Forecasting with Option-Implied Information*

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Abstract

This chapter surveys the methods available for extracting information from option prices that can be used in forecasting. We consider option-implied volatilities, skewness, kurtosis, and densities. More generally, we discuss how any forecasting object which is a twice differentiable function of the future realization of the underlying risky asset price can utilize option-implied information in a well-defined manner. Going beyond the univariate option-implied density, we also consider results on option-implied covariance, correlation and beta forecasting, as well as the use of option-implied information in cross-sectional forecasting of equity returns. We discuss how option-implied information can be adjusted for risk premia to remove biases in forecasting regressions.

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1 Introduction

We provide an overview of techniques used to extract information from derivatives, and document the usefulness of this information in forecasting. The premise of this chapter is that derivative prices contain useful information on the conditional density of future underlying asset returns. This information is not easily extracted using econometric models of historical values of the underlying asset prices, even though historical information may also be useful for forecasting, and combining historical information with information extracted from derivatives prices may be especially effective.

1.1 Options and Other Derivative Securities

A derivative contract is an asset whose future payoff depends on the uncertain realization of the price of an underlying asset. Many different types of derivative contracts exist: futures and forward contracts, interest rate swaps, currency and other plain-vanilla swaps, credit default swaps (CDS) and variance swaps, collateralized debt obligations (CDOs) and basket options, European style call and put options, and American style and exotic options. Several of these classes of derivatives, such as futures and options, exist for many different types of underlying assets, such as commodities, equities, and equity indexes.

Because of space constraints, we are not able to discuss all the available techniques and empirical evidence of predictability for all available derivatives contracts. We therefore use three criteria to narrow our focus. First, we give priority to larger and more liquid markets, because they presumably are of greater interest to the reader, and the extracted information is more reliable. Second, we focus on methods that are useful across different types of securities. Some derivatives, such as basket options and CDOs, are multivariate in nature, and as a result techniques for information extraction are highly specific to these securities. While there is a growing literature on extracting information from these derivatives, the literature on forecasting using this information is as yet limited, and we therefore do not focus on these securities. Third, some derivative contracts such as forwards and futures are linear in the return on the underlying security, and therefore their payoffs are too simple to contain enough useful and reliable information relative to nonlinear contracts. This makes these securities less interesting for our purpose. Other securities, such as exotic options, have path-dependent payoffs, which may make information extraction cumbersome.

Based on these criteria, we mainly focus on European-style options. European-style options hit the sweet spot between simplicity and complexity and will therefore be the main, but not the exclusive, focus of our survey.[1] Equity index options are of particular interest, because the underlying risky asset (a broad equity index) is a key risk factor in the economy. They are among the most liquid exchange-traded derivatives, so they have reliable and publicly available prices. The

[1] Note that for American options the early exercise premium can usually be estimated (using binomial trees for example). By subtracting this estimate from the American option price, a synthetic European option is created which can be analyzed using the techniques we study in this Chapter.

fact that the most often used equity index options are European-style also makes them tractable and computationally convenient.[2] For these reasons, the available empirical literature on equity index options is also the most extensive one.

Forecasting with option-implied information typically proceeds in two steps. First, derivative prices are used to extract a relevant aspect of the option-implied distribution of the underlying asset. Second, an econometric model is used to relate this option-implied information to the forecasting object of interest. For example, the Black-Scholes model can be used to compute implied volatility of an at-the-money European call option with 30 days to maturity. Then, a linear regression is specified with realized volatility for the next 30 days regressed on today's implied Black-Scholes volatility. We will focus on the first step in this analysis, namely extracting various information from observed derivatives prices. The econometric issues in the second step are typically fairly standard and so we will not cover them in any detail.

Finally, there are a great number of related research areas we do not focus on, even though we may mention and comment on some of them in passing. In particular, this chapter is not a survey of option valuation models (see Whaley (2003)), or of the econometrics of option valuation (see Garcia, Ghysels, and Renault (2010)), or of volatility forecasting in general (see Andersen, Bollerslev, Christoffersen, and Diebold (2006)). Our chapter exclusively focuses on the extraction of information from option prices, and only to the extent that such information has been used or might be useful in forecasting.

1.2 Risk Premia

Risk premia play a critical role when forecasting with option-implied information. Here we briefly outline the impact of risk premia, using the simplest possible example of a derivatives contract, a forward. A forward contract requires the seller to deliver to the buyer the underlying asset with current spot price, S_0, on a prespecified future date, T, at a prespecified price, F_0, to be paid at date T. In the absence of taxes, short-sale constraints and assuming that investors can lend and borrow at the same risk-free rate, r, the no-arbitrage price of the forward contract on a financial asset is simply

$$F_0 = S_0 \exp(rT). \tag{1}$$

assuming continuous compounding. The simplicity of this contract makes it well-suited to explore the impact of risk premia. Consider using forward prices as predictors of the future realized spot price, S_T. Again for simplicity, assume that the price evolves as a Brownian motion with risk premium μ and volatility σ. The distribution of the future spot price is log-normal in this case and the expected future spot price is

$$E_0\left[S_T\right] = S_0 \exp\left(\left(r+\mu\right)T\right).$$

[2] Most studies use options on the S&P500 index, which are European. Early studies used options on the S&P100, which was the most liquid market at the time. These options are American.

Using the no-arbitrage condition (1), we get

$$E_0\left[S_T\right] = F_0 \exp\left(\mu T\right) \tag{2}$$

Equation (2) is a very simple example of how a derivatives contract can contain useful information about future values of the underlying security. Similar intuition holds for more complex derivatives, such as options, but the relationship between the derivatives price and the future underlying is more complex. As explained above, the simplicity of (2) is why we do not extensively discuss the extraction of information from forward contracts in this chapter.[3] Equation (1) indicates that the forward price, F_0, is a simple linear function of the underlying asset price, S_0, and therefore information from F_0 is not likely to be more useful than S_0 itself.

Equation (2) illustrates that unless the asset's risk premium μ is equal to zero, the forward price will be a biased forecast of the future spot price of the asset. In standard asset pricing models, the risk premium μ of the asset will be positive if the asset price is positively correlated with overall wealth in the economy, implying that $F_0 < E_0\left[S_T\right]$, so that the forward price will be a downward-biased forecast of the future spot price. In a standard forecasting regression, the bias will show up in the intercept if μ does not change over time. If μ is time-varying, the bias may show up in the slope coefficient. Notice that what is critical here is that μ is not directly observable, so that we cannot easily correct the future spot price for the risk premium.

While for more complex derivatives the relation between the derivatives price, the expected price of the underlying, and the risk premium will typically be more complex, the same intuition holds: the presence of a risk premium will impact on information extracted from options. Also, while our example addresses the first moment of the underlying security, the derivatives-implied estimate of the higher moments of the return on the underlying security are also biased by the presence of risk premia.

While risk premia are not directly observable, they can be estimated, by combining the prices of the derivative and the underlying contract, but this typically requires additional assumptions. We therefore proceed in two steps. In Sections 2-4, we work exclusively under the so-called risk-neutral or pricing measure where risk premia are zero. We subsequently discuss the incorporation of risk premia into option-implied forecasts in Section 5.

[3] In certain markets futures contracts trade in the hours after today's closing and/or before tomorrow's opening of the spot market. In this case the futures price can serve as a more useful predictor of future spot prices than yesterday's closing spot price simply because the futures price is observed more recently. The futures price will still be a biased forecast as equation 2 shows, but the bias will be small if the maturity, T, of the futures contract is small and/or if the risk-premium, μ, is small. If trading in the futures markets can be done more cheaply and efficiently than in the underlying spot market then futures prices may lead spot prices at short horizons as well.

1.3 Chapter Overview

This chapter addresses forecasting future realizations using several option-implied objects: volatility, skewness and kurtosis, and the return density.[4] There is a recent and quite distinct literature on forecasting expected equity returns (over time and in the cross-section) using higher option-implied moments, and we also discuss this evidence in some detail.

Note that the available evidence on forecasting with option-implied volatility is more extensive than the evidence using skewness and kurtosis. There are several reasons for this. On the one hand, it is more straightforward to estimate option-implied volatility compared to option-implied higher moments. More subtly, option-implied forecasts are more likely to be informative about future moments when risk premia are small, and there is growing evidence that volatility risk premia are smaller than the risk premia for higher moments, in particular for skewness.

The chapter proceeds as follows. Section 2 discusses methods for extracting volatility and correlation forecasts from option prices. Section 3 focuses on constructing option-implied skewness and kurtosis forecasts. Section 4 covers techniques that enable the forecaster to construct the entire density, thus enabling event probability forecasts for example. Sections 2-4 cover model-based as well as model-free approaches. When discussing model-based techniques, we discuss in each section the case of two workhorse models, Black and Scholes (1973) and Heston (1993), as well as other models appropriate for extracting the object of interest. Sections 2-4 use the option-implied distribution directly in forecasting the physical distribution of returns. Section 5 discusses the theory and practice of allowing for risk premia and thus converting option-implied forecasts to physical forecasts. Section 6 concludes.

2 Extracting Volatility and Correlation from Option Prices

Volatility forecasting is arguably the most widely used application of option-implied information. When extracting volatility information from options, model-based methods were originally more popular, but recently model-free approaches have become much more important. We will discuss each in turn.

2.1 Model-Based Volatility Extraction

In this section we will review the most commonly used option valuation model for volatility extraction, namely the Black and Scholes (1973) model. The Black-Scholes model only contains one unknown parameter, volatility, which is constant, and so extracting an option-implied volatility forecast from this model is straightforward. We will also review the Heston (1993) model. The Heston model allows for stochastic volatility, which can be correlated with returns, but it contains multiple parameters and so it is more cumbersome to implement. Finally, we will use an argument

[4] The option-implied first moment is equal to the risk-free rate and so not used in forecasting.

from Hull and White (1987) to show how the Black-Scholes model is related to a special case of the Heston model. This relationship suggests why the Black-Scholes model continues to be widely used for extracting option-implied volatility.

2.1.1 Black-Scholes Implied Volatility

Black and Scholes (1973) assume a constant volatility geometric Brownian motion risk-neutral stock price process of the form

$$dS = rSdt + \sigma Sdz$$

where again r is the risk-free rate, σ is the volatility of the stock price, and dz is a normally distributed innovation.[5] Given this assumption, the future log stock price is normally distributed and the option price for a European call option with maturity T and strike price X can be computed in closed form using

$$C^{BS}\left(T, X, S_0, r; \sigma^2\right) = S_0 N(d) - X \exp\left(-rT\right) N\left(d - \sigma\sqrt{T}\right) \tag{3}$$

where S_0 is the current stock price, $N(\cdot)$ denotes the standard normal CDF, and where

$$d = \frac{\ln\left(S_0/X\right) + T\left(r + \frac{1}{2}\sigma^2\right)}{\sigma\sqrt{T}}. \tag{4}$$

European put options can be valued using put-call parity

$$P_0 + S_0 = C_0 + X \exp\left(-rT\right)$$

which can be derived from a no-arbitrage argument alone and so is not model dependent.

The Black-Scholes option pricing formula has just one unobserved parameter, namely volatility, denoted by σ. For any given option with market price, C_0^{Mkt}, the formula therefore allows us to back out the value of σ which is implied by the market price of that option,

$$C_0^{Mkt} = C^{BS}\left(T, X, S_0, r; BSIV^2\right) \tag{5}$$

The resulting option-specific volatility, $BSIV$, is generically referred to as implied volatility (IV). To distinguish it from other volatility measures implied by options, we will refer to it as Black-Scholes IV, thus the $BSIV$ notation.

Although the Black-Scholes formula in (3) is clearly non-linear, for at-the-money options, the relationship between volatility and option price is virtually linear as illustrated in the top panel of Figure 1.

[5] Throughout this chapter we assume for simplicity that the risk-free rate is constant across time and maturity. In reality it is not and the time-zero, maturity-dependent risk-free rate, $r_{0,T}$ should be used instead of r in all formulas. Recently, the overnight indexed swap rate has become the most commonly used proxy for the risk-free rate. See Hull (2011) Chapter 7.

[Figure 1: Black-Scholes Price and Vega]

In general the relationship between volatility and option prices is positive and monotone. This in turn implies that solving for $BSIV$ is quick. The so-called option Vega captures the sensitivity of the option price w.r.t. changes in volatility. In the Black-Scholes model it can be derived as

$$Vega_{BS} = \frac{\partial C_0^{BS}}{\partial \sigma} = S_0 \sqrt{T} N'(d)$$

where d is as defined in (4) and where $N'(d)$ is the standard normal probability density function.

The bottom panel of Figure 1 plots the Black-Scholes Vega as a function of moneyness. Note that the sensitivity of the options with respect to volatility changes is largest for at-the-money options. This in turn implies that changes in at-the-money option prices are the most informative about changes in expected volatility.

In Figure 2 we plot $BSIV$s for out-of-the-money S&P 500 call and put options quoted on October 22, 2009. In the top panel of Figure 2 the $BSIV$s on the vertical axis are plotted against moneyness (X/S_0) on the horizontal axis for three different maturities.

[Figure 2: Black-Scholes Implied Volatility as a Function of Moneyness and Maturity]

The index-option $BSIV$s in the top panel of Figure 2 display a distinct downward sloping pattern commonly known as the "smirk" or the "skew". The pattern is evidence that the Black-Scholes model–which relies on the normal distribution–is misspecified. Deep out-of-the-money (OTM) put options ($X/S_0 << 1$) have much higher $BSIV$s than other options which from Figure 1 implies that they are more expensive than the Black-Scholes model, which assumes lognormality, would suggest. Only a distribution with a fatter left tail (that is negative skewness) would be able to generate these much higher prices for OTM puts. This finding will lead us to consider models that account for skewness and kurtosis in Section 3.

The bottom panel of Figure 2 shows that the $BSIV$ for at-the-money options ($X/S_0 \approx 1$) tends to be larger for long-maturity than short-maturity options. This is evidence that volatility changes over time although it is assumed constant in the Black-Scholes model. We therefore consider models with stochastic volatility next.

2.1.2 Stochastic Volatility

For variances to change over time, we need a richer setup than the Black-Scholes model. The perhaps most widely used stochastic volatility model is Heston (1993),[6] who assumes that the price

[6] Heston's model is based on earlier stochastic volatility models by Hull and White (1987), Scott (1987), Wiggins (1987) and Melino and Turnbull (1990).

of an asset follows the so-called square-root process[7]

$$dS = rSdt + \sqrt{V}Sdz_1 \tag{6}$$
$$dV = \kappa(\theta - V)\,dt + \sigma_V\sqrt{V}dz_2$$

where the two innovations are correlated with parameter ρ.

At time zero, the variance forecast for horizon T can be obtained as

$$VAR_0(T) \equiv E_0\left[\int_0^T V_t dt\right] = \theta T + (V_0 - \theta)\frac{\left(1 - e^{-\kappa T}\right)}{\kappa} \tag{7}$$

The horizon-T variance $VAR_0(T)$ is linear in the spot variance V_0. Notice how the mean-reversion parameter κ determines the extent to which the difference between current spot volatility and long run volatility, $(V_0 - \theta)$, affects the horizon T forecast. The smaller the κ, the slower the mean reversion in volatility, and the higher the importance of current volatility for the horizon T forecast. Recall that in this and the subsequent two sections we set all risk premia to zero and work with the risk-neutral distribution. All expectations are therefore computed using the risk-neutral distribution as well.

Figure 3 shows the volatility term structure in the Heston model, namely

$$\sqrt{VAR_0(T)/T} = \sqrt{\theta + (V_0 - \theta)\frac{\left(1 - e^{-\kappa T}\right)}{\kappa T}} \tag{8}$$

using $\theta = 0.09$ and $\kappa = 2$. $V_0 = 0.36$ (dashed line) corresponds to a high current spot variance and $V_0 = 0.01$ (solid line) corresponds to a low current spot variance.

[Figure 3: Heston Volatility Term Structures]

A similar approach could be taken for the wide range of models falling in the affine class to which the Heston model belongs. Duffie, Pan, and Singleton (2000) provide an authoritative treatment of a general class of continuous time affine models. For examples of discrete time affine models, see Heston and Nandi (2000) and Christoffersen, Heston, and Jacobs (2006).

Hull and White (1987) show that in the case where volatility is uncorrelated with returns ($\rho = 0$),[8] we can think of the stochastic volatility option price as the expected value of the Black-Scholes price

$$C^{SV} = E_0\left[C^{BS}\left(T, X, S_0, r; \frac{1}{T}\int_0^T V_t dt\right)\right]$$

where the conditional expectation is taken over the distribution of future integrated variance. We saw in Figure 1 that the Black-Scholes formula is close to linear in volatility for at-the-money

[7] Christoffersen, Jacobs, and Mimouni (2010) investigate the empirical performance of stochastic volatility models with alternative drift and diffusion specifications.

[8] They also assume that volatility does not carry a separate risk premium. We will discuss the volatility risk premium in Section 5.

options and we therefore have that

$$C^{SV} \approx C^{BS}\left(T, X \approx S_0, S_0, r; E_0\left[\frac{1}{T}\int_0^T V_t dt\right]\right) \tag{9}$$

which holds only approximately because we are using variance and not volatility here.

The important practical implication of this result is that–even when volatility is stochastic–we can invert the Black-Scholes formula for an at-the-money option to easily obtain a decent forecast of the average variance between now and maturity of the option

$$BSIV\left(X \approx S_0\right) \approx \sqrt{E_0\left[\frac{1}{T}\int_0^T V_t dt\right]} \tag{10}$$

when returns and volatility are uncorrelated.

Using only at-the-money options to extract volatility forecasts omits potentially important information from other strikes. Furthermore, we would like to be able to generate option-based forecasts at horizons different from the once corresponding to available option maturities. This can be done by estimating the parameters in the Heston model. Note that whereas the Black-Scholes model only has one parameter, σ, the Heston model has four parameters, namely κ, θ, σ_V, and ρ, in addition to the spot variance, V_0. Estimation of the parameters and spot volatility in the model can be done using a data set of returns, but also using option prices. Bakshi, Cao, and Chen (1997) re-estimate the model daily treating V_0 as a fifth parameter to be estimated along with the structural parameters θ, κ, ρ, and σ_V. Bates (2000) and Christoffersen, Heston, and Jacobs (2009) keep the structural parameters fixed over time. They make use of an iterative two-step option valuation error minimization procedure where in the first step the structural parameters are estimated for a given path of $\{V_t\}_{t=1}^N$. In the second step V_t is estimated each period keeping the structural parameters fixed. Iterating between the first and second step provides the final estimates of structural parameters and spot volatilities. Alternatively, a more formal filtering technique can be used, which is econometrically more complex.

The complications involved in estimating the parameters and filtering the unobserved spot volatility in models such as Heston's–as well as the parametric assumptions required–have motivated the analysis of model-free volatility extraction to which we now turn.

