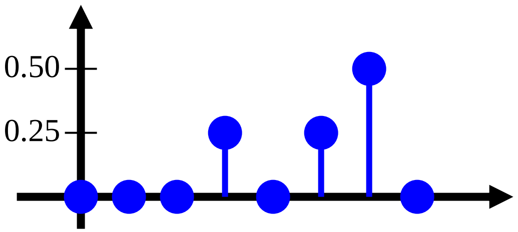
EE 214 PROBABILITY AND RANDOM PROCESSES IN ELECTRICAL ENGINEERING

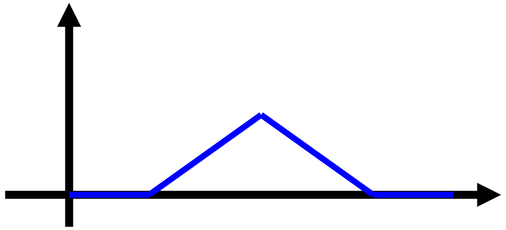
Lecture 2A: Continuous Random Variables

EXAMPLE: DISCRETE VS CONTINUOUS RV

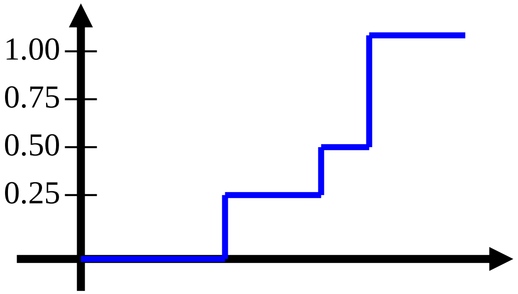
pmf: $p_X(x)$



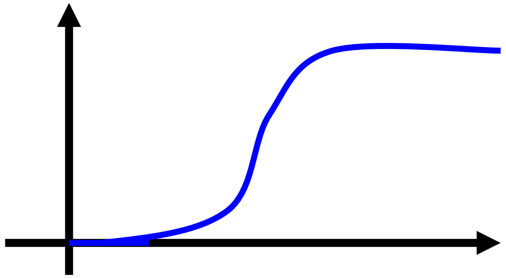
pdf: $f_X(x)$



cdf: $F_X(x)$



cdf: $F_X(x)$



EXAMPLE

In a game of darts, a dart is thrown to a circular board of radius *a* and scored depending on how near to the center of the board the dart hits. Let X be the random variable associated with the score of a single dart thrown.

- Case 1: The board is numbered by concentric, equally spaced regions with values of 100, 90, 80, up to 10 at the outermost annular region of the board.

$S_X = \{10, 20, \ldots, 100\}$ hence X is a discrete random variable.

EXAMPLE

- Case 2: The score of the dart hit is inversely proportional to its distance r (in cm) from the center, given by 1/r. Assume that dart always hits the board.

$S_X = \{x | 1/a < x < \infty\}$, hence X is a continuous random variable.

- Case 3: Scoring for the dart hit is similar to case 2, except that now the dart can hit outside the board area, which will be automatically scored as a zero.

$S_X = \{0, x | 1/a < x < \infty\}$, hence X is a mixed random variable.

