EE 214 PROBABILITY AND RANDOM PROCESSES IN ELECTRICAL ENGINEERING

Lecture 1A: Discrete Random Variables

RANDOM VARIABLES

A random variable is a function that associates a unique numerical value with every outcome of an experiment.

There are two types of random variable - discrete and continuous.

RANDOM VARIABLES

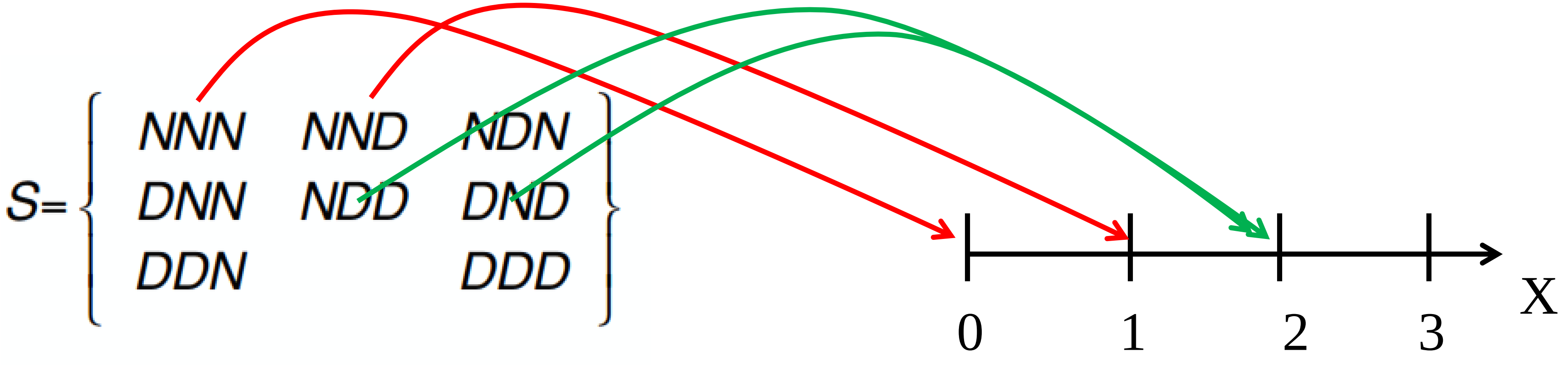
A discrete random variable is one which may take on only a countable number of distinct values such as 0, 1, 2, 3, 4, ... If a random variable can take only a finite number of distinct values, then it must be discrete.

A continuous random variable is one which takes an infinite number of possible values. Continuous random variables are usually measurements. Examples include height, weight, the amount of sugar in an orange, the time required to run a mile.

DISCRETE RANDOM VARIABLE EXAMPLE

Consider the testing of 3 electronic devices (D = defective; N = not defective)

If we count the number of defectives, then we assign values for each outcome



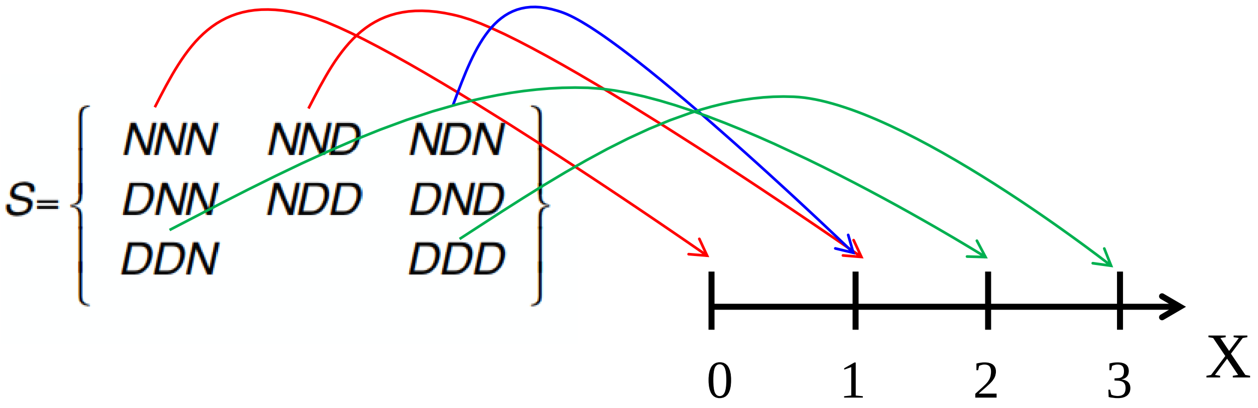
Random variable → capital letter (X)

PROBABILITY DISTRIBUTION

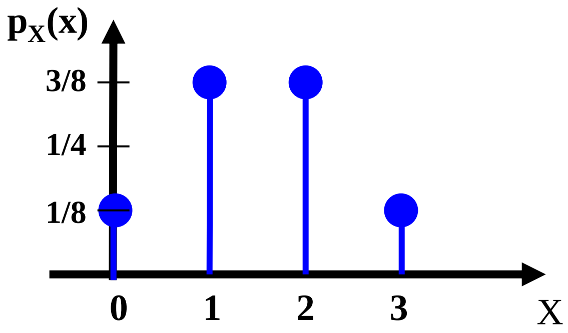
- The probability distribution of a discrete random variable is a list of probabilities associated with each of its possible values. It is called the probability mass function or pmf, denoted by $p_X(x)$.
- More formally, the probability distribution of a discrete random variable X is a function which gives the probability $p(x_i)$ that the random variable equals x_i, for each value x_i:
  - $p(x_i) = P(X=x_i)$