2.2 Model-Free Volatility Extraction

2.2.1 Theory and Implementation

Under the assumptions that investors can trade continuously, interest rates are constant, and the underlying futures price is a continuous semi-martingale, Carr and Madan (1998) show that the expected value of the future realized variance can be computed as

$$E_0\left[\int_0^T V_t dt\right] = 2\int_0^\infty \frac{C_0^F\left(T, X\right) - \max\left(F_0 - X, 0\right)}{X^2} dX, \tag{11}$$

where F_0 is the forward price of the underlying asset and $C^F(T, X)$ is a European call option on the forward contract.

Britten-Jones and Neuberger (2000) show that the relationship also holds when V_t is replaced by the instantaneous squared return

$$E_0\left[\int_0^T (dS_t/S_t)^2\, dt\right] = 2\int_0^{\infty} \frac{C_0^F(T, X) - \max(F_0 - X, 0)}{X^2} dX. \tag{12}$$

Jiang and Tian (2005) generalize this result further and show that (12) holds even if the price process contains jumps.

When relying on options on the underlying spot asset rather than on the forward contract, the expected variance between now and horizon T is

$$VAR_0(T) = 2\int_0^{\infty} \frac{C_0\left(T, e^{-rT}X\right) - \max(S_0 - X, 0)}{X^2} dX.$$

Jiang and Tian (2005, 2007) discuss the implementation of (12). In particular, they discuss potential biases that can arise from

1. Truncation errors: the integration is performed over a finite range of strike prices instead of from 0 to ∞.

2. Discretization errors: the integral over strikes is replaced by a sum.

3. Limited availability of strikes: the range of available strikes is narrow and/or has large gaps.

In practice, a finite range, $X_{\max} - X_{\min}$, of discrete strikes are available. Jiang and Tian (2005) use the trapezoidal integration rule

$$VAR_0(T) \approx \sum_{i=1}^{m}\left\{\frac{\left[C_0^F(T, X_i) - \max(F_0 - X_i, 0)\right]}{X_i^2} + \frac{\left[C_0^F(T, X_{i-1}) - \max(F_0 - X_{i-1}, 0)\right]}{X_{i-1}^2}\right\}\Delta X \tag{13}$$

where $\Delta X = (X_{\max} - X_{\min})/m$, and the discrete (evenly spaced) strikes $X_i = X_{\min} + i\Delta X$.

In order to reduce the discretization error, ΔX needs to be reasonably small. Jiang and Tian (2005) fill in gaps in strikes by applying a cubic spline to the $BSIV$s of traded options, and demonstrate using a Monte Carlo experiment that this approaches works well. To overcome truncation problems, Jiang and Tian (2005) use a flat extrapolation outside of the strike price range, whereas Jiang and Tian (2007) use a linear extrapolation with smooth pasting. Figlewski (2010) proposes further modifications, including: (i) a fourth degree rather than a cubic spline, (ii) smoothing which does not require the interpolation function to fit the traded option prices exactly, and (iii) the application of extreme value functions for the tails of the distribution.

2.2.2 The VIX Volatility Index

The VIX volatility index is published by the Chicago Board of Options Exchange (CBOE). It is probably the best-known and most widely used example of option-implied information. It has become an important market indicator and it is sometimes referred to as "The Investor Fear Gauge" (Whaley (2000)).

The history of the VIX nicely illustrates the evolution in the academic literature, and the increasing prominence of model-free approaches rather than model-based approaches. Prior to 1993, the VIX was computed as the average of the $BSIV$ for four call and four put options just in- and out-of-the-money, with maturities just shorter and longer than thirty days. (See Whaley (2000) for a detailed discussion.) Since 2003, the new VIX relies on a model-free construction, and relies on the following general result.[9]

Consider first a variance swap which is a contract that at time T pays integrated variance between time 0 and T less a strike price, X_{VS}. The strike is set so that the value of the variance swap is zero when written at time 0

$$e^{-rT}E_0\left[\frac{1}{T}\int_0^T V_t dt - X_{VS}\right] = 0$$

Consider next a stock price process with a generic dynamic volatility specification

$$dS = rSdt + \sqrt{V_t}Sdz$$

From Ito's lemma we have

$$d\ln(S) = \left(r - \tfrac{1}{2}V_t\right)dt + \sqrt{V_t}dz$$

so that

$$\frac{dS}{S} - d\ln(S) = \tfrac{1}{2}V_t dt$$

This relationship shows that variance can be replicated by taking positions sensitive to the price, S, and the log price, $\ln(S)$, of the underlying asset.[10] Demeterfi, Derman, Kamal, and Zou (1999) use this expression for the variance to derive the replicating cost of the variance swap as

$$VAR_0\left(T\right) = E_0\left[\int_0^T V_t dt\right] = 2E_0\left[\int_0^T \frac{dS}{S} - d\ln(S)\right] = 2E_0\left[\int_0^T \frac{dS}{S} - \ln\left(\frac{S_T}{S_0}\right)\right] \tag{14}$$

Consider now an arbitrary stock price level, S_*, usually chosen to be close to the forward price. Demeterfi et al. (1999) show that

$$VAR_0\left(T\right) = 2\left[rT - \left(\frac{S_0}{S_*}e^{rT} - 1\right) - \ln\left(\frac{S_*}{S_0}\right) + \int_0^{S_*}\frac{e^{rT}}{X^2}P\left(X,T\right)dX + \int_{S_*}^{\infty}\frac{e^{rT}}{X^2}C\left(X,T\right)dX\right] \tag{15}$$

[9] The VIX calculation assumes a stock price process where the drift and diffusive volatility are arbitrary functions of time. These assumptions encompass for example implied tree models in which volatility is a function of stock price and time. See Dupire (1994) for a discussion of this type of model.

[10] The idea of using log contracts to hedge volatility risk was first introduced by Neuberger (1994).

where P and C are put and call prices. Now, if we let $S_* = F_0$, then because $F_0 = S_0 e^{rT}$ the terms to the left of the integrals cancel and we can write

$$VAR_0(T) = 2e^{rT}\left[\int_0^{F_0} \frac{1}{X^2} P(X,T)\,dX + \int_{F_0}^{\infty} \frac{1}{X^2} C(X,T)\,dX\right]. \tag{16}$$

However, if we set $S_* = X_0$ where X_0 is close to F_0, but not exactly F_0, then a second-order Taylor approximation of the log function around $\frac{F_0}{X_0}$ gives

$$rT - \left(\frac{S_0}{X_0}e^{rT} - 1\right) - \ln\left(\frac{X_0}{S_0}\right) = \left(\frac{F_0}{X_0} - 1\right) + \ln\left(\frac{F_0}{X_0}\right) \approx -\frac{1}{2}\left[\frac{F_0}{X_0} - 1\right]^2 \tag{17}$$

The CBOE implementation of the VIX (CBOE (2009)) is based on (15) and (17), and is given by

$$VIX = 100\sqrt{\frac{2}{T}\sum_i \frac{\Delta X_i}{X_i^2} e^{rT} O(X_i) - \frac{1}{T}\left[\frac{F_0}{X_0} - 1\right]^2} \tag{18}$$

where X_0 is the first strike below F_0, $\Delta X_i = (X_{i+1} - X_{i-1})/2$, and $O(X_i)$ is the midpoint of the bid-ask spread for an out of the money call or put option with strike X_i. Note that the VIX is reported in annual percentage volatility units and that compared to (15), it is an average as opposed to an integrated variance. See Jiang and Tian (2007) for more details on how (18) can be derived from (15).

The CBOE computes VIX using out-of-the-money and at-the-money call and put options. It calculates the volatility for the two available maturities that are the nearest and second-nearest to 30 days. Then they either interpolate, if one maturity is shorter and the other is longer than 30 days, or otherwise extrapolate, to get a 30 day index.

It is noteworthy that the implementation of this very popular index requires several ad hoc decisions which could conceivably affect the results. See for example Andersen and Bondarenko (2007), Andersen and Bondarenko (2009), and Andersen, Bondarenko, and Gonzalez-Perez (2011) for potential improvements to the VIX methodology. The latter paper shows that the time-varying range of strike prices available for the VIX calculation affects its precision and consequently suggests an alternative measure based on corridor variance which uses a consistent range of strike prices over time.

Besides the underlying modeling approach, another important change was made to the computation of the VIX in 1993. Since 1993, the VIX is computed using S&P 500 option prices. Previously, it was based on S&P100 options. Note that the CBOE continues to calculate and disseminate the original-formula index, known as the CBOE S&P100 Volatility Index, with ticker VXO. This volatility series is sometimes useful because it has a price history going back to 1986.

The popularity of the VIX index has spawned the introduction of alternative volatility indexes in the U.S. and around the world. Table 1 provides an overview of VIX-like volatility indexes around the world. Table 1 also contains other option-implied indexes to be discussed below.

[Table 1: Volatility Indexes Around the World]

2.3 Comparison of Option-Implied Volatility Methods

A large number of studies test the ability of option-implied volatility in forecasting the future volatility of the underlying asset. An extensive review of the literature on this topic has been conducted by Poon and Granger (2003). Older studies covered in Poon and Granger (2003) typically used different combinations of $BSIV$s (e.g. ATM, vega-weighted, volume-weighted, etc), whereas more recent studies focus more on model-free estimates, $MFIV$. Overall, the evidence indicates that $BSIV$ and $MFIV$ are biased predictors of the future volatility of the underlying asset.

A plausible reason for the bias is that neither $BSIV$ or $MFIV$ takes into account the non-zero volatility risk premium in option prices. We will discuss this issue in more detail in Section 5. Poteshman (2000) and Chernov (2007) find that the bias disappears when volatility is extracted from option pricing models that allow for volatility risk premium such as Heston (1993), and when jump-diffusion models are used in forecasting. However, there is no evidence that the unbiased estimates outperform $BSIV$ or $MFIV$ in forecasting. Despite the bias, $BSIV$ and $MFIV$ continue to be preferred to the volatility estimate from Heston (1993) or jump-diffusion models in volatility forecasting.

Another possible source of bias is the fact that volatility is the square root of variance so that even if a particular variance forecast is unbiased, then its square root need not be an unbiased forecast of ex-post volatility because of the nonlinearity of the square root function. This is the bias arising from the approximation in (10).

Comparing $BSIV$ and $MFIV$ in their forecasting performance, two recent studies arrive at two opposite conclusions. Jiang and Tian (2005) find that $MFIV$ subsumes all information in $BSIV$ whereas Andersen and Bondarenko (2007) find the opposite result. Thus, there is no consensus on which one of these two predictors work better. It is likely that $BSIV$ will perform relatively well when only a few strike prices are available and when they are close to the current underlying asset price. The $MFIV$ measure is more demanding in terms of the richness of strike prices required in implementation which ultimately may hamper its performance if only a few strike prices are available or if the strike prices far from the current spot price are illiquid. In general, tightly parametric methods are likely to work well when the data is scarce, so long as the parametric assumptions are reasonable. Nonparametric methods are much more likely to do well in data rich environments. This logic is likely to hold in the various applications we consider below in this section and in subsequent sections as well.

2.4 Applications of Option-Implied Volatility Forecasts

In their survey, Poon and Granger (2003) consider papers that compare volatility forecasts from GARCH models and simple historical squared return averages with option-implied volatility forecasts. Out of 34 relevant studies, 26 found that option-implied volatility forecast outperformed simple historical averages. Out of 18 relevant studies, 17 found that option-implied volatility forecasts outperformed GARCH based forecast. The case for option-implied volatility forecasts thus seems strong at least when the alternative is historical models based on daily (or lower-frequency) returns only.

More recently, researchers have compared the forecasting power of option-implied versus realized volatility measures constructed from intraday returns. Table 2 reproduces results from Busch, Christensen, and Nielsen (2011), who regress total realized variance (RV) for the current month on the lagged daily, weekly and monthly realized variance, and subsequently use $BSIV^2$ as a regressor. Realized daily variance is computed using intraday returns. Panel A contains \$/DM FX data for 1987-1999, Panel B contains S&P 500 data for 1990-2002, and Panel C contains Treasury bond data for 1990-2002.

The results in Table 2 are striking. Option-implied variance has an adjusted R^2 of 40.7% for FX, 62.1% for S&P 500 and 35% for Treasury bond data. This compares with R^2 of 26%, 53% and 32.5% respectively for the RV based model.[11] The simple $BSIV^2$ forecast is thus able to compete with some of the most sophisticated historical return-based forecasts. The Treasury bond options contain wild-card features that increase the error in option-implied variance in this market. The fact that $BSIV^2$ performs worse in this case is therefore not surprising.

Table 2 also suggests that there is some scope for forecast combination between option-implied and return-based forecasts. The third row in each panel of Table 2 combines the RV and the $BSIV^2$ forecast. The increase in R^2 compared with the $BSIV^2$-only case is minor for FX, better for S&P 500 and largest for Treasury bond data. These results suggest that when the option data provide biased forecasts–perhaps due to risk premia or early exercise premia–then combining the option-implied forecast with return-based forecasts can be helpful.

[Table 2: Forecasting Realized Variance using Black-Scholes Implied Variance]

Table 3 contains a summary of existing empirical results. We now discuss these empirical results for different underlying assets.

[Table 3: Forecasting with Option-Implied Volatility]

2.4.1 Equity Market Applications

The main market of interest in volatility forecasting has been the equity market, particularly stock market indexes. Most studies find that the option-implied volatility contains useful information over traditional predictors based on historical prices, and that option-implied volatility by itself often outperforms historical volatility.

Almost all studies find that option-implied index volatility is useful in forecasting the volatility of the stock market index, a notable exception being Canina and Figlewski (1993). However,

[11] Busch et al. (2011) also consider forecasting variance using smooth RV and jump components as separate regressors. These models sometimes perform better than the basic RV models.

the evidence is mixed regarding the unbiasedness and efficiency of the option-implied estimates. Fleming, Ostdiek, and Whaley (1995), Fleming (1998), and Blair, Poon, and Taylor (2001) find that $BSIV$ is an efficient, but biased predictor, whereas Day and Lewis (1992) find that $BSIV$ is an unbiased, but inefficient predictor. Christensen and Prabhala (1998) find that $BSIV$ is unbiased and efficient. Busch et al. (2011) find that $BSIV$ is an efficient and unbiased predictor of equity index variance.

Jiang and Tian (2005) find that model-free option-implied volatility ($MFIV$) is biased, but efficient, subsuming all information in $BSIV$. Andersen and Bondarenko (2007) find a different result using a new measure of implied volatility, called Corridor IV (CIV). They compare the forecasting performance of the broad and narrow CIV, which are substitutes of the $MFIV$ and $BSIV$ respectively, and find that the narrow CIV ($BSIV$) is biased, but subsumes the predictive content of the broad CIV ($MFIV$).

Latané and Rendleman (1976), Chiras and Manaster (1978), Beckers (1981), and Lamoureux and Lastrapes (1993) find that $BSIV$ is useful in forecasting the volatility of individual stocks. Swidler and Wilcox (2002) focus on bank stocks, and find that $BSIV$ outperforms historical predictors.

Harvey and Whaley (1992) test if $BSIV$ itself can be predicted and find that $BSIV$ is predictable, but conclude that since arbitrage profits are not possible in the presence of transaction costs, this predictability is not inconsistent with market efficiency. Poon and Granger (2003) provide a comprehensive survey of volatility forecasting in general.

A few studies investigate if option-implied volatility can predict variables other than volatility, such as stock returns and bond spreads. Banerjee, Doran, and Peterson (2007) find that the VIX predicts returns on portfolios sorted on book-to-market equity, size, and beta. Doran, Fodor, and Krieger (2010) find that information in option prices leads analyst recommendation changes.

2.4.2 Other Markets

Fackler and King (1990) and Kroner, Kneafsey, and Claessens (1995) study the forecasting ability of implied volatility in commodity markets. Fackler and King (1990) compare the option-implied distribution with the empirical distribution of the futures prices of corn, live cattle, soybean, and hogs between 1985 and 1988. They find that there are differences among markets in terms of reliability of the option-implied distribution. The option-implied distribution matched the empirical distribution closely for corn and live kettle while the option-implied distribution over-predicted the variability of soybean prices and under-predicted the location of hog prices. Kroner et al. (1995) focuses on volatility forecasting over a 225-day horizon for cocoa, cotton, corn, gold, silver, sugar, and wheat prices. They find that a combination of time-series based predictors and option-implied volatility predicts better than either of the predictors alone.

For currencies, Jorion (1995) and Xu and Taylor (1995) find that $BSIV$ outperforms historical predictors. Pong, Shackleton, Taylor, and Xu (2004) compare $BSIV$ to predictors based on historical intraday data in currency markets, and find that historical predictors outperform $BSIV$ for one-day and one-week horizons, whereas $BSIV$ is at least as accurate as historical predictors for one-month and three-month horizons. Christoffersen and Mazzotta (2005) also find that the implied volatility yields unbiased and accurate forecast of exchange rate volatility.

Cao, Yu, and Zhong (2010) consider the use of option-implied forecasts of volatility for the purpose of credit default swap valuation.

As mentioned above, Busch et al. (2011) investigate assets in three different markets: the S&P 500, the currency market, using the USD/DM exchange rate, and the fixed income market, using the 30-year US Treasury bond. They find that the $BSIV^2$ contains incremental information about future variance in all three markets, relative to continuous and jump components of intraday prices. $BSIV^2$ is an efficient predictor in all three markets and is unbiased in foreign exchange and stock markets (see Table 2). Amin and Ng (1997) also find that implied volatility from Eurodollar futures options forecasts most of the realized interest rate volatility.

2.4.3 Forecasting the Cross-Section of Expected Stock Returns

Ang, Hodrick, Xing, and Zhang (2006) have a very different focus, investigating the performance of the VIX as a pricing factor: they find that the VIX is a priced risk factor with a negative price of risk, so that stocks with higher sensitivities to the innovation in VIX exhibit on average future lower returns. Delisle, Doran, and Peterson (2010) find that the result in Ang et al. (2006) holds when volatility is rising, but not when volatility is falling.

Diavatopoulos, Doran, and Peterson (2008) find that implied idiosyncratic volatility can forecast the cross-section of stock returns.

The cross-sectional forecasting exercises that use option-implied information are potentially very promising, but of course they are fundamentally different from the traditional use of option information in time-series forecasting.

2.5 Extracting Correlations from Option-Implied Volatilities

Certain derivatives contain very rich information on correlations between financial time series. This is especially the case for basket securities, written on a basket of underlying securities, such as collateralized debt obligations (CDOs). As mentioned in the introduction, because of space constraints we limit our survey to options.

We now discuss the extraction of information on correlations for two important security classes, currency and equity. In both cases, some additional assumptions need to be made. Despite the differences in assumptions, in both cases correlations are related to option-implied volatilities. This is not entirely surprising, as correlation can be thought of as a second co-moment. Implied correlation information on equities is particularly relevant, because equity as an asset class is critically important for portfolio management. Table 4 contains a summary of existing empirical results on the use of option-implied correlations in forecasting.

