollar is worth more than the **discounted value** of a dollar in the future.

The interest rate of U.S. Treasury bonds (which are the most secure investment) allows us to determine the potential rate of return of $1 if we were to invest it. That is, if we have $\mathcal{X}$ amount of dollars and the interest rate is r, then in one year we are guaranteed

$$\mathcal{X} \cdot (1 + r)$$

dollars. That means that $\mathcal{X}$ dollars in the present value is worth $\mathcal{X}(1 + r)$ dollars in the future value. Going backwards, if we are guaranteed $\mathcal{Y}$ dollars a year in the future, then the present value of it is

$$\frac{\mathcal{Y}}{1 + r}$$

The present value PV has a simple formula. Say that r_1 is the interest rates this year, r_2 interest rates next year, and so on. If we obtain a stream of payments A_0 immediately, A_1 at the end of year 1, A_2 at the end of year 2, and so on, the present value of the stream of payments is

$$\text{PV} = A_0 + \frac{A_1}{1 + r_1} + \frac{A_2}{(1 + r_1)(1 + r_2)} + \frac{A_3}{(1 + r_1)(1 + r_2)(1 + r_3)} + \dots$$

All financial analyses that assumes this concept of discounted future value of money is called **DCF (Discounted Cash Flow) analysis**.

Example 11. For example, given that $10,000 is invested for one year at 10% interest, the future value of that money is:

$$FV = \$10,000 \times \left(1 + \frac{10\%}{1}\right)^{1\times 1} = \$11,000$$

This means that $10,000 now is equal to $11,000 in the future. The value of $5,000 one year from today, compounded at 7% interest, is:

$$PV = \$5,000 \Big/ \left(1 + \frac{7\%}{1}\right)^{1\times 1} = \$4,673$$

If we increase the rate of compounding, it can change future values:

1. Quarterly Compounding: $FV = \$10,000 \times \left(1 + \frac{10\%}{4}\right)^{4\times 1} = \$11,038$
2. Monthly Compounding: $FV = \$10,000 \times \left(1 + \frac{10\%}{12}\right)^{12\times 1} = \$11,047$
3. Daily Compounding: $FV = \$10,000 \times \left(1 + \frac{10\%}{365}\right)^{365\times 1} = \$11,052$

This means that TVM depends not only on interest rate and time horizon, but also no how many times the compounding calculations are computed each year.

Definition 1.3.18 (Consols)**.** Bonds that pay a fixed amount every year *forever* are known as **consolidated annuities**, or **consols**. The value v of a consol that pays a fixed A dollars every year in an economy with a fixed interest rate r is

$$v = \frac{1}{1+r} + \frac{1}{(1+r)^2} + \frac{1}{(1+r)^3} + \ldots = \frac{1}{1-\frac{1}{1+r}} - 1 = \frac{1}{r}$$

Note that as interest rates decrease, consols become more valuable. For example, at a 5% interest rate, \$1 million paid per year will be worth \$20 million today.

Example 12 (Mortgages). We now fix a monthly interest rate r. We will treat a mortgage as a fixed payment per month for a large number of months (e.g. 360 months for a 30 year mortgage). The present value of these payments over n months is

$$M = \frac{1}{1+r} + \frac{1}{(1+r)^2} + \ldots + \frac{1}{(1+r)^n} = \frac{1}{r}\left(1 - \frac{1}{(1+r)^n}\right)$$

To put this into perspective, at a monthly interest rate of 0.5%, paying \$1 per month for 360 months produces a present value M of \$166.79.

$$M = PV = \frac{1}{1.005} + \frac{1}{1.005^2} + \ldots + \frac{1}{1.005^3 60} \approx 166.79$$

Since \$1 per month for 360 months generates \$166.79 in present value, scaling this up, we can see that \$599.55 per month for 360 months generates \$100,000 in present value. Therefore, to get a current loan of \$100,000, you will be expected to pay back \$599.55 per month for the next 360 months in order to pay back the present value's worth.

Example 13. \$20,000,000 paid right now to you is great. Now, assume that you are paid \$1,000,000 per year for the next 20 years, with the first payment happening immediately. With the interest rate being r, the present value of the \$20 million would be

$$\text{PV} = 1000000 + \frac{1000000}{1+r} + \frac{1000000}{(1+r)^2} + \ldots + \frac{1000000}{(1+r)^{19}}$$

The table below shows what the present value (in thousands) would be at various interest rates:

r	3%	4%	5%	6%	7%	10%
PV	\$15,324	\$14,134	\$13,085	\$12,158	\$11,336	\$9,365

It is clear that interest rates can make a dramatic impact on future value. Cutting them by 25% or even more than 50% in some interest rates.

From looking at a Treasury bill (which offers a one-time payment after n years), we can also calculate the effective interest rate r. For example, say that a n-year bond that will pay \$10,000 is sold at \$9,615.39. We can see that this produces a 4% interest rate by solving the equation

$$10,000 \cdot (1+r)^n = 9,615$$

for r. In general, let the price of the bill be P and its future value (the amount it will pay when the bill matures) is FV. Then

$$P \cdot (1+r)^n = \text{FV} \implies r = \left(\frac{FV}{P}\right)^{1/n} - 1$$

Definition 1.3.19 (Net Present Value of Bonds)**.** Bonds are generally collected semi-annually, but they have a fixed interest rate set at the time of issue during the life of the bond. A n-year $\$\mathcal{X}$ bond at interest rate r with semi-annual payments would have a **net present value** (i.e. how much it is worth now in present value) of

$$\text{NPV} = \frac{nr/2}{(1+r)^{\frac{1}{2}}} + \frac{nr/2}{(1+r)^1} + \frac{nr/2}{(1+r)^{\frac{3}{2}}} + \ldots + \frac{nr/2}{(1+r)^{\frac{2n-1}{2}}} + \frac{nr/2}{(1+r)^n} + \frac{n}{(1+r)^n}$$

For example, a three-year \$10,000 bond at 5% with semi-annual payments would pay \$250 at the end of every half-year for every 3 years, and pay \$10,000 at the end of the three years. The NPV of this bond, with annual interest r (determined by the economy), would be

$$\text{NPV} = \frac{\$250}{(1+r)^{\frac{1}{2}}} + \frac{\$250}{(1+r)^1} + \frac{\$250}{(1+r)^{\frac{3}{2}}} + \frac{\$250}{(1+r)^2} + \frac{\$250}{(1+r)^{\frac{5}{2}}} + \frac{\$250}{(1+r)^3} + \frac{\$10,000}{(1+r)^3}$$

The NPV would be the price of the bond. So, it is important to remember that a bond loses value when interest rates rise. This is because with a rise in interest rates, more attractive bonds with better coupon payments are produced. Therefore, it is good to buy bonds when interest rates are high. When the interest rates drop afterwards, the value of the bond becomes even higher.

Corporate Investment

Definition 1.3.20 (NPV Analysis)**.** Consider an investment project involving spending an investment I and then reaping a return over time. We can represent the **net present value** of this investment as

$$\text{NPV} = -I + \frac{R_1}{1+r} + \frac{R_2}{(1+r)^2} + \frac{R_3}{(1+r)^3} + \ldots$$

where R_1 represents first year revenues, R_2 represents second year revenues, and so on. The investment is made when NPV is positive. A NPV analysis requires three things:

1. Initial investment I must be estimated
2. Revenues $R_1, R_2, \ldots$ must be estimated, which can be heard since new products (new tech) or uncertain outcomes (e.g. oil wells) provide no way of estimating demand
3. An appropriate rate of return r, which is the discount rate that represents the return that could be earned per year on an investment of similar risk

Note that this analysis could be subject to bias, since project leaders would try to minimize I, maximize the R_i's, and minimize r in order to make the project look more attractive to upper management.

Example 14 (Silver Mine). A company looking to develop a silver mine in Mexico estimates that developing the mine would require $4 million per year for four years, Then, it generates a revenue of $1 million per year for 40 years. Then, the net present value (in millions) is

$$\text{NPV} = -4 + \frac{-4}{1.18} + \frac{-4}{1.18^2} + \frac{-4}{1.18^3} + \frac{1}{1.18^4} + \ldots + \frac{1}{1.18^{43}} = -12.697 + 13.377 = 0.81$$

This leads to a revenue of $810,000. In actuality, the problem with mines is that an 18% rate of return makes those future revenues have quite modest present values.

Definition 1.3.21 (Internal Rate of Return Analysis)**.** Similar to NPV analysis, the IRR analysis solves the equation

$$\text{NPV} = 0$$

for the interest rate r, and then the project is undertaken if the rate of return is sufficiently high. However, this equation may have more than one solution or no solutions at all, so it is not transparent what to do in these events.

Definition 1.3.22 (Payback Method)**.** The payback method asks how many years a project must be run before profitability is reached. That is, it solves the equation

$$\text{NPV} = 0$$

for the number of years n.

Note that the NPV approach does a good job when the question is whether to undertake a project or not (better than most approaches in investment decisions), making it the most common approach to investment decisions. However, NPV does a poor job when the question is whether to undertake a project or to delay the project.

Option Value of Investment

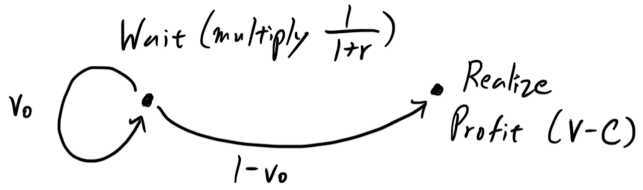
We now deal with the question of whether to undertake an investment project or delay it. When we invest in some products, like oil, the value of this investment can *change* over time. This change leads to *price risk*, which NPV analysis has a hard time predicting.

Say that there is a good $\mathfrak{g}$ that you have spent a constant cost $\mathcal{C}$ investing in (e.g. you've spent $50 on a stock). To normalize this, assume that $0 < \mathcal{C} < 1$. As soon as $\mathcal{C}$ is invested, a value $\mathcal{V}$ is generated, but it varies from time to time in a random manner. Say that the distribution in value (again, normalized) is

$$\mathcal{V} \sim \text{Uniform}[0,1] \implies \mathbb{P}(V \in [a,b]) = |b-a|$$

We can set a **cutoff value** V_0: that is, set a level V_0 and invest in the option if and only if $\mathcal{V} \geq V_0$. The NPV rule simply says $V_0 = C$; that is, invest whenever profitable.

Now, given that we have invested $\mathcal{C}$ into the option (with value represented by the random variable $\mathcal{V}$) with cutoff value V_0, let us model this as a discrete time Markov chain with initial distribution to be at node A with probability V_0 and node B with probability $1 - V_0$.



To elaborate, landing on node A will require us to wait another year to realize the investment, requiring us to discount the value of $\mathfrak{g}$ by $\frac{1}{1+r}$. But when we land on node B (the absorbing state), the investment is automatically realized and we take profit (not discounted yet) defined by random variable

$$\big(\mathcal{V}-\mathcal{C} \mid \mathcal{V}\in[V_0,1]\big)\sim \text{Uniform}[V_0-\mathcal{C},1-\mathcal{C}]$$

However, to make things simpler, we can just compute the expectation of profit (not discounted yet) upon landing on node B. By linearity,

$$\mathbb{E}\big(\mathcal{V}-\mathcal{C} \mid \mathcal{V}\in[V_0,1]\big)=\frac{1+V_0}{2}-\mathcal{C}$$

Since the number of times k that the chain lands on node A before landing on node B (the absorbing state) is a random variable, the discounted profit J behaves like a geometric random variable:

$$\mathbb{P}\left(J=\left(\frac{1}{1+r}\right)^k\left(\frac{1+V_0}{2}-\mathcal{C}\right)\right)=V_0^k\big(1-V_0\big)$$

The expectation of profit J can be found by conditioning on k:

$$\mathbb{E}(J)=\sum_{k=0}^{\infty}\left(\frac{1}{1+r}\right)^k\left(\frac{1+V_0}{2}-\mathcal{C}\right)V_0^k\big(1-V_0\big)=\frac{\big(1-V_0\big)\Big(\frac{1+V_0}{2}-\mathcal{C}\Big)}{1-\dfrac{V_0}{1+r}}$$

We can actually treat this as a function of the cutoff value V_0 to get

$$(\mathbb{E}J)(V_0)=\frac{\big(1-V_0\big)\Big(\frac{1+V_0}{2}-\mathcal{C}\Big)}{1-\dfrac{V_0}{1+r}}$$

Therefore, given that we have a cutoff value V_0, $(\mathbb{E}J)(V_0)$ tells us the firm's expected profit given this V_0. We can get a clue as to what this function looks like. Differentiating it with respect to V_0 gives

$$(\mathbb{E}J)'(V_0)=\frac{1+2r\mathcal{C}+V_0^2-2(1+r)V_0}{2(1+r)\left(1-\frac{V_0}{1+r}\right)^2}$$

which implies that $(\mathbb{E}J)'(\mathcal{C})>0$ and $(\mathbb{E}J)'(1)<0$. This means that there is a maximum at $(C,1)$, which is calculated to be

$$V_0=(1+r)\pm\sqrt{r^2+2r(1-\mathcal{C})}$$

The positive root of the quadratic has $V_0 > 1$, which entails never investing. Therefore, the profit maximizing investment strategy is to invest whenever the value $\mathcal{V}$ exceeds

$$V_0 = (1 + r) - \sqrt{r^2 + 2r(1 - \mathcal{C})}$$

Two things to note:

1. When $r = 0$, then $V_0 = 1$. This makes sense because $r = 0$ corresponds to no discounting, so there is no loss in holding out for the highest possible value.
2. As $r \to \infty$, $V_0 \to \mathcal{C}$. $r \to \infty$ means that the future is valueless, so it is worth investing if the return is anything over its costs. Therefore, the NPV rule only applies if the future is valueless.

We can present the deviation of this option value analysis with NPV analysis. That is, given the cost $\mathcal{C}$, the different curves represent $\mathbb{E}J$ as a function of the interest rate r.

Note that as $r = 0$, it doesn't matter what the cost is; the cutoff value when we should realize the investment is at $V_0 = 1$, since we can keep waiting. But looking at each case individually,

1. For the curve representing $\mathcal{C} = 0$, as the interest rate r increases the future value of the investment gets diminished quickly and so the cutoff value $V_0 = \mathbb{E}J$ will likewise decrease.
2. For $\mathcal{C} = 0.25$, there is an additional cost incurred, meaning that we must have an additional profit, that is, a higher cutoff value V_0, than $\mathcal{C} = 0$, given the same interest rate.
3. Following this line of thought, $\mathcal{C} = 0.5$ will have an higher cutoff value V_0 than that of $\mathcal{C} = 0.25$.

We can draw three additional curves that represent the cutoff value according to NPV analysis: $V_0 = \mathcal{C}$.

We can see that the optimal strategy deviates significantly from the NPV strategy.

Resource Extraction and Harvest

Example 15. Suppose that you believe that the world will run out of oil, so you invest in it by buying it and holding it. If you buy a barrel at \$40 and sell it for \$1000 in 20 years, you get a profit of \$960.

We can interpret this profit in terms of interest rates. More specifically, the **Ramsey rule** implies that the prices of resources that are in fixed supply rise at the interest rate. Therefore, we can view this profit as a constant interest rate that accumulates on the price of oil for 20 years:

$$\$40\,(1 + r)^{20} = \$1000 \implies r = 17.46\%$$

We formulate this mathematically.

Example 16. Let time run in discrete intervals $t = 0, 1, \ldots$ and suppose that the demand for a resource in fixed supply has constant elasticity:

$$p(Q) = aQ^{\frac{1}{\varepsilon}}$$

Suppose that there is a total stock R of the resource with interest rate fixed at r. Let's find the price and consumption of the resource at each time.

