Marwin B. Alejo 2020-20221 EE214_Module3-LabEx1

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I. Simulation of Random Processes and Stationarity

1.a. Stationarity

```
numFxn = 10; % number of sample fxn
samples = 1000; % number of samples/fxn
x = randn(samples,numFxn); % generate ten sample fxns w/ u=0 o=1
t = 0:999; % sample interval

timeMean = mean(x,2);
sampleMean = mean(x',1);

timeVariance = var(x,1,2);
sampleVariance = var(x',1,1);

disp(['1st Moment Stationary- ', num2str(mean([timeMean(:) - sampleMean(:)] .^
 2))]);
disp(['2nd Moment Stationary - ', num2str(mean([timeVariance(:) -
 sampleVariance(:)] .^ 2))]);

% plot the sample fxns
figure; plot(t,x); title('Fig.1: 10 Sample Fxns (Random Process)');

% plot ensemble average and time average
figure; plot(t,sampleMean); title('Fig.2: Ensemble Mean');
figure; plot(t,timeMean); title('Fig.3: Time Mean');
```

```
1st Moment Stationary- 0
2nd Moment Stationary - 0
```

Fig.1: 10 Sample Fxns (Random Process)

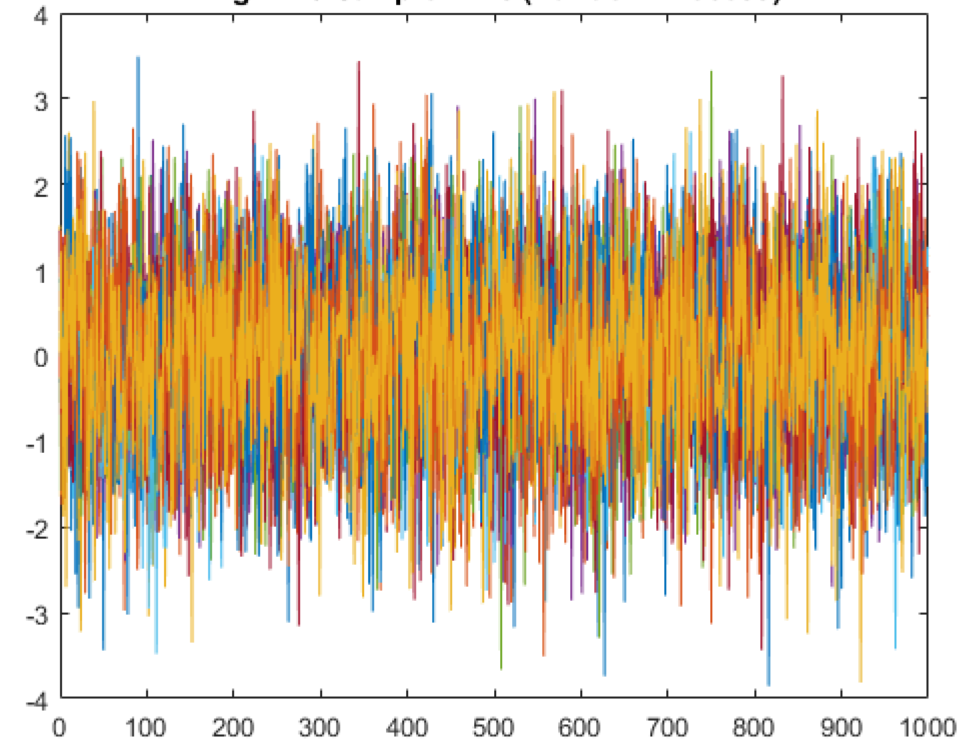
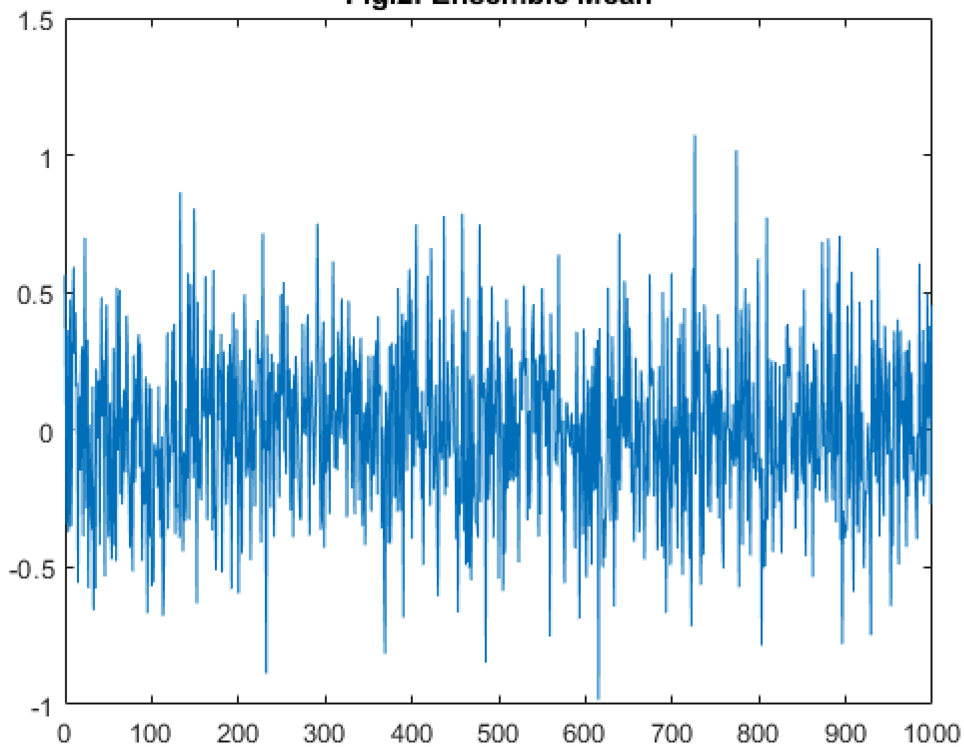
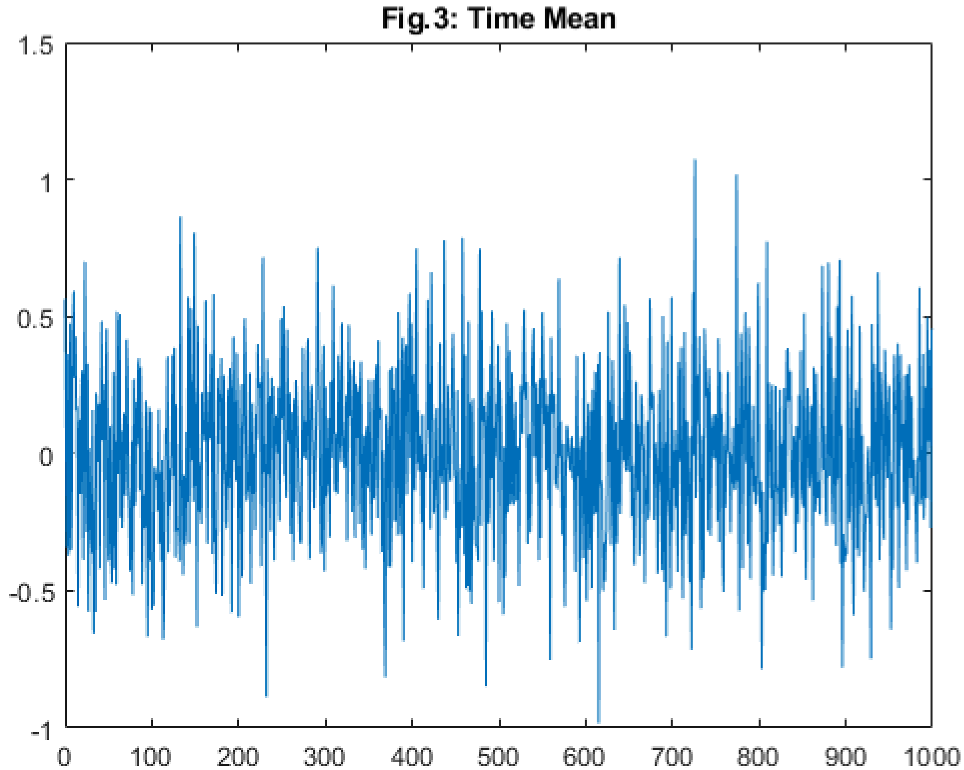