[Table 4: Forecasting with Option-Implied Correlation]

2.5.1 Extracting Correlations From Triangular Arbitrage

Using the U.S. dollar, \$, the British Pound, £, and the Japanese Yen, ¥, as an example, from triangular arbitrage in FX markets we know that

$$S_{\$/£} = S_{\$/¥} S_{¥/£}.$$

From this it follows that for log returns

$$R_{\$/£} = R_{\$/¥} + R_{¥/£}.$$

From this we get that

$$VAR_{\$/£} = VAR_{\$/¥} + VAR_{¥/£} + 2COV(R_{\$/¥}, R_{¥/£})$$

so that the correlation must be

$$CORR(R_{\$/¥}, R_{¥/£}) = \frac{\left(VAR_{\$/£} - VAR_{\$/¥} - VAR_{¥/£}\right)}{2VAR_{\$/¥}^{1/2} VAR_{¥/£}^{1/2}}.$$

Provided we have option-implied variance forecasts for the three currencies, we can use this to get an option-implied covariance forecast. See Walter and Lopez (2000) and Campa and Chang (1998) for applications.

Siegel (1997) finds that option-implied exchange rate correlations for the DM/GBP pair and the DM/JPY pair predict significantly better than historical correlations between 1992 and 1993. Campa and Chang (1998) also find that the option-implied correlation for USD/DM/JPY predicts better than historical correlations between 1989 and 1995. The evidence in Walter and Lopez (2000), however, is mixed. They find that the option-implied correlation is useful for USD/DM/JPY (1990-1997), but much less useful for USD/DM/CHF (1993-1997), and conclude that the option-implied correlation may not be worth calculating in all instances.

Correlations have been extracted from options in fixed income markets. Longstaff, Santa-Clara, and Schwartz (2001) and de Jong et al. (2004) provide evidence that forward rate correlations implied by cap and swaption prices differ from realized correlations.

2.5.2 Extracting Average Correlations Using Index and Equity Options

Skintzi and Refenes (2005) and Driessen, Maenhout, and Vilkov (2009) propose the following measure of average option-implied correlation between the stocks in an index, I,

$$\rho_{ICI} = \frac{VAR_I - \sum_{j=1}^{n} w_j^2 VAR_j}{2\sum_{j=1}^{n-1}\sum_{i>j}^{n} w_i w_j VAR_i^{1/2} VAR_j^{1/2}} \tag{19}$$

where w_j denotes the weight of stock j in the index.

Note that the measure is based on the option-implied variance for the index, VAR_I, and the individual stock variances, VAR_j. Skintzi and Refenes (2005) use options on the DJIA index and its constituent stocks between 2001 and 2002, and find that the implied correlation index is biased upward, but is a better predictor of future correlation than historical correlation. Buss and Vilkov (2011) use the implied correlation approach to estimate option-implied betas and find that the option-implied betas predict realized betas well. DeMiguel, Plyakha, Uppal, and Vilkov (2011) use option-implied information in portfolio allocation. They find that option-implied volatility and correlation do not improve the Sharpe ratio or certainty-equivalent return of the optimal portfolio. However, expected returns estimated using information in the volatility risk premium and option-implied skewness increase both the Sharpe ratio and the certainty-equivalent return substantially. The CBOE has recently introduced an Implied Correlation Index (ICI) for S&P 500 firms based on (19).

3 Extracting Skewness and Kurtosis from Option Prices

The $BSIV$ smirk patterns in Figure 2 revealed that index options imply negative skewness not captured by the normal distribution. As discussed in Rubinstein (1994), prior to 1987, this pattern more closely resembled a symmetric "smile". Rubinstein (1985) documents systematic deviations from Black-Scholes for individual equity options. Other underlying assets such as foreign exchange rates often display symmetric smile patterns in $BSIV$ implying evidence of excess kurtosis rather than negative skewness. In this section we consider methods capable of generating option-implied measures of skewness and kurtosis which can be used as forecasts for subsequent realized skewness and kurtosis. We will begin with model-free methods for higher moment forecasting because they are the most common.

3.1 Model-Free Skewness and Kurtosis Extraction

This section first develops the general option replication approach for which higher-moment extraction is a special case. We will then briefly consider other approaches.

3.1.1 The Option Replication Approach

Building on Breeden and Litzenberger (1978), Bakshi and Madan (2000) and Carr and Madan (2001) show that any twice continuously differentiable function, $H(S_T)$, of the terminal stock price S_T, can be replicated (or spanned) by a unique position of risk-free bonds, stocks and European options. Let $H(S_0) - H'(S_0)\,S_0$ denote units of the risk-free discount bond, which of course is independent of S_T. Let $H'(S_0)$ denote units of the underlying risky stock, which is trivially linear in S_T, and let $H''(X)\,dX$ denote units of (nonlinear) out-of-the-money call and put options with strike price X, then we have

$$H(S_T) = \left[H(S_0) - H'(S_0)S_0\right] + H'(S_0)S_T + \int_0^{S_0} H''(X)\max(X - S_T, 0)\,dX + \int_{S_0}^{\infty} H''(X)\max(S_T - X, 0)\,dX \tag{20}$$

This result is clearly very general. A derivation can be found in Carr and Madan (2001). From a forecasting perspective, for any desired function $H(\cdot)$ of the future realization S_T there is a portfolio of risk-free bonds, stocks, and options whose current aggregate market value provides an option-implied forecast of $H(S_T)$.

Let the current market value of the bond be e^{-rT}, and the current put and call prices be $P_0(T,X)$ and $C_0(T,X)$ respectively, then we have

$$E_0\left[e^{-rT}H(S_T)\right] = e^{-rT}\left[H(S_0) - H'(S_0)S_0\right] + S_0H'(S_0) + \int_0^{S_0} H''(X)P_0(T,X)\,dX + \int_{S_0}^{\infty} H''(X)C_0(T,X)\,dX \tag{21}$$

Bakshi, Kapadia, and Madan (2003) (BKM hereafter) apply this general result to the second, third, and fourth power of log returns. To provide added intuition for the replicating option portfolios we consider higher moments of simple returns instead where $H(S_T) = \left(\frac{S_T - S_0}{S_0}\right)^2$, $H(S_T) = \left(\frac{S_T - S_0}{S_0}\right)^3$, and $H(S_T) = \left(\frac{S_T - S_0}{S_0}\right)^4$.

We can use OTM European call and put prices to derive the quadratic contract as[12]

$$M_{0,2}(T) \equiv E_0\left[e^{-rT}\left(\frac{S_T - S_0}{S_0}\right)^2\right] = \frac{2}{S_0^2}\left[\int_0^{S_0} P_0(T,X)\,dX + \int_{S_0}^{\infty} C_0(T,X)\,dX\right]. \tag{22}$$

The cubic contract is given by

$$M_{0,3}(T) \equiv E_0\left[e^{-rT}\left(\frac{S_T - S_0}{S_0}\right)^3\right] = \frac{6}{S_0^2}\left[\int_0^{S_0}\left(\frac{X - S_0}{S_0}\right)P_0(T,X)\,dX + \int_{S_0}^{\infty}\left(\frac{X - S_0}{S_0}\right)C_0(T,X)\,dX\right] \tag{23}$$

and the quartic contract is given by

$$M_{0,4}(T) \equiv E_0\left[e^{-rT}\left(\frac{S_T - S_0}{S_0}\right)^4\right] = \frac{12}{S_0^2}\left[\int_0^{S_0}\left(\frac{X - S_0}{S_0}\right)^2 P_0(T,X)\,dX + \int_{S_0}^{\infty}\left(\frac{X - S_0}{S_0}\right)^2 C_0(T,X)\,dX\right] \tag{24}$$

Notice how the quadratic contract–which is key for volatility–simply integrates over option price levels. The cubic contract–which is key for skewness–integrates over option prices multiplied by moneyness, $\frac{X - S_0}{S_0} = \frac{X}{S_0} - 1$. The quartic contract–which is key for kurtosis–integrates over the

[12] When using log returns instead, we get

$$M_{0,2}(T) = \int_0^{S_0} \frac{2\left(1 + \ln\left[S_0/X\right]\right)}{X^2} P(T,X)\,dX + \int_{S_0}^{\infty} \frac{2\left(1 - \ln\left[X/S_0\right]\right)}{X^2} C(T,X)\,dX.$$

option prices multiplied by moneyness squared. High option prices imply high volatility. High OTM put prices and low OTM call prices imply negative skewness (and vice versa). High OTM call and put prices at extreme moneyness imply high kurtosis.

We can now compute the option-implied volatility, skewness, and kurtosis (for convenience we suppress the dependence of M on T) as

$$VOL_0(T) \equiv [VAR_0(T)]^{1/2} = \left[e^{rT}M_{0,2} - M_{0,1}^2\right]^{1/2} \tag{25}$$

$$SKEW_0(T) = \frac{e^{rT}M_{0,3} - 3M_{0,1}e^{rT}M_{0,2} + 2M_{0,1}^3}{\left[e^{rT}M_{0,2} - M_{0,1}^2\right]^{\frac{3}{2}}} \tag{26}$$

$$KURT_0(T) = \frac{e^{rT}M_{0,4} - 4M_{0,1}e^{rT}M_{0,3} + 6e^{rT}M_{0,1}^2M_{0,2} - 3M_{0,1}^4}{\left[e^{rT}M_{0,2} - M_{0,1}^2\right]^2} \tag{27}$$

where[13]

$$M_{0,1} \equiv E_0\left[\left(\frac{S_T - S_0}{S_0}\right)\right] = e^{rT} - 1 \tag{28}$$

BKM provide a model-free measure of implied variance, like Britten-Jones and Neuberger (2000) in equation (12). BKM compute the variance of the holding period return, whereas Britten-Jones and Neuberger (2000) compute the expected value of realized variance. These concepts of volatility will coincide if the log returns are zero mean and uncorrelated.

Using S&P 500 index options from January 1996 through September 2009, Figure 4 plots the higher moments of log returns for the one-month horizon using the BKM approach.

[Figure 4: Option-Implied Moments for One-Month S&P 500 Returns]

Not surprisingly, the volatility series is very highly correlated with the VIX index, with a correlation of 0.997 between January 1996 and September 2009. The annualized volatility varied between approximately 0.1 and 0.4 before the subprime crisis of 2008, but its level shot up to an unprecedented level of around 0.8 at the onset of the crisis, subsequently reverting back to its previous level by late 2009. The estimate of skewness is negative for every day in the sample, varying between minus three and zero. Interestingly, skewness did not dramatically change during the subprime crisis, despite the fact that option-implied skewness is often interpreted as the probability of a market crash or the fear thereof. The estimate of kurtosis is higher than three (i.e. excess kurtosis) for every day in the sample, indicating that the option-implied distribution has fatter tails than the normal distribution. Its level did not dramatically change during the sub-prime crisis, but the time series exhibits more day-to-day variation during this period.

The estimation of skewness and kurtosis using the BKM method is subject to the same concerns discussed by Jiang and Tian (2005, 2007), in the context of volatility estimation. Chang, Christoffersen, Jacobs, and Vainberg (2012) present Monte Carlo evidence on the quality of skewness estimates when only discrete strike prices are available. Fitting a spline through the implied

[13] Throughout the chapter $KURT$ denotes raw and not excess kurtosis.

volatilities and integrating the spline, following the methods of Jiang and Tian (2005, 2007), seems to work well for skewness too, and dominates simple integration using only observed contracts.

In February 2011, the CBOE began publishing the CBOE S&P 500 Skew Index. The Skew Index is computed using the methodology in BKM described in this section combined with the interpolation/extrapolation method used in the VIX calculation described in Section 2.2.2. See CBOE (2011) for details.

3.1.2 Other Model-Free Measures of Option-Implied Skewness

Many empirical studies on option-implied skewness use the asymmetry observed in the implied volatility curve in Figure 2, often referred to as the smirk, to infer skewness of the option-implied distribution. There are many variations in the choice of options used to measure the asymmetry of the implied volatility curve. The most popular method involves taking the difference of the out-of-the-money put $BSIV$ and out-of-the-money call $BSIV$. This measure, proposed by Bates (1991), reflects the different extent to which the left-hand tail and the right-hand tail of the option-implied distribution of the underlying asset price deviate from the lognormal distribution. Another approach is to take the difference between the out-of-the-money put $BSIV$ and at-the-money put (or call) $BSIV$ as in Xing, Zhang, and Zhao (2010). This measure only looks at the left-hand side of the distribution, and is often used in applications where the downside risk of the underlying asset is the variable of interest. Another variable that is also shown to be somewhat related to implied skewness is the spread of implied volatility of call and put options with the same maturity and same strike (Cremers and Weinbaum (2010) and Bali and Hovakimian (2009)).

Recently, Neuberger (2011) has proposed a model-free method that extends the variance swap methodology used to compute the VIX index. He shows that just as there is a model-free strategy to replicate a variance swap, a contract that pays the difference between option-implied variance and realized variance, there is also a model-free strategy to replicate a skew swap, a contract that pays the difference between option-implied skew and realized skew.

3.2 Model-Based Skewness and Kurtosis Extraction

In this section we first review two models that are based on expansions around the Black-Scholes model explicitly allowing for skewness and kurtosis. We then consider an alternative model-based approach specifying jumps in returns which imply skewness and kurtosis.

3.2.1 Expansions of the Black-Scholes Model

Jarrow and Rudd (1982) propose an option pricing method where the density of the security price at option maturity, T, is approximated by an alternative density using the Edgeworth series expansion. If we choose the lognormal as the approximating density, and use the shorthand notation for the

Black-Scholes model

$$C_0^{BS}(T,X) \equiv C^{BS}\left(T,X,S_0,r;\sigma^2\right)$$

then the Jarrow-Rudd model is defined by

$$C_0^{JR}(T,X) \approx C_0^{BS}(T,X) - e^{-rT}\frac{(K_3 - K_3(\Psi))}{3!}\frac{d\psi(T,X)}{dX} + e^{-rT}\frac{(K_4 - K_4(\Psi))}{4!}\frac{d^2\psi(T,X)}{dX^2} \tag{29}$$

where K_j is the jth cumulant of the actual density, $K_j(\Psi)$ is the cumulant of the lognormal density, $\psi(T,X)$, so that

$$\begin{aligned}
\psi(T,X) &= \left(X\sigma\sqrt{T2\pi}\right)^{-1}\exp\left\{-\frac{1}{2}\left(d-\sigma\sqrt{T}\right)^2\right\} \\
\frac{d\psi(T,X)}{dX} &= \frac{\psi(T,X)\left(d-2\sigma\sqrt{T}\right)}{X\sigma\sqrt{T}} \\
\frac{d^2\psi(T,X)}{dX^2} &= \frac{\psi(T,X)}{X^2\sigma^2 T}\left[\left(d-2\sigma\sqrt{T}\right)^2 - \sigma\sqrt{T}\left(d-2\sigma\sqrt{T}\right)-1\right]
\end{aligned}$$

and where d is as defined in (4).

In general we have the following relationships between cumulants and moments

$$K_2 = VAR, \qquad K_3 = K_2^{3/2}SKEW, \qquad K_4 = K_2^2(KURT-3)$$

For the lognormal density we have the following moments

$$\begin{aligned}
VAR(\Psi) = K_2(\Psi) &= \exp\left(2\left(\ln(S_0) + \left(r - \frac{1}{2}\sigma^2\right)T\right) + \sigma^2 T\right)\left(\exp\left(\sigma^2 T\right) - 1\right) \\
SKEW(\Psi) = K_3(\Psi)/K_2^{3/2}(\Psi) &= \left(\exp\left(\sigma^2 T\right)+2\right)\sqrt{\exp\left(\sigma^2 T\right)-1} \\
KURT(\Psi) = K_4(\Psi)/K_2^2(\Psi) + 3 &= \exp\left(4\sigma^2 T\right) + 2\exp\left(3\sigma^2 T\right) + 3\exp\left(2\sigma^2 T\right) - 3
\end{aligned}$$

These expressions can be solved for the cumulants, $K_3(\Psi)$ and $K_4(\Psi)$, which are needed in equation (29) above.

The Jarrow-Rudd model in (29) now has three parameters left to estimate, namely, σ, K_3, and K_4 or equivalently σ, $SKEW$ and $KURT$. In principle these three parameters could be solved for using three observed option prices. These parameters would then provide option-implied forecasts of volatility, skewness and kurtosis in the distribution of $\ln(S_T)$. Alternatively they could be estimated by minimizing the option valuation errors on a larger set of observed option prices. Christoffersen and Jacobs (2004) discuss the choice of objective function in this type of estimation problems.

As an alternative to the Edgeworth expansion, Corrado and Su (1996) consider a Gram-Charlier series expansion,[14] in which

$$C_0^{CS}(T,X) = C_0^{BS}(T,X) + Q_3 SKEW + Q_4(KURT-3) \tag{30}$$

[14] See also Backus, Foresi, Li, and Wu (1997).

where

$$Q_3 = \frac{1}{3!} S_0 \sigma \sqrt{T} \left(\left(2\sigma\sqrt{T} - d \right) N'(d) + \sigma^2 T N(d) \right);$$
$$Q_4 = \frac{1}{4!} S_0 \sigma \sqrt{T} \left(\left(d^2 - 1 - 3\sigma\sqrt{T} \left(d - \sigma\sqrt{T} \right) \right) N'(d) + \sigma^3 T^{3/2} N(d) \right)$$

where $N'(d)$ is again the standard normal probability density function. Note that Q_4 and Q_3 represent the marginal effect of skewness and kurtosis respectively and note that d is as defined in (4). In the Corrado-Su model $SKEW$ and $KURT$ refer to the distribution of log return shocks defined by

$$Z_T = \left[\ln S_T - \ln(S_0) - \left(r - \frac{1}{2}\sigma^2 \right) T \right] / \left(\sigma\sqrt{T} \right)$$

Again, option-implied volatility, skewness and kurtosis can be estimated by minimizing the distance between $C_0^{CS}(T, X)$ and a sample of observed option prices or by directly solving for the three parameters using just three observed option prices.

3.2.2 Jumps and Stochastic Volatility

While the Black and Scholes (1973) and stochastic volatility option pricing models are often used to extract volatility, the study of higher moments calls for different models. The Black-Scholes model assumes normality, and therefore strictly speaking cannot be used to extract skewness and kurtosis from the data, although patterns in Black-Scholes implied volatility are sometimes used to learn about skewness.

Stochastic volatility models such as Heston (1993) can generate skewness and excess kurtosis, but fall short in reconciling the stylized facts on physical higher moments with the dynamics of higher option-implied moments (Bates (1996b) and Pan (2002)). Instead, generalizations of the Black and Scholes (1973) and Heston (1993) setup are often used, such as the jump-diffusion model of Bates (1991) and the stochastic volatility jump-diffusion (SVJ) model of Bates (1996b).

In Bates (2000), the futures price F is assumed to follow a jump-diffusion of the following form

$$\begin{aligned} dF/F &= -\lambda \bar{k} dt + \sqrt{V} dz_1 + k dq, \\ dV &= \kappa(\theta - V) dt + \sigma_V \sqrt{V} dz_2 \end{aligned} \tag{31}$$

where q is a Poisson counter with instantaneous intensity λ, and where k is a lognormally distributed return jump

$$\ln(1+k) \sim N\left[\ln\left(1+\bar{k}\right) - \delta^2/2, \delta^2 \right]$$

As in Heston (1993), the return and variance diffusion terms are correlated with coefficient ρ.

