Electrical and Electronics Engineering Institute
University of the Philippines, Diliman
Quezon City, 1101 Philippines

EE 214 Module 5
Lab Exercise 5.1: Spectral Properties of Random processes and LTI systems

General Instructions when submitting machine problems and lab exercises

1. Answers to the questions and additional discussion must be in the form of a written report and submitted in PDF format.
2. Your MATLAB scripts should be executable from the command line with no additional user inputs.
3. Include comments in your code to make your code more understandable
4. Use variable names that are meaningful to make your code more readable
5. Submit all the necessary files (pdf document, m-files, output images) as a zipped file using the following naming convention: **surname_nickname_EE214-LabXX_EE214.zip**
6. Submit the zip file through the UVLe submission bin

I. Sunspot Activity Analysis

1. The matlab script sunspot_demo.m demonstrates the spectral analysis of the sunspot observations to be able to predict the occurrence of sunspot activities. This requires loading the data file sunspot.dat which should be available in your Matlab distribution. This is also discussed in our reference text in Chapter 17 as an example of a WSS process and its spectral analysis. The relevant pages of the text are also uploaded in UVLe for your reference.

    a.) Study the code and the comments in the code give your insights on how the spectral analysis was done, how was the cycle period computed?

    b.) Write your own Matlab code using a periodogram to estimate the sunspot spot activity cycle.

    Discussion: Spectral analysis using PSD for WSS processes.

II. Noise and LTI Systems

1. Generate a Gaussian process, Let $x(t) = X_t$, where X_t is a sequence of independent, identically distributed Gaussian random variables with mean 0 and variance 1. Generate one (1) sample function $x(t, \rho_0)$ of this process from t = 0:999.
    a. We have shown previously that the process is stationary and ergodic but non-correlated, we will not pass this process to a LTI system, a simple low pass filter.
    b. Use x(t) as input to a smoothing filter with impulse response:

$$h(t) = \begin{cases} 0.25 & 0 \le t \le 3 \\ 0, & otherwise \end{cases}$$

The output y(t) is computed as:

$$y(t) = \sum_{t=0}^{\infty} h(\tau)x(t-\tau)$$

Predict the resulting autocorrelation of the output process y(t). Is the output still ergodic? Is it WSS? Explain your answer.

c. To verify the properties of the output process y(t), implement the resulting filter with x(t) s input using the matlab command:
`y = filter([0.25 0.25 0.25 0.25], 1, x);`
where x is a vector containing a sample function of the process x(t).
Estimate the autocorrelation Ry($\boldsymbol{\tau}$) using `xcorr()`, i.e. `ry = xcorr(y,'biased').`
Does the output autocorrelation agree with theory?

d. Let's use another filter defined by the following equation:
`y(t) = 0.9y(t-1) + 0.2x(t)`
Suppose we x(t) as input to this filter and predict the autocorrelation of the output process. Is the output still ergodic? Is it WSS? Justify your answer.

e. To compute the output of the above filter in Matlab, we have:
`y = filter(0.2, [1 -0.9], x);`
where x is a vector containing a sample function of the process x(t).
Estimate the autocorrelation Ry($\boldsymbol{\tau}$) using `xcorr()`, i.e. `ry = xcorr(y,'biased').`
Plot and examine Ry($\boldsymbol{\tau}$) for $-50 \le \boldsymbol{\tau} \le 50$. Does it agree with the theory?

III. Spectral Analysis of Signals with Noise

Included in the lab exercise folder and matlab data files, these are signals with noise at different signal-to-noise ratio (SNR) and different sampling rates. For each matlab data file, try to determine the characteristics of the signal that are buried in noise by performing spectral analysis on each of them.

The table below summarizes the information about the signals in the data files

Matlab file	**Fs**	**Noise level**
signalA_0dB_44100_5Ts.mat	44100	0 dB
signalA_0dB_22000_5Ts.mat	22000	0 dB
signalA_5dB_44100_5Ts.mat	44100	-5 dB

signalA_5dB_22000_5Ts.mat	22000	-5 dB
signalA_10dB_44100_5Ts.mat	44100	-10 dB
signalA_10dB_44100_5Ts.mat	22000	-10 dB

1. Start with the signal with 0 dB noise level and sampling rate at 44100, perform spectral analysis using periodogram. What parameters of the periodogram did you specify (number of FFT points? averaging? one-sided?, ect) ?
    a. Briefly justify the parameters you have chosen for the periodogram.
    b. Were you able to determine the frequencies of the signals buried in noise? What are those frequencies and relative amplitudes?

2. Repeat your spectral analysis the same signal with 0 dB but with a different sampling rate and perform spectral analysis.
    a. What parameters did you use? Are the signal frequencies still discernible?
    b. What is the effect of a lower sampling rate on the spectral analysis?

3. Repeat the process of doing the spectral analysis on the other signals with lower (more negative) SNR and at different sampling rates.
    a. Can you still determine the signal frequencies and relative amplitudes at -5 dB? at -10 dB
    b. What is the effect of increasing noise level and sampling rate on the ability to discern signal frequencies?

4. Assuming we do not have enough data record about the signal due to a shorter observation period, we can simulate this by truncating part of the signal and analysing the rest.
    a. For the signals at -5 dB noise level, at sampling rate of 44100, truncate the signal and retain only 80% of its original length, or set the values of the signals to zero on the truncated part. Perform spectral analysis on the truncated signal. Can you still discern the signal frequencies and relative amplitudes?
    b. Truncate also the signal at -5 dB noise level and sampling rate of 22000 to 80% of its original length. Perform spectral analysis and see if you can still determine the spectral characteristics of the signals
    c. Repeat a) and b) above but this time truncate the signal to 60% of the original length. What can you say about having a shorter data record and the ability to determine the spectral characteristics of the signal?
    d. What is the shortest data record that you can afford but still be able to differentiate the signal frequencies and relative amplitudes for a signal with -5dB noise at 44100 sampling rate?