Fig.2: Ensemble Mean





Discussion: Without further processing and simulations and by just looking with fig1-3, the generated random processes above may be classified as Stationary. Stationary process is a stochastic process with its statistical propoerties don not alter with time. Its joint probability distribution or first/seconds moments are constant. In the case of our given vectors, both the means and variance are constant (independent of time) and as proven by the yielded output '0' of the first and second moments. First and second moments indicate that the vectors (processes) differ by elements but not their mean and variance in terms of time.

1.b. Ergodic

```
numFxn = 1000; % number of sample fxn
samples = 1000; % number of samples/fxn
x = randn(samples,numFxn); % generate ten sample fxns w/ u=0 o=1
t = 0:999; % sample interval

timeMean = mean(x,2);
sampleMean = mean(x,1);

timeVariance = var(x,1,2);
sampleVariance = var(x,1,1);

disp(['1st Moment Ergodicy- ', num2str(mean([timeMean(:) - sampleMean(:)] .^
 2))]);
disp(['2nd Moment Ergodicy - ', num2str(mean([timeVariance(:) -
 sampleVariance(:)] .^ 2))]);

% plot the sample fxns
figure; plot(t,x); title('Fig.4: 1k Sample Fxns (Random Process)');
```

```
% plot ensemble average and time average
figure; plot(t,sampleMean); title('Fig.5: Ensemble Mean');
figure; plot(t,timeMean); title('Fig.6: Time Mean');
```