Bates (2000) derives the n^{th} cumulant for horizon T to be

$$K_n(T) = \left[\frac{\partial^n A(T;\Phi)}{\partial \Phi^n} + \frac{\partial^n B(T;\Phi)}{\partial \Phi^n} V \right]_{\Phi=0} + \lambda T \left[\frac{\partial^n C(\Phi)}{\partial \Phi^n} \right]_{\Phi=0}$$

where

$$
\begin{aligned}
A(T;\Phi) &= -\frac{\kappa\theta T}{\sigma_V^2}\left(\rho\sigma_V\Phi-\kappa-D(\Phi)\right)-\frac{2\kappa\theta}{\sigma_V^2}\ln\left[1+\frac{1}{2}\left(\rho\sigma_V\Phi-\kappa-D(\Phi)\right)\frac{1-e^{D(\Phi)T}}{D(\Phi)}\right],\\
B(T;\Phi) &= \frac{-\left[\Phi^2-\Phi\right]}{\rho\sigma_V\Phi-\kappa+D(\Phi)\left(\frac{1+e^{D(\Phi)T}}{1-e^{D(\Phi)T}}\right)},\ \text{and}\\
C(\Phi) &= \left[\left(1+\overline{k}\right)^{\Phi}e^{\frac{1}{2}\delta^2\left[\Phi^2-\Phi\right]}-1\right]-\overline{k}\Phi,\ \text{and where}\\
D(\Phi) &= \sqrt{\left(\rho\sigma_V\Phi-\kappa\right)^2-2\sigma_V^2\left\{\frac{1}{2}\left[\Phi^2-\Phi\right]\right\}},
\end{aligned}
$$

From the cumulants we can compute the conditional moments for the log futures returns for holding period T using

$$
VAR_0(T)=K_2(T),\quad SKEW_0(T)=K_3(T)/K_2^{3/2}(T),\quad KURT_0(T)=K_4(T)/K_2^2(T)+3.
$$

Besides higher moments such as skewness and kurtosis, this model yields parameters describing the intensity and size of jumps, which can potentially be used to forecast jump-like events such as stock market crashes and defaults.

There is an expanding literature estimating models like (31) as well as more general models with jumps in volatility using returns and/or options. See for instance Bates (2000), Bates (2008), Andersen, Benzoni, and Lund (2002), Pan (2002), Huang and Wu (2004), Eraker, Johannes, and Polson (2003), Broadie, Chernov, and Johannes (2009), Li, Wells, and Yu (2008), and Chernov, Gallant, Ghysels, and Tauchen (2003).

3.3 Comparison of Methods

The list of studies on option-implied skewness and kurtosis is much shorter than those on option-implied volatility. In fact, we do not know of any study that compares different estimation methods. The real challenge in evaluating these methods is the fact that we do not have a good estimate of ex-post realized skewness and kurtosis because higher moments are hard to estimate accurately from time series of prices or returns. For example, the estimates of skewness and kurtosis of the S&P 500 return will be markedly different if we use a sample period that contains a stock market crash compared to one that does not. Without a proper realized measure, it is difficult to compare different methods empirically.

The method that is used most widely at the moment is the model-free methodology of Bakshi et al. (2003). This method has the advantage of being model-free and simple to implement. However, it is sensitive to the choice of extrapolation method applied to the tails of the distribution where there is no traded option price. Another limitation of this method is that the moments are risk-neutral, so the estimates are likely to be biased.

Model-based methods such as Heston (1993) and Bates (2000) allow us to compute physical moments and risk-neutral moments separately. Therefore, the physical moment estimates from these models are potentially unbiased although they can still be biased if the model is misspecified. The disadvantages of these methods are that they are restricted by the model specifications and that they are harder to implement.

3.4 Applications of Option-Implied Skewness and Kurtosis Forecasts

As discussed in Section 2.4, many studies use option-implied volatility to forecast the volatility of the underlying asset. A few studies have used option-implied skewness and kurtosis to forecast the returns on the underlying, as well as cross-sectional differences in stock returns. Table 5 contains a summary of existing empirical results.

[Table 5: Forecasting with Option-Implied Skewness and Kurtosis]

3.4.1 Time Series Forecasting

Bates (1991) investigates the usefulness of jump parameters estimated using a jump diffusion model for forecasting the stock market crash of 1987. He also forecasts using a skewness premium constructed from prices of out-of-the-money puts and calls. Bates (1996a) examines option-implied skewness and kurtosis of the USD/DM and USD/JPY exchange rates between 1984 and 1992, and finds that the option-implied higher moments contain significant information for the future USD/DM exchange rate, but not for the USD/JPY rate. The option-implied higher moments are again estimated both using a model-based approach, using a jump-diffusion dynamic, but also using a model-free measure of the skewness premium.

Navatte and Villa (2000) extract option-implied moments for the CAC 40 index using the Gram-Charlier expansion. They find that the moments contain a substantial amount of information for future moments, with kurtosis contributing less forecasting power than skewness and volatility.

Carson, Doran, and Peterson (2006) find that the implied volatility skew has strong predictive power in forecasting short-term market declines. However, Doran, Peterson, and Tarrant (2007) find that the predictability is not economically significant.

For individual stocks, Diavatopoulos, Doran, Fodor, and Peterson (2012) look at changes in implied skewness and kurtosis prior to earnings announcements and find that both have strong predictive power for future stock and option returns. DeMiguel et al. (2011) propose using implied volatility, skewness, correlation and variance risk premium in portfolio selection, and find that the inclusion of skewness and the variance risk premium improves the performance of the portfolio significantly.

3.4.2 Forecasting the Cross-Section of Expected Stock Returns

Two recent studies investigate if option-implied higher moments of the S&P 500 index help explain the subsequent cross-section of returns. Chang, Christoffersen, and Jacobs (2009) test the cross-section of all stocks in the CRSP database, whereas Agarwal, Bakshi, and Huij (2009) investigate returns on the cross-section of hedge fund returns. Both studies use the model-free moments of BKM described in Section 3.1. Both studies find strong evidence that stocks with higher sensitivity to the innovation in option-implied skewness of the S&P 500 index exhibit lower returns in the future. Agarwal et al. (2009) also find a positive relationship between a stock's sensitivity to innovations in option-implied kurtosis of the S&P 500 index and future returns.

Several recent studies find a cross-sectional relationship between the option-implied skew of individual stocks and their subsequent returns. Xing et al. (2010) define skew as the difference in implied volatilities between out-of-the-money puts and at-the-money calls. They find that steeper smirks are associated with lower future stock returns. Doran and Krieger (2010) decompose the volatility skew into several components. They find that future stock returns are positively related to the difference in volatilities between at-the-money calls and puts, and negatively related to a measure of the left skew of the implied volatility curve. These results are consistent with those found in Cremers and Weinbaum (2010), Bali and Hovakimian (2009), and Xing et al. (2010). More importantly, the results in Doran and Krieger (2010) indicate that different measures of implied skewness can lead to different empirical results on the relationship between implied skewness and the cross-section of future stock returns.

Conrad, Dittmar, and Ghysels (2009) and Rehman and Vilkov (2010) both use the model-free skewness of Bakshi et al. (2003), but report conflicting results on the relationship between implied skewness and the cross-section of future stock returns. Conrad et al. (2009) find a negative relationship while Rehman and Vilkov (2010) find a positive one. One difference between these two empirical studies is that Conrad et al. (2009) use average skewness over the last three months whereas Rehman and Vilkov (2010) use skewness measures computed only on the last available date of each month. Again, these conflicting results indicate that the relationship between equity skews and the cross-section of future stocks returns is sensitive to variations in empirical methodology.

3.4.3 Option-Implied Betas

Section 2.5 above documents how option-implied correlation can be extracted from option data. Given the assumptions, correlations are a function of option-implied volatilities. Chang et al. (2012) provide an alternative approach, assuming that firm-specific risk has zero skewness. In this case it is possible to derive an option-implied beta based on the option-implied moments of firm j and the market index I as follows

$$\beta_j = \left(\frac{SKEW_j}{SKEW_I}\right)^{1/3} \left(\frac{VAR_j}{VAR_I}\right)^{1/2}, \tag{32}$$

where VAR and $SKEW$ can be computed from index options and from equity options for firm j using (25) and (26). Chang et al. (2012) find that, similar to the evidence for implied volatilities, historical betas and option-implied betas both contain useful information for forecasting future betas.

4 Extracting Densities from Option Prices

There are many surveys on density forecasting using option prices. See Söderlind and Svensson (1997), Galati (1999), Jackwerth (1999), Jondeau and Rockinger (2000), Bliss and Panigirtzoglou (2002), Rebonato (2004), Taylor (2005), Bu and Hadri (2007), Jondeau, Poon, and Rockinger (2007), Figlewski (2010), Fusai and Roncoroni (2008), and Markose and Alentorn (2011). We describe the details of only a few of the most popular methods in this section, and refer the readers interested in the details of other methods to these surveys. We start by discussing model-free estimation, and subsequently discuss imposing more structure on the problem using no-arbitrage restrictions or parametric models.

4.1 Model-Free Estimation

Breeden and Litzenberger (1978) and Banz and Miller (1978) show that the option-implied density of a security can be extracted from a set of European-style option prices with a continuum of strike prices. This result can be derived as a special case of the general replication result in (20), see for instance Carr and Madan (2001).

The value of a European call option, C_0, is the discounted expected value of its payoff on the expiration date T. Under the option-implied measure, $f_0(S_T)$, the payoff is discounted at the risk-free rate

$$C_0(T,X) = e^{-rT}\int_0^{\infty} \max\{S_T - X, 0\}\, f_0(S_T)\, dS_T = e^{-rT}\int_X^{\infty} (S_T - X)\, f_0(S_T)\, dS_T \tag{33}$$

We can take the partial derivative of C_0 with respect to the strike price X to get

$$\frac{\partial C_0(T,X)}{\partial X} = -e^{-rT}\left[1 - \tilde{F}_0(X)\right], \tag{34}$$

which yields the cumulative distribution function (CDF)

$$\tilde{F}_0(X) = 1 + e^{rT}\frac{\partial C_0(T,X)}{\partial X} \text{ so that } \tilde{F}_0(S_T) = 1 + e^{rT}\left.\frac{\partial C_0(T,X)}{\partial X}\right|_{X=S_T}. \tag{35}$$

The conditional probability density function (PDF) denoted by $f_0(X)$ can be obtained by taking the derivative of (35) with respect to X.

$$f_0(X) = e^{rT}\frac{\partial^2 C_0(T,X)}{\partial X^2} \text{ so that } f_0(S_T) = e^{rT}\left.\frac{\partial^2 C_0(T,X)}{\partial X^2}\right|_{X=S_T} \tag{36}$$

As noted above, the put-call parity states that $S_0 + P_0 = C_0 + Xe^{-rT}$, so that if we use put option prices instead, we get

$$\tilde{F}_0(S_T) = e^{rT}\left.\frac{\partial P_0(T,X)}{\partial X}\right|_{X=S_T} \text{ and } f_0(S_T) = e^{rT}\left.\frac{\partial^2 P_0(T,X)}{\partial X^2}\right|_{X=S_T}. \tag{37}$$

In practice, we can obtain an approximation to the CDF in (35) and (37) using finite differences of call or put option prices observed at discrete strike prices

$$\tilde{F}_0(X_n) \approx 1 + e^{rT}\left(\frac{C_0(T, X_{n+1}) - C_0(T, X_{n-1})}{X_{n+1} - X_{n-1}}\right) \tag{38}$$

or

$$\tilde{F}_0(X_n) \approx e^{rT}\left(\frac{P_0(T, X_{n+1}) - P_0(T, X_{n-1})}{X_{n+1} - X_{n-1}}\right). \tag{39}$$

Similarly, we can obtain an approximation to the PDF in (36) and (37) via

$$f_0(X_n) \approx e^{rT}\frac{C_0(T, X_{n+1}) - 2C_0(T, X_n) + C_0(T, X_{n-1})}{(\Delta X)^2} \tag{40}$$

$$f_0(X_n) \approx e^{rT}\frac{P_0(T, X_{n+1}) - 2P_0(T, X_n) + P_0(T, X_{n-1})}{(\Delta X)^2}. \tag{41}$$

In terms of the log return, $R_T = \ln S_T - \ln S_0$, the CDF and PDF are

$$\tilde{F}_{0,R_T}(x) = F_0\left(e^{x+\ln S_0}\right) \qquad \text{and} \qquad f_{0,R_T}(x) = e^{x+\ln S_0} f_0\left(e^{x+\ln S_0}\right).$$

The most important constraint in implementing this method is that typically only a limited number of options are traded in the market. This approximation method can therefore only yield estimates of the CDF and the PDF at a few points in the domain. This constraint has motivated researchers to develop various ways of imposing more structure on the option-implied density. In some cases the additional structure exclusively derives from no-arbitrage restrictions, in other cases a parametric model is imposed. We now survey these methods in increasing order of structure imposed.

4.2 Imposing Shape Restrictions

Aït-Sahalia and Duarte (2003) propose a model-free method of option-implied density estimation based on local polynomial regressions that incorporates shape restrictions on the first and the second derivatives of the call pricing function. Again, let $f_0(S_T)$ be the conditional density, then the call option prices are

$$C_0(T, X) = e^{-rT}\int_0^{+\infty} \max(S_T - X, 0)\, f_0(S_T)\, dS_T$$

By differentiating the call price C with respect to the strike X, we get

$$\frac{\partial C_0(T, X)}{\partial X} = -e^{-rT}\int_X^{+\infty} f_0(S_T)\, dS_T.$$

Since $f_0(S_T)$ is a probability density, it is positive and integrates to one. Therefore,

$$-e^{-rT} \le \frac{\partial C_0(T, X)}{\partial X} \le 0. \tag{42}$$

By differentiating the call price twice with respect to the strike price, we obtain as before

$$\frac{\partial^2 C_0(T,X)}{\partial X^2} = e^{-rT} f_0\left(X\right) \geq 0. \tag{43}$$

Two additional restrictions can be obtained using standard no arbitrage bounds of the call option prices,

$$\max\left(0, S_0 - Xe^{-rT}\right) \leq C_0\left(T,X\right) \leq S_0.$$

Li and Zhao (2009) develop a multivariate version of the constrained locally polynomial estimator in Aït-Sahalia and Duarte (2003) and apply it to interest rate options.

4.3 Using Black-Scholes Implied Volatility Functions

The simple but flexible Ad-Hoc Black-Scholes ($AHBS$) model in which the density forecast is constructed from Black-Scholes implied volatility curve fitting is arguably the most widely used method for option-implied density forecasting, and we now describe it in some more detail. The density construction proceeds in two steps.

First, we estimate a second-order polynomial or other well-fitting function for implied Black-Scholes volatility as a function of strike and maturity. This will provide the following fitted values for $BSIV$. We can write

$$BSIV\left(T,X\right) = a_0 + a_1 X + a_2 X^2 + a_3 T + a_4 T^2 + a_5 XT \tag{44}$$

Second, using this estimated polynomial, we generate a set of fixed-maturity implied volatilities across a grid of strikes. Call prices can then be obtained using the Black-Scholes functional form

$$C_0^{AHBS}\left(T,X\right) = C_0^{BS}\left(T, X, S_0, r; BSIV\left(T,X\right)^2\right). \tag{45}$$

Once the model call prices are obtained the option-implied density can be obtained using the second derivative with respect to the strike price.

$$f_0\left(S_T\right) = e^{rT} \left.\frac{\partial^2 C_0^{AHBS}\left(T,X\right)}{\partial X^2}\right|_{X=S_T}.$$

Shimko (1993) was the first to propose this approach to constructing density forecasts from smoothed and interpolated $BSIV$s. Many variations on the Shimko approach have been proposed in the literature, and strictly speaking most of these are not entirely model-free, because some parametric assumptions are needed. The differences between these variations mainly concern three aspects of the implementation (See Figlewski (2010) for a comprehensive review):

1. Choice of independent variable: the implied volatility function can be expressed as a function of strike (X), or of moneyness (X/S), or of option delta. See Malz (1996).

2. Choice of interpolation method: implied volatilities can be interpolated using polynomials (Shimko (1993) and Dumas, Fleming, and Whaley (1998)) or splines, which can be quadratic, cubic (Bliss and Panigirtzoglou (2002)) or quartic (Figlewski (2010)). Malz (1997), Rosenberg (1998), Weinberg (2001), and Monteiro, Tutuncu, and Vicente (2008) propose different alternatives.

3. Choice of extrapolation method: for strikes beyond the range of traded options, one can use extrapolation (Jiang and Tian (2005), Jiang and Tian (2007)), truncation (Andersen and Bondarenko (2007), Andersen and Bondarenko (2009) and Andersen et al. (2011)), or alternatively a parametric density function can be used. For instance Figlewski (2010) and Alentorn and Markose (2008) propose the Generalized Extreme Value distribution. Lee (2004), Benaim and Friz (2008), and Benaim and Friz (2009) derive restrictions on the slope of the implied volatility curve at tails based on the slope's relationship to the moments of the distribution.

Figure 5 shows the CDF and PDF obtained when applying a smoothing cubic spline using $BSIV$ data on 30-day OTM calls and puts on the S&P 500 index on October 22, 2009 together with the CDF and PDF of the lognormal distribution. The model-free estimate of the option-implied distribution is clearly more negatively skewed than the lognormal distribution. Note that we only depict the distribution for available strike prices and thus do not extrapolate beyond the lowest and highest strikes available.

[Figure 5: Option-Implied Distribution from BSIV Curve Fitting vs. Lognormal]

Related approaches are proposed by Madan and Milne (1994), who use Hermite polynomials, and Abadir and Rockinger (2003), who propose the use of hypergeometric functions. Empirical studies using these approaches include Abken, Madan, and Ramamurtie (1996), Jondeau and Rockinger (2001), Flamouris and Giamouridis (2002), Rompolis and Tzavalis (2008), and Giacomini, Härdle, and Krätschmer (2009).

Many alternative approaches have been proposed including 1) Implied binomial trees (Rubinstein (1994)) and its extensions (Jackwerth (1997), Jackwerth and Rubinstein (1996), Jackwerth (2000), and Dupont (2001)); 2) Entropy (Stutzer (1996), Buchen and Kelly (1996)); 3) Kernel regression (Aït-Sahalia and Lo (1998), Aït-Sahalia and Lo (2000)); 4) Convolution approximations (Bondarenko (2003)); and 5) Neural networks (Healy, Dixon, Read, and Cai (2007)). However, Black-Scholes implied volatility curve fitting remains the simplest and most widely used method.[15]

4.4 Static Distribution Models

As discussed above in Section 3.2.1, Jarrow and Rudd (1982) propose an Edgeworth expansion of the option-implied distribution around the lognormal density and Corrado and Su (1996) propose

[15] See for example Christoffersen and Jacobs (2004) and Christoffersen, Heston, and Jacobs (2011).

a related Gram-Charlier expansion which we also discussed above. These methods can be used to produce density forecasts as well as moment forecasts.