Letting Q_t represent the quantity consumed at time t, we can see that discounting future prices gives us

$$p(Q_0)(1+r)^t = p(Q_t)$$

so

$$aQ_0^{-1/\varepsilon}(1+r)^t = p(Q_0)(1+r)^t = p(Q_t) = aQ_t^{-1/\varepsilon} \implies Q_t = Q_0(1+r)^{-t\varepsilon}$$

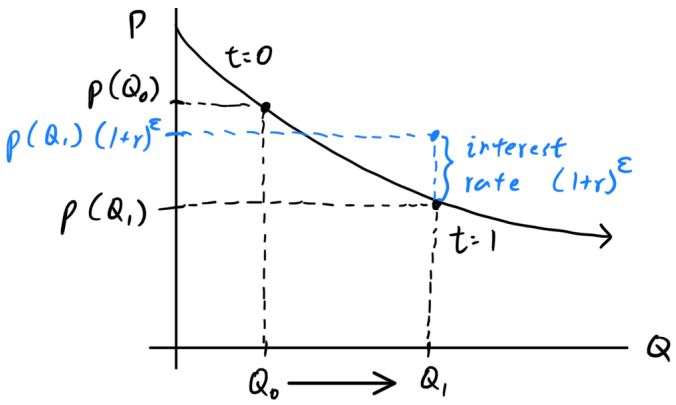
Finally, the resource constraint implies that

$$R = \sum_{i=0}^{\infty} Q_i = Q_0 \sum_{i=0}^{\infty} (1+r)^{-i\varepsilon} = \frac{Q_0}{1-(1+r)^{-\varepsilon}}$$

This solves

$$\begin{aligned} Q_0 &= R\big(1-(1+r)^{-\varepsilon}\big) \\ Q_1 &= Q_0(1+r)^{\varepsilon} \\ Q_2 &= \ldots \end{aligned}$$

To represent this graphically, we can see below:



1.4 Taxation

Definition 1.4.1 (Taxes)**.** To help fund public works and services, and to build and maintain the infrastructures used in a country, the government taxes its individual and corporate residents. Most governments use an agency or department to collect taxes; in the U.S. this is performed by the **Internal Revenue Service (IRS)**. There are several very common types of taxes:

1. **Income Tax** - A percentage of individual earnings filed to the federal government
2. **Corporate Tax** - A percentage of corporate profits taken as tax by the government

3. **Sales Tax** - Taxes levied on certain goods and services
4. **Property Tax** - Taxes based on the value of land and property assets
5. **Tariffs** - Taxes on imported goods
6. **Estate Tax** - Rate applied to the fair market value of property in a person's estate at the time of death
7. **Capital Gains Taxes** - Taxes on income that results from the sale of assets in which the sale price was higher than the purchasing price

The **tax rate** (i.e. the percentage at which an individual/corporation is taxed) can differ widely depending on the type of tax and the country. These taxes are filed as:

1. Single
2. Head of Household
3. Married, filing jointly
4. Married, filing separately

Definition 1.4.2 (Progressive, Regressive Tax System)**.** Like many nations, the U.S. has a **progressive tax system**, through which a higher percentage of tax revenues are collected from high-income individuals/corporations rather rather than from low-income individual earners (as done in a **regressive tax system**). Taxes are imposed at the federal, state, and local levels. Generally,

1. the federal government levies income, corporate, and payroll taxes
2. the state levies sales taxes
3. municipalities or other local governments levy property taxes

Example 17 (2020 U.S. Tax Brackets). There are seven federal tax brackets for the 2020 tax year. For single filers, we have

Tax Rate	Taxable Income Bracket	Tax Owed
10 %	\$0 to \$9,875	10% of taxable income
12%	\$9876 to \$40,125	\$987.50 plus 12% of amount over \$9,875
22%	\$40,126 to \$85,525	\$4,617.50 plus 22% of amount over \$40,125
24%	\$85,526 to \$163,300	\$14,605.50 plus 24% of amount over \$85,525
32%	\$163,301 to \$207,350	\$33,271.50 plus 32% of amount over \$163,300
35%	\$207,351 to \$518,400	\$47,367.50 plus 35% of amount over \$207,350
37%	\$518,401 or more	\$156,235 plus 37% of amount over \$518,400

Note that you don't lose anything by moving up a tax bracket. That is, if you earn \$520,000, then the 37% tax does not apply to the entire \$520,000. You get taxed 10% of the first \$9,875, then 12% of your income past \$9,876 up to \$40,125, and so on.

However, if you are married, you can file jointly. This means that if the husband is only working and the wife is unemployed, then the husband has an advantage in tax benefits since the overhead for every income bracket is increased (in fact, doubled until the 6th bracket). For married pairs, filing jointly, we have

Tax Rate	Taxable Income Bracket	Tax Owed
10 %	\$0 to \$19,750	10% of taxable income
12%	\$19,751 to \$80,250	\$1,975 plus 12% of amount over \$19,750
22%	\$80,251 to \$171,050	\$9,325 plus 22% of amount over \$80,250
24%	\$171,051 to \$326,600	\$29,211 plus 24% of amount over \$171,050
32%	\$326,601 to \$414,700	\$66,543 plus 32% of amount over \$326,600
35%	\$414,701 to \$622,050	\$94,735 plus 35% of amount over \$414,700
37%	\$622,051 or more	\$167,307.50 plus 37% of amount over \$622,050

Example 18 (2020 Sales Taxes by State). Since the state levies sales taxes, we list them for some states.

State	Sales Rate
Alabama	4.00%
California	7.25%
Washington D.C.	6.00%
Florida	6.00%
Illinois	6.25 %
Indiana	7.00%
Massachusetts	6.25 %
North Carolina	4.75%
New Jersey	4.75 %
Washington	6.50%

Example 19 (2020 Capital Gains Tax Rates). Capital taxes are divided into **short-term capital gains** and **long-term capital gains**. The short term tax rates (filing single) are usually taxed at the same rate as your ordinary income:

Tax Rate	Taxable Income Bracket	Tax Owed
10 %	\$0 to \$9,875	10% of taxable income
12%	\$9876 to \$40,125	\$987.50 plus 12% of amount over \$9,875
22%	\$40,126 to \$85,525	\$4,617.50 plus 22% of amount over \$40,125
24%	\$85,526 to \$163,300	\$14,605.50 plus 24% of amount over \$85,525
32%	\$163,301 to \$207,350	\$33,271.50 plus 32% of amount over \$163,300
35%	\$207,351 to \$518,400	\$47,367.50 plus 35% of amount over \$207,350
37%	\$518,401 or more	\$156,235 plus 37% of amount over \$518,400

However, long-term capital gains tax rates (filing single) give better benefits:

Taxable Income Bracket	Capital Gains Tax Rate
\$Up to \$40,000	0%
\$40,001 to \$441,450	15%
Over \$441,450	20%

1.5 Macroeconomics

1.5.1 International Trade

Definition 1.5.1 (Imports, Exports)**.** A product that is sold to the global market is called an **export**. A product that is bought from the global market is an **import**.

Definition 1.5.2 (International Trade)**. International trade**, which is the exchange of goods/services between countries, allows countries to expand their markets and access goods/services that otherwise may not have been available domestically.

Global trade allows wealthy countries to use their resources (e.g. labor, technology, or capital) more efficiently. Different countries are endowed with different assets and natural resources, such as land, labor, capital, resources, and technologies. This allows some countries to have a *comparative advantage* over others, and by taking advantage of trade, everyone can benefit.

In addition to increased efficiency, international trade allows countries to participate in a global economy, encouraging the opportunity for foreign direct investment. This allows economies to grow more efficiently and to become competitive economic participants.

Free Trade vs Protectionism

International trade has two contrasting views regarding the level of control placed on trade between countries.

Definition 1.5.3 (Free Trade)**.** The **free trade** approach, also referred to as **laissez-faire** economics, place absolutely no restrictions on trade. It states that supply and demand factors, operating on a global scale, will ensure that production happens efficiently. Therefore, nothing needs to be done to protect or promote trade and growth because market forces will do so automatically.

Elaborating, laissez-faire says that economic competition constitutes a "natural order," also known as the *invisible hand* (by Adam Smith), that rules the world. Furthermore, it is argued that this hand is the best type of regulation, meaning that there is no need for businesses and industrial affairs to be complicated by government intervention. They oppose any federal involvement in the economy, including minimum wages (price floors), duties, trade restrictions, and corporate taxes), viewing them as a penalty for production.

Definition 1.5.4 (Protectionism)**. Protectionism** holds that regulation of international trade is important to ensure that markets function properly. Advocates of this theory believe that market inefficiencies may hamper the benefits of international trade, and they aim to guide the market accordingly. Protectionists support these federal involvements.

We define some common forms of protectionism.

Definition 1.5.5 (Subsidies)**.** A **subsidy** is a direct or indirect payment to individuals or firms, usually in the form of a cash payment from the government or a targeted tax cut. Subsidies are generally seen as a privileged type of financial aid, as they lessen an associated burden that was previously levied against the receiver, or promote a particular action by providing financial support. It is often considered to be in the overall interest

of the public, given to promote a social good or an economic policy. Some common types of subsidies are:

1. **Welfare payments**, which are government-sponsored assistance programs for individuals and families in need (e.g. health care assistance, food stamps). They are typically funded through taxation. In the U.S., the federal government provides grants to each state through the **Temporary Assistance for Needy Families (TANF)** program. Eligibility for benefits is based on a number of factors, such as income level and family size. Welfare beneficiaries usually receive a biweekly or monthly payment in the form of food stamps, vouchers, or even direct payments.
2. **Unemployment benefits (aka compensation, income)**, which are temporary government payments to unemployed workers who have lost their jobs due to layoffs or other reasons not of their own fault (business closed, etc.). It is to provide a social safety net to individuals who are looking for a new job. They are often calculated as a percentage of the average of the claimant's pay over a recent 52-week period (subject to income tax). Under normal circumstances, most states pay a maximum of 26 weeks of unemployment benefits, but benefits can be extended or augmented during times of economic crisis.
3. **Subsidized student loans**, which are student loans that have a *subsidized interest rate*. This means that the individual pays for the principal amount, but the government pays the accrued interests. This encourages people to further their education.

Historically, the vast majority of corporate subsidies in the U.S. have gone towards four industries: agriculture, financial institutions, oil companies, and utilities companies.

Definition 1.5.6 (Stimulus Package)**.** A **stimulus package** is a coordinated effort to increase government spending, and lower taxes and interest rates, to stimulate an economy and lift it out of depression.

Example 20 (Unemployment Benefits). We list some cases of unemployment benefits:

1. As of 2018, Minnesota had one of the highest maximum weekly benefits amounts at $683 (up to 26 weeks), with Massachusetts at $742 (up to 30 weeks). Florida had $275 maximum weekly benefit for 12 weeks.
2. During the Great Recession, unemployment income payments may last for over 100 weeks. During times of low unemployment, such benefits tend to last for up to 26 weeks in most states.
3. On March 27, 2020, President Trump signed into law a $2 trillion coronavirus emergency stimulus package called the *Coronavirus Aid, Relief, and Economic Security (CARES) Act*, which put provisions in place to provide unemployment benefits to unemployed individuals affected by the pandemic. The law also expanded eligibility to allow those who otherwise don't qualify for benefits, including self-employed people, freelancers, and independent contractors.

Definition 1.5.7 (Tax Deductions, Credits)**.** Other types of subsidies include **tax deductions** and **tax credits**.

1. Tax deductions reduce the amount of taxable income

2. Tax credits reduce the actual amount of tax owed

Example 21 (Affordable Care Act). The Affordable Care Act, also known as *Obamacare*, is a healthcare reform signed into law by President Obama in March 2010. With the enactment of the ACA, a number of U.S. families became eligible for health-care subsidies, based on household income and size. The law includes tax credits that lower monthly health insurance bills, and cost-sharing reductions, which reduce out-of-pocket costs.

Definition 1.5.8 (Economic Sanctions)**.** A **sanction** is a penalty levied on another country, or on individual citizens of another country. It is an instrument of foreign policy and economic pressure with the goal being to force a country to alter its behavior without using military threat (which can be expensive and risky). Some common forms of economic sanctions are:

1. **Tariffs**, which are taxes imposed on goods imported from country B into country A. Upon imposing this tariff, the domestic consumer in A may shy away from the product due to an increase in price, restricting these imports. Two types:

   (a) A **specific/fixed tariff** is levied as a fixed fee based on the type of item, e.g. $1000 on a car.

   (b) An **ad-valorem tariff** is levied based on the item value, such as 10% of the value of the car.

   Tariffs may be imposed to raise revenue or to protect domestic industries from foreign competition. This can lead to hurting domestic consumers (due to higher prices), generating tensions, decreased innovation, and possibly even a **trade war**.

2. **Quotas**, which are limits on how many goods can be either imported from another country or exported to that country.

3. **Embargoes**, which are trade restrictions that prevents a country from trading with another.

4. **Non-Tariff Barriers (NTBs)**, which are non-tariff restrictions on imported goods. Rather than taxing them, governments can require licensing and packaging requirements, product standards, and other requirements.

5. **Asset Freezes (or Seizures)**, which prevents assets owned by a country or individual from being sold or moved.

Unilateral sanctions are acted by a single country, while a **multilateral one** means that a group of block of countries is supporting its use. Multilateral sanctions are considered less risky, but unilateral sanctions can be very effective if enacted by an economically powerful country.

Example 22 (Bill H.R. 850, World vs Iran). Blocking a country's exports through an import sanction increases the possibility that the target country will experience a substantial economic burden. On July 31, 2013, the U.S. passed a sanction that blocked Iran from selling any oil abroad because of its nuclear program. This bill followed a year in which Iran's oil exports had already been cut in half by international sanctions.

Example 23 (U.S. Embargo on Cuba). In 1962, the U.S. embargo on Cuba prevented almost all U.S. exports to Cuba. Since the year 2000, the embargo no longer prohibits the trade of food and humanitarian supplies.

Definition 1.5.9 (WTO)**.** Sometimes, a government can use a sanction as a way to demonstrate resole or to create a distraction from domestic trouble. To mediate these potentially irresponsible behavior, international organization such as the **World Trade Organization (WTO)** have panels that objectively review disputes between countries.

Moreover, decisions on economic sanctions made by the U.S. are often based on mandates by the United Nations. Allied countries frequently band together, making joint agreements to restrict trade with specific nations.

Sometimes, sanctions can have drastic unforeseen impacts.

Example 24 (OAPEC Embargo on U.S.). The *Organization of Arab Petroleum-Exporting Countries (OAPEC)* issued an embargo on oil shipments to the United States in 1973 as a punishment for resupplying Israel with arms. This caused fuel shortages, rationing, and soaring gas prices. OAPEC was using the embargo as a tool of foreign policy, but the effects later spilled over and exacerbated the worldwide stock market crash of 1973-74.

Example 25 (Embargoes after 9/11 Attacks). In the wake of the September 11 terrorist attacks in 2001, U.S. embargoes were increasingly directed against countries with known ties to terrorist organizations. Lately, U.S. embargoes have become more widespread.

Example 26 (Trump vs China). When President Trump began his term in 2016, he pledged to make it easier to consumers to buy American products. He proceeded to slap import taxes on certain goods entering the country, leading some nations, such as China, to hit back with punitive measures of their own.

1.5.2 Unemployment and Inflation

Definition 1.5.10 (U.S. Department of Labor)**.** The **U.S. Department of Labor (DOL)** is a agency responsible for enforcing federal labor standards and promoting workers' well-being. Some of its purposes are:

1. It creates employment opportunities, protects retirement and healthcare benefits, help employers find workers, etc.
2. It enforces many laws, including the *Fair Labor Standard Act*, which establishes minimum wage standards and overtime pay.
3. Oversees the **Bureau of Labor Statistics (BLS)**, which provides important data such as the unemployment rate, CPI, and PPI.

Every month the BLS conducts the **Current Population Survey** using a sample of 60,000 households, or around 110,000 individuals. Unemployment rates and other measures are calculated after collection.

Definition 1.5.11 (Unemployment)**. Unemployment** is persons above a specified age (usually 15) not being in paid employment or self-employment but currently available for

work during the reference period. The **Bureau of Labor Statistics** defines it as:

1. People with jobs are *employed*.
2. People who are jobless, looking for a job, and available for work are *unemployed*.
3. The **labor force** is made up of the employed and the unemployed.
4. People who are neither employed nor unemployed are *not in the labor force*.

Note that those on active duty in the Armed Forces are not considered in the labor force. Students can be classified as employed, unemployed, or neither, whether they are in school on a full or part-time basis.

Definition 1.5.12. The **unemployment rate** is the percentage of the labor force that is jobless. It is a **lagging indicator**, meaning that it generally rises or falls *after* changing economic conditions, rather than anticipating them.

There are six ways that the unemployment rate could be calculated: U-1 through U-6:

1. **U-1** refers to people who have been unemployed for 15 weeks or longer, expressed as a percentage of the labor force:

$$\text{U-1} = \frac{\text{Unemployed 15+ Weeks}}{\text{Labor Force}}$$

2. **U-2** refers to people who lost their jobs, or whose temporary jobs ended

$$\text{U-2} = \frac{\text{Job Losers}}{\text{Labor Force}}$$

3. The **U-3 rate** is the most widely used and cited:

$$\text{U-3} = \frac{\text{Unemployed}}{\text{Labor Force}}$$

4. **U-4** refers to unemployed people, plus discouraged workers

$$\text{U-4} = \frac{\text{Unemployed + Discouraged}}{\text{Labor Force + Discouraged}}$$

   where **discouraged workers** are those who are available to work and would like a job, but have given up actively looking for one.

5. **U-5** refers to unemployed people, plus those who are marginally attached to the labor force

$$\text{U-5} = \frac{\text{Unemployed + Marginally Attached}}{\text{Labor Force + Marginally Attached}}$$

   where **marginally attached** workers include discouraged workers and anyone else who would like a job and has looked for one in the past 12 months but have given up actively searching.