1st Moment Ergodicy- 0.002018
2nd Moment Ergodicy - 0.0043018

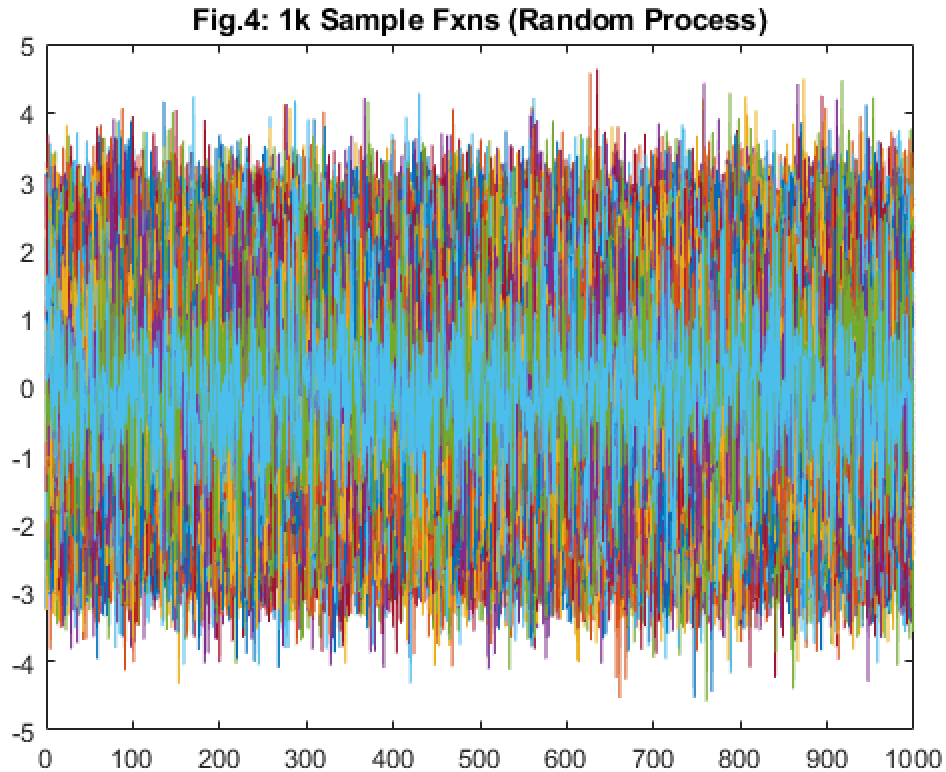


Fig.5: Ensemble Mean

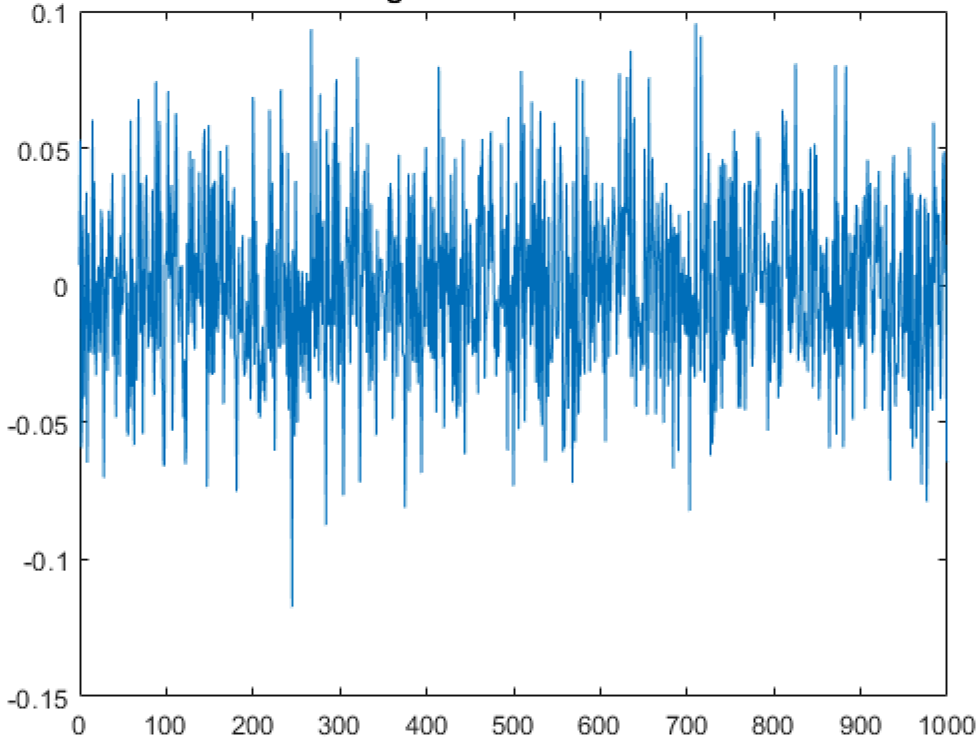
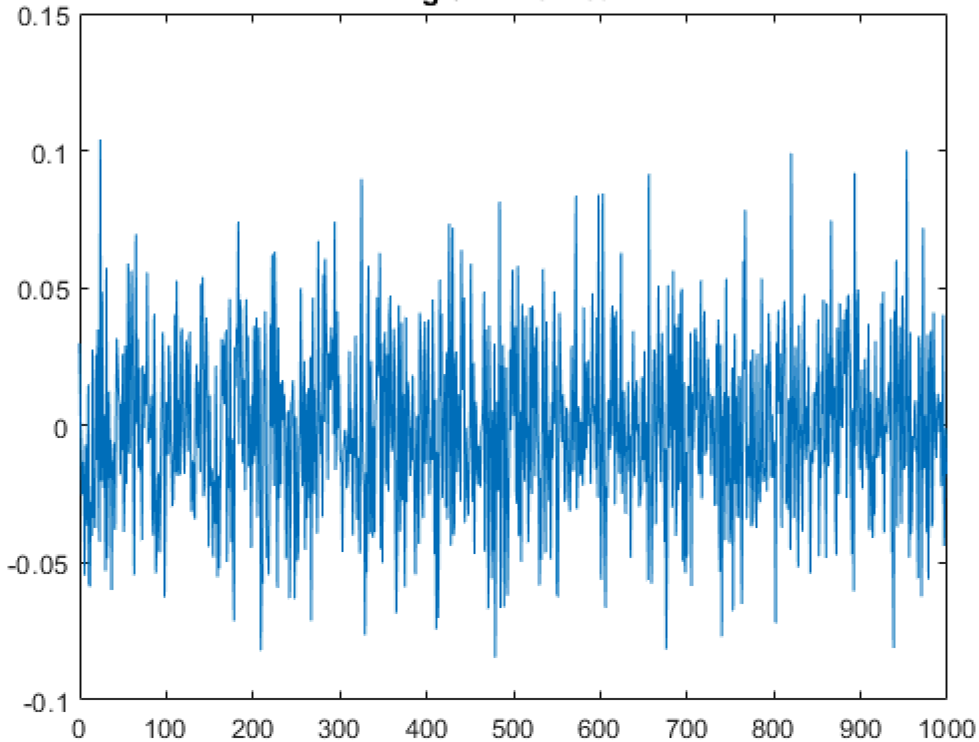