If we alternatively assume that S_T is distributed as a mixture of two lognormals, then we get

$$f_0(S_T) = w\psi(S_T, \mu_1, \sigma_1, T) + (1-w)\psi(S_T, \mu_2, \sigma_2, T) \tag{46}$$

The forward price for maturity T imposes the constraint

$$F_0 = w\mu_1 + (1-w)\mu_2$$

where μ_1 and μ_2 are parameters to be estimated, subject to the above constraint, along with the remaining parameters w, σ_1 and σ_2. The resulting option pricing formula is simply a weighted average of BS option prices.

$$C_0^{Mix}(T, X) = wC^{BS}\left(T, X, \mu_1, r; \sigma_1^2\right) + (1-w)C^{BS}\left(T, X, \mu_1, r; \sigma_2^2\right)$$

Most applications assume a mixture of two or three lognormals. The resulting mixture is easy to interpret, especially when it comes to predicting events with a small number of outcomes. The moments can be obtained from

$$E_0[S_T^n] = w\mu_1^n \exp\left(\tfrac{1}{2}\left(n^2 - n\right)\sigma_1^2 T\right) + (1-w)\mu_2^n \exp\left(\tfrac{1}{2}\left(n^2 - n\right)\sigma_2^2 T\right)$$

The distribution is thus flexible enough to capture higher moments such as skewness and kurtosis. See for instance Ritchey (1990), Bahra (1997), and Melick and Thomas (1997) as well as Taylor (2005).

Alternative parametric distributions have been entertained by Bookstaber and McDonald (1991), who use a generalized beta distribution of the second kind, Sherrick, Garcia, and Tirupattur (1996), who use a Burr III distribution, Savickas (2002), who uses a Weibull distribution, and Markose and Alentorn (2011), who assume a Generalized Extreme Value (GEV) distribution. Other distributions used include generalized Beta functions (Aparicio and Hodges (1998) and Liu, Shackleton, Taylor, and Xu (2007)), generalized Lambda distribution (Corrado (2001)), generalized Gamma distribution (Tunaru and Albota (2005)), skewed Student-t (de Jong and Huisman (2000)), Variance Gamma (Madan, Carr, and Chang (1998)), and Lévy processes (Matache, Nitsche, and Schwab (2004)).

4.5 Dynamic Models with Stochastic Volatility and Jumps

There is overwhelming evidence that a diffusion with stochastic volatility (Heston (1993)), a jump-diffusion with stochastic volatility (Bates (1996b)), or an even more complex model for the underlying with jumps in returns and volatility is a more satisfactory description of the data than a simple Black-Scholes model. Nevertheless, these models are not the most popular choices in forecasting applications. This is presumably due to the significantly higher computational burden, which is especially relevant in a forecasting application which requires frequent re-calibration of the model.

The advantage of assuming a stochastic volatility model with jumps is that the primitives of the model include specification of the dynamic of the underlying at a frequency which can be chosen by the researcher. This not only adds richness to the model in the sense that it allows the multiperiod distribution to differ from the one-period distribution, it also allows consistent treatment of options of different maturities, and it ensures that the estimation results can be related in a straightforward way to estimation results for the underlying security.

In affine SVJ models closed form solutions are available for the conditional characteristic function for the log stock price at horizon T defined by

$$\Upsilon_0 \left(i\phi, T \right) \equiv E_0[\exp \left(i\phi \ln \left(S_T \right) \right)].$$

The characteristic function can be used to provide call option prices as follows:

$$C_0(T, X) = S_0 P_1 \left(T, X \right) - X e^{-rT} P_2 \left(T, X \right), \tag{47}$$

where P_1 and P_2 are obtained using numerical integration of the characteristic function. The cumulative density forecast implied by the model is directly provided by $P_2 \left(T, X \right)$ and the density forecast can be obtained from

$$f_0 \left(S_T \right) = \left. \frac{\partial P_2 \left(T, X \right)}{\partial X} \right|_{X = S_T},$$

which must be computed numerically.[16]

4.6 Comparison of Methods

Many studies have compared the empirical performance of different density estimation methods. Jondeau and Rockinger (2000) compare semi-parametric methods based on Hermite and Edgeworth expansions, single and mixture lognormals, and methods based on jump diffusion and stochastic volatility models, and recommend using the mixture of lognormals model for short-run options, and the jump-diffusion model for long-run options. Coutant, Jondeau, and Rockinger (2001) compare Hermite expansion, maximum entropy, mixture of lognormals, and a single lognormal methods and conclude that all methods do better than a single lognormal method. They favor the Hermite expansion method due to its numerical speed, stability, and accuracy. Bliss and Panigirtzoglou (2002) compare double-lognormal and smoothed implied volatility function, focusing on the robustness of their parameter estimates, and conclude that the smoothed implied volatility function method dominates the double-lognormal method. Campa, Chang, and Reider (1998) and Jackwerth (1999) compare various parametric and nonparametric methods, and conclude that the estimated distributions obtained using different methods are rather similar.

In summary, these and many other papers compare different estimation methods, and arrive at conclusions that are not always consistent with one another. Since the resulting densities are often

[16] In recent work, Andersen, Fusari, and Todorov (2012) estimate SVJ models by minimizinng option implied volatility errors subject to fitting the historical path of realized volatility.

not markedly different from each other when using different estimation methods, it makes sense to use methods that are computationally easy and/or whose results are easy to interpret given the application at hand. Because of computational ease and the stability of the resulting parameter estimates, the smoothed implied volatility function method is a good choice for many purposes. The jump-diffusion model is useful if the event of interest is a rare event such as stock market crash. The lognormal-mixture is particularly useful when dealing with situations with a small number of possible outcomes, such as elections.

4.7 Applications of Option-Implied Density Forecasts

Table 6 contains a summary of the existing empirical studies using option-implied densities (OID) in forecasting. Several early studies focus on the markets for commodities and currencies. Silva and Kahl (1993) extend Fackler and King (1990) and test two hypotheses: (i) the OID becomes more reliable as commodity option markets mature, and (ii) the reliability of the OID can be improved by using a distribution-free approach rather than a model based on lognormal assumption. They find evidence supporting their first hypothesis, and also find that the lognormal approach is better than the distribution-free approach. Melick and Thomas (1997) estimate the distribution of crude oil futures price during the Persian Gulf crisis, assuming a mixture of three lognormal distributions to capture the fact that there were mainly three possible outcomes to the crisis. They find that the OID was consistent with market commentary at the time, which predicted a major disruption in oil prices. They also find that the lognormal mixture model performed better in characterizing the data than the standard lognormal model. More recently, Høg and Tsiaras (2011) compare the performance of (i) OID, (ii) time-series based densities, and (iii) risk-adjusted OID, in forecasting the crude oil prices during 1990-2006 period. They find that the risk-adjusted OID (also referred to as physical or real-world density) outperform the other two approaches.

The OID has also been applied to currency markets. Leahy and Thomas (1996) estimate densities from Canadian dollar futures options around the referendum on Quebec sovereignty in October 1995, using the mixture of lognormals model as in Melick and Thomas (1997). They find that the OID immediately preceding the day of the referendum was consistent with three possible outcomes: no surprise, surprising defeat of the sovereignty proposal, and a surprising victory. They conclude that the use of lognormal rather than the mixture model based densities in circumstances in which there are a few possible outcomes can obscure interesting features of the data. Malz (1997) use model-free OID to explore the forward rate bias. Malz finds that option-implied volatility, skewness, and kurtosis have considerable explanatory power for excess returns on forward exchange rates. Campa and Chang (1996), Malz (1996), and Haas, Mittnik, and Mizrach (2006) examine the information content of exchange rate OIDs around ERM crises of 1992. All three studies find that option-implied measures provide useful information for policy makers. Haas et al. (2006) find that time-series based measures are also useful. Campa, Chang, and Refalo (2002) apply the intensity of realignment and credibility measures developed in Campa and Chang

(1996) to the "crawling peg" between the Brazilian Real and the USD as well as a realignment of the "maxibands" between 1994 and 1999. They conclude that the OID is a better indicator of such events in currency management relative to macroeconomic or interest-rate based indicators. Campa et al. (1998) and Bodurtha and Shen (1999) study the USD/DM and USD/Yen relationship. Campa et al. (1998) find a positive correlation between skewness and the spot rate, which means that the stronger the currency, the more expectations are skewed towards a further appreciation of that currency. Bodurtha and Shen (1999) show that investors should consider both historical and implied volatility covariance parameter estimates in their Value-at-Risk computations.

Using equity options, Gemmill and Saflekos (2000) study the predictive power of OID around four stock market crashes and three British elections between 1987 and 1997 using FTSE-100 index options. They cannot reject the hypothesis that OID reflects market sentiment, but sentiment as measured has little forecasting ability. Mizrach (2006) examines whether the collapse of Enron was expected in option markets, and find that options market remained far too optimistic about the stock until just weeks before their bankruptcy filing. Shackleton, Taylor, and Yu (2010) estimate S&P 500 index densities using various methods, and find that the risk-adjusted OID has the best performance. Recently, Kostakis, Panigirtzoglou, and Skiadopoulos (2011) use estimated densities for portfolio selection. They find that the risk-adjusted OID improves the risk-adjusted return of the portfolio when compared with the portfolio formed based only on historical return distributions.

[Table 6: Forecasting using Option-Implied Densities]

4.8 Event Forecasting Applications

There is a significant and expanding literature on prediction markets. The primary purpose of these markets is to forecast future events, and the contracts are designed to facilitate extracting information used in forecasting. This literature is covered in detail in Snowberg, Wolfers, and Zitzewitz (2011) and is therefore not discussed here. We instead focus on the prediction of events using option data, where the primary function of the traded options is not prediction itself. In this literature, which naturally overlaps with the density forecasting work discussed above, estimation methods vary greatly depending on the events to be forecast. We therefore do not describe details of the estimation methods but instead focus our attention on empirical results. Table 7 contains a summary of relevant empirical studies.

[Table 7: Forecasting with Option-Implied Event Probabilities]

Many stock market events are of great interest from a forecasting perspective, including stock market crashes and individual corporate events such as earnings announcements, stock splits, and acquisitions. Bates (1991) is perhaps the best known study of whether and how stock market index option prices reveal the market's expectation of future stock market crashes. He studies the behavior of S&P 500 futures options prices prior to the crash of October 1987, and finds unusually negative skewness in the option-implied distribution of the S&P 500 futures between October 1986 and August 1987, leading to the conclusion that the crash was expected. He finds, however, that the same indicators do not exhibit any strong crash fears during the two months immediately preceding the crash. There are few other studies investigating if index option prices can predict stock market crashes. Doran et al. (2007) find that the option skew is useful in predicting stock market crashes and spikes, but conclude that the value of this predictive power is not always economically significant. Overall therefore, there is some evidence in favor of predictability, but the evidence is not conclusive.

Mizrach (2006) finds that option prices did not reflect the risk of Enron until just weeks before the firm's bankruptcy filing in 2001. Other studies examine corporate events other than crashes, and the results of these studies are more positive. Jayaraman, Mandelker, and Shastri (1991), Barone-Adesi, Brown, and Harlow (1994), Cao, Chen, and Griffin (2005), and Bester, Martinez, and Roşu (2011) all test the forecasting ability of variables in the option market (e.g. prices, trading volume, etc.) prior to corporate acquisitions. Jayaraman et al. (1991) find that implied volatilities increase prior to the announcement of the first bid for the target firm and decrease significantly at the announcement date, indicating that the market identifies potential targets firms prior to the first public announcement of the acquisition attempt. Cao et al. (2005) find that takeover targets with the largest preannouncement call-volume imbalance increases experience the highest announcement day returns. As for the probability of success and timing of announced acquisitions, Bester et al. (2011) show that their option pricing model yields better predictions compared to a "naive" method, although Barone-Adesi et al. (1994) find no evidence that option prices predict the timing of announced acquisitions.

5 Allowing for Risk Premia

So far in the chapter we have constructed various forecasting objects using the so-called risk-neutral or pricing measure implied from options. When forecasting properties of the underlying asset we ideally want to use the physical measure and not the risk-neutral measure which is directly embedded in option prices. Knowing the mapping between the two measures is therefore required. A fully specified option valuation model provides a physical measure for the underlying asset return as well as a risk-neutral measure for derivatives valuation, and therefore implicitly or explicitly defines the mapping. In this section we explore the mapping between measures. We will use superscript Q to denote the option-implied density used above, and superscript P to denote the physical density of the underlying asset.

5.1 Complete Markets

Black and Scholes (1973) assume a physical stock price process of the form

$$dS = (r + \mu) \, Sdt + \sigma Sdz \tag{48}$$

where μ is the equity risk premium. In the Black-Scholes model a continuously rebalanced dynamic portfolio consisting of one written derivative, C, and $\frac{\partial C}{\partial S}$ shares of the underlying asset has no risk and thus earns the risk-free rate. This portfolio leads to the Black-Scholes differential equation.

In the complete markets world of Black-Scholes the option is a redundant asset which can be perfectly replicated by trading the stock and a risk-free bond. The option price is independent of the degree of risk-aversion of investors because they can replicate the option using a dynamic trading strategy in the underlying asset. This insight leads to the principle of risk-neutral valuation where all derivative assets can be valued using the risk-neutral expected pay-off discounted at the risk free rate. For example, for a European call option we can write

$$C_0(T,X) = \exp(-rT)\, E_0^Q \left[\max\{S_T - X, 0\}\right] \tag{49}$$

Using Ito's lemma on (48) we get

$$d\ln(S) = \left(r + \mu - \tfrac{\sigma^2}{2}\right) dt + \sigma dz$$

which implies that log returns are normally distributed

$$f_0^P(\ln(S_T)) = \frac{1}{\sqrt{2\pi\sigma^2 T}} \exp\left(\frac{-1}{2\sigma^2 T}\left(\ln(S_T) - \ln(S_0) - \left(r + \mu - \tfrac{\sigma^2}{2}\right)T\right)^2\right)$$

Under the risk-neutral measure, $\mu = 0$, and we again have the lognormal density, but now with a different mean

$$f_0^Q(\ln(S_T)) = \frac{1}{\sqrt{2\pi\sigma^2 T}} \exp\left(\frac{-1}{2\sigma^2 T}\left(\ln(S_T) - \ln(S_0) - \left(r - \tfrac{\sigma^2}{2}\right)T\right)^2\right)$$

which can be used to compute the expectation in (49).

In a Black-Scholes world, the option-implied density forecast will therefore have the correct volatility and functional form but a mean biased downward because of the equity premium, μ. Since the risk-neutral mean of the asset return is the risk-free rate, the option price has no predictive content for the mean return.

5.2 Incomplete Markets

The Black-Scholes model is derived in a complete market setting where risk-neutralization is straightforward. Market incompleteness arises under the much more realistic assumptions of market frictions arising for example from discrete trading, transactions costs, market illiquidity, price jumps and stochastic volatility or other non-traded risk factors.

5.2.1 Pricing Kernels and Investor Utility

In the incomplete markets case we can still assume a pricing relationship of the form

$$\begin{aligned} C_0(T,X) &= \exp(-rT)\, E_0^Q\left[\max\{S_T - X, 0\}\right] \\ &= \exp(-rT) \int_X^\infty \max\{\exp(\ln(S_T)) - X, 0\}\, f_0^Q(\ln(S_T))\, dS_T \end{aligned}$$

But the link between the option-implied and the physical distributions is not unique and a pricing kernel M_T must be assumed to link the two distributions. Define

$$M_T = \exp(-rT) \frac{f_0^Q(\ln(S_T))}{f_0^P(\ln(S_T))}$$

then we get

$$\begin{aligned} C_0(T, X) &= \exp(-rT) E_0^Q[\max\{S_T - X, 0\}] \\ &= E_0^P[M_T \max\{S_T - X, 0\}] \end{aligned}$$

The pricing kernel (or stochastic discount factor) describes how in equilibrium investors trade off the current (known) option price versus the future (stochastic) pay-off.

The functional form for the pricing kernel can be motivated by a representative investor with a particular utility function of terminal wealth. Generally, we can write

$$M_T \propto U'(S_T)$$

where $U'(S_T)$ is the first-derivative of the utility function.

For example, assuming exponential utility with risk-aversion parameter γ we have

$$U(S) = -\frac{1}{\gamma} \exp(-\gamma S)$$

so that $U'(S) = \exp(-\gamma S)$, and

$$f_0^Q(S_T) = \exp(rT) M_T f_0^P(\ln(S_T)) = \frac{\exp(-\gamma(S_T)) f_0^P(S_T)}{\int_0^\infty \exp(-\gamma(S)) f_0^P(S) dS}$$

where the denominator ensures that $f_0^Q(S_T)$ is a proper density.

Assuming instead power utility, we have

$$U(S) = \frac{S^{1-\gamma} - 1}{1 - \gamma}, \tag{50}$$

so that $U'(S) = S^{-\gamma}$, and

$$f_0^Q(S_T) = \frac{S_T^{-\gamma} f_0^P(S_T)}{\int_0^\infty S^{-\gamma} f_0^P(S) dS}$$

Importantly, these results demonstrate that any two of the following three uniquely determine the third: 1) the physical density; 2) the risk-neutral density; 3) the pricing kernel. We refer to Hansen and Renault (2010) for a concise overview of various pricing kernels derived from economic theory.

The Black-Scholes model can be derived in a discrete representative investor setting where markets are incomplete. Brennan (1979) outlines the sufficient conditions on the utility function and return distribution to obtain the Black-Scholes option pricing result.

5.2.2 Static Distribution Models

Liu et al. (2007) show that if we assume a mixture of lognormal option-implied distribution as in (46) and furthermore a power utility function with risk aversion parameter γ as in (50) then the physical distribution will also be a mixture of lognormals with the following parameter mapping

$$\begin{aligned}
\mu_i^P &= \mu_i \exp\left(\gamma \sigma_i^2 T\right) \text{ for } i = 1, 2 \\
w^P &= \left[1 + \frac{1-w}{w}\left(\frac{\mu_2}{\mu_1}\right)^{\gamma} \exp\left(\frac{1}{2}\left(\gamma^2 - \gamma\right)\left(\sigma_2^2 - \sigma_1^2\right) T\right)\right]^{-1}
\end{aligned}$$

where σ_1^2 and σ_1^2 do not change between the two measures.

The physical moments can now be obtained from

$$E_0^P\left[S_T^n\right] = w^P \left(\mu_1^P\right)^n \exp\left(\tfrac{1}{2}\left(n^2 - n\right)\sigma_1^2 T\right) + \left(1 - w^P\right)\left(\mu_2^P\right)^n \exp\left(\tfrac{1}{2}\left(n^2 - n\right)\sigma_2^2 T\right).$$

Keeping μ_1, μ_2, w, σ_1^2 and σ_1^2 fixed at their option-implied values, the risk aversion parameter γ can be estimated via maximum likelihood on returns using the physical mixture of lognormals defined by the parameter mapping above. Liu et al. (2007) also investigate other parametric distributions. See Fackler and King (1990) and Bliss and Panigirtzoglou (2004) for related approaches.