WHEEL OF FORTUNE

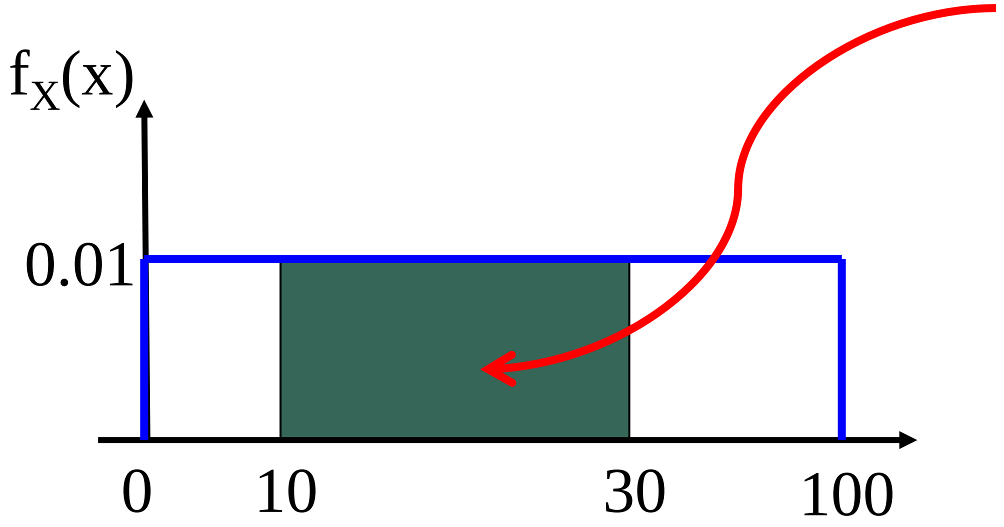
- Continuous RV case: Wheel of all real numbers from 0 to 100

$$\text{PDF } f_X(x) = \begin{cases} 0.01 & 0 < x \leq 100 \\ 0 & \text{elsewhere} \end{cases}$$

$$\text{CDF } F_X(x) = \begin{cases} x/100 & 0 < x \leq 100 \\ 1 & x > 100 \end{cases}$$

QUESTION:

What is the probability of getting a number between 10 and 30?