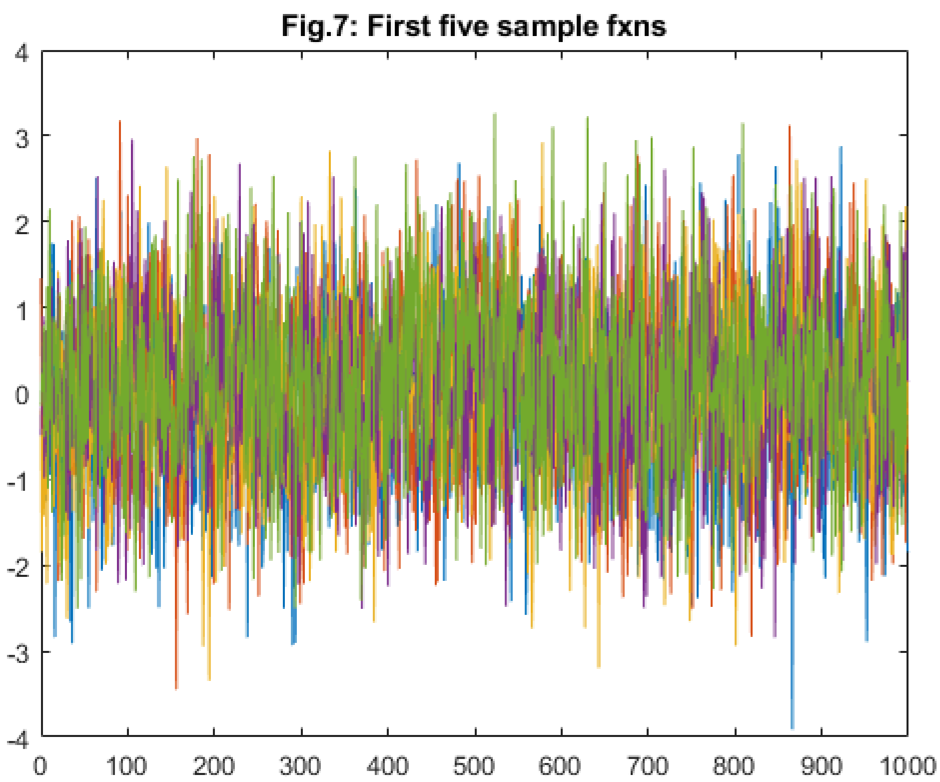
Fig.6: Time Mean



Discussion: Similar to 1a, the vectors produced in 1b may be classified as both Stationary (by mere plot observation) and Ergodic as the moment along the time and sample dimensions are almost equivalent (see first and second moments of ergodicy). Unlike in Stationary that the moment are 'the same as is', in Ergodic, the moments are almost or equivalently the same but not as is, as always. Moreover, a sample is used to represent the entirety of the whole processes. Take note that these moments differ for every time execution as they are taken from particular samples of the processes and not the entire processes.