5.2.3 Dynamic Models with Stochastic Volatility

The Heston model allows for stochastic volatility implying that the option, which depends on volatility, cannot be perfectly replicated by the stock and bond. Markets are incomplete in this case and the model therefore implicitly makes an assumption on the pricing kernel or the utility function of the investor. Heston (1993) assumes that the price of an asset follows the following physical process

$$\begin{aligned}
dS &= (r + \mu V)\, S dt + \sqrt{V} S dz_1 \\
dV &= \kappa^P \left(\theta^P - V\right) dt + \sigma_V \sqrt{V} dz_2
\end{aligned} \tag{51}$$

where the two diffusions are allowed to be correlated with parameter ρ. The mapping between the physical parameters in (51) and the option-implied parameters in (6) is given by

$$\kappa = \kappa^P + \lambda, \;\; \theta = \theta^P \frac{\kappa^P}{\kappa}$$

where λ is the price of variance risk.

Christoffersen et al. (2011) show that the physical and option-implied processes in (51) and (6) imply a pricing kernel of the form

$$M_T = M_0 \left(\frac{S_T}{S_0}\right)^{\gamma} \exp\left(\delta T + \eta \int_0^T V(s) ds + \xi (V_T - V_0)\right) \tag{52}$$

where ξ is a variance preference parameter.[17] The risk premia μ and λ are related to the preference parameters γ and ξ via

$$\mu = -\gamma - \xi\sigma_V\rho$$
$$\lambda = -\rho\sigma_V\gamma - \sigma_V^2\xi$$

In order to assess the implication of the price of variance risk, λ, for forecasting we consider the physical expected integrated variance

$$\begin{aligned} VAR_0^P(T) &= \theta^P T + \left(V_0 - \theta^P\right)\frac{\left(1 - e^{-\kappa^P T}\right)}{\kappa^P} \\ &= \theta\frac{\kappa}{\kappa-\lambda}T + \left(V_0 - \theta\frac{\kappa}{\kappa-\lambda}\right)\frac{\left(1 - e^{-(\kappa-\lambda)T}\right)}{\kappa-\lambda} \end{aligned}$$

Under the physical measure the expected future variance in the Heston (1993) model of course differs from the risk-neutral forecast in (7) when $\lambda \neq 0$. If an estimate of λ can be obtained, then the transformation from option-implied to physical variance forecasts is trivial.

In Figure 6 we plot the physical volatility term structure per year defined by

$$\sqrt{VAR_0^P(T)/T} = \sqrt{\theta\frac{\kappa}{\kappa-\lambda} + \left(V_0 - \theta\frac{\kappa}{\kappa-\lambda}\right)\frac{\left(1 - e^{-(\kappa-\lambda)T}\right)}{(\kappa-\lambda)T}} \tag{53}$$

along with the option-implied term structure from (8). We use parameter values as in Figure 3, where $\theta = 0.09$ $\kappa = 2$, and we set $\lambda = -1.125$ which implies that $\theta/\theta^P = \frac{\kappa-\lambda}{\kappa} = (1.25)^2$ which corresponds to a fairly large variance risk premium. Figure 6 shows the effect of a large volatility risk premium on the volatility term structure. For short horizons and when the current volatility is low then the effect of the volatility risk premium is relatively small. However for long-horizons the effect is much larger.

5.2.4 Model-free Moments

Bakshi et al. (2003) also assume power utility with parameter γ, and show that option-implied and physical moments for the market index are approximately related as follows

$$VAR^Q \approx VAR^P\left(1 - \gamma SKEW^P\left(VAR^P\right)^2\right)$$
$$SKEW^Q \approx SKEW^P - \gamma(KURT^P - 3)\left(VAR^P\right)^2$$

Given a reasonable estimate for γ it is thus possible to convert option-implied estimates for VAR^Q and $SKEW^Q$ into physical moment forecasts, VAR^P and $SKEW^P$ without making explicit assumptions on the functional form of the distribution.

[17] Christoffersen, Elkamhi, Feunou, and Jacobs (2010) provide a general class of pricing kernels in a discrete time setting with dynamic volatility and non-normal return innovations.

5.3 Pricing Kernels and Risk Premia

There is a related literature focusing on estimating pricing kernels from P and Q densities rather than on forecasting. Jackwerth (2000) estimates pricing kernels using Q-densities obtained from one day of option prices, and P-densities using returns for the previous month. Christoffersen et al. (2011) estimate pricing kernels using the entire return sample by standardizing returns by a dynamic volatility measure. Some authors assume that Q is time varying while P is constant over time. Aït-Sahalia and Lo (2000) use four years of data to estimate P when constructing pricing kernels. Rosenberg and Engle (2002) use returns data over their entire sample 1970-1995 to construct estimates of the pricing kernel.

Interestingly, recent evidence suggests that key features of the pricing kernel such as risk premia are useful for forecasting returns. This is not surprising, because as we saw above, the pricing kernel is related to preferences, and therefore changes in the pricing kernel may reflect changes in risk aversion, or, more loosely speaking, in sentiment.

For example, Bollerslev, Tauchen, and Zhou (2009), Bekaert, Hoerova, and Lo Duca (2010), and Zhou (2010) find strong evidence that the variance risk premium, $VAR^Q - VAR^P$, can predict the equity risk premium. Bollerslev, Marrone, Xu, and Zhou (2012) report evidence on this from a set of international equity markets. Bakshi, Panayotov, and Skoulakis (2011) compute the forward variance, which is the implied variance between two future dates, and find that the forward variance is useful in forecasting stock market returns, T-bill returns, and changes in measures of real economic activity. A related paper by Feunou, Fontaine, Taamouti, and Tedongap (2011) find that the term structure of implied volatility can predict both the equity risk premium and variance risk premium.

Risk premia can be estimated in various ways. Parametric models can be used to jointly calibrate a stochastic model of stock and option prices with time-variation in the Q and P densities. For instance, Shackleton et al. (2010) and Pan (2002) calibrate stochastic volatility models to options and asset prices. In recent work, Ross (2011) suggests a methodology for estimating physical volatility using option prices in a representative investor setting. Alternatively, (model-free) option-implied moments can be combined with separately estimated physical moments to compute risk premia. In this case, the question arises how to estimate the physical moments. The literature on the optimal type of physical information to combine with option-implied information is in its infancy. However, we have extensive knowledge about the use of different types of physical information from the literature on forecasting with historical information as chronicled in this and the previous Handbook of Economic Forecasting volume.

6 Summary and Discussion

The literature contains a large body of evidence supporting the use of option-implied information to predict physical objects of interest. In this chapter we have highlighted some of the key tools for extracting forecasting information using option-implied moments and distributions.

Option-implied forecasts are likely to be most useful when

- The options market is highly liquid so that market microstructure biases do not adversely affect the forecasts. In illiquid markets bid-ask spreads are wide and furthermore, option quotes are likely to be based largely on historical volatility forecasts so that the option quotes add little or no new information.

- Many strike prices are available. In this case, model-free methods can be used to extract volatility forecasts without assuming particular volatility dynamics or return distributions.

- Many different maturities are available. In this case the option-implied forecast for any desired horizon can be constructed with a minimum of modeling assumptions.

- The underlying asset is uniquely determined in the option contract. Most often this is the case, but in the case of Treasury bond options, for example, many different bonds are deliverable, which in turn complicates option-based forecasting of the underlying return dynamics.

- Options are European, or in the case of American options ,when the early exercise premium is likely to be easily assessed. The early exercise premium must be estimated and subtracted from the American option price before using the method surveyed in this chapter. Estimating the early exercise premium thus adds noise to the option-implied forecast.

- The underlying asset has highly persistent volatility dynamics. In this case time-series forecasting models may be difficult to estimate reliably.

- The underlying asset undergoes structural breaks. Such breaks complicate the estimation of time-series models and thus heighten the role for option-implied forecasts.

- Interest centers on higher moments of the underlying asset return. The nonlinear payoff structure in option contracts makes them particularly attractive for higher-moment forecasting.

- Risk premia are small. Risk-premia may bias the option-implied forecasts but option-based forecasts may nevertheless contain important information not captured in historical forecasts.

Related to the last bullet point, we have also summarized the key theoretical relationships between option-implied and physical densities, enabling the forecaster to take into account risk-premia and convert the option-implied forecasts to physical forecasts. We hasten to add that it is certainly not mandatory that the option-implied information is mapped into the physical measure to generate forecasts. Some empirical studies have found that transforming option-implied to physical information improves forecasting performance in certain situations (see Shackleton et al. (2010) and Chernov (2007)) but these results do not necessarily generalize to other types of forecasting exercises.

We would expect the option-implied distribution or moments to be biased predictors of their physical counterparts, but this bias may be small, and attempting to remove it can create problems of its own, because the resulting predictor is no longer exclusively based on forward-looking information from option prices, but also on backward-looking information from historical prices as well as on assumptions on investor preferences.

More generally, the existence of a bias does not prevent the option-implied information from being a useful predictor of the future object of interest. Much recent evidence for example on variance forecasting (See Table 2 from Busch et al. (2011)) strongly suggests that this is indeed the case empirically.

Going forward, developing methods for combining option-implied and historical return-based forecasts would certainly be of use–in particular when risk-premia may be present. In certain applications, for example volatility forecasting, combination can be done in a simple regression framework or even by using equal weighting. In other applications, such as density forecasting, more sophisticated approaches are needed. Timmermann (2006) provides a thorough overview of available techniques.

Most of the empirical literature has been devoted to volatility forecasting using simple Black-Scholes implied volatility. Clearly, much more empirical research is needed to assess the more sophisticated methods for computing option-implied volatility as well as the other forecasting objects of interest including skewness, kurtosis, and density forecasting.

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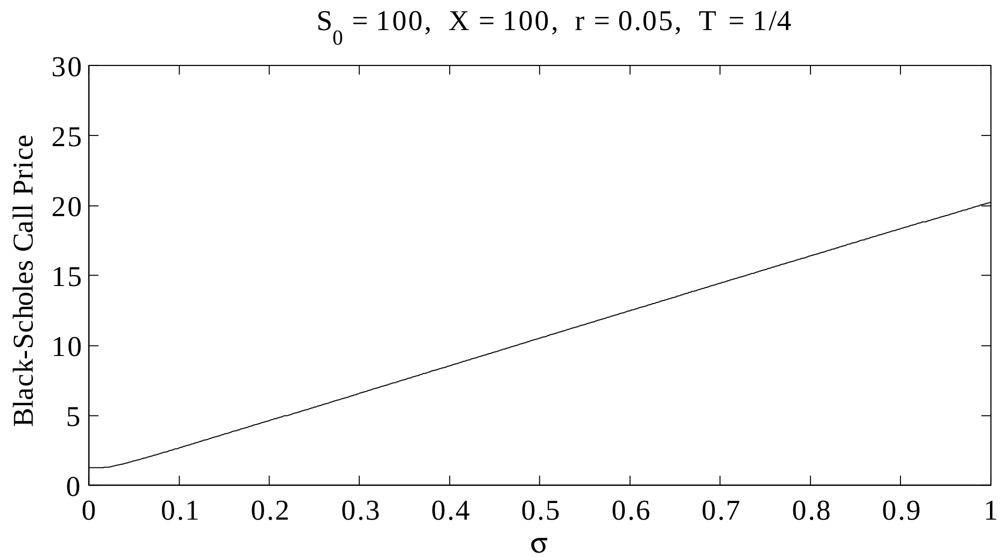
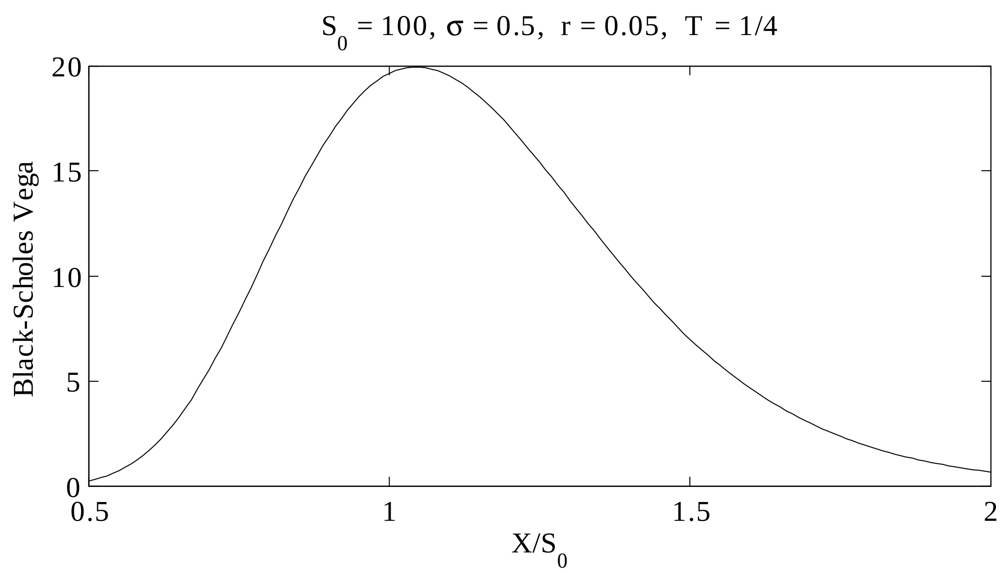
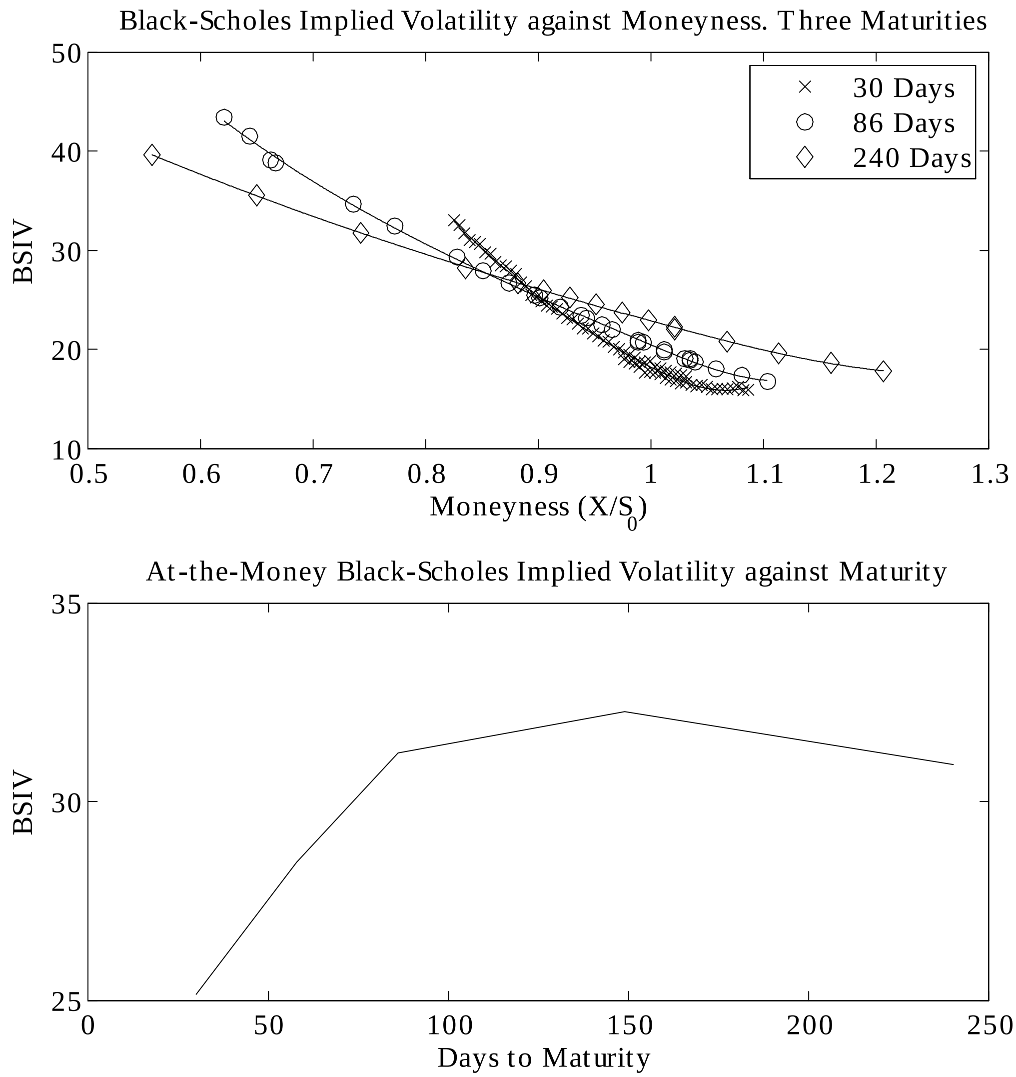
Figure 1: Black-Scholes Price and Vega





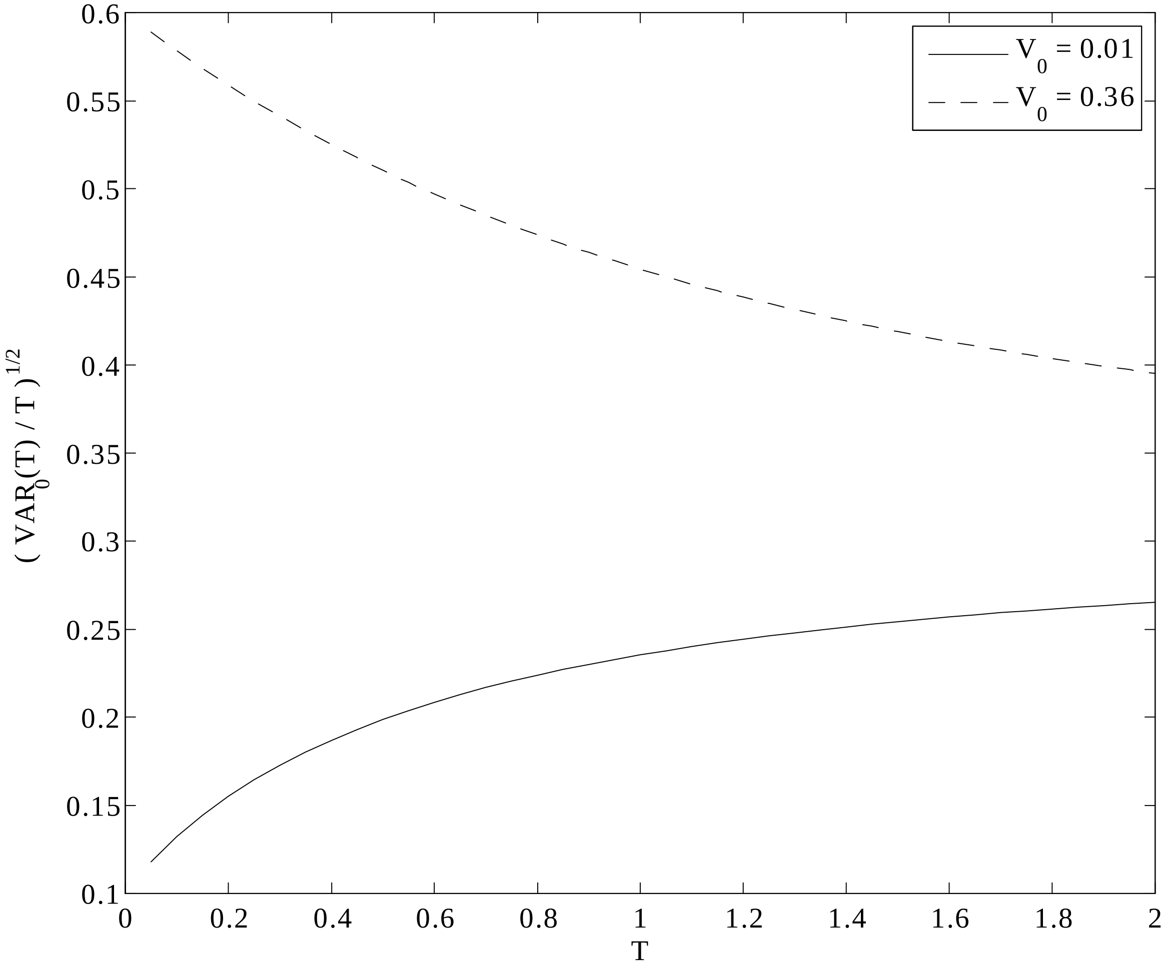
Notes to Figure: In the top panel, we plot the Black-Scholes call price as a function of volatility for an at-the-money option with a strike price of 100 and three months to maturity. The risk-free interest rate is 5% per year. In the bottom panel we plot the Black-Scholes Vega as a function of moneyness for a call option with a volatility of 50% per year.