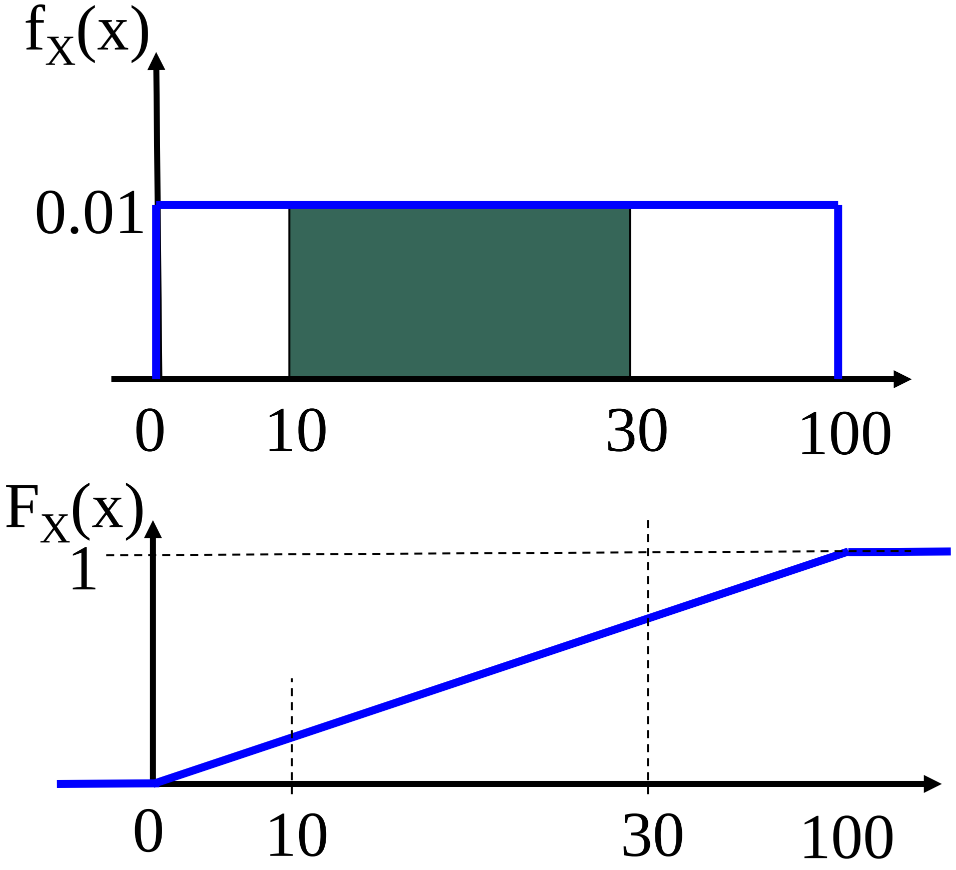
Area under the curve

$$P(10 < x < 30) = \int_{10}^{30} f_X(x)dx$$

$$= \int_{10}^{30} 0.01dx = 0.01(20)$$

$$= 0.2$$

USING CDF



$P(10 < x \leq 30)$

$= P(x \leq 30) - P(x \leq 10)$

$= F_X(30) - F_X(10)$

$= 30/100 - 10/100$

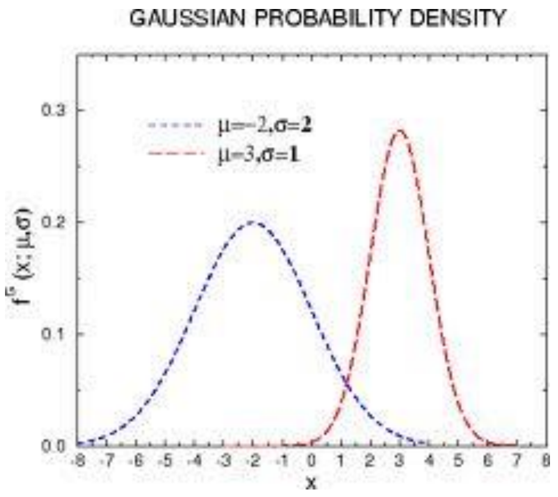
$= 0.3 - 0.1$

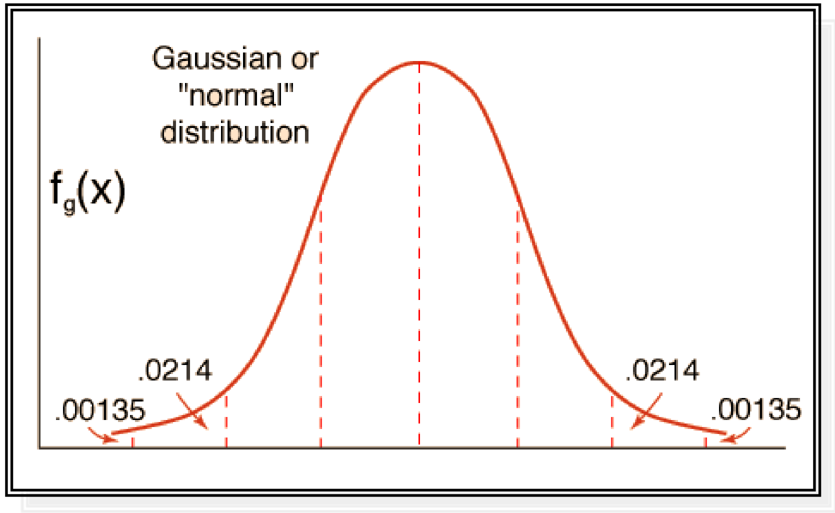
$= 0.2$

GAUSSIAN

- Model for natural random phenomena
- PDF

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-m)^2}{2\sigma^2}}$$

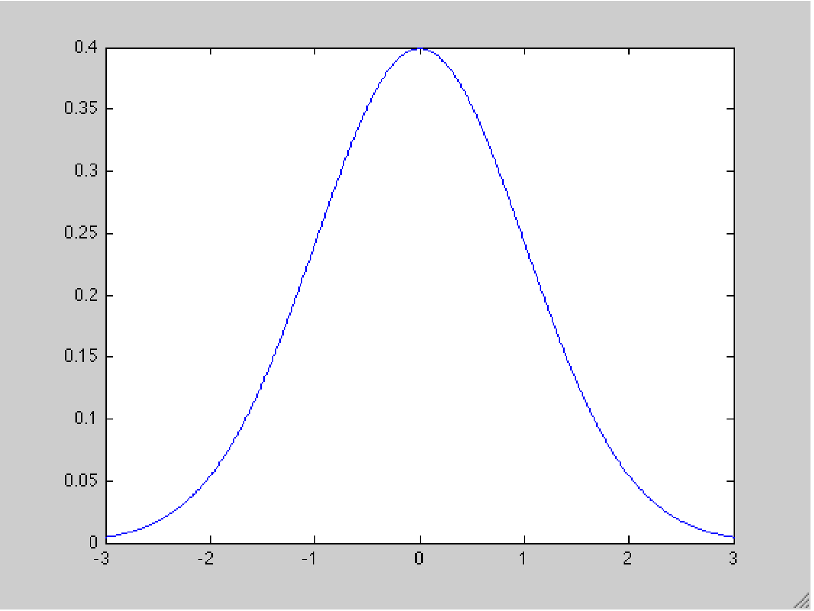




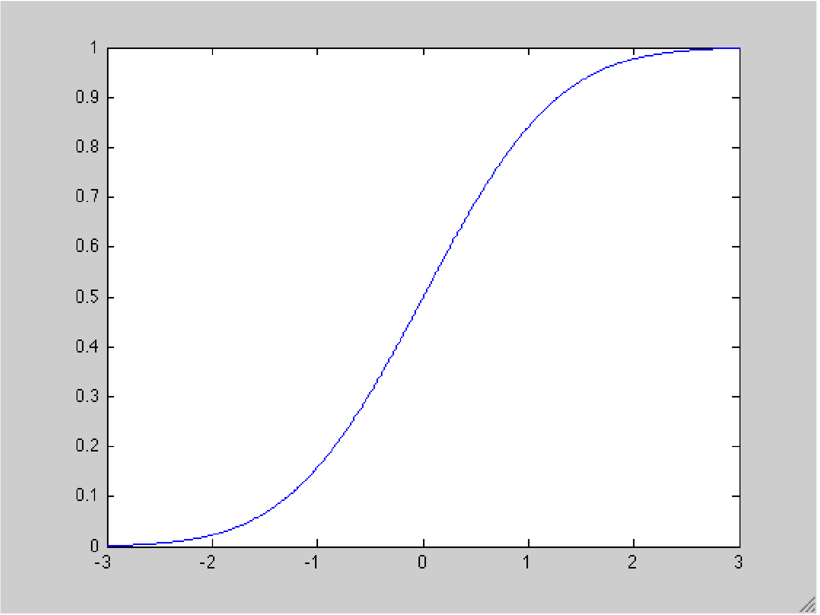
- Mean: m
- Variance: σ^2

GAUSSIAN PDF AND CDF