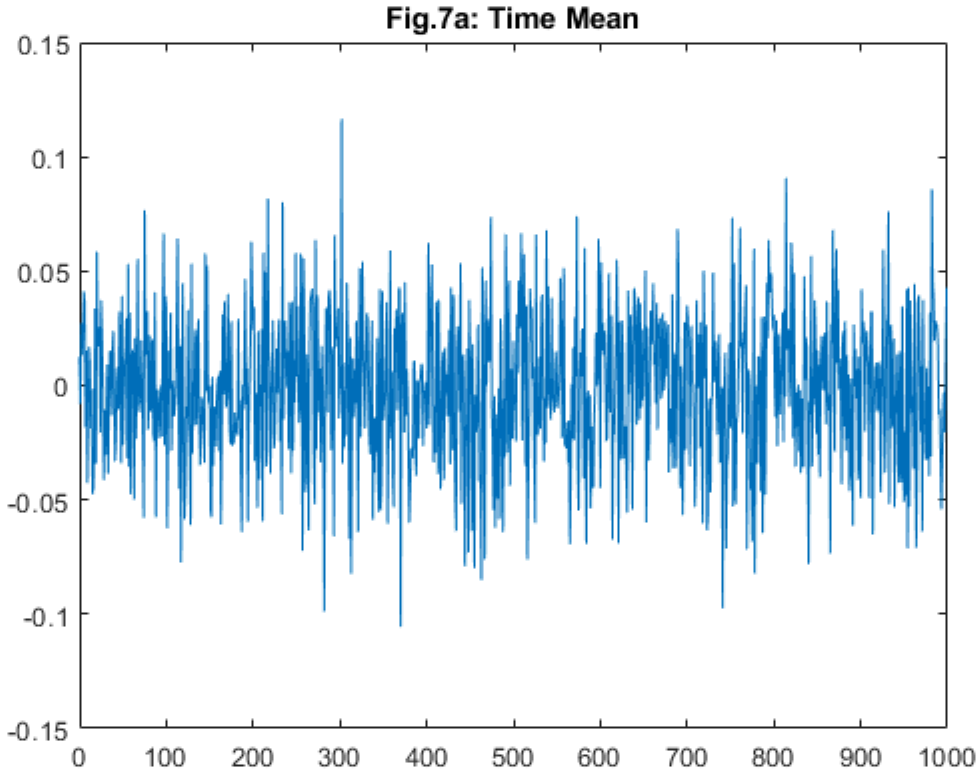
2a-b. $x(t) = 2sin(2 * \pi * 0.002t)$

```
N = randn(1000,1000);
t = 0:999; % sample interval
x = 2*sin(2*pi*0.002*t)+N;

figure; plot(t,x(:,1:5)); title('Fig.7: First five sample fxns');

timeMean = mean(x,2);
sampleMean = mean(x,1);
```



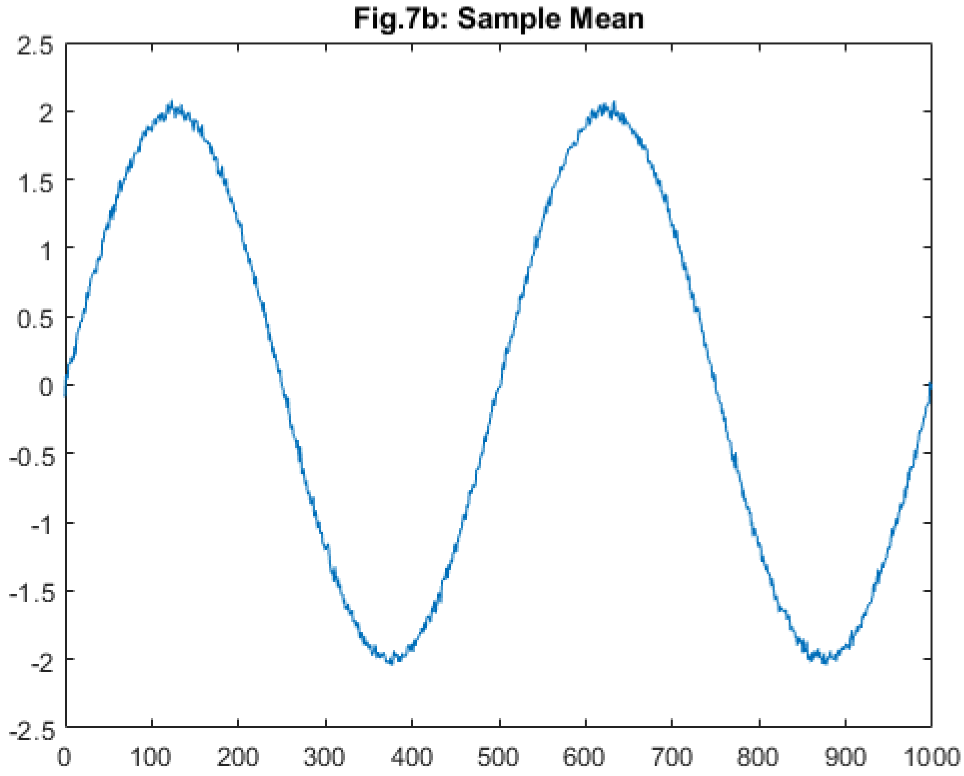
Time mean

```
figure; plot(t,timeMean); title('Fig.7a: Time Mean')
```