6. **U-6** refers to unemployed people, plus people who are marginally attached to the labor force, plus those who are employed part-time for economic reasons.

$$\text{U-6} = \frac{\text{Unemployed + MA + PTER}}{\text{Labor Force + MA}}$$

This metric, also called the *real unemployment rate*, is the BLS's most comprehensive. In addition to the categories included in U-5, it accounts for people who have been forced to settle for part-time even though they want to work full-time. This category is called the **underemployed**.

From this, we can see that by definition

$$\text{U-1} < \text{U-2} < \text{U-3} < \text{U-4} < \text{U-5} < \text{U-6}$$

Definition 1.5.13 (Inflation)**. Inflation** is a measure of the rate of the rising prices of goods and services in an economy. It can occur in nearly any product/services, including need-based expenses such as housing, food, medical care, etc.

There are various factors that can drive inflation:

1. **Cost-Push Inflation** occurs when prices increases due to increases in production costs, such as raw materials and wages. The demand for goods is unchanged while the supply of goods declines due to the higher costs of production. As a result, the added costs of production are passed onto consumers in the form of higher prices for the finished goods.
2. **Demand-Pull Inflation** can be caused by strong consumer demand for a product or service. When there's a surge in demand for goods across an economy, prices increase, and the result is demand-pull inflation. As the demand for a particular good or service increases, the available supply decreases. When fewer items are available, consumers are willing to pay more to obtain the item—as outlined in the economic principle of supply and demand. The result is higher prices due to demand-pull inflation.

   Consumer confidence tends to be high when unemployment is low, and wages are rising—leading to more spending. Economic expansion has a direct impact on the level of consumer spending in an economy, which can lead to a high demand for products and services.

Inflation at a moderate amount is healthy for the economy, but it can be a concern because it erodes a consumer's purchasing power and can even interfere with the ability to retire.

Example 27 (Cost-Push Inflation)*.* One of the signs of possible cost-push inflation can be seen in rising commodity prices such as oil and metals since they're major production inputs. For example, if the price of copper rises, companies that use copper to make their products might increase the prices of their goods. The business will pass on the higher costs of raw materials to consumers. The result is higher prices for consumers without any change in demand (but rather a change in supply).

Wages also affect the cost of production and are typically the single biggest expense for businesses. When the economy is performing well, and the unemployment rate is low, shortages in labor or workers can occur. Companies, in turn, increase wages to attract

qualified candidates, causing production costs to rise for the company. If the company raises prices due to the rise in employee wages, cost-plus inflation occurs.

Natural disasters can also drive prices higher. For example, if a hurricane destroys a crop such as corn, prices can rise across the economy since corn is used in many products.

Example 28 (Demand-Pull Inflation from Corporations). Companies also play a role in inflation, especially if they manufacture popular products. A company can raise prices simply because consumers are willing to pay the increased amount. Corporations also raise prices freely when the item for sale is something consumers need for everyday existence, such as oil and gas. However, it's the demand from consumers that provides the corporations with the leverage to raise prices.

Example 29 (Housing Market). The housing market, for example, has seen its ups and downs over the years. If homes are in demand because the economy is experiencing an expansion, home prices will rise. The demand also impacts ancillary products and services that support the housing industry. Construction products such as lumber and steel, as well as the nails and rivets used in homes, might all see increases in demand resulting from higher demand for homes.

There are two main ways to measure inflation.

Definition 1.5.14 (Consumer Price Index)**.** The **Consumer Price Index (CPI)** is a measure that examines the weighted average of prices of a basket of consumer goods and services, such as transportation, food, and medical care. It is calculated by taking the average change in prices over time that consumers pay for a basket of goods and services, and it is associated with the cost of living.

Definition 1.5.15 (Producer Price Index)**.** The **Producer Price Index (PPI)** is a measure that reports the average price changes from domestic production over time. It is different from the CPI in that is measures costs from the viewpoint of industries that make the products, whereas the CPI measures prices from the perspective of consumers.

Example 30 (PPI of Balloons). The PPI has a base number of 100, which represents no change in prices. If the production of balloons has a PPI of 115 for the month of July, the 115 figure indicates that it cost the balloon manufacturing industry 15% more to produce balloons in July than it did in June.

Note that companies can both benefit and be hurt by inflation. They can charge more for their products as a result of a surge in demand for their goods, increasing their profit margins, or they can lose money by a surge in production costs (and they cannot pass on the higher costs to consumers through higher prices).

1.5.3 Flow of Money

Currency

Definition 1.5.16 (Money Supply)**.** There are many definitions of **money** and what we call the **money supply**, which is the total value of the financial assets that are considered

money. These classifications may differ by country, but the U.S. uses the M0, M1, and M2 classifications.

1. The **monetary base**, or **M0**, is equal to coin currency, physical paper, and central bank reserves.
2. The **M1** covers M0 in addition to checkable bank deposits (checking accounts)and traveler's checks (but does not include savings accounts, bonds)
3. The **M2** covers M1 in addition to savings deposits, time deposits (e.g. CDs), and money market shares (e.g. funds).

Money serves essentially three roles:

1. It is a *medium of exchange*, an asset that individuals use to trade for goods and services rather than for consumption.
2. It is a *store of value*, having a means of holding purchasing power over time (which may decrease).
3. It is a *unit of account*, used to standardize rates and payments.

By March 2021, the U.S. had $18,682.9 billion (1.9 trillion) in M1 and $19,896.2 billion in M2 (2 trillion).

Example 31 (U.S. Monthly Money Supply Measures). These are the money supply measures of the U.S. starting from May 2020, in billions of dollars. Note how they are consistently increasing.

Date	M1	M2
May 2020	16,275.9	17,893.0
Jun. 2020	16,601.7	18,179.6
Jul. 2020	16,792.6	18,320.0
Aug. 2020	16,906.0	18,381.8
Sep. 2020	17,176.3	18,605.0
Oct. 2020	17,367.1	18,751.1
Nov. 2020	17,610.0	18,960.2
Dec. 2020	17,829.6	19,125.8
Jan. 2021	18,109.6	19,378.7
Feb. 2021	18,401.0	19,650.3
Mar. 2021	18,682.9	19,896.2

Definition 1.5.17 (Types of Money)**.** There are actually different types of money:

1. **Commodity money** is a medium of exchange that is an actual good, normally gold or silver, that had intrinsic value in other uses. These alternative uses gave commodity money value independent of its role as a medium of exchange.
2. **Commodity-backed money** is a medium of exchange with no intrinsic value, but whose ultimate value was guaranteed by a promise that it could always be converted into valuable goods on demand. For example, U.S. banks issued private paper bills, which promised to exchange their notes for gold and silver coins on demand.

This was preferred since it tied up fewer valuable resources. Although a note-issuing bank still had to keep some gold/silver on hand, it had to keep only enough to satisfy demands for redemption of its notes. It could rely on the fact that only a fraction of its paper notes would be redeemed on a normal day.

3. **Fiat money** is not even backed by any commodity. Rather, its value arises entirely from the fact that it is generally accepted as a means of payment, a role/policy that is decreed by the government (that is, it exists by government *fiat*). This means that the "promise to pay" for commodity-backed money was replaced with the promise to accept that currency.

Commercial and Central Banks

Definition 1.5.18 (Commercial Banks)**.** A **commercial bank** is what most people refer to as everyday banks. They provide basic banking services to individuals and to small/medium-sized businesses. These include

1. Accepting deposits and offering checking account services
2. Making loans and mortgages
3. Offering basic investment services such as CDs and safe deposit boxes

Banks are businesses, too, and they make money in two ways:

1. They collect service charges and fees, ranging from account fees (maintenance charges, minimum balance, overdraft, etc.), safe deposit box fees, and late fees.
2. They earn money from interest they earn by lending out money to other clients. It is clear that the interest rate paid by the bank on the money they borrow is less than the rate charged on the money they lend (otherwise, they would not profit).

Many banks now operate exclusively online, in contrast to their physical, *brick-and-mortar* locations. Note that their savings account pays interests well below the U.S. Treasury bond rates, and checking accounts generally do not pay interest.

Commercial banks are an important part of the economy because now only do they provide consumers with an essential service, they also help create capital and liquidity in the market. Indeed, they maintain the flow of money by taking money from customers deposit for their savings and lending it out to others. They play a role in the creation of credit, which leads to increase in production, employment, and consumer spending.

Definition 1.5.19 (Bank Runs, Failures)**. Bank runs** happen when a large number of people, out of panic, start making withdrawals from banks because they feat the institutions will run out of money. A **bank panic** occurs when multiple banks endure runs at the same time.

Definition 1.5.20 (Central Bank)**.** A **central bank**, or reserve bank is an institution that manages the currency and monetary policy of a state and oversees their commercial banking system. Functions of a central bank include:

1. **Monetary Policy**: by setting the official interest rate and controlling the money supply

2. **Financial Stability**: Acting as a government's banker and the bankers' bank ("lender of last resort")
3. **Reserve management**: Managing a country's foreign-exchange and gold reserves and government bonds
4. **Banking Supervision**: Regulating and supervising the banking industry
5. **Coins and Notes Issuance**

Definition 1.5.21 (Federal Reserve Structure)**.** The **Federal Reserve**, or the **Fed**, is the central bank of the United States. The Fed is composed of several layers.

1. It is governed by the 7-member presidentially appointed **Federal Reserve Board (FRB)**.
2. 12 regional Federal Reserve Banks, located throughout the nation (based in New York, Boston, Philadelphia, Cleveland, Richmond, Atlanta, Chicago, St. Louis, Minneapolis, Kansas City, Dallas, San Francisco), regulate and oversee privately owned commercial banks.
3. The **Federal Open Market Committee (FOMC)** sets the monetary policy, consisting of the FRB and the 12 regional bank presidents.

These banks protect regional economic interests. Even though they don't operator for profit, they generate income from interest on government securities acquired to Fed monetary policy actions. Unusually, it is a separate entity from the U.S. Department of the Treasury.

Definition 1.5.22 (U.S. Department of the Treasury)**.** The **Department of the Treasury (USDT)** is the national treasury of the Fed. Its purposes are listed:

1. It oversees the **Bureau of Engraving and Printing** and the **U.S. Mint**, the two agencies responsible for printing all paper currency and coins. Note that some printing/minting does occur, but the vast majority of the American money supply is digitally debited and credited to major banks.
2. It executes money circulation in the fiscal system.
3. It collects all federal taxes through the Internal Revenue System (IRS).
4. It manages the U.S. government debt instruments (Treasury bonds).
5. It licenses and supervises banks.

Definition 1.5.23 (FDIC Insurance)**.** Customers find commercial bank investments, such as savings accounts and CDs, attractive because they are insured by the **Federal Deposit Insurance Corporation (FDIC)**, which is an independent federal agency insuring deposits in U.S. banks. This means that the FDIC insures deposits up to $250,000 per depositor as long as the bank is a member firm of the FDIC. Therefore, people with more than $250,000 should spread their savings out in multiple FDIC-insured banks, or across multiple accounts (a checking and multiple savings) in the same FDIC-insured bank.

Reserve Requirements

We can now talk about how the Fed carries out monetary policy to control the money supply. First, we must distinguish the difference between the federal bank and commercial banks.

In order to minimize the possibility of bank runs, the Fed legally requires commercial banks to have a required **reserve ratio** (i.e. the amount of funds that a bank holds in reserve to ensure that it is able to meet liabilities in case of sudden withdrawals). These **federal funds** are stored in the regional Federal Reserve banks. It is based on a percentage of the bank's total deposits (i.e. the total amount of money deposited into the bank):

1. 0% requirement for banks with eligible deposits up to $16 million.
2. 3% requirement for banks up to $122.3 million.
3. 10% requirement for banks greater than $122.3 million.

The end-of-the-day balances in each bank's account, averaged over two-week reserve maintenance periods, are used to determine whether the bank meets its reserve requirements.

Note that as of March 2020, the minimum reserve requirement for all deposit institutions was fixed to 0%. The *International Banking Act of 1978* requires branches of foreign banks operating in the U.S. to follow the same required reserve ratio standards. But preventing bank runs isn't the only purpose of reserve requirements; it is used to influence the money supply in the economy. The higher the reserve requirements, the less money in circulation. **Excess reserves** refer to reserves held by a bank in excess of what is required, serving as a safety buffer.

3 Main Interest Rates

Now, there is a dynamic relationship between the federal bank and the many commercial banks. They can lend money to each other, and depending on who lends who, different interest rates are charged.

Definition 1.5.24 (IOR/IOER)**.** We can think of the reserve requirement as a loan from commercial banks to the central bank. Starting from 2008, the Fed pays interest on these (required and excess) reserves.

1. The **interest on reserves (IOR)** is the rate of interest that banks are paid on required reserves.
2. The **interest on excess reserves (IOER)** is the rate of interest paid on excess reserves.

This rate is directly controlled by the Fed.

Definition 1.5.25 (Discount Rate)**.** Going the other way, if commercial banks borrow money from the central bank, they must pay an interest. For example, banks can borrow funds to keep up their required reserves by taking a loan from the Fed itself at the discount rate. More specifically, there are actually three different discount rates:

1. The **primary credit rate** (mentioned the most) is the basic interest rate charged to most banks (e.g. 0.25%).
2. The **secondary credit rate** is charged to banks that don't meet the primary rate requirements. It is typically half a point higher than the primary credit rate (e.g. 0.75%).
3. The **seasonal discount rate** is for small community banks that need a temporary boost in funds to meet local borrowing needs. This may include loans for farmers, students, resorts, and other seasonal activities.

This rate is directly controlled by the Fed.

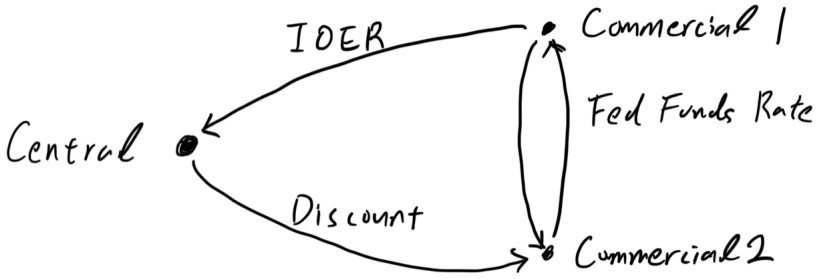
Recognize that each bank wants to hold the optimal amount of federal funds. If it holds too few federal funds, it might not be able to honor its obligations to other banks (e.g., when they show up with checks written by its customers) or might fall below the legal amount of reserves it is required to hold. If it holds too many federal funds, it will earn less interest on its assets than it otherwise would on other investments. Hence, each bank is trying to hold the optimal amount of reserves, given its unique position and the opportunity cost of holding reserves.

Definition 1.5.26 (Federal Funds Rate)**.** So, there will inevitably be banks that hold too many federal funds that can lend money to banks that hold too few federal funds. Therefore, this money can be lent between banks, with the promise that those funds are paid back overnight. This financial market in which interbank lending occurs in the U.S. is called the **federal funds market**, and the interest rate the lending bank can charge is referred to as the **federal funds rate (FFR)**. There are two variations of this rate:

1. The actual interest rate for a loan between two banks is determined through negotiations between them. The weighted average of this rate across all such transactions on a certain night is the **federal funds effective rate (FFER)**.
2. The **federal funds target rate (FFTR)** is a (usually quarter percent) range set by the FOMC (usually 8 times a year) as a guidepost, which they enforce by open market operations and adjustments in IOR/IOERs. The target rate is almost what is meant by the media referring to the Fed "changing interest rates." The actual FFR generally lies within the range of that target rate.

There is not much to talk about the FFER, but it is worth knowing how the Fed indirectly influences the FFER to converge to the target rate using something called open market operations.

All three of these rates can be summarized in the following diagram:



The IOER is the interest on the flow of money from commercial banks to the central bank. The discount rate is the interest on the flow of money from the central bank to commercial banks. The fed funds rate is the interest on the flow of money between commercial banks.

Definition 1.5.27 (Open Market Operation)**.** The Fed can indirectly change the actual FFR to the target rate through **open market operations (OMO)**, which are the purchases and sales of primarily U.S. Treasury securities (bonds) on the open market (and sometimes commercial securities) in order to regulate the supply of money that is on reserve in U.S. banks. In the simplest sense,

1. If the Fed wants the FFR to decrease, then it buys government securities (with possibly newly printed money) from a group of banks. As a result, those banks end up holding fewer securities and more excess cash reserves, which they then lend out in the federal funds market to other banks. This increase in the supply of available reserves causes the federal funds rate to decrease.

2. If the Fed wants the FFR to increase, it does the reverse open market operation of selling government securities to the banks, taking money out of circulation and making loans harder to obtain due to a decreased supply of available reserves. This causes the federal funds rate to rise.

As shown before, the federal funds rate may increase or decrease depending on the overall supply of reserves in the federal funds market. If the supply increases, this surplus of reserves pulls the FFR down, and if the supply decreases, this shortage of reserves pushes the FFR up; therefore, the federal funds rate acts as a catalyst that brings the federal funds market to equilibrium, ensuring that supply satisfies demand with the proper interest rate.