Figure 2: Black-Scholes Implied Volatility as a Function of Moneyness and Maturity



Notes to Figure: In the top panel, we plot Black-Scholes implied volatility ($BSIV$) against moneyness, X/S_0, for various out-of-the-money S&P 500 options quoted on October 22, 2009. In the bottom panel we plot at-the-money $BSIV$ against days to maturity (DTM).

Figure 3: Volatility Term Structures in the Heston Model

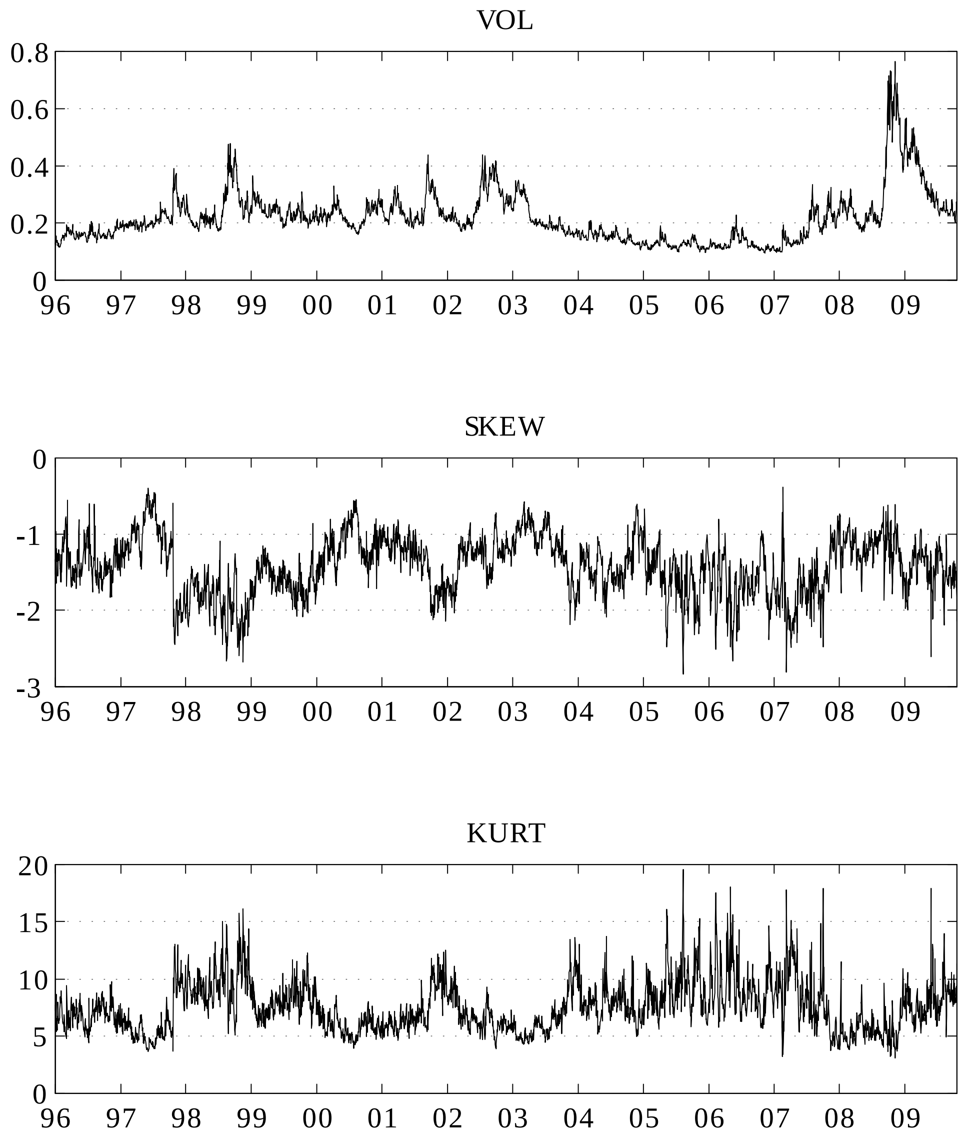


Notes to Figure: We plot the volatility term structure in the Heston model defined as

$$\sqrt{VAR_0\left(T\right)/T} = \sqrt{\theta + \left(V_0 - \theta\right)\frac{\left(1 - e^{-\kappa T}\right)}{\kappa T}}$$

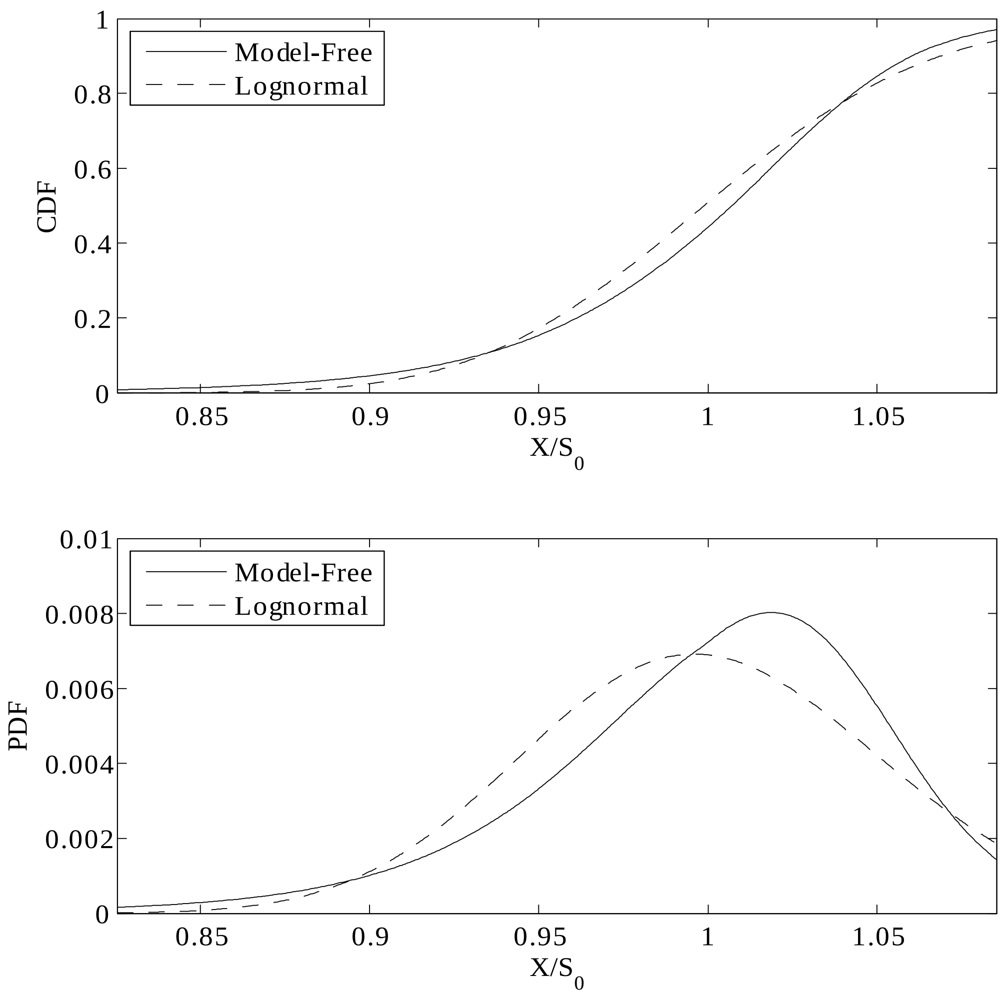
where $\theta = 0.09$ and $\kappa = 2$. $V_0 = 0.36$ (dashed line) corresponds to a high current spot variance and $V_0 = 0.01$ (solid line) corresponds to a low current spot variance.

Figure 4: Option-Implied Moments for One-Month S&P 500 Returns



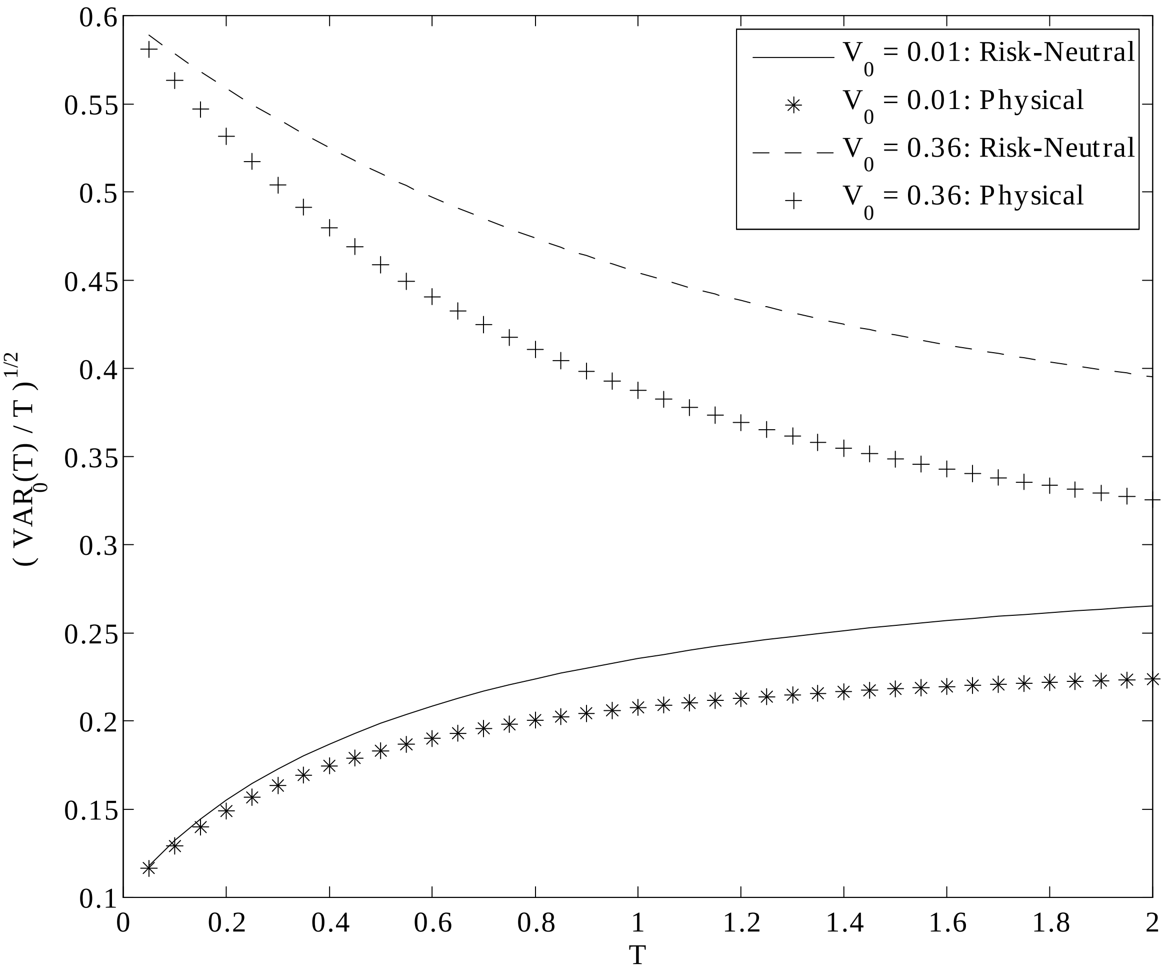
Notes to Figure: We plot the volatility, skewness, and kurtosis implied by S&P 500 index options using the methodology in Bakshi et al. (2003).

Figure 5: Option-Implied Distribution from BSIV Curve Fitting vs. Lognormal



Notes to Figure: We plot the CDF and PDF obtained from applying a cubic spline (solid lines) using data for S&P index options with thirty days to expiration on October 22, 2009, together with the CDF and PDF of the lognormal distribution (dashed lines).

Figure 6: Physical and Option-Implied Volatility Term Structures in the Heston Model



Notes to Figure: We plot the option-implied (from Figure 3) as well as the physical volatility term structure in the Heston model defined as

$$\sqrt{VAR_0^P\left(T\right)/T} = \sqrt{\theta\frac{\kappa}{\kappa-\lambda} + \left(V_0 - \theta\frac{\kappa}{\kappa-\lambda}\right)\frac{\left(1-e^{-(\kappa-\lambda)T}\right)}{\left(\kappa-\lambda\right)T}}$$

where $\theta = 0.09$, $\kappa = 2$, and $\lambda = -1.125$. Lines with '*' and '+' markers denote physical forecasts. The dashed line with $V_0 = 0.36$ shows the option-implied forecast from a a high current spot variance and the solid line with $V_0 = 0.01$ shows the option-implied forecasts from a low current spot variance.

Table 1: Volatility Indexes Around the World

Country	Exchange	Index	Underlying	Maturity	Launch Date	Method
US	Chicago Board Options Exchange (CBOE)	VIX	S&P 500	1 month	Sep 2003 (old index renamed VXO, 1993-)	Demeterfi, Derman, Kamal, and Zou (1999) (VIX methodology)
US	CBOE	VXV	S&P 500	3 months	Nov 2007	VIX
US	CBOE	VXO	S&P 100	1 month	1993	Whaley (1993)
US	CBOE	VXD	DJIA	1 month	Mar 2005	VIX
US	CBOE	VXN	Nasdaq 100	1 month		VIX
US	CBOE	VXAZN, VXAPL, VXGS, VXGOG, VXIBM	Stocks - Amazon, Apple, Goldman Sachs, Google, IBM	1 month	Jan 2011	VIX
US	CBOE	EVZ, GVZ, OVX, VXEEM, VXSLV, VXFXI, VXGDX, VXEWZ, VXXLE	ETFs - EuroCurrency, gold, crude oil, emerging markets, silver, China, gold miners, Brazil, energy sector	1 month	2008	VIX
US	CBOE	ICJ, JCJ, KCJ	S&P 500	As of May 2011, KCJ - Jan 2012, ICJ - Jan 2013, JCJ - Jan 2014, the tickers are to be recycled as they expire	Jul 2009	Skintzi and Refenes (2005)
Australia	Australian Securities Exchange	S&P/ASX 200 VIX (ASX code: XVI)	S&P/ASX 200 (XJO)	1 month	Sep 2010	VIX
Belgium	Euronext	VBEL	BEL 20	1 month	Sep 2007	VIX
Canada	TMX	S&P/TSX 60 VIX (VIXC)	S&P/TSX 60	1 month	Oct 2010	VIX
Europe	Eurex	VSTOXX	Euro STOXX 50	30, 60, 90,..., 360 days	Apr 2005 (30 days); May 2010 (60-360 days)	VIX
France	Euronext	VCAC	CAC 40	1 month	Sep 2007	VIX
Germany	Deutsche Borse	VDAX-NEW	DAX	1 month	Apr 2005 (previously VDAX, Dec 1994)	VIX
Hong Kong	Hong Kong Futures Exchange	VHSI	HSI	1 month	Feb 2011	VIX
India	National Stock Exchange of India	India VIX	NIFTY	1 month	Jul 2010	VIX
Japan	CSFI, Univ. of Osaka	CSFI - VXJ	Nikkei 225	1 month	Jul 2010	VIX
Mexico	Mexican Derivatives Exchange	VIMEX	Mexican Stock Exchange Price and Quotation Index (IPC)	3 months	Apr 2006	Fleming, Ostdiek, and Whaley (1995)
Netherlands	Euronext	VAEX	AEX	1 month	Sep 2007	VIX
South Africa	Johannesburg Stock Exchange	New SAVI	FTSE/JSE Top40	3 months	2010 (previously SAVI, 2007-)	VIX
South Korea	Korea Exchange	V-KOSPI	KOSPI200	1 month	Apr 2009	VIX
Switzerland	Six Swiss Exchange	VSMI	SMI	1 month	Apr 2005	VIX
UK	Euronext	VFTSE	FTSE 100	1 month	Jun 2008	VIX

Table 2: Forecasting Monthly Realized Variance using Black-Scholes Implied Variance

Panel A: Foreign exchange data					
Constant	RV_M	RV_W	RV_D	$BSIV^2$	Adj. R^2
0.0061	0.2186	0.0981	0.1706	-	26.0
(0.0011)	(0.1138)	(0.1438)	(0.0828)		
0.0022	-	-	-	0.8917	40.7
(0.0011)				(0.0884)	
0.0021	-0.1483	0.0769	0.0765	0.8733	41.1
(0.0011)	(0.1178)	(0.1284)	(0.0754)	(0.1419)	
Panel B: S&P 500 data					
Constant	RV_M	RV_W	RV_D	$BSIV^2$	Adj. R^2
0.0053	0.6240	-0.3340	0.6765	-	53.0
(0.0025)	(0.1132)	(0.1039)	(0.1007)		
-0.0050	-	-	-	1.0585	62.1
(0.0027)				(0.0667)	
-0.0052	0.0378	-0.1617	0.3177	0.9513	64.0
(0.0027)	(0.1311)	(0.0943)	(0.1026)	(0.1391)	
Panel C: Treasury bond data					
Constant	RV_M	RV_W	RV_D	$BSIV^2$	Adj. R^2
0.0031	0.3600	0.1112	0.1389	-	32.5
(0.0005)	(0.1106)	(0.1143)	(0.0744)		
0.0023	-	-	-	0.5686	35.0
(0.0006)				(0.0641)	
0.0018	0.0462	0.1835	0.0817	0.3933	40.4
(0.0006)	(0.1254)	(0.1086)	(0.0710)	(0.0882)	

Note: We reproduce parts of Table 1 from Busch, Christensen, and Nielsen (2011), who regress total realized variance (RV) for the current month on the lagged monthly (subscript M), weekly (subscript W) and daily (subscript D) realized variance. Black-Scholes implied variance ($BSIV^2$) is introduced in univariate regressions as well as an additional regression in the RV regressions. Panel A contains $/DM FX data for 1987-1999, Panel B contains S&P 500 data for 1990-2002, and Panel C contains Treasury bond data for 1990-2002.

