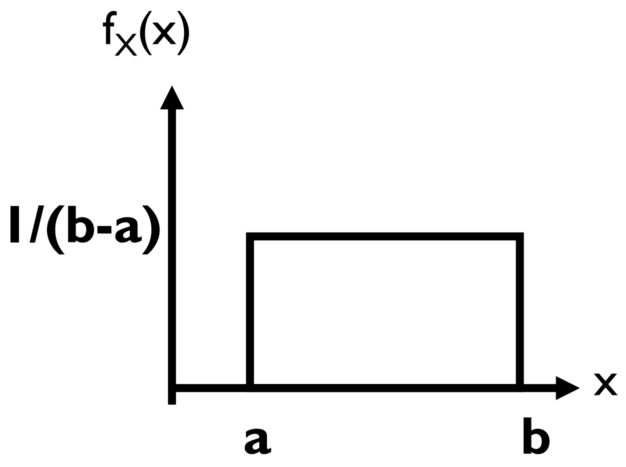
EE 214 PROBABILITY AND RANDOM PROCESSES IN ELECTRICAL ENGINEERING

Lecture 2B: Expected Value of Continuous RV; Variance, Transformation of CRV

UNIFORM RANDOM VARIABLE

- Sub-intervals of the same length are equally likely



$$E[X] = (b+a)/2$$

$$E[X^2] = \int_b^a x^2 f_X(x)dx = \left.\frac{x^3}{3(b-a)}\right|_a^b$$

$$E[X^2] = \frac{b^3 - a^3}{3(b-a)} = \frac{b^2 + ab + a^2}{3}$$

$$\sigma_X{}^2 = \frac{b^2 + ab + a^2}{3} - \left(\frac{b+a}{2}\right)^2 = \frac{(b-a)^2}{12}$$

TRANSFORMATION

- Lex X be a continuous random variable with pdf

$$f_X(x) = \begin{cases} 2x & 0 \le x \le 1 \\ 0 & elsewhere \end{cases}$$

- We define Y = g(X)

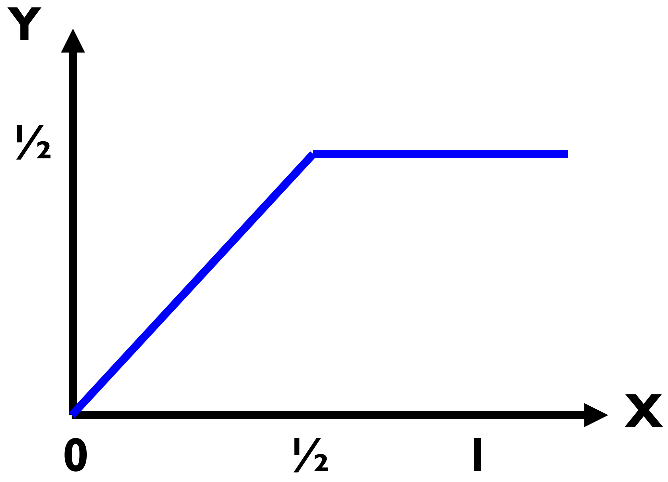
$$Y = \begin{cases} X & 0 \le X \le \frac{1}{2} \\ \frac{1}{2} & X > \frac{1}{2} \end{cases}$$

- Determine $F_Y(y)$

SOLUTION

- Note: R_X=[0,1]

$$Y = \begin{cases} X & 0 \le X \le \frac{1}{2} \\ \frac{1}{2} & X > \frac{1}{2} \end{cases}$$



- Therefore, R_Y=[0,1/2]
  - $F_Y(y) = 0$ $y<0$
  - $F_Y(y) = 1$ $y > ½$

SOLUTION

- For 0<y<1/2

$$f_X(x) = \begin{cases} 2x & 0 \le x \le 1 \\ 0 & elsewhere \end{cases}$$

$$Y = \begin{cases} X & 0 \le X \le \frac{1}{2} \\ \frac{1}{2} & X > \frac{1}{2} \end{cases}$$