We can also categories OMOs as such:

1. **Permanent Open Market Operations (POMOs)** refers to when the central bank actively buys and sells treasury securities in the open market on a continual basis. When any central bank *consistently* uses the open market to buy/sell securities in order to adjust the money supply, it is engaging in POMOs.

2. **Temporary Open Market Operations** refers to when specific quantities of treasury securities are authorized to be purchased or sold for a period to address a financial crisis or other economic emergency. This is used to add or drain reserves available to the banking system on a temporary basis.

Example 32. We list some of the rates from May 2015 to now, with gaps of a few months in between (even though the rates change every month).

Date	FFR	IOER	Discount Rate
05/01/2015	0.12%	0.25%	1%
11/01/2015	0.12%	0.25%	1%
12/01/2015	0.24%	0.37%	1%
02/01/2016	0.38%	0.5%	1%
10/01/2016	0.4%	0.5%	1%
01/01/2017	0.65%	0.75%	1%
04/01/2017	0.9%	1%	1.5%
08/01/2017	1.16%	1.25%	1.75%
01/01/2018	1.41%	1.5%	2%
04/01/2018	1.68%	1.75%	2.25%
08/01/2018	1.91%	1.95%	2.5%
10/01/2018	2.19%	2.2%	2.75%
03/01/2019	2.41%	2.4%	3%
07/01/2019	2.4%	2.35%	3%
11/01/2019	1.55%	1.55%	2.25%
04/01/2020	0.05%	0.1%	0.25%

From looking at the data, all of these interest rates tend to rise and fall together, and usually

$$\text{FFR} < \text{IOER} < \text{Discount Rate}$$

This makes sense because the Fed would encourage commercial banks to borrow from each other rather than hold excess reserves, which is preferred over borrowing money directly from the Fed. But in extreme situations, the above inequality may not hold for brief periods.

Fractional Reserve Banking and the Money Multiplier

Definition 1.5.28 (The Credit Market Funnel)**.** Suppose the U.S. Treasury prints $10 billion in new bills, and the Fed credits an additional $90 billion in readily liquifiable accounts. At first, it might seem like the economy just received a monetary influx of $100 billion, but that is actually a very small percentage of the actual money creation.

Nearly all of that $100 billion enters banking reserves, but with the legally required reserve ratio of 10%, the new $100 billion in bank reserves could potentially result in a nominal monetary increase of $1 trillion.

Definition 1.5.29 (Money Multiplier Effect)**.** When the central bank creates monetary reserves and sends those to commercial banks, banks can lend much of that money to consumers, who will also deposit most into other banks.

Example 33. For example, if the Fed issues $1 billion in reserves to a bank, the bank can then lend $900 million to borrowers. These borrowers will then ultimately deposit those funds back to the banking systems (either directly or indirectly from people paid with the loaned money), which can then be loaned out at 90%—so if that $900 million is deposited, an additional $810 million may be deposited. Ultimately, through this money multiplier effect, the $1 billion in reserves will turn into $10 billion in new credit money in the economy.

Monetary Policy

Setting borrowing costs is how the Fed does its job; steering the world's largest economy between recession and overheating. They determine how "hot" the economy is by looking at various measures. The two most important ones are:

1. Inflation, measured by the CPI/PPI, and usually targeted to be at 2%.
2. Unemployment rate, measured by the CPS, which should be minimized.

We have already seen how the Fed uses open market operations to adjust the FFR. But how does this exactly affect other rates? In the simplest terms,

1. A decreased FFR means that banks can borrow money easily at low interest rates, meaning that they can then use the reserves that they have obtained at lower rates to offer loans at lower interest rates to consumers/businesses. The cheaper credit in turn causes consumers/businesses to spend and invest, boosting sales and economic activity (an **expansionary goal**).

   Furthermore, an decreased FFR, which implies lower yields, tend to push away investment capital from investors abroad seeking higher returns on bonds and interest-rate products. This makes the U.S. dollar weaker.

2. An increased FFR means that banks borrow less money due to higher interest rates, meaning that they give loans at even higher ones to consumers. This increase in the cost of credit through the funds rate curbs demand and reduces consumers/-firms taking out loans to spend and invest, leading to an economic cooldown (a **contractionary goal**).

   Furthermore, an increased FFR, which implies higher yields, tend to attract investment capital from investors abroad seeking higher returns on bonds and interest-rate products. This makes the U.S. dollar stronger.

Note that the FFR is the *benchmark interest rate* of the economy, since all other interest rates on loans from commercial banks are dependent on it. That is, a higher FFR will mean that commercial banks will have to raise interest rates on all of their loans (to profit accordingly), and a lower FFR will mean that banks will lower their interest rates on loans. Therefore consumers should care about the FFR since it influences how much they pay to borrow and how much they're paid to save.

Let's talk about what aspects of the economy the FFR affects:

1. Bond market. Remember that when interest rates rise, bond prices generally fall (in the secondary market), and when interest rates fall, bond prices generally rise. However, in the primary market, how does the original coupon rate get established?
2. **Prime Rates**, or the **Bank Prime Loan Rate**, is the credit rate that banks extend to their most credit-worthy customers, leading to a rise in every other interest rate. Since the FFR determines the minimum interest rate that banks must put on a loan to make a profit, it is clear that

   $$\text{Increased Fed funds rate} \implies \text{Increased prime rate}$$

   In fact, prime rates are pegged at 300 points (3%) over the target rate. Since prime rates represent the base rate at which banks loan out to consumers, an increase in

this base rate will increase every other rate, such as

(a) Adjustable-rate mortgages

(b) Auto loans

(c) Variable-rate credit card expenses

(d) CDs, savings accounts, and money market accounts

depending on factors such as one's assets, liabilities, and creditworthiness.

3. If global capital flows are moving into dollar-denominated assets, chasing higher rates of return, the U.S. dollar strengthens, causing it to be more expensive in the exchange rate.
4. **Exports**: Due to the dollar strength, U.S. export sales may decline since foreign countries will need to pay more for U.S. goods.
5. **Imports, Inflation:** A strong dollar makes foreign imports cheaper, since the price of foreign currency is relatively cheap. This results in cheaper products at U.S. stores (since domestic companies have to keep prices low to compete with cheap foreign imports), and these low prices translate to low inflation.

On the extreme end, zero and negative rate environments benefit the economy through easier borrowing. In an extreme negative rate environment, borrowers even receive interest payments.

Definition 1.5.30 (Other Monetary Policy Tools)**.** Most of the time, the Fed funds rate is unchanged or changed incrementally by 0.25%, but in extreme cases, the rate can change drastically. However, in less extreme cases, the Fed can also adjust these rates too to influence monetary policy.

1. *It can change the bank reserve requirements.* If they are higher, more money is "locked in" the required reserve and the money supply shrinks. If they are lower, more money moves from the required reserve to the excess reserve, allowing better trading rates.
2. *It can change the discount rates.* If the discount rates are higher, it will discourage commercial banks from loaning money from the Fed, and if rates are lower, commercial banks have more of an incentive to loan money from central banks.
3. *It can change the IOR/IOER.* If the IOER rate is low, banks would prefer to lend funds out since it will be more profitable to gain interest according to the FFR rather than the IOER. This increases cash circulation and makes it easier to borrow money. If the IOER rate is high, then banks would prefer to keep more money at the Fed rather than lend to a potentially risky borrower. This chokes the circulation of money since banks won't want to take money out of their excess reserves.

1.5.4 Gross Domestic Product

Definition 1.5.31 (Gross Domestic Product)**.** If we think of a nation as a giant firm that takes in inputs and produces outputs, then the **gross domestic product (GDP)** is the total monetary or market value of all the finished goods and services produced within

a country's borders in a specific time period. As a broad measure of overall domestic production, it functions as a comprehensive scorecard of a given country's economic health within the international market. It is typically calculated on an annual basis (*annualized GDP*), and sometimes on a quarterly one. There are multiple ways we can measure GDP:

1. **Nominal GDP** does not take into account inflation. It is an assessment of economic production in terms of the current prices of goods and services.
2. **Real GDP** does take inflation into account, allowing it to be measured in current dollars.
3. **GDP Growth** compares the year-over-year change in a country's economic output in order to measure how fast an economy is growing, expressed as a percentage rate. If GDP growth rates accelerate, it may be a signal that the economy is "overheating" and the central bank may seek to raise interest rates. Conversely, a shrinking (or negative) GDP growth rate is a signal that rates should be lowered and that stimulus may be necessary.
4. **GDP Purchasing Power Parity (PPP)** is a method to see how one country's GDP measures up in "international dollars" using a method that adjusts for differences in local prices and costs of living in order to make cross-country comparisons.
5. **GDP per capita** is a measurement of the GDP per person in a country's population. It indicates the amount of output or income per person in an economy and can indicate average productivity or average living standards. GDP per capita can be stated in nominal, real, or PPP terms.

Ways of Calculating GDP

There are three ways of calculating the GDP of a nation, all of which should theoretically add to the same value.

1. Production Approach
2. Income Approach
3. Expenditure Approach

Definition 1.5.32 (Expenditure Approach)**.** The **expenditure/spending approach** calculates spending by the different groups that participate in the economy. It can be calculated using the following formula:

$$GDP = C + G + I + NX$$

where

1. C = Consumption; This refers to private consumption expenditures or consumer spending. Consumers spend money to acquire goods and services, such as groceries and haircuts (which are goods and services produced). Consumer spending is the biggest component of GDP, accounting for more then 2/3 of the U.S. GDP. Consumer confidence, therefore, has a very significant bearing on economic growth.
2. G = Government Spending; This represents government consumption expenditure and gross investment. For example, governments spend money on equipment, infrastructure, and payroll.

3. I = Investment; This refers to private domestic investment or capital expenditures (i.e. company expenditures). Businesses spend money in order to invest in their business activities, such as buying machinery.

4. NX = Net exports (which is Exports−Imports) All expenditures by companies located in a given country, even if they are foreign companies, are included in this calculation.

There is another important economic measurement of a country.

Definition 1.5.33 (Gross National Income)**.** The difference between GDP and **Gross National Income (GNI)** is that GDP defines its scope according to location, while GNI defines its scope according to ownership. That is, GNI is product produced by enterprises owned by a country's citizens, even in foreign countries.

It is clear to see by definition

$$\begin{aligned} GNI = GDP &+ (\text{income receipts from the rest of the world}) \\ &- (\text{income payments from the rest of the world}) \end{aligned}$$

Chapter 2

Financial Markets and Assets

Definition 2.0.1 (Securities)**.** A **security** refers to a fungible (able to be traded), negotiable financial instrument that holds some type of monetary value. It can represent:

1. an ownership position in a publicly-traded corporation via stock (*equity*)
2. a creditor relationship with a governmental body or a corporation represented by owning that entity bond (*debts*)
3. or rights to ownership as represented by an option.

Generally, they can be divided into **equity** and **debts**.

Definition 2.0.2 (IOU)**.** An **IOU** is a phonetic acronym of the words "I owe you," and is a written document that acknowledges the existence of a debt. It is conventionally used for less formal documents.

Definition 2.0.3 (Counterparty Risk)**. Counterparty risk** is the likelihood or probability that one of those involved in a transaction might default on its contractual obligation. It can exist in credit, investment, and trading transactions.

Definition 2.0.4 (Credit Quality)**. Credit quality** is a measurement of an individual's or company's creditworthiness, or the ability to repay its debt. It can also be seen as a measure of the level of counterparty risk to the lender/creditor. They are measured by private independent **credit rating agencies**, the most popular being

1. Standard & Poor's
2. Moody's
3. Fitch

each with its own designations, ranging from

1. high **investment-grade**: AAA to AA
2. medium **investment-grade**: A to BBB
3. low **junk, high-yield, or subprime**: BB, B, CCC, CC, C
4. Already in default: D

Credit scores can be assigned to multiple things:

1. **Bond ratings** are used to determine the risk that the bond issuer will default on its payments.
2. **Corporate credit ratings** are based on a firm's financial statements, including the specific company's capital structure, credit payment history, revenue, and earnings.
3. **FICO scores** determine an individual's credit quality for credit card expenses, mortgages, auto loans, etc. It ranges from 300-850.

Investing in securities issued by the U.S. treasury is considered the absolute safest way to invest in bonds, then bluechip corporations, and then smaller companies.

Note that if a security has a lower credit quality, the risk of default is greater and these securities may have greater potential profits (e.g. pay more interest, better dividends, great potential, etc.)

2.1 The Primary Market vs Secondary Market

Financial markets such as the stock market or the bond market is generally segmented into two markets.

1. Securities are bought from an issuing body on the primary market
2. Securities trade between consumers on the secondary market

Definition 2.1.1 (Primary Market)**.** The **primary market** is referred to as the "new issues" market in which transactions strictly occur directly between the bond issuers and the bond buyers. That is, the primary market yields the creation of brand new debt securities that have not previously been offered to the public.

It is where companies, governments, and other groups to go to obtain financing through debt-based or equity-based securities. Primary markets are facilitated by **underwriting groups**. All issues on the primary market are subject to strict regulation. Companies must file statements with the **Securities and Exchange Commission (SEC)** and other securities agencies must wait until their filings are approved before they can offer them for sale.

Underwriting and Investment Banking

Definition 2.1.2 (Underwriters)**.** An **underwriting group** is a temporary association of investment bankers and broker-dealers who wish to purchase a new issue of securities from an issuer in order to distribute the issue to investors at a profit. The underwriting group shares the risk and aids in the successful distribution of the new securities issue once the issuance goes public.

1. The group buys the issuance (of new stocks/bonds) from the firm first at a specified price and then sells it to the public, as opposed to the company selling the shares directly to investors.
2. The underwriting group resells the issues to investors in order to make a profit. The profit is referred to as the **underwriting spread**.

For the issuing company, having an underwriting group means that they are paid upfront for the shares they are issuing. As a result, a significant amount of risk is removed from the issuing company and taken on by the underwriting group. The issuing company no longer has to sell the inventory of its stock directly to investors. The profit or loss for the group is determined by how the new stock performs on the market.

The underwriter has two options for distributing shares to initial investors:

1. **Book-building**, in which shares can be awarded to investors of their choosing. (Usually this happens)
2. **Auctions**, in which investors who are willing to bid above the offer price receive the shares. (Rare, but a notable example is Google's 2004 IPO)

The commission fee per share ranges anywhere from 3% to 7%, which may be worth having the safety and expertise of underwriters.

Once the initial sale is complete, further trading is conducted on the secondary market, where the bulk of exchange trading occurs each day. Stock exchanges represent secondary markets. Note that investors typically pay less for securities on the primary market than on the secondary market.

Definition 2.1.3 (Types of Primary Market Issues)**.** We list some common types:

1. **IPO**: An **initial public offering (IPO)** occurs when a private company sells shares of stock to the public for the first time, a process known as **going public**. The process, including the original price of the new shares, is set by a designated investment bank, hired by the company to do the initial underwriting for a particular stock.

   For example, a company (the "Company") hires five underwriting firms to determine the financial details of its IPO. The underwriters detail that the issue price of the stock will be $15. Investors can then buy the IPO at this price directly from the issuing company. This is the first opportunity that investors have to contribute capital to a company through the purchase of its stock. A company's equity capital is comprised of the funds generated by the sale of stock on the primary market.

2. **Direct Listing**: Companies that can't afford underwriting or don't want share dilution choose the **direct listing** (aka. Direct Placement or Direct Public Offerings) process rather than an IPO. Without an intermediary, however, there is no safety net ensuring the shares sell. In a direct listing, no new shares are created and only existing, outstanding shares are sold with no underwriters involved.

3. A **rights offering (issue)** permits companies to raise additional equity through the primary market after already having securities enter the secondary market. Current investors are offered prorated (distributed) rights based on the shares they currently own, and others can invest anew in newly minted shares.

4. **Private placement** allows companies to sell directly to more significant investors such as hedge funds and banks without making shares publicly available.

5. **Preferential allotment** offers shares to select investors (usually hedge funds, banks, and mutual funds) at a special price not available to the general public.

Definition 2.1.4 (Investment Banks)**.** When companies want to issue stocks and bonds to raise capital, executing these transactions requires special expertise in fields such as

1. pricing financial instruments (coupon payments, stock price, etc.)
2. navigating regulatory requirements

This is done by an investment bank. In essence, IBs are a bridge between large enterprises and the investor. The primary goal of an investment bank is to advise businesses and governments on how to meet their financial challenges. Investment banks help their clients with financing, research, trading and sales, wealth management, asset management, IPOs, mergers, securitized products, hedging, and more.