- Bell-shaped and even symmetric
- Mean = 0, standard deviation = 1



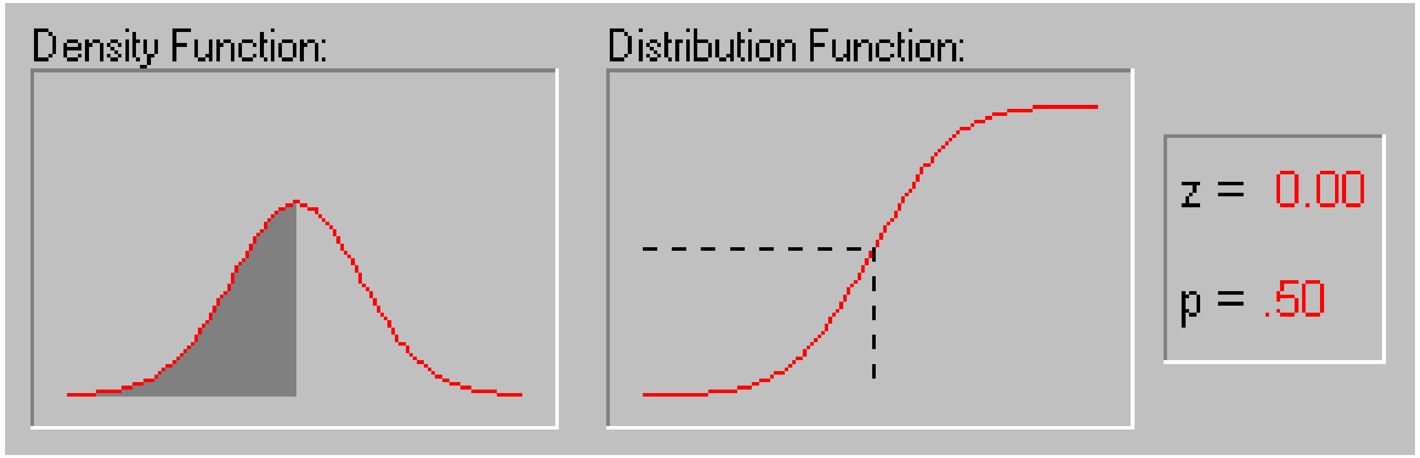
Gaussian PDF



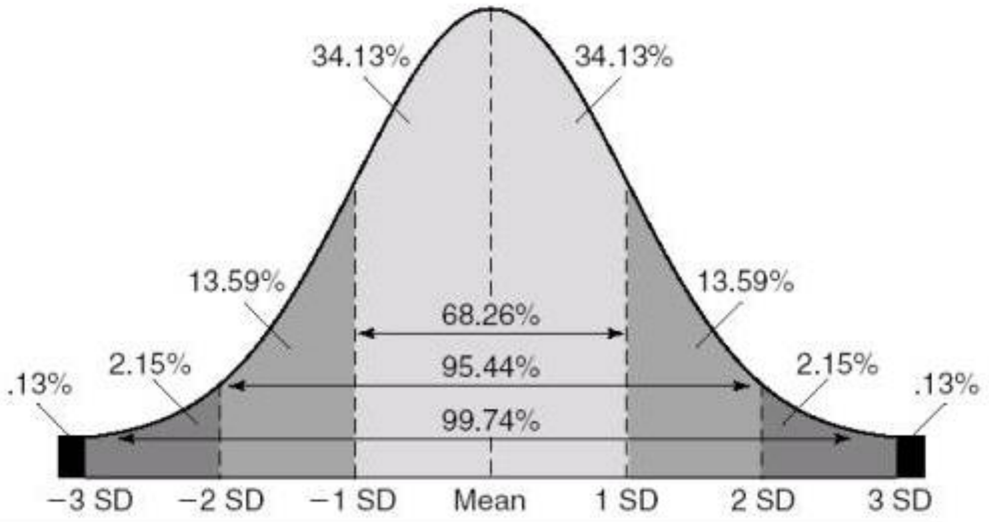
Gaussian CDF

PDF AND CDF

- Most important of all distributions
  - Scientific error measurements
  - noise models (AWGN)
  - Scores on tests



PERCENTILE SCORES



z-scores	−3	−2	−1	0	1	2	3
Percentile ranks	0.1	2	16	50	84	98	99.9
IQ	55	70	85	100	115	130	145
SAT	200	300	400	500	600	700	800

NORMALIZED GAUSSIAN

- Gaussian RV with m=0 and σ=1
  - Standard normal distribution
- Any Gaussian RV may be expressed in standard form

$$F_X(x) = G\left(\frac{x-m}{\sigma}\right)$$

NORMALIZED GAUSSIAN TABLE

x_{norm}	$G(x_{norm})$	x_{norm}	$G(x_{norm})$	x_{norm}	$G(x_{norm})$
0	0.50	0.6	0.73	1.4	0.92
0.1	0.54	0.7	0.76	1.6	0.94
0.2	0.58	0.8	0.79	1.8	0.96
0.3	0.62	0.9	0.82	2.0	0.98
0.4	0.66	1.0	0.84	2.5	0.99
0.5	0.69	1.2	0.88	3.0	1.00

EXERCISES

A Gaussian random variable Y has a mean of 10 and a standard deviation 3. Find:

- $P(7 < Y)$
- $P(Y < 11.5)$
- $P(7 < Y < 11.5)$

EXERCISES

Inductance values of coils made on a production line are Gaussian with a mean of 1.2mH and a standard deviation of 0.1mH. What is the probability of selecting an inductor from this line having a value within 1.0mH $\pm$10%?