- P(Y=1/2) = P(X>1/2)

$$F_{Y1}(y) = \int_0^y 2x\,dx = x^2\Big|_0^y = y^2$$

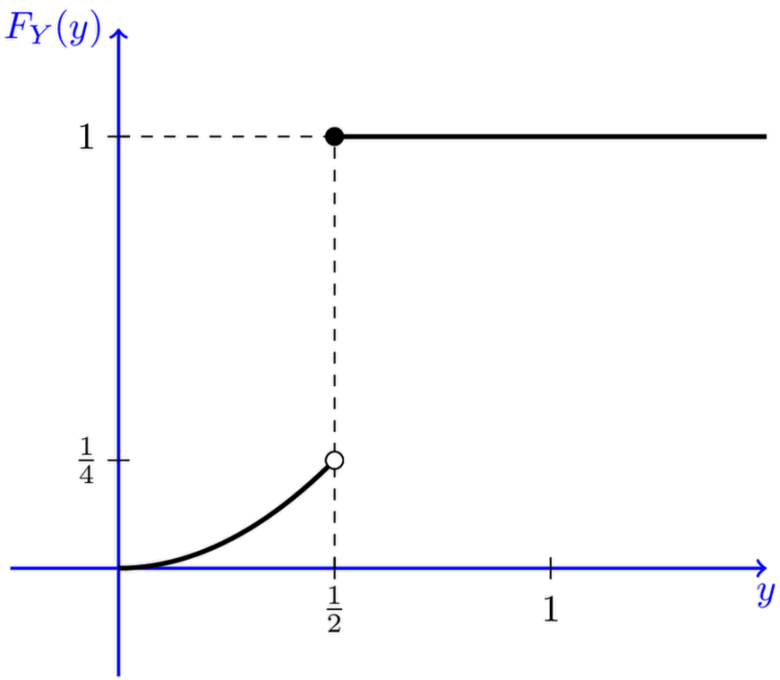
$$F_{Y2}\left(\frac{1}{2}\right) = \int_{1/2}^{1} 2x\,dx = x^2\Big|_{1/2}^{1} = \frac{3}{4}$$

SOLUTION

$$F_Y(y) = F_{Y1}(y) + F_{Y2}(y)$$

$$F_{Y1}(y) = \begin{cases} 0 & y < 0 \\ y^2 & 0 \le y < 1/2 \\ 1 & y > 1/2 \end{cases}$$

$$F_{Y2} = \frac{3}{4} \qquad y = 1/2$$



CHANGING UNITS

- Question: Suppose that a measurement of the temperature of an object in ^{0}F is a random variable X with a certain pdf. What would happen to the pdf if we instead want the measurement Y in ^{0}C?

To convert X°F to Y°C

$$Y = \frac{5}{9}\left(X - 32\right)$$

FUNCTIONS OF RANDOM VARIABLES

- Then we take the P(Y≤y) and replace Y by the function of X

$$F_Y(y) = P(Y \le y) = P\left(\frac{5}{9}\left(X - 32\right) \le y\right)$$

$$F_Y(y) = P\left(X \le \frac{9}{5}y + 32\right)$$

$$F_Y(y) = F_X\left(\frac{9}{5}y + 32\right)$$

FUNCTIONS OF RANDOM VARIABLES

- To get the pdf, we differentiate the expression and apply the chain rule

$$f_Y(y) = \frac{d}{dy} F_Y(y) = \frac{d}{dy} F_X\left(\frac{9}{5}y + 32\right)$$

$$f_Y(y) = f_X\left(\frac{9}{5}y + 32\right) \bullet \frac{d}{dy}\left(\frac{9}{5}y + 32\right)$$

$$f_Y(y) = \frac{9}{5} f_X\left(\frac{9}{5}y + 32\right)$$

FUNCTIONS OF RANDOM VARIABLES

General Steps:

- Express $F_Y(y) = P(Y \leq y)$ as a probability statement in X
- Express the probability in terms of the CDF or pdf of X
- Differentiate to get $f_Y(y)$, using chain rule if necessary

EXAMPLE

- The temperature achieved in a certain reaction varies from experiment to experiment. Measurements in Fahrenheit seem to follow a Gaussian distribution with a mean of 320 °F and standard deviation of 20 °F. Describe the pdf of the temperature in Celsius.

EXAMPLE

$$f_Y(y) = \frac{9}{5} f_X\left(\frac{9}{5}y + 32\right) \qquad f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\overline{X})^2}{2\sigma^2}}$$

$$f_Y(y) = \frac{9}{5}\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{\left(\frac{9}{5}y+32-\overline{X}\right)^2}{2\sigma^2}}$$

$$f_Y(y) = \frac{1}{\left(5\sigma/9\right)\sqrt{2\pi}} e^{-\frac{\left(y+[32-\overline{X}]5/9\right)^2}{2[5\sigma/9]^2}} = \frac{1}{11.11\sqrt{2\pi}} e^{-\frac{(y-160)^2}{2(11.11)^2}}$$

EXAMPLE

- A random variable X is uniformly distributed over the interval (0,6). If another random variable is formed such that $Y = 2(X - 3)^2 - 4$, find the density of Y.