1. The bank can recommend the best way to raise funds, whether it be by selling an ownership stake in the company through a stock offer or borrowing from the public through a bond issue. IBs can help determine how to price these instruments by utilizing sophisticated financial models.
2. In a stock offering, financial analysts will look at factors, e.g. earnings potential and management team strength, to estimate how much a share of a company is worth. In a bond offering, the bank will look at prevailing interest rates for similarly rated businesses to figure out how much it will have to compensate borrowers.
3. IBs can offer advice in a Merger and Acquisition (M&A) scenario. They can advise its management team on how much the company is worth and how to structure M&A deals.
4. They must create the documentation that must go to the SEC before the company can sell shares. This means compiling financial statements and other documents.

In reality, the task of underwriting securities often falls on more than one bank. In larger offerings, the managing underwriter will often form a **syndicate** (temporary alliance of businesses that manages a large transaction) of other banks that sell a portion of the shares. Some of the largest investment banks are:

1. JPMorgan Chase
2. Goldman Sachs
3. BofA Securities
4. Morgan Stanley
5. Citigroup
6. Deutsche Bank

Example 34. Suppose Acme Water Filter Company hopes to obtain $1 million in an initial public offering. Based on a variety of factors, including the firm's expected earnings over the next few years, Federici Investment Bankers determines that investors will be willing to pay $11 each for 100,000 shares of the company's stock. As the sole underwriter of the issue, Federici buys all the shares at $10 apiece from Acme. If it manages to sell all 100,000 at $11, the bank makes a nice $100,000 profit (100,000 shares x $1 spread).

However, depending on its arrangement with the issuer, Federici may be on the hook if the public's appetite is weaker than expected. If it has to lower the price to an average

of $9 a share to liquidate its holdings, it's lost $100,000. Therefore, pricing securities can be tricky. Investment banks generally have to outbid other institutions who also want to handle the transaction on behalf of the issuer. But if their spread isn't big enough, they won't be able to squeeze a healthy return out of the sale.

Definition 2.1.5 (Secondary Market)**.** The **secondary market** deals with securities that have already been sold in the primary market, which are then bought and sold at later dates. Investors can purchase these bonds from a broker, who acts as an intermediary between the buying and selling parties.

2.2 Stocks

Stocks represent another type of security and are the shares into which the ownership of a corporation is divided. That is, owning a stock is equivalent to owning its relative position in the company and its profits, and possibly **voting rights**. **Dividends** are payments that may be issued to shareholders of a company every quarter.

Definition 2.2.1 (Common, Preferred, Executive Stocks)**.** Both **common stocks** and **preferred stocks** represent an ownership in a company. However, there are some differences between them:

1. Preferred stock gives no voting rights to shareholders while common stocks does.
2. Preferred shareholders have priority over a company's income, meaning that they are paid dividends before common shareholders.
3. Common stock shareholders receive dividends in proportion to the amount of profit generated each year, while preferred stock dividends are fixed. An exception is made if there simply is not enough profit to cover dividend obligations. In that case, all dividend payments are deferred.
4. Common stockholders are last in line when it comes to company assets, which means they will be paid out after creditors, bondholders, and preferred shareholders (upon liquidation).
5. Common stocks are more liquid than preferred stocks.

Executive shares can give the owner priority voting rights, typically multiple votes per share. Companies typically issue these to ensure that the directors and owners retain control of the company even after putting its stock on the public market.

Classification of Shares

Some stocks can have special qualities, which are listed below.

1. **Nonvoting shares**, which the owner has no right to vote in corporate governance.
2. **Deferred shares**, which the owner is set on a lower priority for dividends and corporate assets. If the company pays a dividend but doesn't have enough money to pay all shareholders, deferred shareholders will not receive payment.

3. **Cumulative (preferred) Shares**, which offer the right to accumulate (fixed) dividend payments.
4. **Non-cumulative (preferred) shares**, which offer no back payment of deferred dividend payments
5. **Participating (preferred) shares**, which offer higher-than-normal dividends when profits are higher-than-normal
6. **Convertible (preferred) shares**, which give the option to convert shares into common stock if desired

Definition 2.2.2 (Class A, B, C, F Stocks)**.** Another classification of stocks are often referred to as **classes**, which may be accompanied by more/less voting rights or dividends. Companies can define these classes in almost whatever way they want, but an example would be:

1. **Class A** shares refer to a classification of common stock that is usually accompanied by more voting rights than class B shares. However, there is no legal requirement that companies structure their share classes this way.
2. **Class B** shares can be issued as a class of common stock with fewer voting rights and lower dividend priority than Class A shares.
3. **Class C** shares can refer to preferred stocks.
4. **Class F** shares can refer to executive stocks, having 100 voting rights each.

A detailed description of a company's different stock classes is included in the company's **bylaws** (rules that govern how a company is run) and **charter** (a legal document that formally establishes a corporation, issued by governments, granting it legal rights, privileges, and responsibilities and may define/limit its scope of operations).

Definition 2.2.3 (RSUs)**. Restricted stock units** are a form of stock-based employee compensation. RSUs are restricted during a **vesting period** that may last several years, during which time they cannot be sold. Once **vested**, the RSUs are just like any other shares of company stock. For tax purposes, the entire value of vested RSUs must be included as ordinary income in the year of vesting. RSUs don't have voting rights nor dividends until actual shares get issued to an employee at vesting. If an employee leaves before the conclusion of their vesting schedule, they forfeit the remaining shares to the company.

RSUs give an employee an incentive to stay with a company long term and help it perform well so that their shares increase in value. If an employee decides to hold their shares until they receive the full vested allocation, the employee receives the capital gain minus the value of the shares withheld for income taxes and the amount due in capital gains taxes. RSUs also allow a company to defer issuing shares until the vesting schedule is complete, which helps delay the dilution of its shares.

2.2.1 Stock Issuance

Companies can issue stocks in order to raise capital. In order to do this, we must know what the par value of a stock is.

Definition 2.2.4 (Par Value of a Stock)**.** Corporations are required to specify its stock's **par value** upon its issuance. Par value is the nominal value that a corporation allocates to common stock. Legally, par value denotes the least amount that a corporation can receive for each share of stock it issues. This means that shareholders must pay at the minimum the par value of the stock.

Most corporate founders issue stock with a par value between \$0.001 and \$1, because the government assumes a par value of \$100 when no par value is given (and few corporations can risk a default assessment of \$100 per share of par value).

Definition 2.2.5 (Issuing Shares)**.** Corporate founders can determine the amount of money they need from investors to accomplish company objectives. For example, they may estimate that the corporation will need \$75,000, which will come from equity.

They can determine how many shares they will issue and the share price in order to raise \$75,000. For example, they can issue 10,000 shares at a share price of \$7.50, 25,000 shares at a share price of \$3, or 1,000 shares for a stock price of \$75.

Definition 2.2.6 (Paid-in-Capital, in excess of par)**.** In the balance sheet, corporations must split the amount of capital raised between par value and additional paid-in capital. For example, if a company issued 10,000 shares of stock with a par value of \$0.50 at \$7.50 per share, it would record the transaction as

1. Paid-in-capital: \$5,000
2. Paid-in-capital in excess of par: \$70,000

in the company balance sheet.

Definition 2.2.7 (Stock Buyback)**.** For some reasons, a company can buy back its own stock from the market, which is referred to as a **stock buyback**. These bought shares owned by the company itself are called **treasury stock**, which do not collect dividends and has no voting rights.

Definition 2.2.8. A couple terms to know:

1. **Authorized shares** refers to the maximum number of shares that a corporation is legally permitted to issue.
2. **Outstanding shares** refer to a company's stock currently held by all its shareholders, including shares held by institutional investors and RSUs owned by company insiders. It does not include treasury stocks.
3. **Floating shares** are the shares considered available for the general public. Moreover, the **floating percentage** represents the portion of outstanding shares that are floating.
$$\text{Floating Percentage} = \frac{\text{Floating Shares}}{\text{Shares Outstanding}}$$
4. **Shares issued** refers to the total number of shares issued, including all public shares, private shares, and treasury shares.

This is nicely summarized by the equation

$$\begin{aligned}\text{Shares Issued} &= \big(\text{Floating Shares} + \text{Private Shares}\big) + \text{Treasury Shares} \\ &= \text{Outstanding Shares} + \text{Treasury Shares}\end{aligned}$$

Definition 2.2.9 (Dilution)**.** Additional stock issues **dilute** the ownership of existing shareholders, since the amount of stocks owned by an individual remains the same, while the total number of shares issued increases.

Publicly traded companies often experience a drop in their share prices when they announce additional share issues. If management can invest the additional funds to generate higher profits, share prices could rise, which benefits all shareholders. Dilution also leads to loss of control. This is particularly relevant for small-business owners, because by issuing additional shares, they are effectively giving up some control to the new shareholders.

Definition 2.2.10 (Stock Split, Reverse-Split)**.** A **stock split** is when a company divides the existing shares of its stock into multiple new shares to boost the stock's liquidity (lowering the price allows more people to buy it). Although the number of shares outstanding increases by a specific multiple, the total dollar value of the shares remains the same compared to pre-split amounts, because the split does not add any real value.

Reverse stock splits are effectively the opposite transaction, where a company divides, instead of multiplies, the number of shares that stockholders own, raising the market price accordingly.

2.2.2 Derivatives

Definition 2.2.11 (Derivatives)**.** A derivative is a financial security with a value that is reliant upon or derived from, an underlying asset or group of assets. The derivative itself is a contract between two or more parties, and the derivative derives its price from fluctuations in the underlying asset. Derivatives can be used to hedge a position, speculate on the directional movement of an underlying asset, or give leverage to holdings.

The most common underlying assets for derivatives are:

1. stocks
2. bonds
3. commodities
4. currencies
5. interest rates
6. market indexes

They can be traded either

1. over-the-counter (OTC), which is the process of how securities are traded for companies not listed on a formal exchange (mainly traded via an unregulated dealer network)
2. on a regulated exchange

OTC derivatives constitute a greater proportion of the derivatives market, but exchange-traded derivatives have many advantages, such as standardization, liquidity, and elimination of default risk.

Originally, derivatives were used to ensure balanced exchange rates for goods traded internationally. With the differing values of national currencies, international traders needed a system to account for differences.

Example 35. Imagine a European investor, whose investment accounts are all denominated in euros (EUR). This investor purchases shares of a U.S. company through a U.S. exchange using U.S. dollars (USD). Now the investor is exposed to exchange-rate risk while holding that stock. Exchange-rate risk the threat that the value of the euro will increase in relation to the USD. If the value of the euro rises, any profits the investor realizes upon selling the stock become less valuable when they are converted into euros.

To hedge this risk, the investor could purchase a currency derivative to lock in a specific exchange rate. This kind of derivative is called a currency future or currency swap.

Futures Contracts

Definition 2.2.12. Futures contracts are an agreement between two parties for the purchase and delivery of an asset at an agreed upon price at a future date. Futures trade on an exchange, and the parties involved in the futures transaction are *obligated to fulfill* a commitment to buy or sell the underlying asset. This means that the purchase *must* be made, regardless of the current market price at the expiration date. The amount you pay for the actual futures contract is called the **premium**.

Example 36. Say that a trader wants to speculate on the price of crude oil by entering into a futures contract in May with the expectation that the price will be higher by year-end. The December crude oil futures contract is trading at $50 and the trader locks in the contract.

Since oil is traded in increments of 1,000 barrels, the investor now has a position worth $50,000 of crude oil (1,000 x $50 = $50,000). However, the trader will only need to pay a fraction of that amount up-front—the initial margin that they deposit with the broker.

From May to December, the price of oil fluctuates as does the value of the futures contract. If oil's price gets too volatile, the broker may ask for additional funds to be deposited into the margin account—a **maintenance margin**.

In December, the end date of the contract is approaching, which is on the third Friday of the month. The price of crude oil has risen to $65, and the trader sells the original contract to exit the position. The net difference is cash-settled, and they earn $15,000, less any fees and commissions from the broker ($65 - $50 = $15 x 1000 = $15,000). However, if the price oil had fallen to $40 instead, the investor would have lost $10,000 ($40 - $50 = negative $10 x 1000 = negative $10,000).

Example 37 (Oil Futures). Say that on Nov. 6, 2019, Company A buys a futures contract for oil at a price of $62.22 per barrel that expires Dec. 19, 2019. The company does this because it needs oil in December and is concerned that the price will rise before the company needs to buy. Buying an oil futures contract hedges the company's risk because

the seller on the other side of the contract is obligated to deliver oil to Company-A for $62.22 per barrel once the contract has expired. Assume oil prices rise to $80 per barrel by Dec. 19, 2019. Company-A can accept delivery of the oil from the seller of the futures contract, but if it no longer needs the oil, it can also sell the contract before expiration and keep the profits.

In this example, it is possible that both the futures buyer and seller were hedging risk. Company-A needed oil in the future and wanted to offset the risk that the price may rise in December with a long position in an oil futures contract. The seller could be an oil company that was concerned about falling oil prices and wanted to lock in a deal.

For example, the futures contract for West Texas Intermediate (WTI) oil trades on the CME represents 1,000 barrels of oil. If the price of oil rose from $62.22 to $80 per barrel, the trader with the long position—the buyer—in the futures contract would have profited $17,780 [($80 − $62.22) × 1000 = $17,780]. The trader with the short position—the seller—in the contract would have a loss of $17,780.

Not all futures contracts are settled at expiration by delivering the underlying asset. Many derivatives are cash-settled, which means that the gain or loss in the trade is simply an accounting cash flow to the trader's brokerage account.

Example 38 (Stock Futures). Suppose individual A believes that the stock price for AMC Entertainment is going to go up and buys a futures contract of 100 shares for $3 per share which will expire in a month. He will need to pay a small portion of the contract as an initial payment now, but when a month passes, he pays the total $300. If the price of AMC is $7, then he can immediately sell the 100 shares he bought to make a profit of $4 × 100 = $400. If the price of AMC is lower, then he has a net loss.

Swaps

Definition 2.2.13. Swaps is a derivative contract through which two parties exchange the cash flows or liabilities from two different financial instruments. One cash flow is generally fixed, while the other is variable and is based on a benchmark interest rate, floating currency exchange rate, or index price. The most common type of swap is an **interest rate swap**.

Swaps don't trade on exchanges, and retail investors do not generally engage in swaps. Rather, swaps are over the counter contracts primarily between businesses that are customized to the needs of both parties.

Example 39 (Interest Rate Swap). A trader might use an interest rate swap to switch from a variable interest rate loan to a fixed interest rate loan, or vice versa.

Imagine that Company XYZ has borrowed $1,000,000 and pays a variable rate of interest on the loan that is currently 6%. XYZ may be concerned about rising interest rates that will increase the costs of this loan or encounter a lender that is reluctant to extend more credit while the company has this variable rate risk.

Assume that XYZ creates a swap with Company QRS, which is willing to exchange the payments owed on the variable rate loan for the payments owed on a fixed rate loan of 7%. That means that XYZ will pay 7% to QRS on its $1,000,000 principal, and QRS will

pay XYZ 6% interest on the same principal. At the beginning of the swap, XYZ will just pay QRS the 1% difference between the two swap rates.

If interest rates fall so that the variable rate on the original loan is now 5%, Company XYZ will have to pay Company QRS the 2% difference on the loan. If interest rates rise to 8%, then QRS would have to pay XYZ the 1% difference between the two swap rates. Regardless of how interest rates change, the swap has achieved XYZ's original objective of turning a variable rate loan into a fixed rate loan.

Some examples of swaps can be about:

1. interest rates
2. mortgage bonds
3. exchange rates

Options Contracts

Definition 2.2.14 (Options Contracts)**.** An **options contract** is similar to a futures contract in that it is an agreement between two parties to buy or sell an asset at a predetermined future date for a specific price. The key difference between options and future is that, with an option, the buyer is not obliged to exercise their agreement to buy or sell. It is an opportunity only, not an obligation. The amount you pay for the actual option contract is called the **premium**. There are two types of options:

1. A **put option** is a contract that gives the right to sell an asset at a specific price. It is typically a bearish bet on the market, since you can sell it at a higher price when the price goes down.
2. A **call option** is a contract that gives the right to buy an asset at a specific price. It is typically a bullish bet on the market, since you can buy it at a lower price when the price goes up.

Note that for stock options, a single contract covers 100 shares of the underlying stock.

Example 40 (Company Options). Company ABC's shares trade at $60, and a call writer is looking to sell calls at $65 with a one-month expiration. If the share price stays below $65 and the options expire, the call writer keeps the shares and can collect another premium by writing calls again.

If the share price appreciates to a price above $65, referred to as being in-the-money, the buyer calls the shares from the seller, purchasing them at $65. The call-buyer can also sell the options if purchasing the shares is not the desired outcome.

Example 41 (Stock Put Options). If you wanted to buy a put option on Intel stock at a strike price of $48 per share, expecting the stock to go down in price in six months to sit around at $45 or $46, you could make a decent profit by exercising your put option and selling those shares at a higher price if the market price goes down.