Sample mean

```
figure; plot(t,sampleMean); title('Fig.7b: Sample Mean')
```




```
timeVariance = var(x,1,2);
sampleVariance = var(x,1,1);

disp(['1st Moment - ', num2str(mean([timeMean(:) - sampleMean(:)] .^ 2))]);
disp(['2nd Moment - ', num2str(mean([timeVariance(:) - sampleVariance(:)] .^
 2))]);
```

```
1st Moment - 2.0034
2nd Moment - 4.0193
```

Discussion: By mere observation of the given expression and not Fig.7, it can be implied that the processes are neither stationary nor ergodic for x(t) rely on time t. It is not stationary for its variance and mean are not constant and time-variant. It is not ergodic for its time mean is not equal to or tend to the ensemble average. Observe the values generated with first and second moments and their difference in values. Furthermore, it is observable that the sample mean and variance mean are equivalent to each other.

3. Stationarity of randomly phased sinusoids.

```
Fs = 8000;
F1 = 100;
t = 0:1/Fs:5/F1-(1/Fs);
A = 1; K = 10;

% Generate K realizations of the phase
Ph = -pi+(2*pi).*rand(1,K);

% Generate freq mtx
```

```
Fmat = t'*F1*ones(1,K);

% Generate the phase mtx
Phmat = ones(size(t'))*Ph;

% Generate the sinusoid
Snk = A*sin(2*pi*Fmat+Phmat);

% Plot K relaizations (different phase) of the sinusoids
figure; plot(Snk); title('Fig.8: K realizations of sine');

% Visualize the random process in 3D
figure; mesh(Snk); title('Fig.9: Random Processes');

% compute the time average of each realization
Tave = mean(Snk);

% Ensemble mean
Eave = mean(Snk');

% plot autocorrelation
figure; plot(xcorr(Snk)); title('Fig.10: Autocorrelation of Snk')

timeMean = mean(Snk,2);
sampleMean = mean(Snk',1);

timeVariance = var(Snk,1,2);
sampleVariance = var(Snk',1,1);

disp(['1st Moment WSS- ', num2str(mean([timeMean(:) - sampleMean(:)] .^ 2))]);
disp(['2nd Moment WSS- ', num2str(mean([timeVariance(:) -
 sampleVariance(:)] .^ 2))]);
```

```
1st Moment WSS- 0
2nd Moment WSS- 0
```