Table 3: Forecasting with Option-Implied Volatility

Authors	Year	Market	Predictor	To Predict	Method	Conclusion
Fackler and King	1990	Commodity	Mean, vol	Mean, vol	Average of just OTM put and call IV	Corn and live cattle reliable, but overstate volatility of soybean and understate location of hog prices
Kroner, Kneafsey, and Claessens	1995	Commodity	Vol	Vol	Barone-Adesi and Whaley (1987)	Combination of IV and historical outperform
Jorion	1995	Currency	Vol	Vol	Black (1976) at the money	IV outperform historical, but biased
Taylor and Xu	1995	Currency	Vol	Vol	Barone-Adesi and Whaley (1987)	IV outperform historical
Pong, Shackleton, Taylor, and Xu	2004	Currency	Vol	Vol	OTC quotes	IV as accurate as historical at 1 and 3 month horizons, but not better
Christoffersen and Mazzotta	2005	Currency	Vol, density, interval	Vol, density, interval	Malz (1997)	Unbiased and accurate forecast
Day and Lewis	1992	Equity	Vol	Vol	Dividend adjusted BS + Whaley (1982)	Add IV to GARCH and EGARCH. Both are unbiased, but inconclusive as for the relative performance.
Harvey and Whaley	1992	Equity	Vol	Vol	Cash-dividend adjusted binomial	IV predicts, but arbitrage profits are not possible, thus consistent with market efficiency
Canina and Figlewski	1993	Equity	Vol	Vol	Binomial tree adjusting for dividends and early exercise	IV does not predict
Fleming, Ostdiek, and Whaley	1995	Equity	Vol	Vol	Cash-dividend adjusted binomial, old VIX (Whaley (1993))	Biased, but useful for forecasting
Christensen and Prabhala	1998	Equity	Vol	Vol	BS	IV outperform historical
Fleming	1998	Equity	Vol	Vol	Modified binomial model of Fleming and Whaley (1994)	IV is an upward biased forecast, but contains relevant information.
Blair, Poon, and Taylor	2001	Equity	Vol	Vol	VIX	VIX forecasts best and high-frequency intraday returns add no incremental information.
Poon and Granger	2003	Equity	Vol	Vol	N/A	Review of volatility forecasting, table with summary of literature
Jiang and Tian	2005	Equity	Vol	Vol	Britten-Jones and Neuberger (2000), cubic spline	Model-free IV subsumes all information in BS IV and historical volatility.
Ang, Hodrick, Xing, and Zhang	2006	Equity	Vol	Cross-section of stock returns	VIX	Innovation in VIX is a priced risk factor with a negative price of risk.
Andersen and Bondarenko	2007	Equity	Vol	Vol	Corridor implied volatility (CIV)	Broad CIV related to model-free IV. narrow CIV related to BS IV. narrow IV is a better volatility predictor than model-free IV or BS IV.
Bollerslev, Tauchen, and Zhou	2009	Equity	Variance risk premium	Equity risk premium	VIX	VRP predicts stock market return
Bekaert, Hoerova, and Lo Duca	2010	Equity	Variance risk premium	Equity risk premium	VIX	A lax monetary policy decreases risk aversion after about five months. Monetary authorities react to periods of high uncertainty by easing monetary policy.
Zhou	2010	Equity	Variance risk premium	Equity risk premium	VIX	VRP predicts a significant positive risk premium across equity, bond, and credit markets in the short-run (1-4 months)

Table 3 (continued): Forecasting with Option-Implied Volatility

Bakshi, Panayotov, Skoulakis	2011	Equity	Forward variances	(i) Growth in measures of real economic activity, (ii) Treasury bill returns, (iii) stock market returns, and (iv) changes in variance swap rates	Forward variances extracted from the prices of exponential claims of different maturities (Carr and Lee (2008))	The forward variances predict (i) growth in measures of real economic activity, (ii) Treasury bill returns, (iii) stock market returns, and (iv) changes in variance swap rates
Fenou, Fontaine, Taamouti, and Tedongap	2011	Equity	Term structure of implied voaltility	Equity risk premium, variance risk premium	VIX	Term structure of implied volatility predicts both equity risk premium and variance risk premium
DeLisle, Doran, and Peterson	2011	Equity	Vol	Cross-section of stock returns	VIX	Result in Ang et al. (2006) holds when volatility is rising, but not when volatility is falling.
Latane and Rendleman	1976	Equity (individual)	Vol	Vol	Vega-weighted average of individual stock option BS IVs	Outperform historical
Chiras and Manaster	1978	Equity (individual)	Vol	Vol	BS	Risk-free return using option trading strategies
Beckers	1981	Equity (individual)	Vol	Vol	Weighted average BS IV vs. at-the-money BS IV	At-the-money BS IV predicts better than weighted average of BS Ivs.
Sheikh	1989	Equity (individual)	Vol	Split announcement and ex-dates	Roll (1977), American option with dividends	No relative increase in IV of stocks announcing splits, but increase is detected at the ex-date.
Lamoureux and Lastrapes	1993	Equity (individual)	Vol	Vol	Hull and White (1987), stochastic volatility option pricing model	IV contains incremental information to historical
Swidler and Wilcox	2002	Equity (individual)	Vol	Bank stock volatility	Old VIX	IV Outperform historical
Banerjee, Doran, and Peterson	2007	Equity (individual)	Vol	Return of characteristic-based portfolios	VIX	Strong predictive ability
Diavatopoulos, Doran, and Peterson	2008	Equity (individual)	Idiosyncratic volatility	Future cross-sectional stock returns		Strong positive link
Doran, Fodor, and Krieger	2010	Equity (individual)	Vol	Abnormal return after analyst recommendation change	Simulate Bates (1996b) model of SVJ	Information in option markets leads analyst recommendation changes
Demiguel, Plyakha, Uppal, and Vilkov	2011	Equity (individual)	Vol, skew, correlation, variance risk premium	Portfolio selection	Bakshi, Kapadia, Madan (2003) for volatility and skew, Driessen, Maenhout, and Vilkov (2009) for correlation, Bollerslev, Gibson, Zhou (2011) for VRP	Exploiting information contained in the volatility risk premium and option-implied skewness increases substantially both the Sharpe ratio and certainty-equivalent return
Amin and Ng	1997	FI	Vol	Volatility of interest rate	Heath, Jarrow, and Morton (1992)	Predicts well. show how to combine IV with historical.
Busch, Christensen, and Nielsen	2011	FI, equity, currency	Vol	Realized volatility, jump	Numerical inversion of Black (1976)	Prediction in all three markets

Table 4: Forecasting with Option-Implied Correlation

Authors	Year	Market	Predictor	To Predict	Method	Conclusion
Siegel	1997	Currency	Correlation	Correlation: USD/DM/pound	Garman and Kohlhagen (1983)	Outperform historical
Campa and Chang	1998	Currency	Correlation	Correlation between USD/DM and USD/YEN	From relationship between implied volatilities of three exchange rates	Outperforms forecast based on historical correlation
Walter and Lopez	2000	Currency	Correlation	Correlation: USD/DEM/JPY, USD/DEM/JPY	From relationship between implied volatilities of three exchange rates	Useful for USD/DEM/JPY, not for USD/DEM/JPY, so may not be useful in general
Skintzi and Refenes	2005	Equity (individual)	Correlation	Correlation	Implied correlation based on IV of index and individual stocks	Although the implied correlation index is a biased forecast of realized correlation, it has a high explanatory power, and it is orthogonal to the information set compared to a historical forecast.
Driessen, Maenhout and Vilkov	2009	Equity (individual)	Correlation	Correlation	Stock prices follow a geometric Brownian motion with constant drift and possibly stochastic diffusion. Assume that a single state variable drives all pairwise correlations	The entire index variance risk premium can be attributed to the high price of correlation risk.
Buss and Vilkov	2011	Equity (individual)	Correlation and factor betas	Factor betas	Stock prices follow a multifactor model. Assume that a single state variable drives all pairwise correlations	Most efficient and unbiased predictor of beta
Chang, Christoffersen, Jacobs, and Vainberg	2012	Equity (individual)	Beta, moments	Beta	Formula based on implied vol and skew of index and individual stocks	Forecast future beta
Demiguel, Plyakha, Uppal, and Vilkov	2011	Equity (individual)	Vol, skew, correlation, variance risk premium	Portfolio selection	Bakshi, Kapadia, Madan (2003) for volatility and skew, Driessen, Maenhout, and Vilkov (2009) for correlation, Bollerslev, Gibson, Zhou (2011) for VRP	Exploiting information contained in the volatility risk premium and option-implied skewness increases substantially both the Sharpe ratio and certainty-equivalent return
Longstaff, Santa-Clara, and Schwartz	2003	FI	Correlation	Correlation	Caps and swaptions	Implied correlation is lower than historical correlation. Significant mispricings detected.
De Jong, Driessen, and Pelsser	2004	FI	Correlation	Correlation	Caps and swaptions	Implied correlation is different from historical correlation. Significant mispricings detected.

Table 5: Forecasting with Option-Implied Skewness and Kurtosis

Authors	Year	Market	Predictor	To Predict	Method	Conclusion
Bates	1996	Currency	Skew, kurt	Skew, kurt of USD/DM, USD/YEN, 1984-1992	Jump diffusion	The implicit abnormalities (e.g. moments) predict future abnormalities in log-differenced $/DM futures prices, but not S/yen.
Bates	1991	Equity	Skew premium, jump-diffusion parameters	Crash of 1987	Jump diffusion	Risk-neutral distribution of stock market return negatively skewed one year before the crash, but no strong crash fears during 2 months immediately preceding the crash
Navatte and Villa	2000	Equity	Vol, skew, kurt	Moments of CAC 40 index	Gram-Charlier	Implied moments contain substantial amount of information, which decreases with the moment's order
Doran, Carson, and Peterson	2006	Equity	Skew	Crash	Barone-Adesi and Whaley (1987)	Implied volatility skew has significant forecast power for assessing the degree of market crash risk
Agarwal, Bakshi, and Huij	2009	Equity	Vol, skew, kurt	Cross-section of hedge fund returns	Bakshi, Kapadia, Madan (2003)	Innovations in implied market vol, skew, kurt are all priced risk factors for hedge fund returns.
Chang, Christoffersen, and Jacobs	2011	Equity	Vol, skew, kurt	Cross-section of stock returns	Bakshi, Kapadia, Madan (2003)	Negative relationship between the sensitivity to innovation in option-implied skewness of the S&P 500 index and future cross-section of stock returns
Doran, Peterson and Tarrant	2007	Equity (individual)	Skew	Crash and spikes upward	Barone-Adesi and Whaley (1987)	Reveal crashes and spikes with significant probability, but not economically significant
Diavatopoulos, Doran, Fodor, and Peterson	2008	Equity (individual)	Skew, kurt prior to earnings announcements	Stock and option returns	Bakshi, Kapadia, Madan (2003)	Both have strong predictive power
Bali and Hovakimian	2009	Equity (individual)	Realized-implied volatility, call-put IV spread	Cross-section of stock returns		Negative relationship with realized-implied volatility. positive relationship with call-put IV spread.
Conrad, Dittmar and Ghysels	2009	Equity (individual)	Vol, skew, kurt	Cross-section of stock returns	Bakshi, Kapadia, Madan (2003)	Negative relationship between volatility and skew with future return, and positive relationship between kurt and future return
Cremers and Weinbaum	2010	Equity (individual)	Call-put IV spread	Cross-section of stock returns	BS IV	Positive relationship
Doran and Krieger	2010	Equity (individual)	Volatility skew	Return	Five skew measures based on ATM, ITM, OTM IV and traded volume	Discourage the use of skew-based measures for forecasting equity returns without fully parsing the skew into its most basic portions.
Rehman and Vilkov	2010	Equity (individual)	Skew	Return	Bakshi, Kapadia, Madan (2003)	Positive relationship
Xing, Zhao and Zhang	2010	Equity (individual)	Volatility skew	Cross-section of stock returns	OTM put IV - ATM call IV (highest volume, highest open-interest, or volume or open-interest weighted)	Stocks with steepest smirks underperform stocks with least pronounced smirks
Demiguel, Plyakha, Uppal, and Vilkov	2011	Equity (individual)	Vol, skew, correlation, variance risk premium	Portfolio selection	Bakshi, Kapadia, Madan (2003) for volatility and skew, Driessen, Maenhout, and Vilkov (2009) for correlation, Bollerslev, Gibson, Zhou (2011) for VRP	Exploiting information contained in the volatility risk premium and option-implied skewness increases substantially both the Sharpe ratio and certainty-equivalent return

Table 6: Forecasting using Option-Implied Densities

Authors	Year	Market	Predictor	To Predict	Method	Conclusion
Silva and Kahl	1993	Commodity	PDF	PDF of soybean, corn	Log-normal vs. linear interpolation of CDF	Option-implied density of soybean became more reliable from 1985-1987 to 1988-1990 as option market matured.
Melick, Thomas	1997	Commodity	PDF from American options	Crude oil price during gulf crisis	Mixture of two & three lognormals	Option-implied density was consistent with the market commentary at the time, which predicted a major disruption in oil price. Lognormal mixture worked better than lognormal.
Hog and Tsiaras	2011	Commodity	PDF	PDF and various intervals and regions of interest of crude oil	Generalized Beta of the second kind (GB2), risk-neutral to physical using statistical recalibration	Outperform historical
Leahy, Thomas	1996	Currency	PDF from American options	Canadian dollar around Quebec referendum	Mixture of three lognormals	Option prices were consistent with the market commentary.
Campa, Chang	1996	Currency	PDF, Intensity of realignment and credibility measures	ERM target-zone credibility and devaluation	Arbitrage bounds	Devaluation predicted with different time lags from a week to a year
Malz	1996	Currency	PDF from American options	PDF and realignment probability of sterling/DM	Jump diffusion	Useful for defense of target zones against speculative attack
Malz	1997	Currency	PDF, moments	Excess return puzzle in currency markets	IV in function of ATM IV, risk reversal price, strangle price, etc.	Tests International CAPM using risk-neutral moments as explanatory variables and show that they have considerably greater explanatory power for excess returns in currency markets.
Campa, Chang, Reider	1998	Currency	PDF from American options	PDF of USD/DM and USD/YEN	Compare cubic splines, Implied binomial tree, mixture of normals	Use trimmed binomial tree
Bodurtha and Shen	1999	Currency	PDF of correlation	Covariance VaR of mark and yen	Whaley (1982)	Implied correlation provides incremental explanatory power over historical-based correlation estimates
Campa, Chang, Refalo	2002	Currency	PDF, Intensity and credibility measure of Campa and Chang (1996)	PDF and realignment probability of Brazilian Real/USD, 1991-1994	Shimko (1993) IV in quadratic function of strike	Anticipate realignments of exchange rate bands
Haas, Mittnik, and Mizrach	2006	Currency	PDF	PDF, central bank credibility during ERM	Mixture of normals	Both historical and option based forecasts are useful
Gemmill and Saflekos	2000	Equity	PDF	PDF, four crashed, three British elections	Mixture of two lognormals	Little forecasting ability
Mizrach	2006	Equity	PDF	Enron's collapse	Mixture of lognormals	Market remained too optimistic until just weeks before the collapse
Shackleton, Taylor, Yu	2010	Equity	PDF	PDF, S&P 500 index return	Calibration of jump-diffusion model, statistical transformation from risk-neutral to physical	Compare historical, risk-neutral, and risk-transformed physical PDF. Performance depends on the forecast horizon.
Kostakis, Panigirtzoglou, Skiadopoulos	2011	Equity	PDF	Portfolio selection	Smoothed IV smile + Barone-Adesi and Whaley (1987)	Improved portfolio

Table 7: Forecasting with Option-Implied Event Probabilities

Authors	Year	Market	Predictor	To Predict	Method	Conclusion
Melick, Thomas	1997	Commodity	PDF from American options	Crude oil price during gulf crisis	Mixture of two & three lognormals	Option prices were consistent with the market commentary. Use of lognormal model would have overestimated the prob. and price impact of major disruption.
Leahy, Thomas	1996	Currency	PDF from American options	Canadian dollar	Mixture of three lognormals	Option prices were consistent with the market commentary.
Campa, Chang	1996	Currency	PDF, Intensity of realignment and credibility measures	ERM target-zone credibility and devaluation	Arbitrage bounds	Devaluation predicted with different time lags from a week to a year
Malz	1996	Currency	PDF from American options	PDF and realignment prob. of sterling/DM	Jump diffusion	Useful for defense of target zones against speculative attack
Campa, Chang, Refalo	2002	Currency	PDF, Intensity of realignment and credibility measures	PDF and realignment prob. of Brazilian Real/USD, 1991-1994	Shimko (1993) IV in quadratic function of strike	Anticipate realignments of exchange rate bands
Haas, Mittnik, and Mizrach	2006	Currency	PDF	PDF, central bank credibility during ERM	Mixture of normals	Both historical and option based forecasts are useful
Bates	1991	Equity	Jump-diffusion parameters	Crash of 1987	Jump diffusion	Risk-neutral distribution negatively skewed one year before the crash, but no strong crash fears during 2 months immediately preceding the crash
Gemmill and Saflekos	2000	Equity	PDF	PDF, four crashed, three British elections	Mixture of two lognormals	Little forecasting ability
Doran, Carson, and Peterson	2006	Equity	Skew	Crash	Baron-Adesi and Whaley (1987)	Implied volatility skew has significant forecast power for assessing the degree of market crash risk
Mizrach	2006	Equity	PDF	Enron's collapse	Mixture of lognormals	Market remained too optimistic until just weeks before the collapse
Sheikh	1989	Equity (individual)	Vol	Split announcement and ex-dates	Roll (1977), American option with dividends	No relative increase in IV of stocks announcing splits, but increase is detected at the ex-date.
Jayaraman, Mandelker, and Shastri	1991	Equity (individual)	Premia on call options	Information leakage prior to merger announcement	BS	Implied volatility increases prior to the announcement of the first bid for the target firm and decrease significantly at the announcement date.
Barone-Adesi, Brown and Harlow	1994	Equity (individual)	Probability and timing of acquisitions	Probability and timing of acquisitions	Propose model of probability weighted IV	Cannot predict success or timing of acquisition
Cao, Chen, and Griffin	2005	Equity (individual)	Call volume imbalance	Takeover announcement day return	N/A	Takeover targets with the largest preannouncement call-volume imbalance increases experience the highest announcement day returns
Bester, Martinez, and Rosu	2011	Equity (individual)	Merger success probability	Merger success probability	New option pricing model with merger	Better prediction compared to naive method