Example 42 (Stock Put Options). If an investor owns a put option to sell XYZ at $100, and XYZ's price falls to $80 before the option expires, the investor will gain $20 per share,

minus the cost of the premium. If the price of XYZ is above $100 at expiration, the option is worthless and the investor loses the premium paid upfront.

Example 43 (Stock Call Options). Suppose you buy a call option contract of 100 shares of AMC to be bought at $3 per share, expiring in 6 months, at a premium of $100. Within the next 6 months, if the price of $AMC jumps to $10, then you can exercise the call option to get 100 AMC shares at $3 per share, even though the market price is $10 per share. You can immediately sell these shares to get a profit of $\$7 \times 100 = \$700$, minus the premium price of $100, for a final profit of $600.

Example 44 (Stock Call Options). For example, an investor opens a call option to buy stock XYZ at a $50 strike price sometime within the next three months. The stock is currently trading at $49. If the stock jumps to $60, the call buyer can exercise the right to buy the stock at $50. That buyer can then immediately sell the stock for $60 for a $10 profit per share.

Alternatively, the option buyer can simply sell the call and pocket the profit, since the call option is worth $10 per share. If the option is trading below $50 at the time the contract expires, the option is worthless. The call buyer loses the upfront payment for the option, called the premium.

2.3 Stock Analysis

There are generally two types of analysis that investors use when trading stocks:

1. Fundamental, or Value, Analysis, which focuses on the fundamental health of the company to buy stocks.
2. Technical Analysis, which focuses on the price movements of the stocks.

Technical analysis is inherently riskier, but it has potential to bring greater gains in shorter terms. It is used by well known investors like Jim Simons of Renaissance Technologies LLC, while fundamental investing is used by Benjamin Graham and Warren Buffet. Both have been exploited successfully, but the general public may be better off with value investing.

Most of the tools introduced in this section is about technical analysis. The main tools of fundamental analysis can be found in the accounting chapter.

Definition 2.3.1 (Support, Resistance Levels)**.**

Moving Averages

A **n-day moving average** of a stock price is the average price of the stock over the past n days (commonly $n = 10, 20, 50, 100, 200$). It smooths out price trends by filtering out the "noise" from random short-term price fluctuations. Generally speaking,

1. If the stock price is above a moving average, the trend is up.
2. If the stock price is below a moving average, the trend is down.

This is because a moving average can act as a support or resistance, but this does not work every time; it is just a general rule of thumb.

Definition 2.3.2 (Simple Moving Average)**.** An **n-day simple moving average (SMA)** averages the n most recently daily closing prices to create a new average each day. Each average is connected to the next, creating the singular flowing line.

Definition 2.3.3 (Exponential Moving Average)**.** The **exponential moving average (EMA)** applies more weighting to the most recent prices. This means that the EMA reacts more quickly to price changes than the SMA does.

Note that the moving average length plays an important part to the investor. An MA with a short time frame will react much quicker to price changes than an MA with a long lookback period, resulting in less lag. Therefore, a 20-day MA may be more beneficial to a short-term trader, while a 100-day MA is better suited to the long-term trader. However, a 20-day MA is more susceptible to noise, meaning that it may produce more reversal signals.

Definition 2.3.4 (Trading Strategy: Crossover)**.**

Technical Analysis Indicators

2.4 Bonds

Definition 2.4.1 (Bond)**. Bonds** are units of corporate debt issued by companies and securitized as tradeable assets. It is a **fixed income instrument** since bonds are traditionally paid a fixed interest rate (coupon) to debtholders).

When investors buy bonds, they essentially lend bond issuers money. In return, bond issuers agree to pay investors interest on bonds through the life of the bond and to repay the face value of bonds upon maturity. They initial price of most bonds is typically set **at par** (meaning set at nominal, or face value), usually \$100 or \$1000 face value per individual bond. A bond has the following components

1. the **face value**, which is the money amount the bond will be worth at maturity
2. the **coupon rate**, i.e. the rate of interest the bond issuer will pay on the face value of the bond, e.g. a rate of 5%. We can view the coupon rate as

$$\text{Coupon Rate} = \frac{\text{Annual Coupon Payment}}{\text{Bond Face Value}}$$

3. **coupon dates**, which are the dates on which the bond issuer will make interest payments. Payments can be made in any interval, but the standard is in semiannual payments.
4. the **maturity date**, which is when the bond will mature and the bond issuer will pay the bondholder the face value of the bond
5. the **issue price**, which is the price at which the bond issuer originally sells the bonds

When an investor buys a bond at face value of $1000 from a corporation/government at a semiannual interest rate of 3% with a maturity date of 30 years, then every 6 months, the bondholder will receive $30 for 30 years. After the 30 years, the bond will mature and the investor will receive the face value of the bond back. Therefore, the bondholder will have $2,800.

Definition 2.4.2 (Categories of Bonds)**.** There are four primary categories of bonds sold in the markets, excluding foreign bonds.

1. **Corporate bonds** are issued by companies. Companies issue bonds rather than seek bank loans for debt financing in many cases because bond markets offer more favorable terms and lower interest rates.
2. **Municipal bonds** are issued by states and municipalities (local governments). Some municipal bonds offer tax-free coupon income for investors.
3. **Government bonds**, or sometimes called **treasuries**, are issued by the U.S. Treasury. Government bonds issued by national governments may be referred to as sovereign debt.
   (a) Those with a year or less to maturity are called **Bills**
   (b) Those with 1-10 years to maturity are called **Notes**
   (c) Those with more than 10 years to maturity are called **Bonds**
4. **Agency bonds** are those issued by government-affiliated organizations (such as Fannie Mae or Freddie Mac).

Definition 2.4.3 (Varieties of Bonds)**.** The bonds available for investors come in different varieties, separated by the rate or type of interest or coupon payment, being recalled by the issuer, or other attributes.

1. **Zero-coupon bonds** do not pay coupon payments and instead are issued at a discount to their par value that will generate a return once the bondholder is paid the full face value when the bond matures. U.S. Treasury bills are a zero-coupon bond.
2. **Convertible bonds** are debt instruments with an option that allows bondholders to convert their debt into stock (equity) at some point, depending on certain conditions like the share price.

   For example, imagine a company that needs to borrow $1 million to fund a new project. They could borrow by issuing bonds with a 12% coupon that matures in 10 years. However, if they knew that there were some investors willing to buy bonds with an 8% coupon that allowed them to convert the bond into stock if the stock's price rose above a certain value, they might prefer to issue those. The convertible bond may be the best solution for the company because they would have lower interest payments while the project was in its early stages. If the investors converted their bonds, the other shareholders would be diluted, but the company would not have to pay any more interest or the principal of the bond.
3. **Reverse convertible bonds** are like convertible bonds, but now the power to convert debts into stock lies in the hands of the bond issuers. Issuers generally exercise

the reverse convertible bond's option if the underlying shares have fallen below a set price, often referred to as the knock-in level, in which case the bondholders will receive the stock rather than the principal and any additional coupons. Their yields are generally higher, though.

Say XYZ bank issues a reverse convertible bond on the bank's own debt with a built-in put option on the shares of ABC Corp., a blue chip company. The bond may have a stated yield of 10% to 20%, but if the shares in ABC decrease substantially in value, the bank holds the right to issue the blue-chip shares to the bondholder, instead of paying cash at the bond's maturity.

4. **Callable bonds** is one that can be "called" back by the company before it matures.

   Assume that a company has borrowed $1 million by issuing bonds with a 10% coupon that mature in 10 years. If interest rates decline (or the company's credit rating improves) in year 5 when the company could borrow for 8%, they will call or buy the bonds back from the bondholders for the principal amount and reissue new bonds at a lower coupon rate.

5. **Puttable bonds** allows the bondholders to put or sell the bond back to the company before it has matured. This is valuable for investors who are worried that a bond may fall in value, or if they think interest rates will rise and they want to get their principal back before the bond falls in value.

2.4.1 The Bond Market

Bond Pricing and Interest Rates

As hinted by the title of this section, a bond does not need to be held until maturity. A bondholder can sell their bonds in the open market, where the price can fluctuate. When talking about interest rates, we can set a lot of standards, but usually the standard is to look at the interest rates of a 10-year U.S. treasury bond.

Definition 2.4.4 (Premium and Discount)**.** If a security is trading for a price higher or lower than its face value, it trades at a **premium** or **discount**, respectively.

Definition 2.4.5 (Current Yield)**.** The **current yield of a bond** is defined

$$\text{Current Yield} = \frac{\text{Annual Cash Inflows}}{\text{Market Price}}$$

Since the annual coupon payments are fixed (since it is a fixed percentage of the principal value), the current yield is dependent on whether the bond had been purchased at a premium (lower yield) or discount (higher yield).

The price of a (fixed rate) bond changes in response to changes in the interest rates in the economy. To see why, observe. Say that you have bought a corporate bond at face value of $1000 with an annual coupon rate of 10%. So, the issuer will pay the bondholder $100 each year. Say that prevailing interest rates are also 10% at the time that this bond is issued, as determined by the rate on a short-term government bond. An investor would be indifferent investing in the corporate bond or the government bond since both would return $100.

	Price	Annual Return (Fixed)	Current Yield
Corporate B.	\$1000	\$100	10%
Treasury B.	\$1000	\$100	10%

Imagine that the economy has taken a turn for the worse and interest rates dropped to 5%. Now, the investor can receive \$50 from the government bond, but would still receive \$100 from the corporate bond.

	Price	Annual Return (Fixed)	Current Yield
Corporate B.	\$1000	\$100	10%
Treasury B.	\$1000	\$50	5%

The difference makes the corporate bond much more attractive, increasing its demand. Market forces will bit up the price of the bond until it trades at a premium that **equalizes the prevailing interest rate environment**. That is, in this case, the bond will trade at a price of \$2000 so that the \$100 coupon represents 5%.

	Price	Annual Return (Fixed)	Current Yield
Corporate B.	\$2000	\$100	5%
Treasury B.	\$1000	\$50	5%

Likewise, if interest rates soared to 15%, then an investor could make \$150 from the government bond and would not pay \$1000 to earn just \$150, decreasing the demand of the corporate bond. The bond will be sold until it reached a price that equalized the yields, in this case to a price of \$666.67.

	Price	Annual Return (Fixed)	Current Yield
Corporate B.	\$666.67	\$100	15%
Treasury B.	\$1000	\$150	15%

Therefore, we can see that **a bond's price varies inversely with interest rates works**. When interest rates go up, bond prices fall in order to have the effect of equalizing the current yield on the bond with prevailing rates, and vice versa.

2.4.2 Yield to Maturity (YTM)

Yield to maturity is the total return anticipated on a bond if the bond is held until it matures, expressed as an annual rate. It is similar to current yield, which divides annual cash inflows from a bond by the market price of that bond to determine how much money one would make by buying a bond and holding it for one year.

Unlike current yield, YTM accounts for the **present value** of a bond's future coupon payments (using discounted cash flow analysis). That is, it factors in the time value of money, whereas a simple current yield calculation does not, making it a more thorough means of calculating.

Definition 2.4.6 (YTM of a Discount Bond)**.** The YTM of a zero-coupon bond with n years to maturity (that is issued at the discounted price) gives us the simplest formula:

$$\text{YTM} = \sqrt[n]{\frac{\text{Face Value}}{\text{Current Price}}} - 1$$

Note that this could be written in the form

$$\text{Face Value} = \text{Current Price} \times \big(1 + \text{YTM}\big)^n \iff \text{Current Price} = \frac{\text{Face Value}}{(1 + \text{YTM})^n}$$

That is, the YTM is a constant interest rate that represents what an investor would earn at the time of maturity.

YTM is the interest rate an investor would earn by reinvesting every coupon payment from the bond at a constant interest rate until the bond's maturity date. This means that the present value of all the future cash flows equals to the bond's market price. That is, the YTM must satisfy

$$\text{Bond Price} = \frac{\text{Coupon 1}}{(1 + \text{YTM})^1} + \frac{\text{Coupon 2}}{(1 + \text{YTM})^2} + \ldots + \frac{\text{Coupon n}}{(1 + \text{YTM})^n} + \frac{\text{Face Value}}{(1 + \text{YTM})^n}$$

Since the coupon payments are usually the same, we can solve this with the formula

$$\text{Bond Price} = \left(\text{Coupon} \times \frac{1 - \frac{1}{(1+\text{YTM})^n}}{\text{YTM}}\right) + \left(\text{Face Value} \times \frac{1}{(1 + \text{YTM})^n}\right)$$

This formula is consistent with the one above for the zero-coupon bond, since the Bond price is just equal to the face value discounted for n years by the YTM.

For example say that we have a bond with \$1000 face value and 10% interest rate which matures in 3 years. If it is currently priced at par (at \$1000), then we can see that by substituting YTM= 10%,

$$\$1000 = \frac{\$100}{1 + 0.1} + \frac{\$100}{(1 + 0.1)^2} + \frac{\$100}{(1 + 0.1)^3} + \frac{\$1000}{(1 + 0.1)^3}$$

Note that this balance with the YTM being exactly the interest rate and the current price being the face value is not a coincidence. That is, if we took the present value of all the future cash flows coming from the coupon payments and the principal payment discounted at the rate of the interest, it would be exactly the face value of the bond itself.

If the bond is priced at a discount, then we would require a higher YTM to "balance" the equation. For example, YTM must be 14.331% if the current price is \$900.

$$\$900 = \frac{\$100}{1 + 0.14331} + \frac{\$100}{(1 + 0.14331)^2} + \frac{\$100}{(1 + 0.14331)^3} + \frac{\$1000}{(1 + 0.14331)^3}$$

This means that rather than requiring a 10% discount rate, we need a higher discount rate (YTM) such that the present value of all future cash inflows equals \$900. In other words, if we buy this bond and reinvest all coupon payments at the same rate as the bond's current yield (\$100/\$900 = 11.11%), then we would get an average annual return of 14.331%, which is better than 10%.

If the same bond is priced at a premium of \$1100, then we would require a lower YTM of 6.242%.

$$\$1100 = \frac{\$100}{1 + 0.06242} + \frac{\$100}{(1 + 0.06242)^2} + \frac{\$100}{(1 + 0.06242)^3} + \frac{\$1000}{(1 + 0.06242)^3}$$

Therefore, we need a lower discount rate than 10% such that the present value of all future cash inflows equals \$1100. That is, if we buy this bond and reinvest all coupon payments at the same rate as the bond's current yield (\$100/\$1100 = 9.11%), then we would get an average annual return of 6.242%, which is worse than 10%.

Therefore,

1. Bond priced at discount $\implies$ YTM > Interest Rate
2. Bond priced at premium $\implies$ YTM < Interest Rate

2.5 Other Financial Assets

Definition 2.5.1 (Alternative Investment)**.** An **alternative investment** is a financial asset that does not fall into one of the conventional investment categories. Conventional categories include stocks, bonds, and cash. Alternative investments include private equity, venture capital, hedge funds, managed futures, art and antiques, commodities, derivatives contracts, and real estate.

2.5.1 Loan-Backed Securities

Definition 2.5.2 (Securitization)**.** Assets can be created by pooling individual loans and selling shares in that pool, a process called **securitization**. Securitization has been applied to mortgages, student loans, credit card loans, and auto loans (up to thousands of them in a single security). But with so many loans packaged together, it can be difficult to assess the true quality of the asset.

Definition 2.5.3 (MBS)**. Mortgage backed securities (MBS)** is an investment similar to a bond that is made up of a bundle of home loans bought from the banks that issued them. Investors in MBS receive periodic payments similar to bond coupon payments. The process is as such:

1. A bank (the "Bank") pays for an individual's (the "Individual") home and receives regular payments from the Individual as a mortgage with a fixed interest rate.
2. The Bank pools together mortgages exclusively, which are debts, and sells them at a discount to be packaged as a mortgage-backed security to an investor (the "Investor"), through possibly a broker. This MBS is a type of collateralized bond.
3. The Investor now owns the debt security, and an MBS is as safe as the mortgage loans that back it up. The bank takes no risk nor damages if the Individual defaults, since the situation is essentially equivalent to the Investor paying for the Individual's home.

Essentially, the mortgage-backed security turns the bank into an intermediary between the Individual and the Investor. A bank can grant mortgages to its customers and then sell them at a discount for inclusion in an MBS. The bank records the sale as a plus on its balance sheet and loses nothing if the homebuyer defaults sometime down the road.

In order to be sold on the markets today, an MBS must have received one of the two top ratings issued by an accredited credit rating agency.

2.5.2 Funds

Definition 2.5.4 (Funds)**.** A **fund** is simply a pool of money that is allocated for a specific purpose. Funds can be established for any purpose whatsoever. We can generally divide funds into three categories:

1. Individual funds
2. Investment funds
3. Government funds

Individual Funds

Definition 2.5.5 (Emergency Funds)**. Emergency funds** are personal savings vehicles created by individuals used to cover period of financial hardships.