Fig.8: K realizations of sine

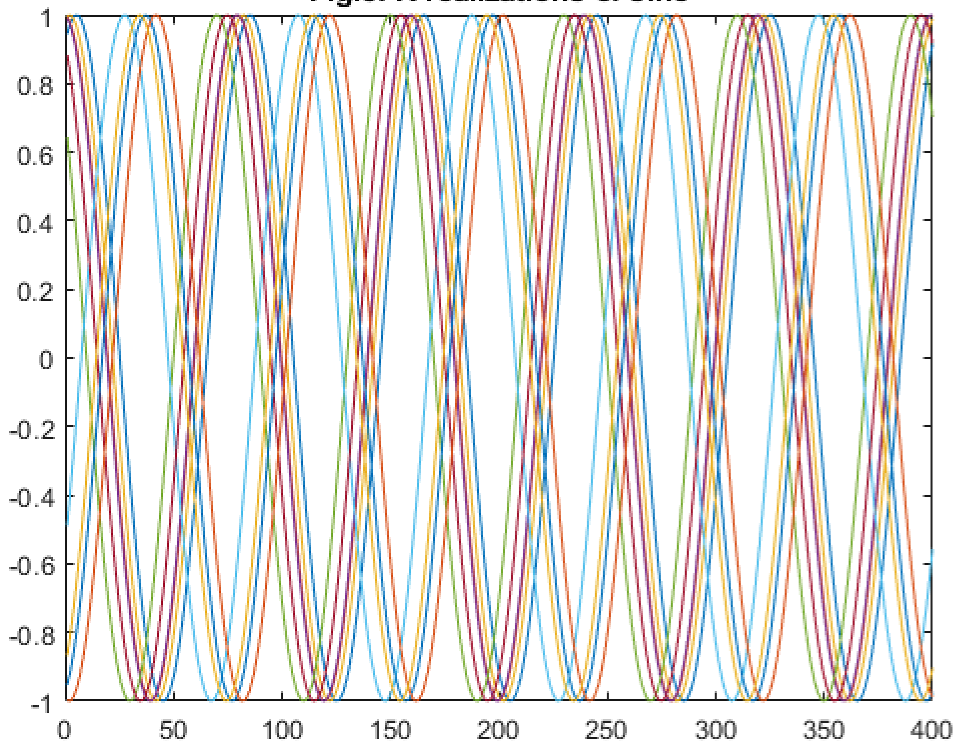
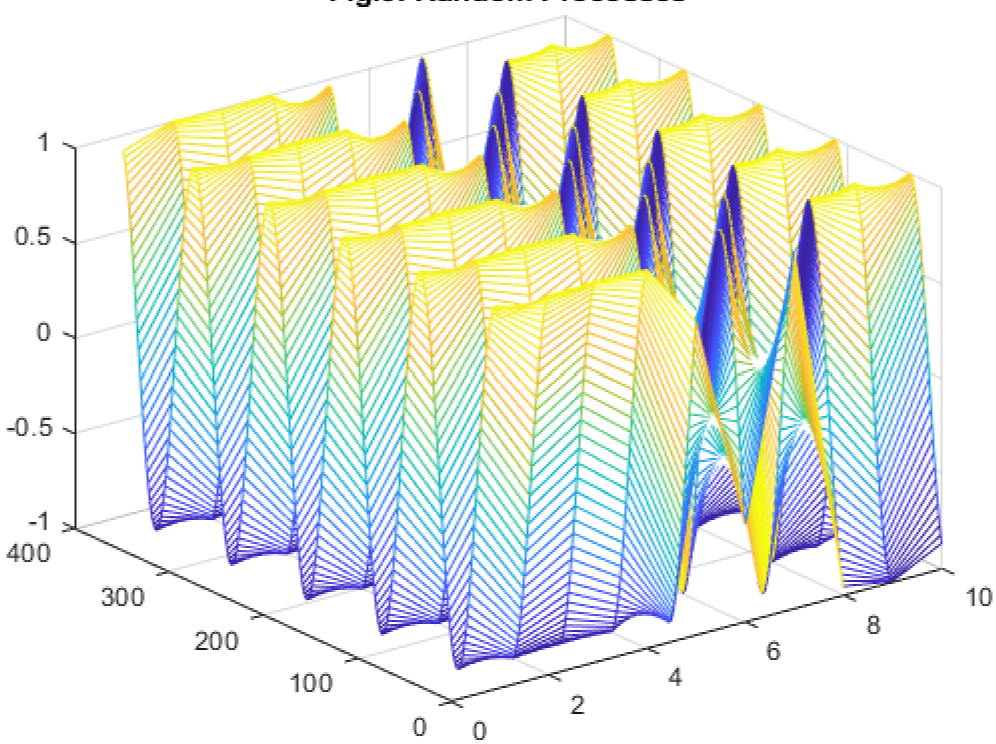
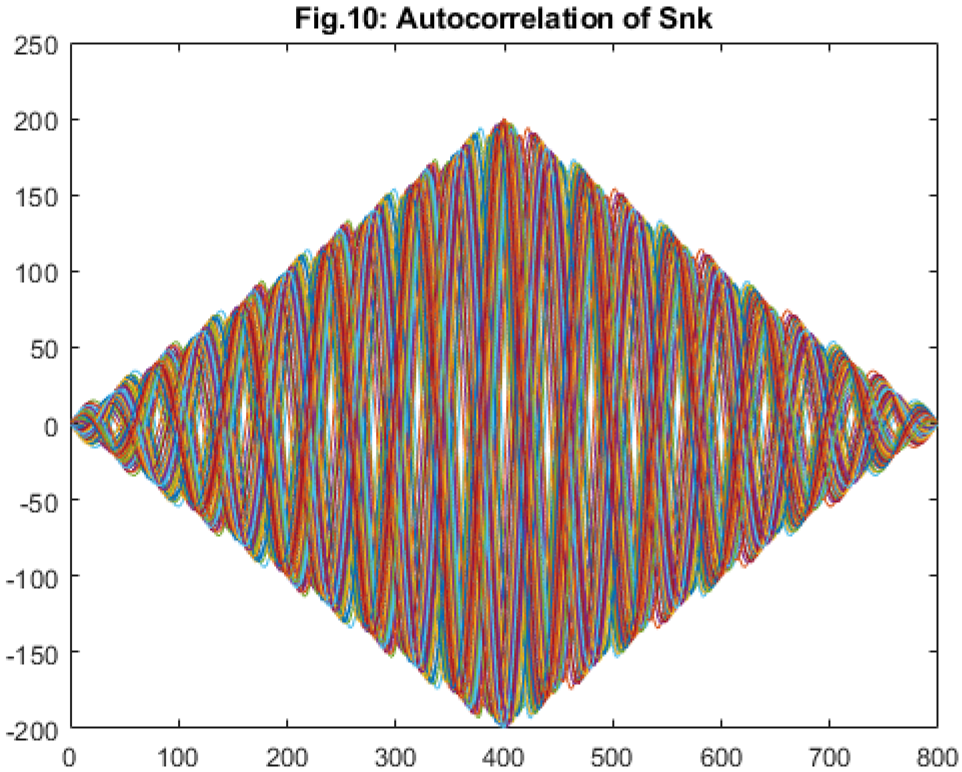