PROBABILITY MASS FUNCTION

X = number of defective devices out of 3



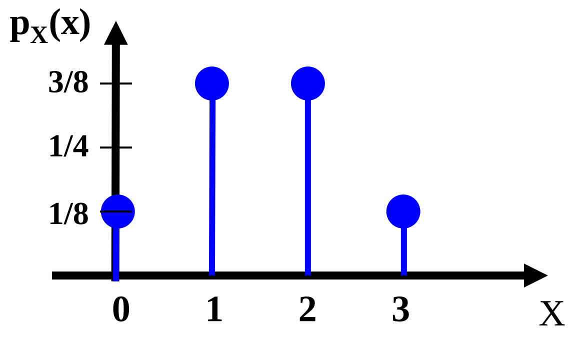
x	0	1	2	3
P(X = x)	1/8	3/8	3/8	1/8

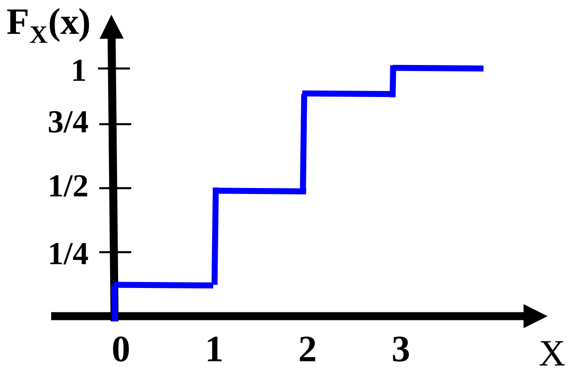


CUMULATIVE DISTRIBUTION FUNCTION

All random variables (discrete and continuous) have a **cumulative distribution function** or cdf, denoted by F(x). It is a function giving the probability that the random variable X is less than or equal to x, for every value x.

$$F(x) = P(X \leq x)$$





PROPERTIES

- $p_X(x)$ satisfies axioms of probability
- $\Sigma p_X(x_i) = 1$

- Nondecreasing

  If $x_1 < x_2$ then $F_X(x_1) \le F_X(x_2)$
- Extreme Limits

  $F_X(+\infty) = 1 \quad F_X(-\infty) = 0$
- Complement

  $P(X>x) = 1 - F_X(x)$

EXAMPLE

In a game of darts, a dart is thrown to a circular board of radius *a* and scored depending on how near to the center of the board the dart hits. Let X be the random variable associated with the score of a single dart thrown.

- The board is numbered by concentric, equally spaced regions with values of 100, 90, 80, up to 10 at the outermost annular region of the board.

$$S_X = \{10, 20, \ldots, 100\}$$ hence X is a discrete random variable.

WHEEL OF FORTUNE

- Discrete RV case: Equal intervals marked 1,2,…,100

PMF $p_X(x) = 1/100 = 0.01$

CDF $F_X(x) = \begin{cases} 0 & x \leq 0 \\ [x]/100 & 0 < x \leq 100 \\ 1 & x > 100 \end{cases}$

EXAMPLE

Two independent rolls of a fair tetrahedral die

- F: outcome of the first throw
- S: outcome of the second throw
- X = min(F,S)

x	$p_X(x)$
1	7/16
2	5/16
3	3/16
4	1/16

S = second roll				
4	(1,4)	(2,4)	(3,4)	(4,4)
3	(1,3)	(2,3)	(3,3)	(4,3)
2	(1,2)	(2,2)	(3,2)	(4,2)
1	(1,1)	(2,1)	(3,1)	(4,1)
	1	2	3	4
	F = first roll			

BINOMIAL PROBABILITY

- We have n independent coin tosses, with P(H) = p

Determine:

- P(HTHHT)
- P(a toss sequence)
- P(k heads)

BINOMIAL PROBABILITY

- We have n independent coin tosses, with P(H) = p

Determine:

- P(HTHHT) $= P(H)*P(T)*P(H)*P(H)*P(T)$

$$= P(H)^3 * P(T)^2$$

$$= (p)^3 (1-p)^2$$

BINOMIAL PROBABILITY

- We have n independent coin tosses, with P(H) = p

Determine:

- $P(HTHHT) = (p)^3 (1\text{-}p)^2$
- P(a toss sequence): assuming k heads (and n-k tails)

$$= (p)^k (1\text{-}p)^{(n\text{-}k)}$$

BINOMIAL PROBABILITY

- We have *n* independent coin tosses, with P(H) = *p*

Determine:

- $P(HTHHT) = (p)^3 (1-p)^2$
- $P(\text{a toss sequence}) = (p)^k (1-p)^{(n-k)}$ for a sequence with k heads
- P(k heads)

$$\sum \mathrm{P(sequence\ with\ k\ heads)} = \binom{n}{k} p^k (1-p)^{(n-k)}$$

BERNOULLI RANDOM VARIABLE

- Used to represent an experiment with only 2 possible outcomes
- Probability mass function (PMF)

Outcome	X	$p_X(x)$
Failure	0	$1 - p$
Success	1	p

- The experiment is called a Bernoulli trial

BINOMIAL RANDOM VARIABLE

- The number of successes in n Bernoulli trials
- Probability mass function

$$p_X(k) = \binom{n}{k} p^k (1-p)^{n-k}, k = 0, 1, 2, \ldots, n$$

  - n: number of Bernoulli trials
  - k: number of successes
  - p: probability of success