Definition 2.5.6 (College Funds)**. College funds** are usually tax-advantaged savings plans set up by families to allocate funds for their children's college expenses.

Definition 2.5.7 (Trust Funds)**. Trust funds** are legal arrangement set up by a **grantor** who appoints a third-party **trustee** (usually a trust bank) to administer valuable assets for the benefit of a listed beneficiary for a period of time, after which all or a portion of the funds are released to the beneficiary.

The primary motivation for establishing a trust fund is for an individual to create a vehicle that sets terms for the way assets are to be held, gathered, or distributed in the future. Generally, the grantor is creating an arrangement that, for a variety of reasons, is carried out after they are no longer mentally competent or alive. The fund can contain nearly any asset imaginable, such as cash, stocks, bonds, property, etc.

Some types of trust funds include:

1. A **revocable/living trust** lets a grantor better control assets during the grantor's lifetime. The grantor places assets into a trust that can then transfer to any number of designated beneficiaries after the grantor's death. Living trusts are not made public, meaning an estate is distributed with a high level of privacy. While the grantor is still living and not incapacitated, the trust details can be changed or revoked.
2. An **irrecovable trust** is very difficult to change or revoke, but there can be considerable tax benefits for the grantor to effectively give away control of the assets to the trust funds.
3. An **asset protection trust (APT)** is created to protect a person's assets from claims of future creditors.
4. A **blind trust** is created so the beneficiary is not aware of who holds power of attorney for the trust (generally the trustee).

Definition 2.5.8 (Retirement Funds)**. Retirement funds** are savings vehicles used by individuals saving for retirement. Retirees receive monthly income or pensions from retirement funds.

Investment Funds

Definition 2.5.9 (Mutual Funds)**. Mutual funds** are investment funds managed by professional managers who allocate the funds received from individual investors into stocks, bonds, and/or other assets. These funds are managed through stock picking and market timing (that is, choosing securities to invest in and strategizing when to buy and sell them). This means that maintenance fees can be high.

Definition 2.5.10 (Mutual Fund Family)**.** A **family of mutual funds** is a group of funds that are marketed under one or more brand names, usually having the same distributor (the company which handles selling and redeeming shares of the fund in transactions with investors). Some examples of fund families and their **assets under management (AUM)** are:

Fund Family	AUM	Fund Name
BlackRock	$8.676 trillion	iShares
Vanguard	$7.1 trillion	Vanguard
Fidelity	$4.9 trillion	Fidelity
State Street Global Advisors (SSGA)	$3.5 trillion	SPDR
Charles Schwab	$3.25 trillion	Schwab
Goldman Sachs	$2.1 trillion	GS
Invesco	$1.349 trillion	Powershares
Wilshire Associates	$1 trillion	Wilshire
T. Rowe Price	$991.1 billion	T. Rowe Price
Morgan Stanley	$715 billion	MS
AllianceBernstein	$697 billion	AB
WisdomTree Investments	$41.8 billion	WisdomTree

These companies manage these funds.

Definition 2.5.11 (Management Expense Ratio)**.** The **management expense ratio (MER)** is a measure of a mutual fund's operating costs relative to assets. It is calculated:

$$\text{Expense Ratio} = \frac{\text{Total Fund Costs}}{\text{Total Fund Assets}}$$

The MER refers to how much of a fund's assets are used towards administrative and other operating expenses. It is important because it lets an investor know how much they are paying in costs by investing in a specific fund and how much their returns will be reduced by. The lower the expense ratio the better because it means that an investor is receiving higher returns on their invested capital.

Operating expenses can vary depending on the fund, but the expenses within the fund remain relatively stable. The biggest portion of fund expenses is the fee paid to a fund's investment manager, but other costs include recordkeeping, custodial services, taxes, legal expenses, and accounting/auditing fees. Generally,

1. Funds investing in large companies should not have an MER over 1%
2. Funds investing in smaller companies should not have an MER over 1.25%

Any funds with higher MERs are either really expensive or should have a reason that justify their costs.

Example 45 (Vanguard S&P500 ETF MER). The *Vanguard S&P500 ETF* has one of the lowest expense ratios in the industry at 0.03% annually; investors are charged just $3 per year for every $10,000 invested. The *Fidelity Contrafund* is one of the largest actively managed funds in the marketplace with an expense ratio of 0.86%, or $86 per $10,000 per year.

Example 46 (MERs of Index vs Active Funds). Due to the lower management level of index funds, the expense ratios of index funds would be significantly smaller than those of mutual ones. This is indeed the case.

1. The *AB Large Cap Growth Fund* is an actively managed fund with a net MER of 0.61%. It's management fee is 0.50%, meaning that 0.11% of the total fund assets goes to non-management expenses.
2. The *T. Rowe Price Equity Index 500 Fund* is a passive fund designed to replicate the S&P500. It's net MER is 0.19%. It's management fee of 0.06%, meaning that 0.13% of the total fund assets goes to non-management expenses.

Definition 2.5.12 (Money-Market Funds)**. Money-market funds** are highly liquid mutual funds purchased to earn interest for investors through short-term interest-bearing securities such as Treasury bills and commercial paper.

Definition 2.5.13 (Exchange-Traded Funds)**. Exchange Traded Funds (ETFs)** are similar to mutual funds but traded on the public exchanges like stocks.

Example 47 (S&P500 Funds). There are many funds whose portfolio of stocks are designed to track those of the S&P 500 due to its popularity as a barometer of U.S. equity markets, including both mutual funds and exchange-traded funds (ETFs). Different fund families offer different funds that mimick the S&P500 differently. They include:

1. Fidelity 500 Index Fund (FXAIX)
2. Schwab S&P500 Index Fund (SWPPX)
3. Vanguard 500 Index Fund Investor Shares (VFINX)
4. State Street S&P500 Index Fund Class N (SVSPX)
5. SPDR S&P500 ETF (SPY)

The SPDR funds are also managed by State Street Global Advisors.

Definition 2.5.14 (Index Funds)**.** An **index fund** is a type of mutual fund or an ETF with a portfolio constructed to match or track the components of a financial market index, such as the S&P500 Index. It is said to provide broad market exposure, low operating expenses, and low portfolio turnover. Index funds seek to match the risk and return of the market, on the theory that in the long-term, the market will outperform any single investment. The most popular index funds track the S&P500, and other examples include

1. Vanguard 500, made to track the S&P500 faithfully, in composition and performance.
2. Russell 2000, made up of small-cap company stocks
3. Wilshire 5000 Total Market Index, the largest U.S. equities index
4. MSCI EAFE, consisting of foreign stocks from Europe, Australasia, and the Far East
5. Bloomberg Barclays U.S. Aggregate Bond Index, which follows the total bond market
6. Nasdaq Composite, made up of 3000 stocks listed on the Nasdaq exchange
7. Dow Jones Industrial Average (DJIA) consisting of 30 large-cap companies

Definition 2.5.15 (Hedge Funds)**. Hedge funds** are investment vehicles for high-net-worth individuals or institutions designed to increase the return on investors' pooled funds by incorporating high-risk strategies such as short selling, derivatives, and leverage. A few things to note:

1. They are more expensive compared to conventional investment funds, and will often restrict investment to high-net-worth or sophisticated investors.
2. The number of hedge funds throughout the world had exploded, with approximately 11,000 currently and 5500 in North America.
3. Even though they were successful in the 1990s and early 2000s, many have underperformed (especially after fees and taxes).
4. They are less liquid, since they require a minimum commitment of 1 year, along with specific time periods when money can be withdrawn (e.g. quarterly or biannually).
5. Rather than charging an expense ratio only, hedge funds charge both an expense ratio and a performance fee, known as the *Two and Twenty*: a 2% asset management fee and then a 20% cut of any gains generated.
6. They have much less regulations than all other funds, so they can do pretty much do what they want as long as they disclose the strategy upfront to investors.

It's ironic because the word "hedging" is actually the practice of attempting to reduce risk, but the goal of most hedge funds is to maximize return on investment.

Definition 2.5.16 (Government Bond Funds)**.** Government bond funds are for investors looking to put their money away in low-risk investments through Treasury securities, such as Treasury bonds, or agency-issued debt. Both alternatives are backed by the U.S. government.

Government Funds

Definition 2.5.17 (Debt-Service Funds)**. Debt-service funds** are allocated to repay the government's debt.

Definition 2.5.18 (Capital Projects Funds)**. Capital projects fund** resources are used to finance the capital projects of a country, such as purchasing, building, or renovating equipment, structures, and other capital assets.

Definition 2.5.19 (Permanent Funds)**. Permanent funds** are investments and other resources that the government is not allowed to cash or spend. However, the government normally has the right to spend any revenue these investments generate on appropriate functions of government.

2.5.3 Retirement Plans

2.6 Private Equity

Definition 2.6.1 (Types of Investors)**.** There are three types of investors:

1. A **retail investor**, also known as an **individual investor**, is a non-professional investor who generally invests smaller amounts than larger, institutional investors. The retail investment market is enormous since it includes retirement accounts, brokerage firms, online trading, and robo-advisors.
2. **Accredited investors** refer to investors who are financially sophisticated and have a reduced need for the protection provided by regulatory disclosure filings. They are allowed to buy and invest in unregistered securities as long as they satisfy requirements regarding income, net worth, asset size, governance status, or professional experience.
3. An **institutional investor** is a company or organization that buys, sells, and manages investment securities on behalf of its clients, members, or shareholders. They are considered the whales of Wall Street and are subject to less regulatory oversight. Broadly speaking, there are six types of institutional investors:
   (a) Endowment funds
   (b) Commercial banks
   (c) Mutual funds
   (d) Hedge funds
   (e) Pension funds
   (f) Insurance companies

   Buying and selling of large positions by institutional investors can create supply and demand imbalances that result in sudden price moves in stocks, bonds, or other assets.

Definition 2.6.2 (Private Equity)**. Private Equity** is an alternative investment class that consists of capital that is not listed on a public exchange. It is composed of funds and investors that directly invest in private companies. Private equity investment comes primarily from institutional and accredited investors, who can dedicate substantial sums of money for extended time periods. The process for investment is as such:

1. A **private equity firm**, also known as the **general partner (GP)**, gathers investors, known as the **limited partners (LP)**, who would like to invest.
2. The firm then invests it in some private business or other investment vehicle. This investment is known as the **private equity fund**.
3. Note that the private equity fund is really owned by the limited partners, but it managed by the general partner. In fact, the LP typically own 99% of shares in the fund, while the GP owns 1%.
4. Unlike public investments, holding periods for private equity investments last much longer, usually 10+ years. This is to ensure a turnaround for distressed companies or to enable liquidity events such as an IPO or sale to a public company.

Some advantages and disadvantages of private equity include:

1. Advantage: It is favored by companies because it allows them access to liquidity as an alternative to conventional financial mechanisms, such as high interest bank loans or listing on public markets.
2. Advantage: For private companies, private equity financing can help them attempt unorthodox growth strategies away from the glare of public markets.
3. Advantage: The pressure of quarterly earnings dramatically reduces the time frame available to senior management to turn a company around or experiment with new ways to cut losses or make money.
4. Disadvantage: It can be difficult to liquidate holdings in private equity because, unlike public markets, a ready-made order book that matches buyers with sellers is not available. A firm has to undertake a search for a buyer in order to make a sale of its investment.
5. Disadvantage: Pricing of shares for a company in private equity is determined through negotiations between buyers and sellers and not by market forces.
6. Disadvantage: The rights of private equity shareholders are generally decided on a case-by-case basis through negotiations instead of a broad governance framework.

2.6.1 Buyouts

Definition 2.6.3 (Controlling Interest)**.** A **controlling interest** is when a shareholder, or a group acting in kind, holds a majority of a company's *voting* stock (at least 50%), giving it significant influence over any corporate actions. Note that this mainly means common stock, not preferred ones.

Definition 2.6.4 (Proxy Fight)**.** One way to gain a controlling interest is through a **proxy fight**, which is the act of a group of shareholders joining forces and attempting to gather enough shareholder proxy votes to win a corporate vote. This could result in replacing corporate management or the board of directors.

Definition 2.6.5 (Buyout)**.** A **buyout** is the acquisition of a controlling interest in a company. That is, the company's shares are bought by an entity to gain a controlling interest in that company.

1. If the stake is bought by the firm's management, it is known as a **management buyout (MBO)**.
2. A **friendly buyout/takeover** is a scenario in which a target company is willingly acquired by another company. Such action is subject to approval by both the target company's shareholders and the U.S. Department of Justice. In situations where the DOJ fails to grant approval, it's typically because the deal violates antitrust (anti-monopoly) laws.

   In a friendly takeover, a public offer of stock/cash is made by the acquiring firm. The board of the target firm will publicly approve the buyout terms, which must then be approved by shareholders (this is usually not a problem is the board agrees to it) and regulators. The acquiring company usually strives to extend fair buyout terms, offering to buy shares at a premium to the current market price.
3. A **hostile takeover** is when the target company is unwillingly acquired by another company. This is accomplished by directly going to the company's shareholders or fighting to replace management to get the acquisition approved. It can be done through either a tender offer or a proxy fight.

Definition 2.6.6 (Leveraged Buyouts)**.** Many buyouts are performed as **leveraged buyouts (LBOs)**. Usually, the acquiring company may not want to commit too much capital to buy the target company. So, the acquiring company partners up with a private equity firm, which raises funds that covers 50% to 90% of the total cost. These funds can come in multiple ways:

1. Bank Financing: The sponsor uses borrowed funds from a bank or a group of banks, called a syndicate.
2. Private Placement of Bonds: Bonds are offered through a private placement (an offering of debt instruments to pre-selected investors). Investors pay cash upfront for the face value of the bond and in return, get paid, an interest rate until the maturity date of the bond.
3. Subordinated/Mezzanine Debt: This is a method of obtaining funding without offering collateral, but often requires a higher interest rate or other benefits.

The private equity sponsor loans these funds to the acquiring company (with an interest rate, of course), the acquiring company buys the target company, and the acquisition is complete.

Notice that this is a high-risk high-reward strategy, where the acquisition has to realize high returns and cash flows in order to pay the interest on the debt. The target company's assets are typically provided as collateral for the debt, and buyout firms sometimes sell parts of the target company to pay down the debt. In come cases, the acquiring company is also provided as additional collateral. Therefore, in the case where the acquiring company defaults, the private equity firm would get the target company and possibly the acquiring company.

Firms that specialize in funding and facilitating buyouts (**buyout firms**), act alone or together on deals, and are usually financed by institutional investors, wealthy individuals, or loans.

Going Private

The term **going private** refers to a transaction of series of transactions that convert a publicly traded company to a private entity. Once a company goes private, its shareholders are no longer able to trade their shares in the open market. The three most common types of going private transactions are:

1. private equity leveraged buyouts
2. management buyouts
3. tender offers

Definition 2.6.7 (Tender Offer)**.** A **tender offer** occurs when a company or individual makes a public offer to buy most or all of a company's shares. This is another type of buyout and may or may not be leveraged.

Sometimes, tender offers are made (and accepted) even when the current management team of the target company does not want the company to be sold. In this situation, this would be a hostile takeover.

2.6.2 Venture Capital

Definition 2.6.8 (Venture Capital, VC Rounds)**. Venture capital (VC)** is a form of private equity financing that is provided by venture capital firms or funds to startups and early-stage companies that have been deemed to have high long-term growth potential (over decades). Venture capital firms or funds invest in these early-stage companies in exchange for equity, or an ownership stake in the hopes that some of the firms will become successful.

Generally, a company would gain investments in certain rounds, or waves:

1. A **pre-seed** or **Angel round** is the earliest infusion of capital by founders, supporters, and angel investors, to build a prototype and discover initial product-market fit.
2. The **seed round** is generally the first formal equity round with an institutional lead. Investors purchase preferred stock at a valuation set by the lead investor or a convertible note. Companies at this stage may or may not have revenues. Note that companies a this stage have yet to meet requirements for quality for bank loans.
3. The **Series A round** is the name typically given to a company's first significant round of venture capital financing. The name refers to the class of preferred stock sold to investors in exchange for their investment. Series A preferred stock is often convertible into common stock in certain cases such as an IPO or sale of the company.

   Series A rounds typically purchase 10% to 30% of the company, and the capital raised during a series A is usually intended to capitalize the company for 6 months to 2 years as it develops products, performs initial marketing/branding, etc.

   Series B, Series C, etc. rounds are additional VC financing rounds, and the progression/price of stock at these rounds is an indication that a company is progressing as expected. However, too much money in too many rounds may be seen as a sign of delayed progress.

4. **Series A', Series B', etc.** indicate small follow-on rounds that are integrated into the preceding round, generally on the same terms, to raise additional funds.
5. **Series AA, BB, etc.** denotes a new start after a crunchdown or **downround** (i.e. the company failed to meet growth objectives) and is essentially starting again under the umbrella of a new group of funders.
6. Mezzanine finance rounds, bridge loans, and other debt instruments are used to support a company between venture rounds or before its IPO.