Fig.9: Random Processes





Proof of non-ergodic: if we increase K to 100...

```
Fs = 8000;
F1 = 100;
t = 0:1/Fs:5/F1-(1/Fs);
A = 1; K = 100;
Ph = -pi+(2*pi).*rand(1,K);
Fmat = t'*F1*ones(1,K);
Phmat = ones(size(t'))*Ph;
Snk = A*sin(2*pi*Fmat+Phmat);
Tave = mean(Snk);
Eave = mean(Snk');
timeMean = mean(Snk,2);
sampleMean = mean(Snk',1);
timeVariance = var(Snk,1,2);
sampleVariance = var(Snk',1,1);
disp(['1st Moment Ergodicy Test- ', num2str(mean([timeMean(:) -
 sampleMean(:)] .^ 2))]);
disp(['2nd Moment Ergodicy Test- ', num2str(mean([timeVariance(:) -
 sampleVariance(:)] .^ 2))]);
```

```
1st Moment Ergodicy Test- 6.2249e-34
2nd Moment Ergodicy Test- 2.1008e-32
```

a-b. In a generalized statement, the randomly geenrated sinusoids above may be classified as a Wide-Sense Stationary (WSS) random process. The reasoning for this claim is similar to the general description of a stationary random process in section 1 above. Moreover, it is evident that the generated sinusoidal processes are time-invariant with its moments yield a constant '0' for the first and second moment test (see 1st/2nd moment WSS above).

The randomly generated sinusoid may also be classified as an Ergodic Random Process. Ofcourse, to suffice the definition of ergodicity, consider an increased realization K to 100. Generating the sinusoids time mean and sample mean yield approximately equivalent results as shown in the *proof-section above*. Without further words and from this observation, it can be inferred that this random process is an Ergodic RP.

Given Fig.10 or the autocorrelation plot of the generated random process with 10 realizations, aside from the generic stationary properties, it suffices the ff. WSS properties among others: **(1)** $R_x(t)$ is an even function given $R_x(t) = R_x(-t)$ if we consider the x=400 of fig.10 as our time origin. **(2)** It is clear that the generated random process is a WSS given that its autocorrelation plot achieve a maximum realization at time origin (t=0) which is 400 in fig.10.

Discussion: Regardless if the random processes are generated using sin/cos fxns or randn/rand, a random process is stationaru if **(1)** its mean is a contant (time-invariant) value $u_x(t) = ux$; **(2)** its mean square value is also a constant value; **(3)** its variance is constant value $\sigma_x^2(t) = \sigma_x^2$; **(4)** its autocorrelation depends on the time distance between two samples $R_x(t_1, t_2) = R_x(T)$; **(5)** its autocovariance depends on time distance between two samples at time t_1 and $t_2 : K_x(t_1, t_2) = K_x(T)$ -- *be noted that autocovariance and autocorrelation depends on time but the entire functions of the process itself* .

4. Real-life realization of Random Process

a. Paying monthly electricity bill. Paying monthly electric bill requires an individual to perform certain processes according to their preferred mode of payment. Considering the traditional electric bill over-the-counter payment, an individual requires to fall-in-line prior to his/her bill to be received by the counter and the volume of the line of people paying their bill over-the-counter depend on the time of the day. In general, the randomness of this process (electric bill payment) is the number/volume of people paying their monthly electric bill over-the-counter varies each time(minutes,hours,days,weeks,monthly). This volume of bill payers may be averaged for a set of time and may be used to statistically determine the possible volume of electric bill customers who are going to pay through over-the-counter payment mode for a particular time.

b. Listening or watching the daily Covid-19 news cases. Similar to 4a above, the randomness of this process are the numbers or volume of observed/reported Covid-19 cases per day and when averaged by a factor of month, semi-annual, or annual. This averaged number may be used as an activation value for determining the possible volume of case for a particular time signature through the observed parameters.

c. Observations of people walking and vehicles passing in a busy intersection. Similar to 4a and 4b, the volume of people and vehicles passing through an intersection differ in all moments of time. Given a day, the number or volume of people crossing the pedestrians of the intersection and the number of cars passing-through the same intersection differ every hour for a total number of 24 random processes for a day. A collection of this record for a month might be used to create a predicitve model for the later months which would allow the possible numbers of people and cars passing through the same intersection in each hour of the following months and times -- ofcourse in consideration of the observed parameters.

Why is each process a random one? Although it is considered that in real-life, nothing may be done twice or exactly the same under the same time but only a particular process may occupy a particular moment of time per function. Furthermore, there are processes that may be considered to as stationary or repetitive, the values or function performed under a certain still vary from one process execution to another hence random.

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