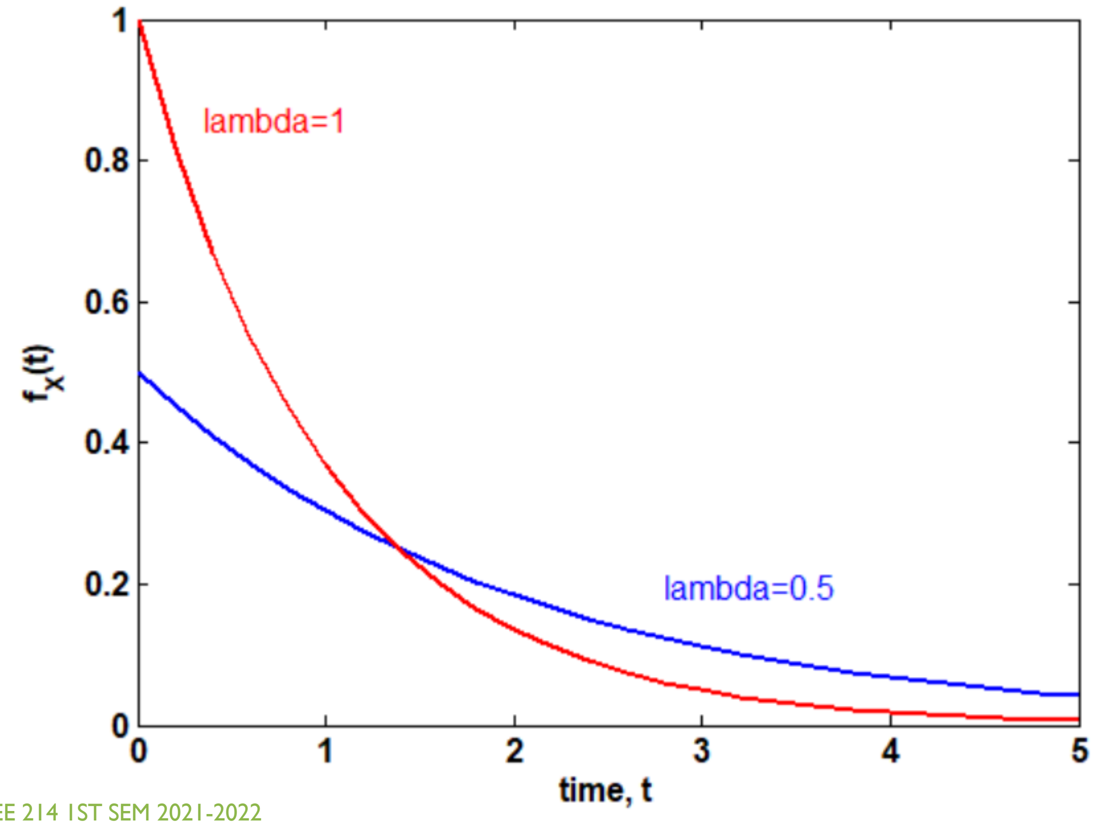
EXPONENTIAL RANDOM VARIABLES

- model for waiting time (i.e., radioactive particle to arrive at counter, packet to arrive at router, etc.)

$$f_X(t) = \lambda e^{-\lambda t} \quad ; \lambda \geq 0$$

- mean: $1/\lambda$
- variance: $1/\lambda^2$

EXPONENTIAL RV



$$P[X \geq a] = \int_a^{\infty} \lambda e^{-\lambda x} dx = e^{-\lambda a} \leq \frac{1}{\lambda a}$$

EXAMPLE

- The time until a small meteorite first lands anywhere in the Sahara desert is modelled as an exponential random variable with a mean of 10 days. The time is currently midnight. What is the probability that a meteorite first lands some time between 6am and 6pm of the first day?

SOLUTION

- Let: X = time (in days) elapsed until first landing

$$f_X(t) = \lambda e^{-\lambda t} \quad ; \lambda = 0.1$$

$$Z = P(0.25 \le X \le 0.75)$$

$$= P(X \ge 0.25) - P(X \ge 0.75)$$

$$= e^{-0.25*0.1} - e^{-0.75*0.1} = e^{-0.025} - e^{-0.075}$$

$$Z = 0.0476$$