Venture capitalists provide this financing in the interest of generating a return through an eventual "exit" event, such as the company selling shares to the public for the first time in an IPO or disposal of shares happening via a merger or sale.

Definition 2.6.9 (Due Diligence)**. Due diligence** is the investigation or exercise of care that a reasonable business/person is normally expected to take before entering into an agreement or contract with another party.

Definition 2.6.10 (Parties in a Venture Round)**.** The parties in a VC round are:

1. **Founders/stakeholders**, which introduce the companies to the investors
2. The company being funded
3. A **lead investor**, typically the best known or most aggressive venture capital firm that is participating in the investment or contributing the largest sum of cash. The lead typically oversees most of the negotiation, legal work, due diligence, etc. It may also introduce the company to other investors.
4. **Co-investors** are other major investors who contribute alongside the lead investor.
5. **Piggyback investors**, who are typically angel investors, rich individuals, institutions, and others who contribute money but take a passive role in the investment.
6. Law firms and accountants who are typically retained by all parties to advise, negotiate, and document the transaction.

Stages in a Venture Round

There are several stages in a venture round.

1. *Introduction.* Investors and companies seek each other out through formal and informal business networks, personal connections, paid or unpaid finders, researchers and advisers, and the like. Because there are no public exchanges listing their securities, private companies meet venture capital firms and other private equity investors in several ways, such as speed dating for capital.
2. *Offering.* The company provides the investment firm a confidential business plan to secure initial interest.
3. *Negotiation of Terms.* Non-binding term sheets, letters of intent, and the like are exchanged back and forth as negotiation documents.
4. **Signed term sheet**. Once the parties agree on terms, they sign the term sheet as a non-binding expression of commitment in good faith. These sheets may contain

some procedural promises of limited (30 to 60-day) duration like confidentiality, exclusivity on the part of the company (the company will not seek funding from other sources), and standstill provisions (e.g. the company will not undertake any major business changes or enter agreements that would make the transaction infeasible).

5. **Definitive documents**. The legal papers that document the final transaction are finalized. They generally include:
   (a) *Stock purchase agreements.* The primary contract by which investors exchange money for newly minted shares of preferred stock.
   (b) *Buy-sell agreements, co-sale agreements, right of first refusal, etc.* Agreements by which company founders and other owners of common stock agree to limit their individual ability to sell their shares in favor of the new investors.
   (c) *Investor rights agreements.* Covenants the company makes to new investors, generally includes promises with respect to board seats, negative covenants not to obtain additional financing, sell the company, or make other specified business/financial decisions without the investors' approval, and positive covenants such as inspection rights and promises to provide ongoing financial disclosures.
   (d) *Articles of Incorporation.* Formalize issues like authorization and classes of shares and certain investor protections.
6. **Due Diligence**. While negotiating the definitive agreements, the investors examine the financial statements and books and records of the company, and all aspects of its operations. They may require changes to be made.
7. Final agreement occurs when the parties execute all of the transaction documents, completing the deal and funding is announced (although there are often rumors and leaks).
8. Closing occurs when the investors provide the funding and the company provides stock certificates to the investors. Ideally this would be simultaneous, and contemporaneous with the final agreement. However, conventions in the venture community are fairly lax with respect to timing and formality of closing, and generally depend on the goodwill of the parties and their attorneys.
9. *Post-closing.* After the closing a few things may occur:
   (a) Conversion of convertible notes.
   (b) Securities filing with relevant state/federal regulators.
   (c) Filing of amended Articles of Incorporation
   (d) Preparation of closing binder, containing documentation of entire transaction.

Definition 2.6.11 (Rights and Privileges)**.** Venture investors obtain special privileges that are not granted to holders of common stock. These are embodied in the various transaction documents, but common rights include:

1. Anti-dilution protection: If the company ever sells a significant amount of stock at a price lower than the investor paid, then to protect investors against stock dilution they are issued additional shares

2. Guaranteed board seats
3. Registration rights: The investors have special rights to demand registration of their stock on public exchanges, and to participate in an IPO and subsequent public offerings
4. Liquidation preferences: In any liquidation event such as a merger or acquisition, the investors get their money back, often with interest and/or at a multiple, before common stock is paid any funds from liquidation.
5. Dividends that are given before any common stock

2.6.3 Angel Investing

Definition 2.6.12 (Angel Investor)**.** An **angel investor** is a high-net-worth individual who provides (non-leveraged) financial backing for small startups or entrepreneurs, typically in exchange for ownership equity or convertible debt. Angel investing is often the primary source of funding for many startups, and it fosters innovation in startups. However, these types of investments are risky and usually do not represent more than 10% of the angel investor's portfolio.

Angel investors usually provide more favorable terms compared to other lenders, since they usually invest in the entrepreneur starting the business rather than the viability of the business. They are focused on helping startups take their first steps, rather than the possible profit they may get from the business. Essentially, angel investors are the opposite of venture capitalists. A company that needs money for operations but is not yet ready for venture capital will typically seek angel capital.

They typically use their own money, unlike venture capitalists who pool money from other investors and place them in a managed fund. Additionally, while angel investors are usually individuals, the entity that actually provides the funds may be a LLC, a business, a trust, or an investment fund.

The effective internal rate of return for a successful portfolio for angel investors is approximately 22%. Cheaper sources of financing such as banks are not usually available for such business ventures.

Chapter 3

Business

3.1 Corporate Structure

There is a difference between *ownership* and *management* of a company. Before the 20th century, many companies were small, family-owned, and family-run. However, many are large international conglomerates that trade publicly on multiple exchanges.

Definition 3.1.1 (Corporate Hierarchy)**.** In order to create a corporation in which stockholders' interest are looked after, many firms have implemented a two-tier **corporate hierarchy**, or a **chain of command**. The most common corporate structure in the U.S. consists of a board of directors and a management team. Generally,

1. the management looks after the day-to-day operations
2. the board ensures that the shareholders are adequately represented.

Note that an individual may be only in the board, only in the management team, or in both. In fact, many boards include current and past members of the management team, including the CEO, the CFO, the COO, along with the retired CEO, family members, etc. Generally, the hierarchy is divided into the following groups:

1. Board of directors, with the chairman at the top
2. Executive officers, lead by the CEO, with CFO, COO, CIO
3. Vice presidents, directors
4. Management of smaller departments
5. Employees

Definition 3.1.2 (Board of Directors)**.** The **board of directors** are elected by the shareholders. Every member in the board falls into one of three categories:

1. The **board chair** is technically the leader of the corporation and is responsible for running the board smoothly and effectively. Their duties include
   (a) maintaining strong communication with the CEO and high-level executives
   (b) formulating the company's business strategy

(c) representing management and the board to the general public and shareholders

(d) maintaining corporate integrity

The chair is elected from the board of directors.

2. **Inside directors** are chosen from within the company and are individuals that work in the company daily. They are responsible for

   (a) approving high-level budgets prepared by upper-management

   (b) implementing and monitoring business strategy

   (c) approving core corporate initiatives and projects

   Inside directors are either shareholders or high-level managers from within the company.

3. **Outside directors** have the same responsibilities as the inside directors in determining strategic direction and corporate policy. However, they are chosen externally and are considered independent of the company. Their purpose is to provide unbiased perspectives on issues brought to the board.

Definition 3.1.3 (Management Team)**.** The **management team** is directly responsible for the company's day-to-day operations and profitability. The **C-Suite** refers to the group of a corporation's top senior executives, with positions all starting with the letter 'C' for 'Chief.'

1. The **chief executive officer (CEO)** is typically responsible for the corporation's entire operations and is the main liaison between the board of directors and corporate operations. It is the CEO's responsibility to implement board decisions and initiatives, as well as the maintain the smooth operation of the firm with senior management's assistance.

   Because the board is in charge of executive functions, and the CEO is responsible for integrating company policy into day-to-day operations, the CEO often fills the role of the board chair.

2. The **chief operations officer (COO)** is responsible for the corporation's operations and looks after issues related to marketing, sales, production, and personnel. The COO is often more hands-on than the CEO.

3. The **chief financial officer (CFO)** is responsible for analyzing and reviewing financial data, reporting financial performance, preparing budgets, and monitoring expenditures and costs. The CFO is required to present this information to the board and to the SEC.

4. The **chief risk officer (CRO)** is an executive in charge of managing risks to the company, including complying with government regulations, managing investments, guarding intellectual property, and others.

5. The **chief information officer (CIO)** is the company executive responsible for the management, implementation, and usability of information and computer technologies.

6. The **chief technology officer (CTO)** is the executive in charge of an organization's technological needs as well as its R&D.

Definition 3.1.4. An **interim** position refers to a person appointed by a company's board of directors to assume a certain role during a time of transition of as the result of the sudden departure of the company's previous individual. e.g. an **Interim CEO**

Definition 3.1.5 (President)**.** The role of **president** is quite loose and can be used differently depending on the corporation. The president is generally referred to as the leader of a company's executive group. Generally, the board of directors sets the policy, the president executes the policy and reports back to the board, and then the board reports back to the shareholders, the ultimate owners.

1. The president is the leader of a segment or a critical part of the overall company (rather than the leader of the overall company).
2. In smaller businesses, the president may be the CEO or may even be the owner of the company.
3. The president often holds the position of the COO, since they lead the day-to-day operations. The president has several vice presidents for different parts of the company reporting to them.

Note that the CEO is not always the chair of the board, and the president is not always the COO.

3.2 Expansion and Integration

Companies can expand in multiple ways, including M&As, etc. We will elaborate more on this.

Horizontal vs Vertical Integration

Definition 3.2.1 (Value Chain)**.** A **value chain** is a step-by-step business model that describes the full range of activities needed to create a product or service. For companies that produce goods, a value chain comprises the steps that involve bringing a product from conception to distribution, and everything in between, such as procuring raw materials, manufacturing functions, and marketing activities.

A company can conduct **value-chain analysis** by evaluating the detailed procedures involved in each step of its business: that is, to maximize value at each specific point in a firm's processes. The purpose of a value-chain analysis is to increase production efficiency while minimizing costs.

Definition 3.2.2 (Distribution Model)**.** A **distribution model**, or **distribution channel**, is the manner in which goods move from the manufacturer to the outlet where the consumer purchases them. The conventional distribution model has three levels:

$$\text{Producer} \implies \text{Wholesaler} \implies \text{Retailer}$$

There are many ways a business can construct a distribution model, which may get much more complicated.

Definition 3.2.3 (Vertical Integration)**. Vertical integration** is a strategy whereby a company owns or controls its suppliers, distributors, or retail locations to control its value of supply chain. Vertical integration benefits companies by allowing them to control processes, reduce costs, and improve efficiencies. However, it has its disadvantages, including the significant amounts of capital investment required.

1. **Backward integration** is when a company expands backward on the production path into manufacturing, meaning a retailer buys the manufacturer of the product.
2. **Forward integration** is when a company expands by purchasing and controlling the direct distribution of supply of its products. This cuts out the middleman and increases profits. For example, a clothing manufacturer can open its own retail location to sell its products.

Example 48 (Netflix). Netflix started as a DVD rental business before moving into online streaming of films and movies licensed from major studios. Then, Netflix executives realized they could improve their margins by producing their own original content. Now, Netflix uses its distribution model to promote its original content.

Definition 3.2.4 (Horizontal Integration)**. Horizontal integration** is the acquisition of a business operating at the same level of the value chain in the same industry. This is in contrast to vertical integration, where firms expand into upstream or downstream activities, which are at different stages of production.

The advantages of horizontal integration include creating economies of scale and reducing competition. However, this can lead to oligopolies or even monopolies, which is why horizontal mergers are heavily scrutinized under antitrust laws.

Example 49 (Disney, Pixar). Disney's 2006 acquisition of Pixar in the entertainment media industry is an example of horizontal integration.

Parent Companies, Subsidiaries

In M&As, it is either company A acquires company B, or A and B merge as equals. When company A acquires company B, company A is called the *parent company* while company B is called the *subsidiary*.

Definition 3.2.5 (Spinoff)**.** Rather than performing a buyout on an independent company, the company can *create* a new independent company through a **spinoff**, which is the creation of an independent company through the sale or distribution of new shares of a parent company.

The spin off will have a separate management structure and a new name, but it will retain the same assets, intellectual property, and human resources. Spinoffs may occur for many reasons, such as allocation of resources or a divergence of plans.

Definition 3.2.6 (Parent Company)**.** A parent company holds a controlling stake (at least 50%) of its subsidiaries, giving it control of its operations. Parent companies can be either hands-on or hands-off owners of its subsidiaries, depending on the amount of managerial control given to subsidiary managers, but will always maintain a certain level of active control. There are some types of parent companies:

1. A **conglomerate** is a corporation that is made up of a number of different, sometimes unrelated businesses. In a conglomerate, one parent company owns a controlling stake in a number of smaller companies all of whom conduct businesses separately and independently.

   Conglomerates allows businesses to diversify their portfolio, but the large size of conglomerates can actually hurt the value of their stock (known as the *conglomerate discount*). The financial health of a conglomerate can be difficult to discern by investors sine the management is too complicated.

2. A **holding company**, or an **umbrella company**, is a passive business entity that doesn't manufacture anything, sell any products or service, or conduct any other business operations. Rather, holding companies only hold the controlling stock in other companies. It may oversee the company's management decisions, it does not actively participate in running a business's day-to-day operations. Holding companies are protected from losses accrued by subsidiaries and receive tax benefits.

Because parent companies own more than 50% of the voting stock in a subsidiary, they have to produce financial statements that combine the parent and subsidiary financial statements into one larger set of financial statements.

Example 50 (Berkshire Hathaway). Warren Buffet's Berkshire Hathaway is a conglomerate having a majority stake in over 50 companies as well as minority holdings in dozens more companies. It successfully manages companies involved in everything from plane manufacturing and textiles to insurance and real estate.

Example 51 (General Electric). General Electric (GE), originally an electronics company and innovation lab, the company has expanded to own firms working in energy, real estate, finance, media, and healthcare.

Definition 3.2.7 (Subsidiary)**.** The **subsidiary** is the company that is owned, with a controlling interest, by a parent company. If a subsidiary is 100% owned by another firm, the subsidiary is referred to as a **wholly owned subsidiary**.

Definition 3.2.8 (Alphabet)**.** Alphabet is one of the world's largest technology conglomerates, with a market capitalization of $1.4 trillion as of Feburary, 2021. Its flagship subsidiary is Google, but owns other companies such as

1. Nest: sells smart-home products
2. DoubleClick: ad management and ad serving solutions
3. Looker: business intelligence software and data analytics
4. Youtube: online video-sharing platform
5. Waze: mobile navigation app
6. Fitbit Inc.: wearable fitness devices and app

Chapter 4

Accounting

4.1 Accounting Oversight and Regulations

4.2 The Balance Sheet

Definition 4.2.1 (Receivables, Payables)**. Receivables**, or **accounts receivable** of a company (the "Company") are debts owed to the Company by its customers for goods or services that have been delivered or used but not yet paid for.

Payables, or **accounts payable** of a company (the "Company") are debts owed to the customers by the Company for goods or services that have been paid for but not yet delivered or used.

The financial condition of every enterprise can be divided up into its assets and liabilities. The balance sheet provides a snapshot of the assets and liabilities that a company has.

Definition 4.2.2 (Assets)**. Assets** represent the things that the enterprise has. Some examples of **current assets** (assets expected to be used through standard business operations within one year) are:

1. Cash and cash equivalents (cash or highly-liquid assets)
2. Inventories
3.

Examples of **non-current assets** (not expected to be used within a year) are:

1. Property, plant, and equipment (PPE)
2. Land and real estate
3. Intangible Assets, such as goodwill, brand recognition, customer loyalty, intellectual property, patents, etc.

Definition 4.2.3 (Liabilities)**. Liabilities** represent the debts and responsibilities that it needs to pay off. Some examples are:

1. Accounts payable (liabilities payable to suppliers)
2. Salaries payable (future compensation owed to employees)
3. Deferred revenue (payment by customer that has not yet been earned by the company)

Examples of **non-current liabilities** are:

1.

Definition 4.2.4 (Shareholder's Equity, or Book Value)**.** If the corporation suddenly collapses we can measure how much it is worth by taking its total assets and subtracting from it the total liabilities. The remaining value is called the **book value** of the company.

$$\text{Book Value} = \text{Total Assets} - \text{Total Liabilities}$$

In other words, if the company were to liquidate all of its assets and pay off all of its liabilities, the amount of money remaining would be the "value" of the company. Since this remaining value would be distributed to the shareholder's of the company (since again, they own the company), this book value is also called the **shareholder's equity**.

4.3 Earnings Sheet

4.4 Cash Flow Statement

4.5 Financial Audit and Taxation

Chapter 5

Corporations

Definition 5.0.1 (Shell, Shelf Companies)**.** A **shell company** is a corporation existing only on paper